Cosmic Censorship in Higher dimension II

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Generalizing earlier results on dust collapse in higher dimensions, we show here that cosmic censorship can be restored in gravitational collapse with tangential pressure present if we take the spacetime dimension to be $N \geq 6$. This is under conditions to be motivated physically, such as the smoothness of initial data from which the collapse develops. The models considered here incorporating a non-zero tangential pressure include the Einstein cluster spacetime.

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I. INTRODUCTION

We pointed out recently ([1], to be referred to here as paper I), that the naked singularities of dust collapse which were suggested by the analytic work of Christodoulou, and numerical considerations by Eardley and Smarr [3], can be removed when one goes to a higher spacetime dimension. These naked singularities arise as collapse end state when one considers the gravitational collapse of dust developing from a smooth initial data with various other restrictions. This would thus restore the cosmic censorship, at least for collapsing dust, when one allows for the possibility that spacetime has a sufficiently higher dimension, and when one can motivate various restrictions such as smoothness of the initial data, possibly through various considerations on what is a physically realistic model for gravitational collapse. Several subcases of dust collapse in higher dimensions have also been discussed by various authors [2].

There is a considerable motivation provided in recent years for considering the possibility for the spacetime to have higher dimensions, which mainly arises from the string theoretic and other related considerations. However, an immediate important question that comes up is whether the results such as those in paper I would generalize when we allow the collapsing matter to have a nonzero pressure, rather than having strictly the idealized form of dust where pressures necessarily vanish. Clearly, any realistic collapsing configuration must take non-zero pressures into account while figuring out an issue such as possible final endstates for gravitational collapse.

There have been extensive studies of gravitational collapse models in recent years, particularly from the perspective of investigating collapse end states in terms of either black holes or naked singularity formation, and to examine the validity or otherwise of the cosmic censorship conjecture, which is one of the most fundamental issues in black hole physics today [4]. The generic result of such studies has been, depending on the nature of the regular initial data from which the collapse evolves, either a black hole or a naked singularity develops as the collapse end state within the usual framework of four-dimensional spacetime. In order to ask a question as to what happens to such naked singularities of collapse when one goes to higher spacetime dimensions, it would be appropriate and necessary to examine the case of gravitational collapse where pressures have been included explicitly, even if in a somewhat restricted manner. A well-known example of gravitational collapse with pressure is the so called Einstein cluster, where the pressure is purely tangential, and the radial pressures vanish identically [5]. This model has been studied extensively to find the end state of a continual collapse [6], and it is shown that both black holes or naked singularities do result as final outcome of collapse.

Our purpose here is to examine the Einstein cluster model, and some other collapse configurations with purely tangential pressure, in a higher dimensional spacetime. We show that the results such as those in paper I on the avoidance of naked singularity in a higher spacetime dimension do generalize to this case as well. It is thus seen that even when the gravitational collapse with a tangential pressure is considered, rather than just the pressureless dust, we can still remove the naked singularity and restore the cosmic censorship by going to a higher spacetime dimension. We consider the collapse with tangential pressure in N dimensions, and consider only smooth initial profiles. Two different tangential pressure models are explicitly discussed to demonstrate that the gravitational collapse from smooth initial profiles would necessarily restore the cosmic censorship in higher dimensions ($N \geq 6$), and that the collapse endstate will be necessarily a black hole.

In particular, as pointed out above, the Einstein cluster model has been analyzed extensively towards examining the final end state of collapse in terms of deducing the black hole or naked singularity formation, and is known to provide a useful counter-example to cosmic censorship. It is hence interesting that the naked singularities of this model can be removed, and cosmic censorship restored, by going to higher dimensions. Both the models

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discussed here, which have radial pressure vanishing but a non-zero tangential pressure present, include dust as a special case. Thus the considerations here generalize the results of paper I, and thus the conclusions derived in the case of dust there are generalized to the case when a non-zero pressure is included in the collapse scenario.

The outline of the paper is as follows. In Section II we discuss the collapse equations and the regularity conditions. In Section III, non-static Einstein cluster model is discussed and it is demonstrated how the given sets of initial value parameters, such as the initial density and angular momentum values, decide the singularity curve for the collapse. In section IV we construct one more tangential pressure model with specific choice of one of the metric functions. The dependence of nature of singularity on the number of dimensions for both these models is examined in Section V. Some conclusions are given in Section VI.

II. EINSTEIN EQUATIONS, REGULARITY AND ENERGY CONDITIONS

Let us consider a general spherically symmetric metric in $N \geq 4$ dimensions which can be written as,

$$ds^2 = -e^{\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t,r)d\Omega^2_{N-2} \quad (1)$$

where,

$$d\Omega^2_{N-2} = \sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](d\theta^i)^2 \quad (2)$$

is the line element on $(N-2)$ sphere. Also let us assume the above frame is a *comoving* coordinate system, *i.e.* the energy-momentum tensor for a *Type I* matter field [7] has the form,

$$T^t_t = -\rho;\ T^r_r = p_r;\ T^\theta_\theta = T^\phi_\phi = p_\theta \quad (3)$$

We also take the matter field to satisfy the *weak energy condition*, that is, the energy density measured by any local timelike observer be non-negative, and so for any timelike vector V^i we have,

$$T_{ik}V^iV^k \geq 0 \quad (4)$$

which amounts to,

$$\rho \geq 0;\ \rho + p_\theta \geq 0 \quad (5)$$

In the case of a finite collapsing cloud, there is a finite boundary $0 < r < r_b$, outside which it is matched to an asymptotically flat exterior. The range of the coordinates for the metric is then $0 < r < r_b$, and $-\infty < t < t_s(r)$ where $t_s(r)$ corresponds to the epoch where the shell labeled r reached the spacetime singularity. The dynamical evolution of the system is determined by the Einstein equations, and for the metric (1) these are given as,

$$\rho = \frac{(N-2)F'}{2R^{N-2}R'},\ \ p_r = -\frac{(N-2)\dot{F}}{2R^{N-2}\dot{R}} \quad (6)$$

$$\nu'(\rho + p_r) = (N-2)(p_\theta - p_r)\frac{R'}{R} - p'_r \quad (7)$$

$$-2\dot{R}' + R'\frac{\dot{G}}{G} + \dot{R}\frac{H'}{H} = 0 \quad (8)$$

$$G - H = 1 - \frac{F}{R^{N-3}} \quad (9)$$

where,

$$G(t,r) = e^{-2\psi}(R')^2; \quad H(t,r) = e^{-2\nu}(\dot{R})^2 \quad (10)$$

Here $F = F(t,r)$ is called the *mass function* of the collapsing cloud which is interpreted as the total mass within the shell of comoving radius r. The energy condition then implies $F' \geq 0$. It follows from the above expression for density that there is a spacetime singularity at $R = 0$ and at $R' = 0$. The later are called *shell-crossing* singularities, which occur when successive shells of matter cross each other. These have not been considered generally to be genuine spacetime singularities, and possible extensions of spacetime have been investigated through the same [8]. On the other hand, the singularity at $R = 0$ is where all matter shells collapse to a zero physical radius, and hence this has been known as a *shell-focusing* singularity. The nature of this singularity has been investigated extensively in four-dimensional spacetimes (see e.g. references in [4]). In particular, it is known for the case of four dimensional spherical collapse of tangential pressure models that this singularity can be naked or covered, depending on the nature of the initial data from which the collapse develops [9].

We now use the scaling independence of the comoving coordinate r to write (see e.g. [10]),

$$R(t,r) = r\,v(t,r) \quad (11)$$

and we have,

$$v(t_i,r) = 1 \quad ; \quad v(t_s(r),r) = 0 \quad ; \quad \dot{v} < 0 \quad (12)$$

where t_i and t_s stand for the initial and the singular epochs respectively. The coordinate r has been scaled in such a way that at the initial epoch we have $R = r$, and at the singularity $R = 0$. The fact that we deal here with only collapse models gives the condition $\dot{R} < 0$, or equivalently $\dot{v} < 0$. It should be noted that we have $R = 0$ both at the regular center $r = 0$ of the cloud, and at the spacetime singularity, where all matter shells collapse to a zero physical radius. The regular center is then distinguished from the singularity by a suitable behaviour of the mass function $F(t,r)$ so that the density remains finite and regular there at all times till the singular epoch. The introduction of the parameter v as above then allows us to distinguish the spacetime singularity from the regular center, with $v = 1$ at the initial epoch, including at the center $r = 0$, which then decreases monotonically with time as collapse progresses to the value $v = 0$ at the singularity $R = 0$.

We shall consider here the models where the radial component of the pressure necessarily vanishes ($p_r = 0$), but the tangential pressure can be non-zero. In order to ensure the regularity of the initial data, and for the case of vanishing radial pressure, it is evident from the equation (6) that at the initial epoch the function $F(t, r)$ must have the following form,

$$F = r^{(N-1)}\mathcal{M}(r) \tag{13}$$

From equation (7) we get at the initial epoch,

$$\nu_0(r) = \int_0^r \frac{2p_{\theta_0}}{r\rho_0} dr \tag{14}$$

Now, to preserve the regularity of the initial data it is evident that the tangential pressures at the center should also vanish at any non-singular epoch, *i.e.* $p_{\theta_0}(0) = 0$. Then we can see that $\nu_0(r)$ has the form,

$$\nu_0(r) = r^2 g(r) \tag{15}$$

where $g(r)$ is at least a C^1 function of r at $r = 0$, and at least a C^2 function for $r > 0$. Let us now define a suitably differentiable function $A(r, v)$ in the following manner,

$$\nu'(r, v) = A(r, v)_{,v} R' \tag{16}$$

That is, $A(r, v) \equiv \nu'/R'$. Then from equation (7) we have the equation of state given as,

$$p_\theta = \frac{1}{N-2} A_{,v} R\rho \tag{17}$$

Now using equation (16) we can integrate (8) to get,

$$G = b(r)e^{2rA} \tag{18}$$

Here $b(r)$ is another arbitrary function of the comoving coordinate r. Following a comparison with dust collapse models we can write,

$$b(r) = 1 + r^2 b_0(r) \tag{19}$$

where $b_0(r)$ is the energy distribution function for the collapsing shells. Finally, using equations (16), (18) and (19) in (9) we have,

$$R^{\frac{N-3}{2}}\dot{R} = -e^\nu \sqrt{(1 + r^2 b_0)R^{N-3}e^{2rA} - R^{N-3} + r^{N-1}\mathcal{M}} \tag{20}$$

Again, defining a new function $h(r, v)$ as,

$$h(r, v) = \frac{e^{2rA} - 1}{r^2} \tag{21}$$

we can finally integrate the equation (9) to get,

$$t(v, r) = \int_v^1 \frac{v^{\frac{N-3}{2}} dv}{\sqrt{e^{2(\nu + rA)} b_0 v^{N-3} + e^{2\nu}(v^{N-3}h + \mathcal{M})}} \tag{22}$$

The time of singularity for a shell at a comoving coordinate radius r is the time when the physical radius $R(r, t)$ becomes zero, and is given as $t_s(r) = t(0, r)$. The shells collapse consecutively, that is one after the other to the center as there are no shell-crossings. Taylor expanding the above function around $r = 0$, we get,

$$t(v, r) = t(v, 0) + r \left.\frac{dt(v, r)}{dr}\right|_{r=0} + \frac{r^2}{2!} \left.\frac{d^2 t(v, r)}{d^2 r^2}\right|_{r=0} \tag{23}$$

Let us denote,

$$\mathcal{X}_n(v) = \left.\frac{d^n t(v, r)}{dr^n}\right|_{r=0} \tag{24}$$

We shall now assume that the initial density, pressure and energy functions $\rho(r)$, $p_{\theta 0}(r)$ and $b_0(r)$ are smooth and even, ensuring their analytic nature. We note that the Einstein equations as such do not impose any such restriction, which are to be physically motivated, and it implies a certain mathematical simplicity in arguments to deal with a dynamical collapse. It follows that $\mathcal{M}(r)$, $p_{\theta 0}(r)$ and $b_0(r)$ are now smooth C^∞ functions, which means the Taylor expansions of these functions around the center must be of the following form,

$$M(r) = M_{00} + M_{02}r^2 + M_{04}r^4 + \cdots \tag{25}$$

$$p_{\theta 0}(r) = p_{\theta_{02}} r^2 + p_{\theta_{04}} r^4 + \cdots \tag{26}$$

$$b_0(r) = b_{00}r^2 + b_{02}r^4 + \cdots \tag{27}$$

This means that, all odd terms in r vanish in these expansions, and the presence of only even terms would ensure smoothness. We shall now investigate two different well-known tangential pressure models with smooth initial profiles to show that the naked singularities arising in gravitational collapse in usual four dimensions are removed when we make a transition to higher dimensional ($N \geq 6$) spacetimes.

III. COLLAPSE OF EINSTEIN CLUSTER

The Einstein cluster model [5, 6] has been studied extensively towards examining the final end state of a gravitational collapse in terms of either a black hole or naked singularity. This is a system in which the radial pressure is vanishing, but a non-zero tangential pressure is present. It is a spherically symmetric cluster of rotating particles where the motion of the particles is sustained by an angular momentum which has an average effect of creating a non-zero tangential pressure within the cloud. Neighbouring shell particles are counter-rotating such that spherical symmetry is preserved. In four dimensions it is known to show naked singularity as one of the possible end states of collapse when smooth initial profiles are taken into account [11].

We consider a non-static cluster of gravitating particles moving along circular paths around the center of symmetry in N dimensions. The neighboring shells are counter-rotating so that in any small volume their total angular momentum would be zero. For the non-static Einstein cluster models, we have equation of state as given by,

$$p_\theta = \frac{1}{N-2}\left(\frac{L^2}{R^2+L^2}\right)\rho \tag{28}$$

where $L(r)$ is a function of the radial coordinate r only and is known as the *specific angular momentum*. A comparison with equation (17) gives,

$$A_{,v} = \frac{L^2}{R(R^2+L^2)} \tag{29}$$

We can integrate the above equation to get,

$$e^{2rA} = \frac{R^2}{R^2+L^2} \tag{30}$$

Considering initial density, pressure and energy profiles to be smooth would ensure $L(r)$ also to be smooth and it can be seen from equations (25), (26) and (28) that it is in the form,

$$L^2(r) = L_{04}r^4 + L_{06}r^6 + \cdots \tag{31}$$

Since we have,

$$\nu = \int \frac{(v+rv')L^2(r)}{rv(L^2(r)+r^2v^2)}dr \tag{32}$$

we see that around the regular center $r=0$ the function ν can be expanded as,

$$\nu \sim \nu_{02}(v)r^2 + \nu_{04}(v)r^4 + \cdots \tag{33}$$

From equations (30) and (31) we see that,

$$e^{2rA} = A_{00} + A_{02}r^2 + A_{04}r^4 + \cdots \tag{34}$$

Now in this case we can write the function $t(v,r)$ as,

$$t(v,r) = \int_v^1 \frac{v^{\frac{N-3}{2}}\sqrt{v^2+\frac{L^2}{r^2}}\,dv}{e^\nu\sqrt{b_0v^{N-1} - \left(\frac{L^2}{r^4}\right)v^{N-3} + \mathcal{M}\left(v^2+\frac{L^2}{r^2}\right)}} \tag{35}$$

As we have taken the initial data with only even powers of r, the first derivatives of all the functions appearing in above equation vanish at $r=0$, hence we have for the quantity $\mathcal{X}$ which was defined in equation (24),

$$\mathcal{X}_1(v) = 0 \tag{36}$$

The time for the central shell to reach the singularity is given as

$$t_{s0} = \int_0^1 \frac{v^{\frac{N-3}{2}}\,dv}{e^{\nu_0}\sqrt{b_0v^{N-3}+\mathcal{M}}} \tag{37}$$

Also, for the $t_s(0)$ to be defined one must have the condition,

$$b_0v + \mathcal{M}_0 > 0 \tag{38}$$

The time for other shells close to the center to reach the singularity, *i.e.* the equation for the singularity curve can now be given by,

$$t_s(r) = t_{s_0} + r^2\frac{\mathcal{X}_2(0)}{2} + \cdots \tag{39}$$

Here we see that the value of the quantity $\mathcal{X}_2(0)$ depends on the different functional forms of the free functions $L(r)$ and $\mathcal{M}(r)$, which corresponds to the initial data for this model. In order to determine the visibility or otherwise of the singularity at $R=0$, we need to analyze the causal structure of the trapped surfaces and the nature and behaviour of null geodesics in the vicinity of the same. If there exist future directed null geodesics with past end point at the singularity, which go out to faraway observers in the spacetime, then the singularity is naked. In the case otherwise, we have a black hole resulting as the end state of a continual collapse. We shall discuss this in section V.

IV. COLLAPSE WITH $\nu = c(t) + \nu_0(R)$

Now we construct another explicit example of a collapse model with a vanishing radial but non-vanishing tangential pressure in N dimensions, with smooth initial data. Let us assume,

$$\nu(t,r) = c(t) + \nu_0(R) \tag{40}$$

We note that the above again includes dust as a special case, which corresponds to $\nu_0 = 0$. A comparison with equation (17) gives,

$$A_{,v} = \nu_{0,R} \tag{41}$$

Also using equation (40) in equation (8), we have,

$$G(t,r) = b(r)e^{2\nu_0(R)} \tag{42}$$

Also we can integrate the equation (14) and get,

$$\nu_0(R) = p_{\theta_2}R^2 + \frac{(p_{\theta_4} - \rho_2 p_{\theta_2})}{2}R^4 + \cdots \tag{43}$$

Using equations (13),(40) and (42) in equation (9), we have,

$$R^{\frac{N-3}{2}}\dot{R} = -a(t)e^{\nu_0(R)}K(r,R) \tag{44}$$

where we have defined,

$$K(r,R) = \sqrt{(1+r^2b_0)R^{N-3}e^{2\nu_0} - R^{N-3} + r^{N-1}\mathcal{M}} \tag{45}$$

Here $a(t)$ is a function of time. By a suitable scaling of the time coordinate, we can always make $a(t) = 1$. The negative sign is due to the fact that $\dot{R} < 0$, which is the collapsing cloud condition. Let us define a function $h(R)$ as,

$$h(R) = \frac{e^{2\nu_0(R)} - 1}{R^2} = 2g(R) + \mathcal{O}(R^2) \tag{46}$$

Using equation (46) in equation (44), we have after simplification,

$$v^{\frac{N-3}{2}}\dot{v} = -\sqrt{e^{4\nu_0}v^{N-3}b_0 + e^{2\nu_0}\left(v^{N-1}h(rv) + \mathcal{M}\right)} \tag{47}$$

Integrating the above equation, we get,

$$t(v,r) = \int_v^1 \frac{v^{\frac{N-3}{2}}dv}{\sqrt{e^{4\nu_0}v^{N-3}b_0 + e^{2\nu_0}\left(v^{N-1}h(rv) + \mathcal{M}\right)}} \tag{48}$$

As we have taken the initial data with only even powers of r, the first derivatives of the functions appearing in above equation vanish at $r = 0$, hence we have,

$$\mathcal{X}_1(v) = 0 \tag{49}$$

Again, the time for other shells close to the center to reach the singularity can now be given by the equation,

$$t_s(r) = t_{s_0} + r^2\frac{\mathcal{X}_2(0)}{2} + \cdots \tag{50}$$

V. BEHAVIOUR OF THE APPARENT HORIZON

The outcome of a gravitational collapse, in terms of either a black hole or a naked singularity, is determined by the causal behaviour of non-spacelike curves in the vicinity of the singularity. If there exist future directed families of non-spacelike curves which reach the far away observers in the future, and which have past end point at the singularity, then the singularity forming as collapse endstate will be visible. In the case otherwise, the horizon forms early enough and the outcome is a black hole. To determine this, we can analyze the behaviour of the apparent horizon within the spacetime, which is the boundary of the trapped surfaces forming as the collapse develops.

This boundary of the trapped region of the space-time is given within the collapsing cloud by the equation,

$$\frac{F}{R^{N-3}} = 1 \tag{51}$$

which is the equation for the apparent horizon. If the neighborhood of the center gets trapped earlier than the singularity, then it is covered, otherwise it is naked with families of non-spacelike future directed trajectories escaping away from it. For example, it follows from the above equation that along the singularity curve $t = t_s(r)$ (which corresponds to $R = 0$), for any $r > 0$ we have $F(r)$ going to a constant positive value, whereas the area radius $R \to 0$. Hence it follows that trapping already occurs before the singularity develops at any $r > 0$ along the singularity curve $t_s(r)$ whenever a suitable energy condition is satisfied.

What we need to determine now is when there will be families of non-spacelike paths coming out of the central singularity at $r = 0, t = t_s(0)$, reaching outside observers, and when there will be none. The visibility or otherwise of the singularity is decided accordingly. By determining the nature of the singularity curve, and its relation to the initial data, we are able to deduce whether the trapped surface formation in collapse takes place before or after the central singularity. It is this causal structure that determines the possible emergence or otherwise of non-spacelike paths from the singularity, and settles the final outcome in terms of either a black hole or naked singularity. From equation (51), we have,

$$v_{ah}(r) = [r^2\mathcal{M}(r)]^{\frac{1}{N-3}} \tag{52}$$

Using the above equation in (35) and (48) we get the following results. In case of Einstein cluster, the equation of apparent horizon in (t,r) plane as,

$$t_{ah}(r) = t_s(r) - B_1(r) \tag{53}$$

$$B_1(r) = \int_0^{v_{ah}} \frac{v^{\frac{N-3}{2}}\sqrt{v^2 + \frac{L^2}{r^2}}\,dv}{e^{\nu}\sqrt{b_0 v^{N-1} - \left(\frac{L^2}{r^4}\right)v^{N-3} + \mathcal{M}\left(v^2 + \frac{L^2}{r^2}\right)}} \tag{54}$$

whereas in case of the second model we have the equation of apparent horizon in (t,r) plane given as,

$$t_{ah}(r) = t_s(r) - B_2(r) \tag{55}$$

$$B_2(r) = \int_0^{v_{ah}} \frac{v^{\frac{N-3}{2}}dv}{\sqrt{e^{4\nu_0}v^{N-3}b_0 + e^{2\nu_0}\left(v^{N-1}h(rv) + \mathcal{M}\right)}} \tag{56}$$

As we are considering the behaviour of the apparent horizon close to the central singularity at $r = 0, R = 0$ (all other points $r > 0$ on the singularity curve are already covered), therefore the upper limit of integration in the above equation is small, and hence we can expand the integrand in a power series in v, and keep only the leading order term, which for both the models amounts to,

$$t_{ah}(r) = t_{s_0} + r^2\frac{\mathcal{X}_2(0)}{2} + \cdots - r^{\frac{N-1}{N-3}}\frac{2}{N-1}\mathcal{M}_0^{\frac{1}{N-3}} \tag{57}$$

It is now possible to analyze the effect of the number of dimensions on the nature and shape of the apparent horizon. Firstly, note when we work in four dimensions, and if $\mathcal{X}_2$ is non-zero positive, then the second term in the above equation dominates over the last negative term,

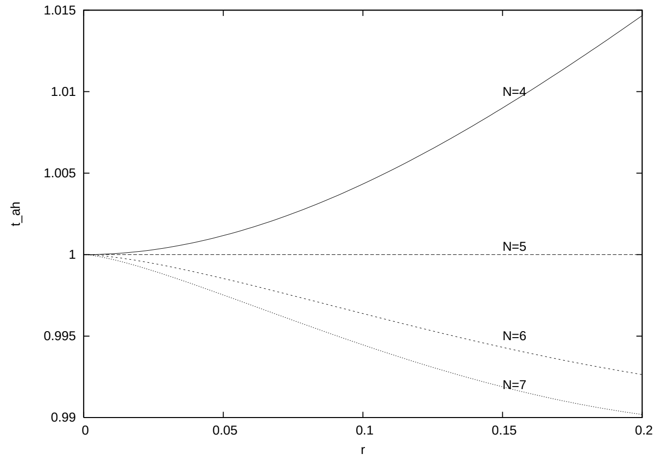

FIG. 1: The apparent horizon in different spacetime dimensions. Here $\mathcal{X}_2 = 0.5$ and apparent horizon curves are given for dimensions 4 to 7.

and the apparent horizon curve is increasing as we move away from the origin, which allows for the possibility that the singularity may be naked. On the other hand, as we increase the number of dimensions and go to dimensions higher than five, the negative term in equation (57) starts dominating, thus advancing the trapped surface formation in time. We thus see that for a smooth initial data and for dimensions higher than five, the apparent horizon becomes a decreasing function of r near the center. This implies that the neighborhood of the center gets trapped before the central singularity and the central singularity is then always covered. To be specific, suppose there is a future directed outgoing null geodesic coming out from the central singularity at $R = 0, r = 0$. If (t_1, r_1) is an event along the same, then $t_1 > t_{s_0}$ and $r_1 > 0$. But for any such r_1, the trapped region already starts before $t = t_{s_0}$, hence the event (t_1, r_1) is already in the trapped region and the geodesic cannot be outgoing. Thus, there are no outgoing paths from the central singularity, making it covered. It follows that the collapse outcome will be necessarily a black hole in the dimensions $N \geq 6$. Thus the naked singularities developing in the Einstein cluster collapse and also in the other tangential collapse model with a smooth initial data are removed, and the cosmic censorship is restored when we go to higher dimensions, thus generalizing the dust results obtained in paper I.

VI. CONCLUSION

We give several concluding remarks in this section.

1. We have shown that the naked singularities forming in the well-known Einstein cluster model in four dimensions are removed when we go to higher dimensions. It follows that the results of paper I, obtained for dust collapse, can be preserved even when tangential pressure is included in the collapse.

2. In five dimensions we have an interesting scenario arising (see also [2]). As it is clearly seen from the equation (57), we have a critical value of $\mathcal{X}_2(0)$, below which the apparent horizon is decreasing and we will get a black hole end state. However, in the case otherwise, a naked singularity can result.

3. It is interesting to note also that the results obtained above are valid, even if the initial profiles, instead of being absolutely smooth C^∞ functions, are taken to be only sufficiently differentiable (*i.e.* at least C^2 functions).

4. We have of course not shown that the cosmic censorship is restored for *all* possible collapse models which have a tangential pressure non-vanishing. However, this result shows the interesting effect that changing the number of dimensions has on the behaviour of the apparent horizon curve, and hence on the visibility or otherwise of the resultant spacetime singularity.

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