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LQR based optimal tuning of PID controller for trajectory tracking of Magnetic Levitation System

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Abstract

In this paper, we consider the stabilization and trajectory tracking of magnetic levitation system using PID controller whose controller gains are determined via Linear Quadratic Regulator (LQR) approach. Firstly, the nonlinear mathematical model of the system is obtained from the first principles. Then by applying Taylor's series, the non linear equation of motion is linearized around the equilibrium point to implement the stabilizing controller. Finally, the gains of the PID controller to achieve the desired response are determined using the LQR theory. Based on the natural frequency and damping ratio of the closed loop system, a new criterion for selecting the weighting matrices of LQR is proposed in this paper. Experiments are conducted on a Quanser magnetic levitation system to evaluate the performance of the proposed methodology and the experimental results prove that the proposed control strategy is effective not only in stabilizing the ball but also in rejecting the disturbance present in the system.

Keywords: Trajectory tracking, Magnetic levitation, PID controller, Linear Quadratic Regulator, Weighting Matrices, Feed forward control

1. Introduction

Magnetic levitation systems have received wide attention recently because of their practical importance in many engineering systems such as high-speed maglev passenger trains, frictionless bearings, levitation of wind tunnel models, vibration isolation of sensitive machinery, levitation of molten metal in induction furnaces, and levitation

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of metal slabs during manufacturing [1]. Magnetic levitation (maglev) technology reduces the physical contact between moving and stationary parts and in turn eliminates the friction problem. Maglev systems are inherently nonlinear, unstable and are described by highly nonlinear differential equations which present additional difficulties in controlling these systems. So the design of feedback controller for regulating the position of the levitated object is always a challenging task. In recent years, a lot of works have been reported in the literature for controlling magnetic levitation systems. The feedback linearization technique has been used to design control laws for magnetic levitation systems [2, 3]. The input-output, input-state, and exact linearization techniques have been used to develop nonlinear controllers [4, 5]. Other types of nonlinear controllers based on nonlinear methods have been reported in the literature [6]. Control laws based on the gain scheduling approach [7], linear controller design [8], and neural network techniques [9] have also been used to control magnetic levitation systems. Classical optimal control theory has evolved over decades to formulate the well known Linear Quadratic Regulators which minimizes the excursion in state trajectories of a system while requiring minimum controller effort [10]. This typical behavior of LQR has motivated control designers to use it for the tuning of PID controllers [11]. PID controllers are most common in process industries due to its simplicity, ease of implementation and robustness. Using the Lyapunov's method, the optimal quadratic regulator design problem reduces to the Algebraic Riccati Equation (ARE) which is solved to calculate the state feedback gains for a chosen set of weighting matrices. These weighting matrices regulate the penalties on the deviation in the trajectories of the state variables (x) and control signal (u). Indeed, with an arbitrary choice of weighting matrices, the classical state-feedback optimal regulators seldom show good set-point tracking performance due to the absence of integral term unlike the PID controllers. Thus, combining the tuning philosophy of PID controllers with the concept of LQR allows the designer to enjoy both optimal set-point tracking and optimal cost of control within the same design framework. The objective of this paper is to present a novel methodology which tunes a PID controller with an LQR based dominant pole placement method for trajectory tracking of magnetic levitation system.

Nomenclature

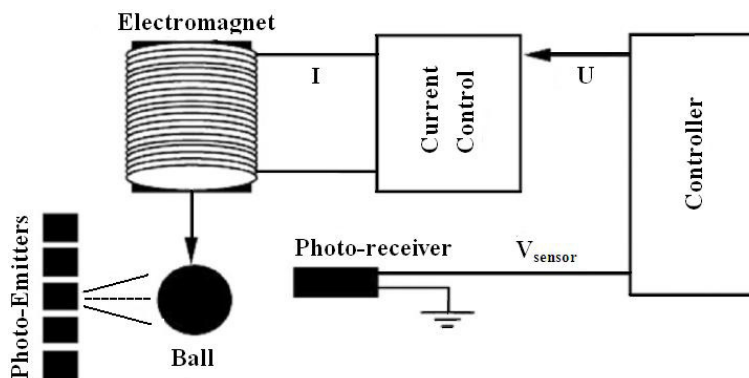|			
V_c	coil voltage	ζ	damping ratio
V_s	supply voltage	ω_n	natural frequency of oscillation
K_p	proportional gain	J	cost function
K_i	integral gain	Q, R	LQR weighting matrices
K_d	derivative gain	A	system matrix
K_{ff}	feed forward gain	B	input matrix

2. System Model

Fig. 1. Magnetic levitation system diagram

Magnetic levitation system is used to levitate a steel ball in air by the electromagnetic force created by an electromagnet. The maglev system consists of an electromagnet, a steel ball, a ball post, and a ball position sensor. The schematic diagram of the magnetic levitation system is shown in Fig. 1. The entire system is encased in a rectangular enclosure which contains three distinct sections. The upper section contains an electromagnet, made of a solenoid coil with a steel core. The middle section consists of a chamber where the ball suspension takes place. One of the electro magnet poles faces the top of a black post upon which a one inch steel ball rests. A photo sensitive sensor embedded in the post measures the ball elevation from the post. The last section of maglev system houses the signal conditioning circuitry needed for light intensity position sensor. The entire system is decomposed

into two subsystems, namely, mechanical subsystem and electrical subsystem. The coil current is adjusted to control the ball position in the mechanical system, where as the coil voltage is varied to control the coil current in an electrical system [12]. Thus, the voltage applied to the electromagnet indirectly controls the ball position. In the following section, we obtain the nonlinear mathematical model of the maglev system and linearize it around the operating region in order to design a stabilizing controller.
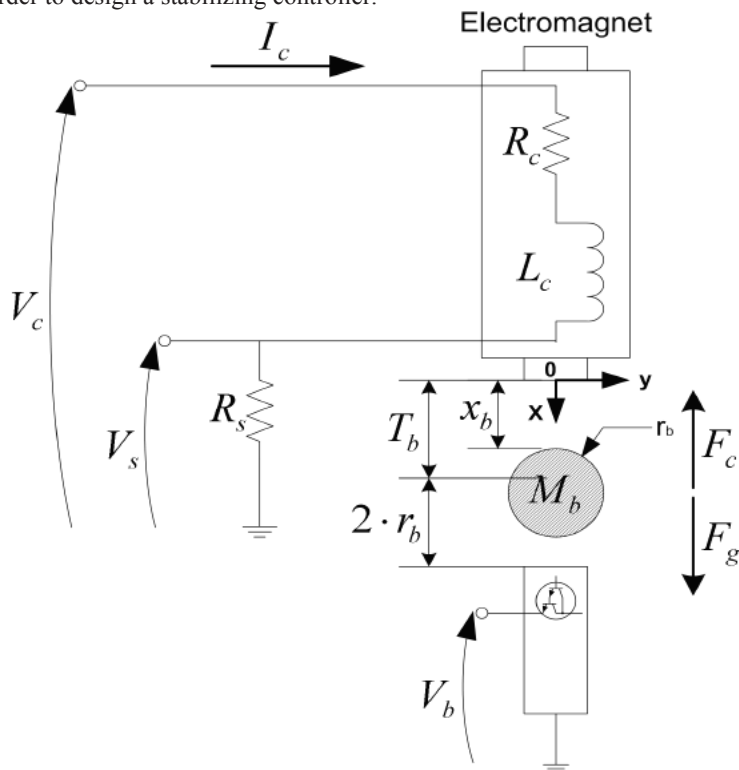

Fig. 2. Schematic of the Maglev plant

Table 1 System parameters

Symbol	Description	Value
L_c	Coil inductance	412.5mH
R_c	Coil resistance	10Ω
N_c	Number of turns in the coil wire	2450
l_c	Coil length	0.0825m
r_c	Coil steel core radius	0.008m
R_s	Current sense resistance	1Ω
K_m	Electromagnet force constant	6.5308E-005 N.m^2/A^2
r_b	Steel ball radius	1.27E-002 m
M_b	Steel ball mass	0.068kg
K_b	Ball position sensor sensitivity	2.83E-003 m/V
g	Gravitational constant	9.81 m/s^2

Applying Kirchoff's voltage law to the electrical circuit shown in Fig. (2)

$$V_c = (R_c + R_s)I_c + L\frac{d}{dt}I_c \tag{1}$$

The transfer function of the circuit can be obtained by applying Laplace transform to Eq. (1)

$$G_c(s) = \frac{I_c(s)}{V_c(s)} = \frac{K_c}{\tau_c s + 1} \tag{2}$$

Where $K_c = \dfrac{1}{R_c + R_s}$ and $\tau_c = \dfrac{L_c}{R_c + R_s}$

2.1 Equation of motion (EOM) of a ball

The force applied on the ball due to gravity can be expressed by

$$F_g = M_b g \tag{3}$$

Force generated by the electromagnet is given by

$$F_c = -\frac{1}{2}\frac{K_m I_c^2}{x_b^2} \tag{4}$$

The total force experienced by the ball is the sum of F_g and F_c

$$F_c + F_g = -\frac{1}{2}\frac{K_m I_c^2}{x_b^2} + M_b g \tag{5}$$

Applying Newton's second law, the nonlinear equation of motion can be obtained as

$$\frac{d^2 x_b}{dt^2} = -\frac{1}{2}\frac{K_m I_c^2}{M_b x_b^2} + g \tag{6}$$

At equilibrium point, all the time derivative terms are set to zero.

$$-\frac{1}{2}\frac{K_m I_c^2}{M_b x_b^2} + g = 0 \tag{7}$$

From Eq. (7), the coil current at equilibrium position, I_{c0}, can be expressed as a function of x_{b0} and K_m.

$$I_{c0} = \sqrt{\frac{2 M_b g}{K_m}}\, x_{b0} \tag{8}$$

The electromagnet force constant, K_m, as a function of the nominal pair (x_{b0}, I_{c0}), can be obtained from (7)

$$K_m = \frac{2 M_b g x_{b0}^2}{I_{c0}^2} \tag{9}$$

The nominal coil current I_{c0} for the electromagnet ball pair can be obtained at the system's static equilibrium. The static equilibrium at a nominal operating point (x_{b0}, I_{c0}) is characterized by the ball being suspended in air at a stationary point x_{b0} due to a constant attractive force created by I_{c0}.

2.2 Linearization of EOM

In order to design a linear controller, the system must be linearized around equilibrium point, the point at which the system will converge as time tends to infinity. The nonlinear system equations are linearized around the operating point using Taylor's series. Applying the Taylor's series approximation about (x_{b0}, I_{c0}) to Eq. (6)

$$\frac{d^2 x_{b1}}{dt^2} = -\frac{1}{2}\frac{K_m I_{c0}^2}{M_b x_{b0}^2} + g + \frac{K_m I_{c0}^2 x_{b1}}{M_b x_{b0}^3} - \frac{K_m I_{c0} I_{c1}}{M_b x_{b0}^2} \tag{10}$$

Substituting Eq. (9) in Eq. (10)

$$\frac{d^2 x_{b1}}{dt^2} = \frac{2 g x_{b1}}{x_{b0}} - \frac{2 g I_{c1}}{I_{c0}} \tag{11}$$

Applying Laplace transform to Eq. (11)

$$G_b(s) = -\frac{K_b \omega_b^2}{s^2 - \omega_b^2} \qquad (12)$$

Where $K_b = \dfrac{x_{b0}}{I_{c0}}$ and $\omega_b = \sqrt{\dfrac{2g}{x_{b0}}}$

The open loop transfer function of a maglev system is a type zero, second order system. The two open loop poles of the system are located at $s = \pm \omega_b$ which indicates that the open loop system is unstable due to location of poles on the right half of the s plane. Thus, the feedback controller is necessary for stabilizing the system. In this work, a pole placement controller using PID and Feed forward approach is designed to not only levitate the ball but also to make the ball follow the reference trajectory.

3. PID plus Feed forward Controller design using pole placement approach

The objective of the control strategy is to regulate and track the ball position in mid-air. The ball position controller is to be designed such that in response to a desired ± 1mm square wave position set point, the ball position behavior should satisfy the following performance requirements.
1. Percentage overshoot $\leq 15\%$
2. Settling time ≤ 1s

The proposed control scheme, as shown in Fig. 3, consists of a PID controller with a feed forward component. The controller gains of both PID and feed forward controller are determined by selecting the closed loop poles, which satisfy the performance specification, via root locus. Three separate gains are used in PID controller design, which introduces two zeros and a pole at origin so that the entire system becomes a Type 1 system, allowing for zero steady state error. The objective of the feed forward control action is to compensate for the gravitational bias. When the PID controller compensates for dynamic disturbances around the linear operating point (x_{b0}, I_{c0}), the feed forward control action eliminates the changes in the force created due to gravitational bias.
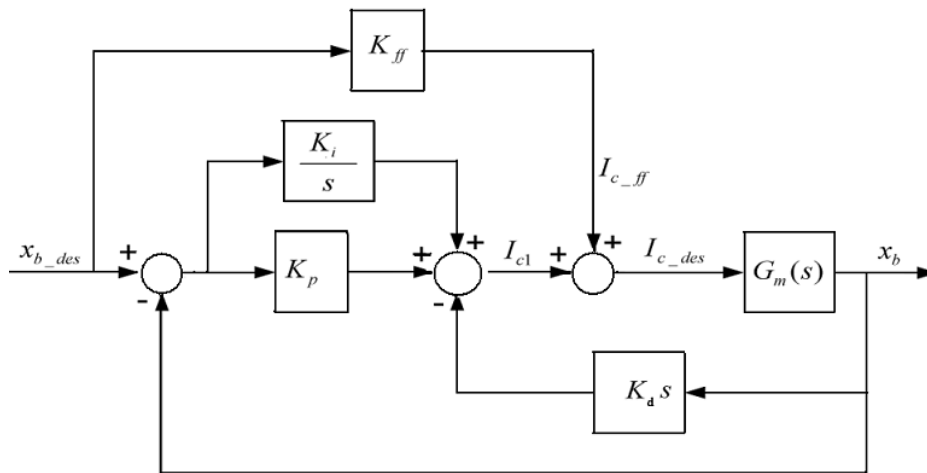

Fig. 3. PID plus Feed forward control loop for ball position control

The open loop transfer function $G_m(s)$ takes the dynamics of the electromagnet current loop into account and it is given by

$$G_m(s) = \frac{x_b(s)}{I_{c_{des}}(s)} = \frac{-2g/I_{c0}}{s^2 - 2g/x_{b0}} \qquad (13)$$

The current feed forward action is represented by

$$I_{c\_ff} = K_{ff} x_{b\_des} \qquad (14)$$

and

$$I_c = I_{c1} + I_{c\_ff}$$ (15)

At equilibrium point, $x_b = x_{b0}$ and $I_c = I_{c0}$. Thus, the feed forward gain is given by

$$K_{ff} = \frac{I_{c0}}{x_{b0}}$$ (16)

The closed loop transfer function of the system is

$$G_c(s) = \frac{x_b(s)}{x_{bdes}(s)} = \frac{2g((K_{ff} + K_p)s + K_i)}{I_{c0}(s^3 - \frac{2gK_d}{I_{c0}}s^2 + \left[\frac{-2g}{x_{b0}} - \frac{2gK_p}{I_{c0}}\right]s - \frac{2gK_i}{I_{c0}})}$$ (17)

The normalized characteristic equation of electromechanical system is

$$s^3 - \frac{2gK_d}{I_{c0}}s^2 + \left[\frac{-2g}{x_{b0}} - \frac{2gK_p}{I_{c0}}\right]s - \frac{2gK_i}{I_{c0}} = 0$$ (18)
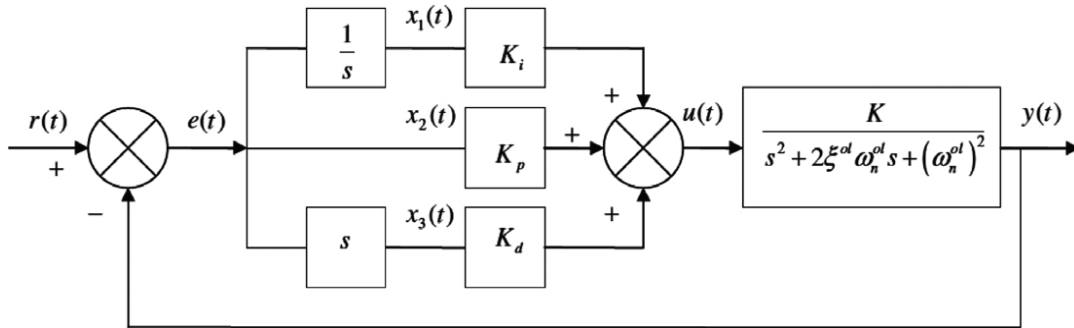
3. LQR based Optimal PID tuning

Fig. 4 LQR based PID tuning of second order process

In this section the gain parameters of PID controller determined using the LQR approach. Suman et al. [10] have given a formulation for tuning the PID controller gains via LQR approach with guaranteed pole placement. In this work, the idea has been extended to a magnetic levitation system which has two control schemes such as PID controller and feed forward controller. Here, the points which are important for determining the controller gain alone are explained and the further detail can be referred in [10]. In this approach, the error, error rate and integral of error are considered as state variables to obtain the optimal controller gains of the PID regulator.
Let the state variables be

$$x_1(t) = \int e(t)dt \quad x_2(t) = e(t) \quad x_3(t) = \frac{d}{dt}e(t)$$ (19)

From Fig.4,

$$\frac{Y(s)}{U(s)} = \frac{K}{s^2 + 2\zeta^o\omega_n^o s + (\omega_n^o)^2} = \frac{-E(s)}{U(s)}$$ (20)

In the state feedback regulator design, the external set point does not affect the controller design, so the reference input r(t)=0 in Fig. 4. When there is no change in the set point, the relation y(t)=-e(t) is valid for standard regulator problem. Thus, Eq. (20) becomes,

$$\left[s^2 + 2\zeta^o\omega_n^o s + (\omega_n^o)^2\right]E(s) = -KU(s)$$ (21)

Applying inverse Laplace transform,

$$\ddot{e} + 2\zeta^o\omega_n^o\dot{e} + (\omega_n^o)^2 e = -Ku$$ (22)

Thus the state space representation of the above system is of the form

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & (-\omega_n^o)^2 & (-2\zeta^o \omega_n^o) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -K \end{bmatrix} u \tag{23}
$$

From Eq. (23), the system matrices are

$$
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & (-\omega_n^o)^2 & (-2\zeta^o \omega_n^o) \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ -K \end{bmatrix} \tag{24}
$$

In order to obtain an optimal performance of system (24) through LQR formulation, the following quadratic cost function should be minimized.

$$
J = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t) \right] dt \tag{25}
$$

The minimization of above cost function gives the optimal control input as

$$
u(t) = -R^{-1}B^T Px(t) = -Fx(t) \tag{26}
$$

Where P is the symmetric positive definite solution of the Continuous Algebraic Riccatti equation given by

$$
A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{27}
$$

The weighting matrix Q is a symmetric positive definite and the weighting factor R is a positive constant. In general, the weighting matrix Q is varied, keeping R fixed, to obtain optimal control signal from the linear quadratic regulator. The corresponding state feedback gain matrix is

$$
F = R^{-1}B^T P = R^{-1} \begin{bmatrix} 0 & 0 & K \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12} & P_{22} & P_{23} \\ P_{13} & P_{23} & P_{33} \end{bmatrix} = R^{-1}K \begin{bmatrix} P_{13} & P_{23} & P_{33} \end{bmatrix} = -\begin{bmatrix} K_i & K_p & K_d \end{bmatrix} \tag{28}
$$

The corresponding expression for the control signal is

$$
u(t) = -Fx(t) = -\begin{bmatrix} -K_i & -K_p & -K_d \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = K_i \int e(t)dt + K_p e(t) + K_d \frac{d}{dt}e(t) \tag{29}
$$

The third row of symmetric positive definite matrix P can be obtained in terms of PID controller gains from Eq. (36)

$$
P_{13} = \frac{K_i}{R^{-1}K} \quad P_{23} = \frac{K_p}{R^{-1}K} \quad P_{33} = \frac{K_d}{R^{-1}K} \tag{30}
$$

The closed loop system matrix for the system (24) with state feedback gain matrix (28) is

$$
A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ (-R^{-1}K^2 P_{13}) & ((-\omega_n^o)^2 - R^{-1}K^2 P_{23}) & (-2\zeta^o \omega_n^o - R^{-1}K^2 P_{33}) \end{bmatrix} \tag{31}
$$

The corresponding characteristic polynomial for the closed loop system is

$$
\Delta(s) = |sI - A_c| = 0
$$

$$
s^3 + s^2(2\zeta^o \omega_n^o + R^{-1}K^2 P_{33}) + s((\omega_n^o)^2 + R^{-1}K^2 P_{23}) + R^{-1}K^2 P_{13} = 0 \tag{32}
$$

The characteristic polynomial of closed system in terms of the desired damping ration and natural frequency is given by,

$$
(s^3 + s^2(2+m)\zeta^c \omega_n^c + s((\omega_n^c)^2 + 2m(\zeta^c)^2(\omega_n^c)^2) + m\zeta^c(\omega_n^c)^3) = 0 \tag{33}
$$

Now, equating the coefficients of Eq. (32) with Eq. (33),

$$\left. \begin{array}{l} (2\zeta^o \omega_n^o + R^{-1}K^2 P_{33}) = (2+m)\zeta^c \omega_n^c \\ (\omega_n^o)^2 + R^{-1}K^2 P_{23} = (\omega_n^c)^2 + 2m(\zeta^c)^2(\omega_n^c)^2 \\ R^{-1}K^2 P_{13} = m(\zeta^c)^2(\omega_n^c)^3 \end{array} \right\} \tag{34}$$

The elements of the third row of matrix P is solved as in (35) by knowing the open loop process characteristics (ζ^o, ω_n^o, K) and desired closed loop dynamics (ζ^c, ω_n^c, K).

$$\left. \begin{array}{l} P_{13} = \dfrac{m(\zeta^c)^2(\omega_n^c)^3}{R^{-1}K^2} \\[2mm] P_{23} = \dfrac{(\omega_n^c)^2 + 2m(\zeta^c)^2(\omega_n^c)^2 - (\omega_n^o)^2}{R^{-1}K^2} \\[2mm] P_{33} = \dfrac{(2+m)\zeta^c \omega_n^c - 2\zeta^o \omega_n^o}{R^{-1}K^2} \end{array} \right\} \tag{35}$$

The remaining unknown elements of 'P' matrix can be determined by solving the following Algebraic Riccati Equation. With the known third row elements of P matrix the other elements of P and Q matrices can be obtained as follows.

$$\left. \begin{array}{l} P_{11} = (\omega_n^o)^2 P_{13} + R^{-1}K^2 P_{13} P_{23} \\ P_{12} = 2\zeta^o \omega_n^o P_{13} + R^{-1}K^2 P_{13} P_{23} \\ P_{22} = 2\zeta^o \omega_n^o P_{23} + R^{-1}K^2 P_{23} P_{33} + (\omega_n^o)^2 P_{33} - P_{13} \end{array} \right\} \tag{36}$$

$$\left. \begin{array}{l} Q_1 = R^{-1}K^2 P_{13}^2 \\ Q_2 = R^{-1}K^2 P_{23}^2 - 2(P_{12} - (\omega_n^o)^2 P_{23}) \\ Q_3 = R^{-1}K^2 P_{33}^2 - 2(P_{23} - 2\zeta^o \omega_n^o P_{33}) \end{array} \right\} \tag{37}$$

Design steps

Step 1: Specify both the open loop characteristics (ζ^o, ω_n^o, K) and the desired closed loop system dynamics (ζ^c, ω_n^c, K).

Step 2: Choose the weighting factor R in LQR and determine the weighting matrix Q using Eq. (37)

Step 3: Obtain the solution of ARE using Eq. (35) and Eq. (36)

Step 4: Calculate the system matrices A and B as specified in Eq. (24)

Step 5: Determine the solution of state feedback control using Eq. (28) and obtain the PID controller gains.

4. Experimental Results

The experimental set up, as shown in Fig. 5, consists of a personal computer and a Quanser's Magnetic levitation plant. The proposed control algorithm is realized in the PC using the real time algorithm, QUARC, which is similar to C like language. In order to attenuate the high frequency noise current, a simple low pass filter of cut off frequency 80Hz is added to the ball position sensor output. Furthermore, by differentiating the ball position, the ball vertical velocity is estimated.The open loop parameters of magnetic levitation system are K=7, $\omega_n^o = 1.8$, and the desired parameters of the closed loop system which satisfy the controller specification given in section 3 are $\zeta^c = 0.8$, $\omega_n^c = 1.4$, and m=9 .

Fig.5. Snap shot of Experimental set up

The corresponding system matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3.24 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}$$

The resultant state feedback gain matrix is

$$F = \begin{bmatrix} 65.8 & 28.14 & 5.03 \end{bmatrix} = -\begin{bmatrix} K_i & K_p & K_d \end{bmatrix}$$

4.1 Trajectory tracking

In order to test the controller's trajectory tracking capability, three test cases such as square, sinusoidal, and ramp signals with a frequency of 1 Hz is chosen. Fig. 6, 7, and 8 show the results for trajectory following. Fig. 9 shows the response of coil current, which tracks the specified command to make the ball follow the reference trajectory. The response of coil voltage is shown in Fig. 10 and it is worth to note that the coil voltage does not reach a saturation value during the trajectory following and it is well below 9V. Fig.11 shows the tracking error, which is the difference between actual trajectory and reference trajectory. The performance indices IAE, IATE, ISE and ITSE are considered for the performance evaluation of the controller, and they are given in Table2.
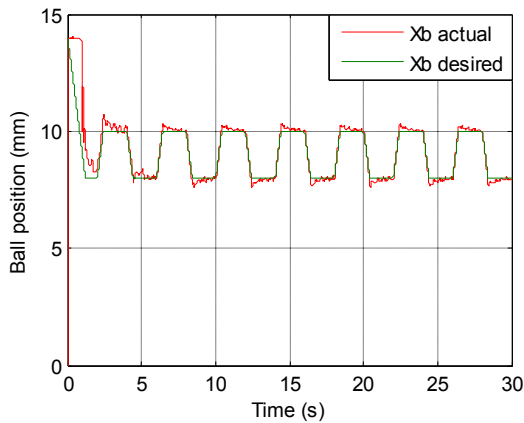

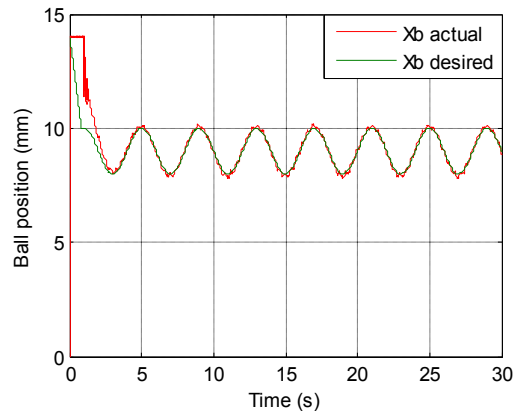Fig. 6 Square wave trajectory

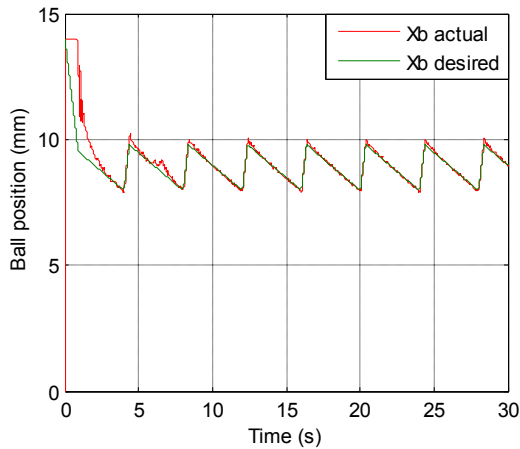Fig.7 Sine wave trajectory

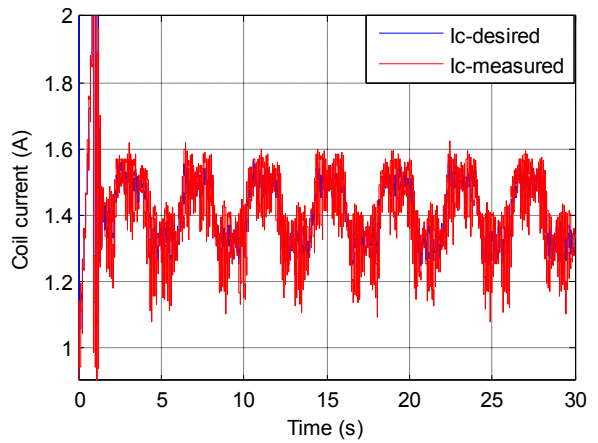Fig. 8 Saw tooth trajectory

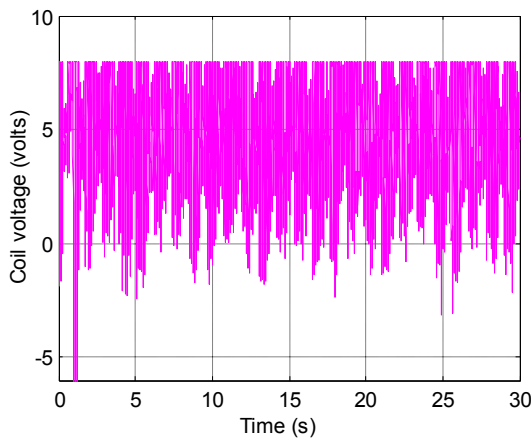Fig. 9 Coil current response

Fig. 10 Coil voltage response
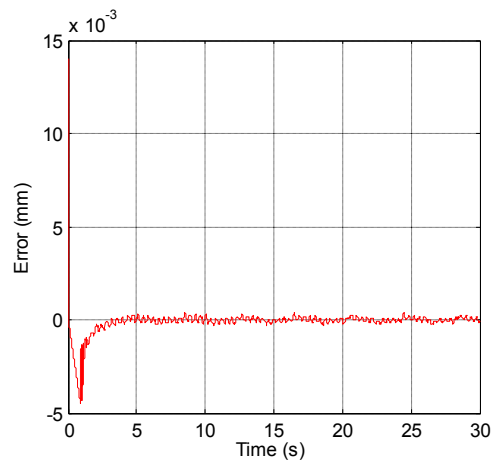
Fig. 11 Trajectory tracking error

Table 2. Performance Indices of LQR based PID tuning method

IAE	IATE	ISE	ITSE
0.0041	0.1252	0.000017	0.000015

5.   Conclusion

The modeling and control of 1DOF magnetic levitation system has been investigated in this paper. The conventional PID controller is combined with the feed forward controller in order to nullify the effect of gravitational bias existing in the magnetic levitation system. The non linear mathematical model of the plant from fundamental physical laws has been obtained and the non linear equation has been linearized around the equilibrium point using Taylor's series. Combining the tuning philosophy of PID controllers with the concept of LQR theory, the simple mathematical gain formulae to obtain the satisfactory response has been obtained. The experimental results demonstrated the effectiveness of the proposed approach not only in stabilizing the ball but also in tracking the various reference trajectories given as an input.

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