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Design of State Estimation Based Model Predictive Controller for a Two Tank Interacting System

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Abstract

Model Predictive Control (MPC) schemes are now widely used in process industries for the control of key unit operations. In this paper, a state estimation based model predictive controller for nonlinear system has been proposed. The model predictive controller is designed by considering a state space model and an extended Kalman filter to predict the future behaviour of the system. The efficacy of the proposed MPC scheme has been demonstrated by conducting simulation studies on a two tank interacting system, a MIMO system.The analysis of the extensive dynamic simulation studies revealed that, the MPC scheme formulated produces satisfactory performance for servo operation.

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Keywords: Model Predictive Control; State Estimation; Extended Kalman Filter; Two Tank Interacting System; Servo operation.

1. Introduction

Model Predictive Control is an increasingly significant and popular control approach because of its use of a possibly nonlinear multivariable process model and its ability to handle constraints on inputs, states and outputs.It uses open loop constrained optimization of finite horizon control criteria in a receding horizon approach. A model is used to predict the future behavior of the system up to the horizon, starting from its current state and a constrained optimization based on the prediction yields an optimal open loop control sequence over the complete

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horizon. Only the first element in this sequence is applied to the plant. New measurements available at the next sample time permit the calculation of an updated initial state value and the optimization is then resolved [6].

Nomenclature

F_{in1}, F_{in2}	Inflow rates of tank 1 and tank 2
h_1, h_2	Height of water in tank 1and tank 2
b_1, b_2	Valve coefficient of valve 1and valve 2
a	Cross section area of connecting pipes
s	Coefficient of connecting pipes
h_{max}	Maximum permissible height of water levels
g	Acceleration due to gravity
A_1, A_2	Area of tank 1 and tank 2
N_2	Prediction horizon
N_u	Control horizon
W_E	Weight matrix for output error
W_U	Weight matrix for control input
Q	Covariance matrix of state noise
R	Covariance matrix of measurement noise

All the states of a system are not measurable in practice, hence state estimation is presented. When the prediction of future behavior is done based on state estimation, there exist many advantages which include accurate result, suppression of noise, increased robustness and the estimator acts as model based filter. In state estimation based MPC, the current control action is obtained online by solving a finite horizon open loop optimal control problem from the state estimate of the system.The need to achieve control of nonlinear process has led to more general MPC formulation. The survey on nonlinear control of chemical processes by Bequette [2] summarizes different NMPC algorithms. The advantages of using state estimation in model predictive control has been reported by Ricker [3].

The main contribution of this paper is to demonstrate the development of model predictive control scheme for a two tank interacting system using extended Kalman filter [4]. The paper is organized as follows. Section 2 discusses about the modeling of two tank interacting system. Section 3 presents the conventional MPC algorithm. Section 4 describes the state estimation based MPC algorithm. Extensive simulation results and analysis are presented in Section 5 and the conclusion in Section 6.

2. Mathematical Modeling Of Two Tank Interacting System

2.1. Description

The system shown in Fig 1 consists of two interacting tanks connected to each other through connecting pipes of circular cross section provided with a valve. The valves 1 and 2 introduce nonlinearity in the system. For the dynamic model, the incoming mass flows F_{in1} and F_{in2} are defined as inputs, while the two measurements $h_1(t)$ and $h_2(t)$ i.e. the height of fluid in tank are considered as outputs. The dynamic model is derived using the incoming and outgoing mass flows and is described by the following differential equations (1) and (2).

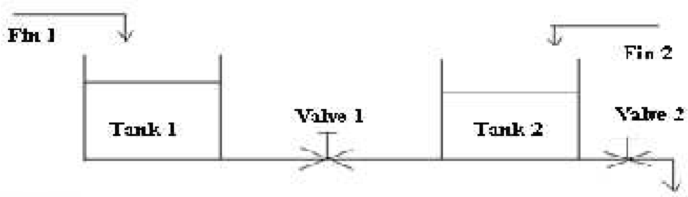

Fig.1. Two Tank Interacting System

For Tank 1,

$$A_1\frac{dH_1}{dt} = F_{in1} - b_1\sqrt{H_1 - H_2} \tag{1}$$

For Tank 2,

$$A_2\frac{dH_2}{dt} = F_{in2} + b_1\sqrt{H_1 - H_2} - b_2\sqrt{H_2} \tag{2}$$

where the valve coefficients of valve1 and valve2 are $b_1=s_1.a_1.\sqrt{2g}$ and $b_2=s_2.a_2.\sqrt{2g}$

The physical parameters of the two tank process are given in Table 1.

Table 1 Physical Parameters of Two Tank Process

Description	Values
Area of the tanks (A_1,A_2)	0.0154 m^2
Acceleration due to gravity	9.81 m/sec^2
Maximum permissible height of water levels (h_{max})	0.63 m
Cross section of the connecting pipes (a)	0.005 m^2
Co-efficient of the connecting pipes (s)	0.45
Nominal operating conditions h_1 = 0.4 m h_2 = 0.3 m	Nominal inflow rate F_{in1} = 0.00315 m^3/sec F_{in2} = 0.00231 m^3/sec

2.2 Linearization

The nonlinear equations are linearized using the Jacobian matrices to get the ABCD parameters. The matrices are given below:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} \\ \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} \end{bmatrix};\ B = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} \end{bmatrix};\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

where f_1 and f_2 are the differential equations (1) and (2) respectively. q_1 and q_2 are the inflow rates. After substituting and simplifying, the discrete matrices are obtained as

$$A = \begin{bmatrix} 0.9071 & 0.0901 \\ 0.0901 & 0.8551 \end{bmatrix};\ B = \begin{bmatrix} 6.1817 & 0.3056 \\ 0.3056 & 6.0052 \end{bmatrix}$$

3. Conventional MPCAlgorithm

The model predictive control is a strategy that is based on the explicit use of some kind of system model to predict the controlled variables over a certain time horizon, the prediction horizon. The control strategy can be described as follows [6]:

1. At each sampling time, the value of the controlled variable y(t+k) is predicted over the prediction horizon k=1,……N_2. This prediction depends on the future values of the control variable u(t+k) within a control horizon k=1,…Nu, where $N_u \leq N_2$. If $N_u < N_2$, then $u(t+k)=u(t+N_u)$, $k=N_u+1,\ldots N_2$.
2. A reference trajectory r(t+k), k=1,….N is defined which describes the desired system trajectory over the prediction horizon.
3. The vector of future controls u(t+k) is computed such that a cost function, usually a function of the errors between the reference trajectory and the predicted output of the model, is minimised.
4. Once minimisation is achieved, the first optimised control action is applied to the plant and the plant outputs are measured. Use this measurement of the plant states as the initial states of the model to perform the next iteration.

Steps 1 to 4 are repeated at each sampling instant; this is called receding horizon strategy.
The block diagram of a model predictive controller is shown in Fig 2.

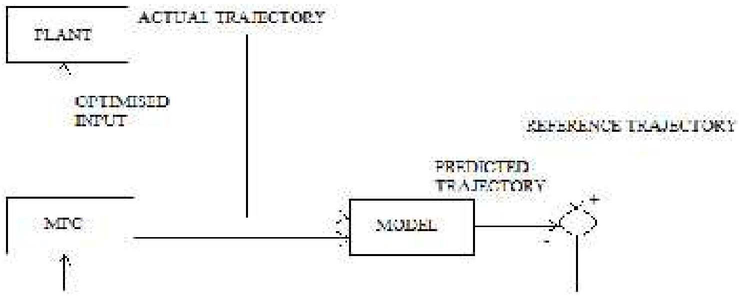

Fig. 2. Block Diagram of MPC controller

As the control variables in a MPC controller are calculated based on the predicted output, the model thus needs to be able to reflect the dynamic behaviour of the system as accurately as possible.

4. State Estimation Based MPC

Conventional MPC techniques are based on the use of linear models. The need to achieve tighter control of strong non-linear process has led to more general MPC formulation in which nonlinear dynamic model is used for prediction [1]. When the prediction of future behaviour is done based on state estimation, there exist many advantages which include accurate result, suppression of noise, increased robustness and the estimator acts as model based filter [3]. In state estimation based MPC, the current control action is obtained online by solving a finite horizon open loop optimal control problem from the state estimate of the system. Fig 3 shows the block diagram of state estimation based MPC.

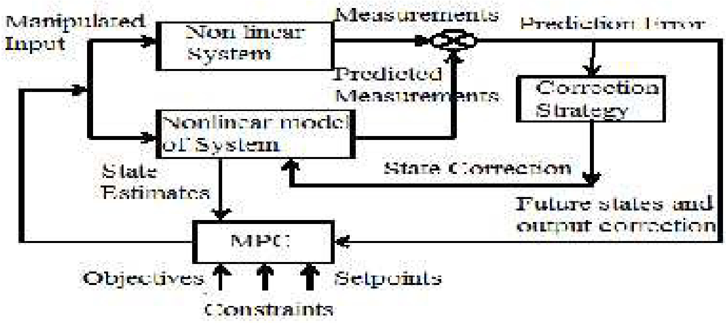

Fig.3. Block Diagram of State Estimation Based MPC

4.1 Extended Kalman Filter

In this section, the EKF algorithm to estimate the internal states and future behaviour of the system has been proposed. The extended Kalman Filter (EKF) is probably the most widely used nonlinear filter[7]. For nonlinear problems, the Kalman Filter is not strictly applicable since linearity plays an important role in its derivation and performance as an optimal filter. The EKF attempts to overcome this difficulty by using a linearized approximation where the linearization is performed about the current state estimate [7].The basic framework for the EKF involves the estimation of the state of a nonlinear dynamic system given by (3) and (4)

$$x(k) = \left[\int_{t_{k-1}}^{t_k} F[x(\tau), u(k)]d\tau\right] + w(k) \quad (3)$$

$$y(k) = H[x(k)] + v(k) \quad (4)$$

In the above equation, $x(k)$ represents the unobserved state of the system, $u(k)$ is a known exogenous input and $y(k)$ is the only observed signal. We have assumed $w(k)$ and $v(k)$ as zero mean Gaussian white noise sequences with covariance matrices Q and R respectively. The symbols F and H represent an n-dimensional function vector and are assumed known. EKF involves the recursive estimation of the mean and covariance of the state under maximum likelihood condition. The function Fcan be used to compute the predicted state from the previous estimate and similarly the function H can be used to compute the predicted measurement from the predicted state. However, F and H cannot be applied to the covariance directly. Instead a matrix of partial derivatives (Jacobian) is computed at each time step with current predicted state and evaluated. This process essentially linearizes the non-linear function around the current estimate.

The predicted state estimates are obtained as

$$\hat{x}(k|k-1) = \int_{t_{k-1}}^{t_k} F[x(\tau), u(k-1)]d\tau \quad (5)$$

The covariance matrix of estimation errors in the predicted estimates is obtained as

$$P(k|k-1) = \varphi(k)P(k-1|k-1)\varphi(k)^T + Q \quad (6)$$

where $\varphi(k)$ is nothing but Jacobian matrix of partial derivatives of F with respect to x

$$\varphi(k) = \left[\frac{\partial F}{\partial x}\right]_{[\hat{x}(k-1|k-1), u(k-1)]} \quad (7)$$

Note that the extended Kalman filter (EKF) computes covariances using the linear propagation. The measurement prediction, computation of innovation and covariance matrix of innovation are as follows

$$\hat{y}(k|k-1) = H[\hat{x}(k|k-1)] \quad (8)$$

$$\gamma(k|k-1) = y(k) - \hat{y}(k|k-1) \quad (9)$$

$$V(k) = C(k)P(k|k-1)C(k)^T + R \quad (10)$$

where $C(k)$ is the Jacobian matrix of partial derivatives of H with respect to x.

$$C(k) = \left[\frac{\partial H}{\partial x}\right]_{[\hat{x}(k-1|k-1), u(k-1)]} \quad (11)$$

The Kalman gain is computed using the following equation

$$K(k) = P(k|k-1)C(k)^T V^{-1}(k) \quad (12)$$

The updated state estimates are obtained using the following equation

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\gamma(k|k-1) \quad (13)$$

The covariance matrix of estimation errors in the updated state estimates is obtained as

$$P(k|k) = [1 - K(k)C(k)]P(k|k-1) \quad (14)$$

4.2 Model Predictive Controller Based on EKF

The concept of model predictive control involves the repeated optimization of a performance objective such as (15) over a finite horizon extending up to a prediction horizon N_2. The control variable u(k+j), over the control horizon N_u , is obtained from solving the cost function [5].

$$J = \sum_{i=1}^{N_2} ||\hat{y}(t+i) - \omega(t+i)||^2_{W_E} + \sum_{j=1}^{N_u} ||\Delta u(t+j-1)||^2_{W_U} \quad (15)$$

$\hat{y}(t+i)$is the predicted outputs vector, $\omega(t+i)$ is the reference and Δu is the increment of input vectors. $W_E$andW_U are the weighting matrices and N_2,N_u must be tuned as controller parameters.

The controller is based on a nonlinear state space model that utilizes an extended Kalman filter (EKF) for estimate of the system states. By using the estimated state variables of the system derivation of some equations, predicted outputs are obtained. The predicted output at j step ahead is as (16).

$$y(t+j) = CA^j x(t) + \sum_{i=0}^{j-1} CA^{j-i-1} Bu(t+i) \quad (16)$$

By applying the expectation function to (16), it becomes:

$$y(t+j) = CA^j E[x(t)] + \sum_{i=0}^{j-1} CA^{j-i-1} Bu(t+i) \quad (17)$$

$\hat{Y}$is vector including one step ahead to N_2 step ahead predicted outputs.

$$\hat{Y} = \begin{bmatrix} \hat{y}(t+1) \\ \hat{y}(t+2) \\ . \\ . \\ \hat{y}(t+N_2) \end{bmatrix} = \begin{bmatrix} CAE[x(t)] + CBu(t) \\ CA^2E[x(t)] + \sum_{i=0}^{1} CA^{1-i}Bu(t+i) \\ . \\ . \\ CA^{N_2}E[x(t)] + \sum_{i=0}^{N_2-1} CA^{1-i}Bu(t+i) \end{bmatrix} \quad (18)$$

Closed form of the above equation is as (19).

$$\hat{Y} = F\hat{x}(t) + GU \quad (19)$$

$$\hat{x}(t) = E[x(t)] \quad (20)$$

The estimates of the states are computed using extended Kalman filter and other parameters are defined as follows:

$$F = \begin{bmatrix} CA \\ CA^2 \\ . \\ . \\ CA^{N_2} \end{bmatrix}$$

N is identity matrix and G, H are block lower triangular matrices that nonzero elements are$(G)_{ij} = CA^{i-j}B$.

By considering the performance index,

$$\hat{Y}_{N12} = F_{N12}\hat{x}(t) + G_{N12}U_{N_u}$$

$$\hat{Y}_{N12} = [\hat{y}(t+1)^T \ldots . \hat{y}(t+N_2)^T]^T$$

$$U_{N_u} = [u(t)^T \ldots u(t+N_u-1)^T]^T$$

And by solving $\frac{\partial J}{\partial u} = 0$, the control law is obtained as in (21):

$$\Delta U_{N_u} = (G_{N12}^T W_E G_{N12} + W_U)^{-1} G_{N12}^T W_E (\omega - F_{N12}\hat{x}(t)) \quad (21)$$

$$u(t) = u(t-1) + (\Delta U_{N_u})_{(1,1)}$$

The response of this controller is appropriate if model of the system was well known. But in general case, uncertainties of the model may cause to bad behaviour of the controller and a robust controller must be designed. Also, the MPC schemes have been implemented in a moving horizon framework i.e. only the first move u(k|k) is implemented on the plant and the optimization problem is reformulated at the next sampling instant based on the updated information from the plant [1].

5. Simulation Results and Analysis

In all the simulation runs of this section, the process is simulated using the state space model. MPC scheme for two tank interacting system has been developed with the sampling time of 0.1 second, prediction horizon of $N_2 = 4$, and control horizon of $N_u = 1$. The error weighting matrix and the controller weighting matrix used in the MPC formulation are $W_E = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$ and $W_U = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$. The following constraint on the manipulated inputs (inflow rates) is imposed i.e. $0<u_1<0.023\ m^3/s$ and $0<u_2<0.023\ m^3/s$.

5.1 Closed loop simulation study of two tank interacting system

To propose the efficiency of state estimation based MPC, a set point variation is introduced in the level of fluid in the tanks as shown in Fig 4. Simulation is carried out in the following four conditions and the performance indices are compared.

1. Conventional MPC when measurement noise is not included in the system.
2. Conventional MPC when measurement noise is included in the system.
3. State Estimation based MPC when measurement noise is not included in the system.
4. State Estimation based MPC when measurement noise is included in the system.

a. *Conventional MPC when measurement noise is not included*.

Fig 4 shows the closed loop response of the system with conventional MPC when measurement noise is not included. The reference for h1 (0.4 to 0.6) and for h2 (0.3 to 0.5) are denoted by continuous lines and the outputs (h1 and h2) are denoted by dotted lines. Fig 5 shows the corresponding variation in the control inputs.

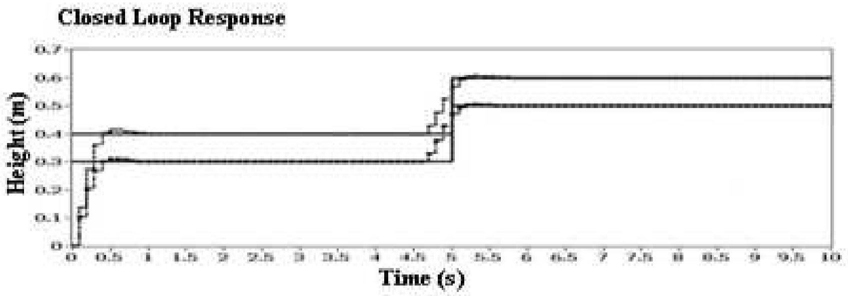

Fig.4. Closed Loop Responses - Conventional MPC without measurement noise

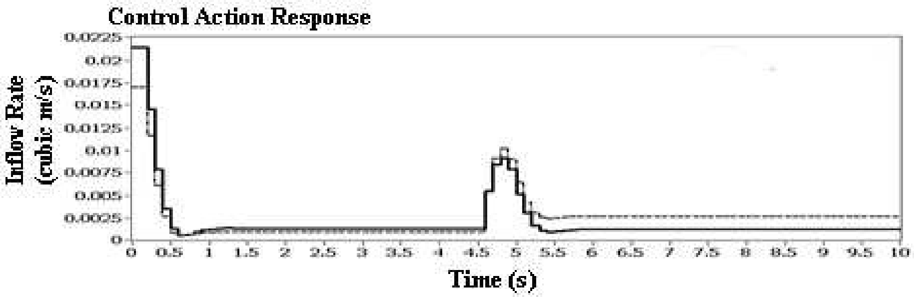

Fig.5. Control Inputs- Conventional MPC without measurement noise.

b. Conventional MPC when measurement noise is included

Fig 6 and 7 show the closed loop response of the system with conventional MPC when measurement noise is included and the corresponding variation in the control inputs respectively.

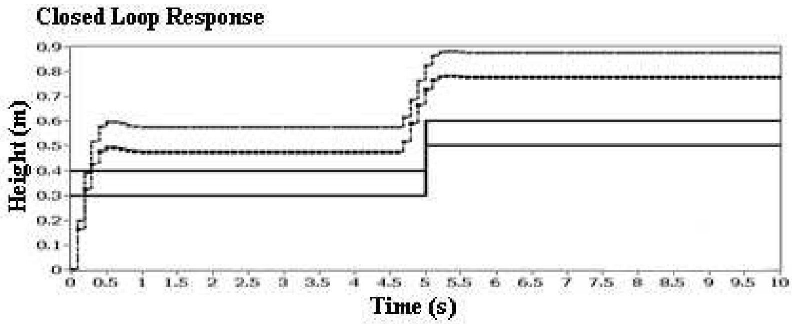

Fig. 6.Closed Loop Responses - Conventional MPC with measurement noise

Fig 6 shows the deviation of the output from the set point due to measurement noise in the system.

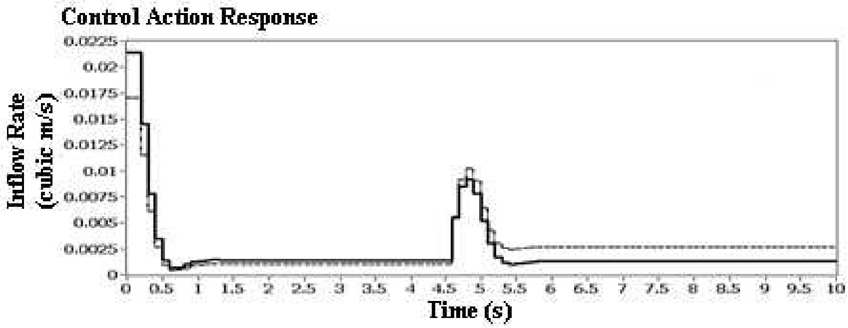

Fig.7. Control Inputs – Conventional MPC with measurement noise

c. State Estimation based MPC when measurement noise is not included

Fig 8 and 9 show the closed loop response of the system with state estimation based MPC when measurement noise is not included and the corresponding variation in the control inputs respectively. The state estimator used is EKF. Random errors are assumed to be present in the measurements (h_1 and h_2) as well as in the inflow rates. The covariance matrices of measurement noise and state noise are assumed as

$$R = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix} \qquad Q = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$$

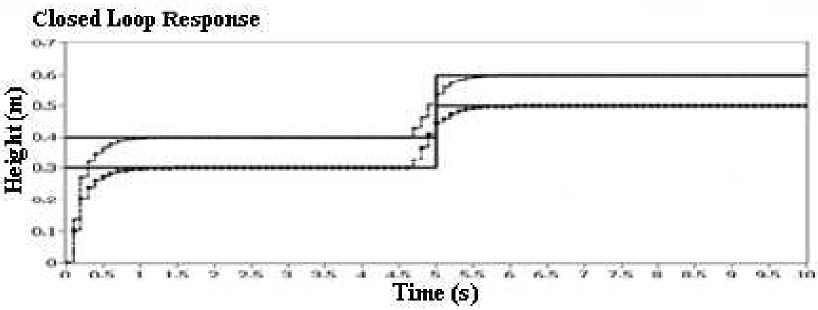

Fig.8. Closed Loop Response – State Estimation based MPC without measurement noise

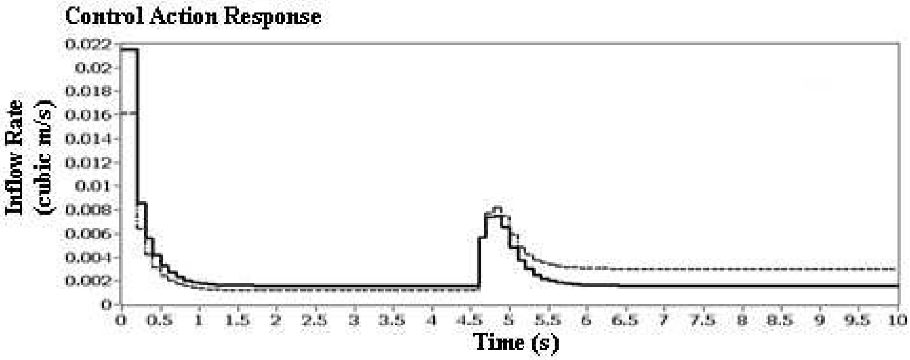

Fig.9. Control Inputs – State Estimation based MPC without measurement noise

d. State Estimation Based MPC when measurement noise is included

Fig 10 and 11 show the closed loop response of the system with state estimation based MPC when measurement noise is included and the corresponding variation in the control inputs respectively.

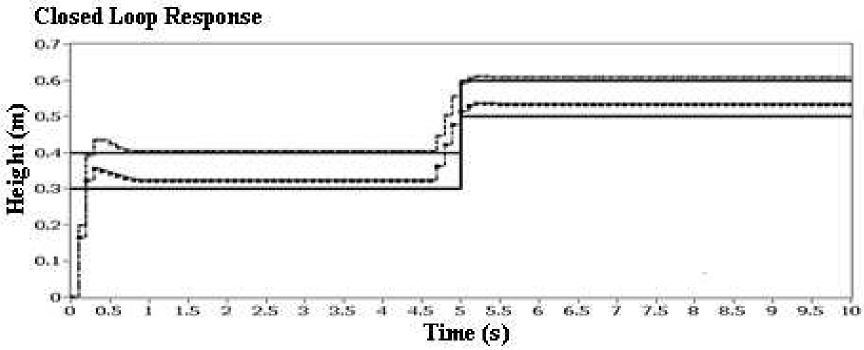

Fig.10. Closed Loop Response – State Estimation based MPC with measurement noise

By comparing Fig 6 and 10, it can be seen, the state estimation based MPC filters the noise more effectively and the system settles with less offset.

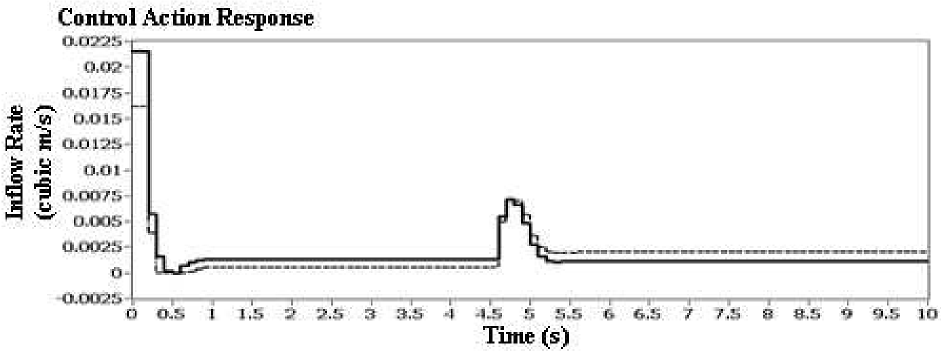

Fig.11. Control Inputs – State Estimation based MPC with measurement noise

From Fig 5, 7, 9 and 11 it can be clearly seen that the control inputs follow the constraints in all the four cases.

5.2 Performance index

Integral Square Error (ISE) is used as performance index to evaluate the performance of conventional and state estimation based MPC. Table 2 shows the comparative study.

Table 2 Comparison between conventional MPC and state estimation based MPC

Controller/ Performance index	ISE
Conventional MPC when measurement noise is not included	2.3404
Conventional MPC when measurement noise is included	13.5121
State Estimation based MPC when measurement noise is not included	2.2831
State Estimation based MPC when measurement noise is included	2.4707

From Table 2, it is clear that the state estimation based MPC shows better performance than the conventional MPC whether the measurement noise is considered or not. Moreover it shows that the state estimation based MPC has better noise suppressing capability.

6. Conclusion

In this paper, a MPC scheme based on EKF has been formulated and applied to two tank interacting system. The EKF is used to predict the future behavior of the system. Further, a comparative simulation has been done between conventional MPC and state estimation based MPC formulations. The performance of both the controllers has been evaluated based on the performance index ISE. From the extensive simulation studies on two tank system and the performance index, it can be inferred that state estimation based MPC is able to achieve satisfactory performance following the constraints and can suppress noise.

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