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Neighborhoods of certain subclasses of analytic functions of complex order with negative coefficients

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Abstract

By making use of the familiar concept of neighborhoods of analytic functions, the authors prove several inclusion relations associated with the (n, δ)-neighborhoods of certain subclasses of analytic functions of complex order, which are introduced here by means of the Ruscheweyh derivatives.

Keywords: Analytic functions; Ruscheweyh derivatives; Starlike functions; Convex functions; (n, δ)-neighborhoods; Inclusion relations

1. Introduction and definitions

Let $\mathcal{A}(n)$ denote the class of functions f of the form:

$$f(z) = z - \sum_{k=n+1}^{\infty} a_k z^k \quad \left(a_k \geqq 0; k \in \mathbb{N} \setminus \{1, \ldots, n\}; n \in \mathbb{N}; \mathbb{N} := \{1, 2, 3, \ldots\}\right), \tag{1.1}$$

which are analytic in the open unit disk

$$\mathbb{U} := \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

Following the earlier investigations by Goodman [1] and Ruscheweyh [2], we define the (n, δ)-neighborhood of a function $f \in \mathcal{A}(n)$ by (see also [3–5] and [6])

$$N_{n,\delta}(f) := \left\{ g : g \in \mathcal{A}(n), g(z) = z - \sum_{k=n+1}^{\infty} b_k z^k \text{ and } \sum_{k=n+1}^{\infty} k|a_k - b_k| \leqq \delta \right\}. \tag{1.2}$$

In particular, for the *identify* function

$$e(z) = z, \tag{1.3}$$

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we immediately have

$$N_{n,\delta}(e) := \left\{ g : g \in \mathcal{A}(n),\, g(z) = z - \sum_{k=n+1}^{\infty} b_k z^k \text{ and } \sum_{k=n+1}^{\infty} k|b_k| \leqq \delta \right\}. \tag{1.4}$$

The above concept of (n, δ)-neighborhoods was extended and applied recently to families of analytically multivalent functions by Altintaş et al. [7], and to families of meromorphically multivalent functions by Liu and Srivastava ([8] and [9]) (see also the more recent works [10] and [11]). The main object of the present paper is to investigate the (n, δ)-neighborhoods of several subclasses of $\mathcal{A}(n)$ of *normalized* analytic functions in $\mathbb{U}$ with negative and missing coefficients, which are introduced here by making use of the Ruscheweyh derivative operator defined by (1.11) or (1.12) below.

First of all, we say that a function $f \in \mathcal{A}(n)$ is *starlike of complex order* γ $(\gamma \in \mathbb{C} \setminus \{0\})$; that is, $f \in \mathcal{S}_n^{\star}(\gamma)$, if it also satisfies the following inequality:

$$\Re\left(1 + \frac{1}{\gamma}\left[\frac{zf'(z)}{f(z)} - 1\right]\right) > 0 \quad (z \in \mathbb{U};\ \gamma \in \mathbb{C} \setminus \{0\}). \tag{1.5}$$

Furthermore, a function $f \in \mathcal{A}(n)$ is said to be *convex of complex order* γ $(\gamma \in \mathbb{C} \setminus \{0\})$; that is, $f \in \mathcal{C}_n(\gamma)$ if it also satisfies the following inequality:

$$\Re\left(1 + \frac{1}{\gamma}\left[\frac{zf''(z)}{f'(z)}\right]\right) > 0 \quad (z \in \mathbb{U};\ \gamma \in \mathbb{C} \setminus \{0\}). \tag{1.6}$$

The classes $\mathcal{S}_n^{\star}(\gamma)$ and $\mathcal{C}_n(\gamma)$ stem essentially from the classes of starlike and convex functions of complex order, which were considered earlier by Nasr and Aouf [12] and Wiatrowski [13], respectively (see also [14–16] and [17]).

Let $\mathcal{S}_n(\gamma, \alpha, \mu, \beta)$ denote the subclass of the function class $\mathcal{A}(n)$ consisting of functions $f(z)$ which satisfy the following inequality:

$$\left|\frac{1}{\gamma}\left(\frac{\alpha\mu z^3 f'''(z) + (2\alpha\mu + \alpha - \mu)z^2 f''(z) + zf'(z)}{\alpha\mu z^2 f''(z) + (\alpha - \mu)zf'(z) + (1 - \alpha + \mu)f(z)} - 1\right)\right| < \beta$$
$$(z \in \mathbb{U};\ \gamma \in \mathbb{C} \setminus \{0\};\ 0 \leqq \mu \leqq \alpha;\ 0 < \beta \leqq 1). \tag{1.7}$$

Suppose also that $\mathcal{R}_n(\gamma, \alpha, \mu, \beta)$ denotes the subclass of the function class $\mathcal{A}(n)$ consisting of functions $f(z)$ which satisfy the following inequality:

$$\left|\frac{1}{\gamma}\left(\alpha\mu z^2 f'''(z) + (2\alpha\mu + \alpha - \mu)zf''(z) + f'(z) - 1\right)\right| < \beta$$
$$(z \in \mathbb{U};\ \gamma \in \mathbb{C} \setminus \{0\};\ 0 \leqq \mu \leqq \alpha;\ 0 < \beta \leqq 1). \tag{1.8}$$

The classes $\mathcal{S}_n(\gamma, \alpha, \mu, \beta)$ and $\mathcal{R}_n(\gamma, \alpha, \mu, \beta)$ were studied recently by Orhan and Kamali [17].

Next, for the functions $f_j(z)$ $(j = 1, 2)$ given by

$$f_j(z) = z + \sum_{k=2}^{\infty} a_{k,j} z^k \quad (j = 1, 2), \tag{1.9}$$

we denote by $(f_1 \star f_2)(z)$ the Hadamard product (or convolution) of $f_1(z)$ and $f_2(z)$, defined by

$$(f_1 \star f_2)(z) := z + \sum_{k=2}^{\infty} a_{k,1} a_{k,2} z^k =: (f_2 \star f_1)(z). \tag{1.10}$$

Thus the Ruscheweyh derivative operator

$$D^{\lambda} : \mathcal{A} \to \mathcal{A} \quad \big(\mathcal{A} := \mathcal{A}(1)\big)$$

is defined by

$$D^{\lambda} f(z) := \frac{z}{(1 - z)^{\lambda+1}} \star f(z) \quad (\lambda > -1;\ f \in \mathcal{A}) \tag{1.11}$$

or, equivalently, by

$$D^{\lambda} f(z) := z - \sum_{k=2}^{\infty} \binom{\lambda + k - 1}{k - 1} a_k z^k \quad (\lambda > -1;\ f \in \mathcal{A}) \tag{1.12}$$

for a function $f \in \mathcal{A}$ of the form (1.1). Here, and in what follows, we make use of the following standard notation for a binomial coefficient:

$$\binom{\kappa}{n} := \frac{\kappa(\kappa - 1) \cdots (\kappa - n + 1)}{n!} \quad (\kappa \in \mathbb{C};\ n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}). \tag{1.13}$$

In particular, we have

$$D^n f(z) = \frac{z \left(z^{n-1} f(z)\right)^{(n)}}{n!} \quad (n \in \mathbb{N}_0). \tag{1.14}$$

Finally, in terms of the Ruscheweyh derivative operator D^{λ} $(\lambda > -1)$ defined by (1.11) or (1.12) above, let $\mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$ denote the subclass of the function class $\mathcal{A}(n)$ consisting of functions $f(z)$ which satisfy the following inequality:

$$\left| \frac{1}{\gamma} \left(\frac{\alpha\mu z^3 \left(D^{\lambda} f(z)\right)''' + (2\alpha\mu + \alpha - \mu) z^2 \left(D^{\lambda} f(z)\right)'' + z \left(D^{\lambda} f(z)\right)'}{\alpha\mu z^2 \left(D^{\lambda} f(z)\right)'' + (\alpha - \mu) z \left(D^{\lambda} f(z)\right)' + (1 - \alpha + \mu) D^{\lambda} f(z)} - 1 \right) \right| < \beta$$

$$(z \in \mathbb{U};\ \gamma \in \mathbb{C} \setminus \{0\};\ \lambda > -1;\ 0 < \beta \leqq 1;\ 0 \leqq \mu \leqq \alpha). \tag{1.15}$$

Also, let $\mathcal{R}_n(\gamma, \lambda, \alpha, \mu, \beta)$ denote the subclass of the function class $\mathcal{A}(n)$ consisting of functions $f(z)$ which satisfy the following inequality:

$$\left| \frac{1}{\gamma} \left(\alpha\mu z^2 \left(D^{\lambda} f(z)\right)''' + (2\alpha\mu + \alpha - \mu) z \left(D^{\lambda} f(z)\right)'' + \left(D^{\lambda} f(z)\right)' - 1 \right) \right| < \beta$$

$$\left(z \in \mathbb{U};\ \gamma \in \mathbb{C} \setminus \{0\};\ \lambda > -1;\ 0 < \beta \leqq 1;\ 0 \leqq \mu \leqq \alpha\right). \tag{1.16}$$

Various *further* subclasses of the function class $\mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$ with

$$\gamma = 1 \quad \text{and} \quad \alpha = \mu = 0$$

were studied in many earlier works (*cf.*, *e.g.*, [18] and [19]; see also the references cited in each of these earlier works). Clearly, in these cases of (for example) the class $\mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$, we have the following relationships:

$$\mathcal{S}_n(\gamma, 0, 0, 0, 1) \subset \mathcal{S}_n^{\star}(\gamma) \quad \text{and} \quad \mathcal{S}_n(\gamma, 0, 1, 0, 1) \subset \mathcal{C}_n(\gamma) \quad (n \in \mathbb{N};\ \gamma \in \mathbb{C} \setminus \{0\}). \tag{1.17}$$

2. Inclusion relations involving the (n, δ)-neighborhood $N_{n,\delta}(e)$

In our investigation of the inclusion relations involving the (n, δ)-neighborhood $N_{n,\delta}(e)$, we shall require the following lemmas.

Lemma 1. *Let $f \in \mathcal{A}(n)$ be defined by (1.1). Then f is in the class $\mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$ if and only if*

$$\sum_{k=n+1}^{\infty} \binom{\lambda + k - 1}{k - 1} \eta(k) a_k \leqq \beta |\gamma|, \tag{2.1}$$

where

$$\eta = \eta(k) := \Big(\alpha\mu k^3 + (\alpha - \mu - 2\alpha\mu + \alpha\mu\beta|\gamma|) k^2 + (\alpha\mu - 2\alpha - 2\mu + 1 + (\alpha - \mu - \alpha\mu)\beta|\gamma|)\, k + (1 - \alpha + \mu)(\beta|\gamma| - 1)\Big).$$

Proof. We first suppose that $f \in \mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$. Then, by appealing to the condition (1.15), we readily obtain

$$\Re\left(\frac{\alpha\mu z^3\left(D^\lambda f(z)\right)''' + (2\alpha\mu+\alpha-\mu)z^2\left(D^\lambda f(z)\right)'' + z\left(D^\lambda f(z)\right)'}{\alpha\mu z^2\left(D^\lambda f(z)\right)'' + (\alpha-\mu)z\left(D^\lambda f(z)\right)' + (1-\alpha+\mu)D^\lambda f(z)} - 1\right) > -\beta|\gamma| \quad (z \in \mathbb{U}) \tag{2.2}$$

or, equivalently,

$$\Re\left(\frac{-\sum_{k=n+1}^{\infty}\binom{\lambda+k-1}{k-1}\left[\alpha\mu k^3 + (\alpha-\mu-2\alpha\mu)k^2 + (\alpha\mu-2\alpha+2\mu+1)k - (1-\alpha+\mu)\right]a_k z^k}{z - \sum_{k=n+1}^{\infty}\binom{\lambda+k-1}{k-1}\left[\alpha\mu k^2 + (\alpha-\mu-\alpha\mu)k + (1-\alpha+\mu)\right]a_k z^k}\right) > -\beta|\gamma| \quad (z \in \mathbb{U}), \tag{2.3}$$

where we have made use of (1.12) and the definition (1.1). We now choose values of z on the real axis and let $z \to 1-$ through *real* values. Then the inequality (2.3) immediately yields the desired condition (2.1).

Conversely, by applying the hypothesis (2.1) and letting $|z| = 1$, we find that

$$\begin{aligned}
&\left|\frac{\alpha\mu z^3\left(D^\lambda f(z)\right)''' + (2\alpha\mu+\alpha-\mu)z^2\left(D^\lambda f(z)\right)''' + z\left(D^\lambda f(z)\right)'}{\alpha\mu z^2\left(D^\lambda f(z)\right)'' + (\alpha-\mu)z\left(D^\lambda f(z)\right)' + (1-\alpha+\mu)D^\lambda f(z)} - 1\right| \\
&= \left|\frac{\sum_{k=n+1}^{\infty}\binom{\lambda+k-1}{k-1}\left[\alpha\mu k^3 + (\alpha-\mu-2\alpha\mu)k^2 + (\alpha\mu-2\alpha+2\mu+1)k - (1-\alpha+\mu)\right]a_k z^k}{1 - \sum_{k=n+1}^{\infty}\binom{\lambda+k-1}{k-1}\left[\alpha\mu k^2 + (\alpha-\mu-\alpha\mu)k + (1-\alpha+\mu)\right]a_k z^k}\right| \\
&\leqq \frac{\beta|r|\left[1 - \sum_{k=n+1}^{\infty}\binom{\lambda+k-1}{k-1}\left[\alpha\mu k^2 + (\alpha-\mu-\alpha\mu)k + (1-\alpha+\mu)\right]a_k\right]}{1 - \sum_{k=n+1}^{\infty}\binom{\lambda+k-1}{k-1}\left[\alpha\mu k^2 + (\alpha-\mu-\alpha\mu)k + (1-\alpha+\mu)\right]a_k} \\
&\leqq \beta|\gamma|.
\end{aligned} \tag{2.4}$$

Hence, by the *maximum modulus principle*, we have

$$f \in \mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta),$$

which evidently completes the proof of Lemma 1. □

Similarly, we can prove the following result.

Lemma 2. *Let the function $f \in \mathcal{A}(n)$ be defined by (1.1). Then f is in the class $\mathcal{R}_n(\gamma, \lambda, \alpha, \mu, \beta)$ if only if*

$$\sum_{k=n+1}^{\infty}\binom{\lambda+k-1}{k-1}\left[\alpha\mu k^3 + (\alpha-\mu-\alpha\mu)k^2 + (1-\alpha+\mu)k\right]a_k \leqq \beta|\gamma|. \tag{2.5}$$

Remark 1. A special case of Lemma 1 when

$$n = 1, \qquad \mu = \alpha = 0, \qquad \gamma = 1 \quad \text{and} \quad \beta = 1 - c \quad (0 \leqq c < 1)$$

was given by Ahuja [20]. Furthermore, in Lemma 1 with

$$n = 1, \qquad \mu = \alpha = 0, \qquad \gamma = 1 \quad \text{and} \quad \beta = 1 - c \quad (0 \leqq c < 1),$$

if we set

$$\lambda = 0 \quad \text{and} \quad \lambda = 1,$$

we obtain the relatively more familiar results of Silverman [21].

Our first main result is given by Theorem 1 below.

Theorem 1. *If*

$$\delta := \frac{(n+1)\beta|\gamma|}{\binom{\lambda+n}{n}\rho}, \tag{2.6}$$

then

$$\mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta) \subset N_{n,\delta}(e), \tag{2.7}$$

where

$$\begin{aligned}\rho := \Big[&\alpha\mu(n+1)^3 + (\alpha\mu\beta|\gamma| + \alpha - \mu - 2\alpha\mu)(n+1)^2 \\ &+ \big((\alpha - \mu - \alpha\mu)\beta|\gamma| + 1 - 2\alpha + 2\mu + \alpha\mu\big)(n+1) \\ &+ (1 - \alpha + \mu)(\beta|\gamma| - 1)\Big].\end{aligned} \tag{2.8}$$

Proof. For a function $f \in \mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$ of the form (1.1) and for ρ defined already by (2.8), Lemma 1 immediately yields

$$\binom{\lambda+n}{n}\rho \sum_{k=n+1}^{\infty} a_k \leqq \beta|\gamma|,$$

so that

$$\sum_{k=n+1}^{\infty} a_k \leqq \frac{\beta|\gamma|}{\binom{\lambda+n}{n}\rho}. \tag{2.9}$$

On the other hand, we also find from (2.1) that

$$\binom{\lambda+n}{n}\tau \sum_{k=n+1}^{\infty} k a_k \leqq \beta|\gamma|,$$

where

$$\begin{aligned}\tau = \Bigg[&\alpha\mu(n+1)^2 + (\alpha\mu\beta|\gamma| + \alpha - \mu - 2\alpha\mu)(n+1) + ((\alpha - \mu - \alpha\mu)\beta|\gamma| + 1 - 2\alpha + 2\mu + \alpha\mu) \\ &+ \left(\frac{(1 - \alpha + \mu)(\beta|\gamma| - 1)}{n+1}\right)\Bigg],\end{aligned} \tag{2.10}$$

that is, that

$$\sum_{k=n+1}^{\infty} k a_k \leqq \frac{\beta|\gamma|(n+1)}{\binom{\lambda+n}{n}\rho} := \delta, \tag{2.11}$$

which, in view of the definition (1.4), proves Theorem 1. □

Similarly, by applying Lemma 2 instead of Lemma 1, we can prove Theorem 2 below.

Theorem 2. *If*

$$\delta := \frac{\beta|\gamma|}{\binom{\lambda+n}{n}\left[\alpha\mu(n+1)^2 + (\alpha - \mu - \alpha\mu)(n+1) + (1 - \alpha + \mu)\right]}, \tag{2.12}$$

then

$$\mathcal{R}_n(\gamma, \lambda, \alpha, \mu, \beta) \subset N_{n,\delta}(e).$$

3. Neighborhood properties for the function classes $\mathcal{S}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta)$ and $\mathcal{R}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta)$

In this section, we determine the neighborhood for each of the function classes

$$\mathcal{S}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta) \quad \text{and} \quad \mathcal{R}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta),$$

which we define here as follows.

Definition 1. A function $f \in \mathcal{A}(n)$ is said to be in the class $\mathcal{S}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta)$ if there exists a function $g \in \mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$ such that the following inequality holds true:

$$\left| \frac{f(z)}{g(z)} - 1 \right| < 1 - b \quad (z \in \mathbb{U}; 0 \leqq b < 1). \tag{3.1}$$

Definition 2. A function $f \in \mathcal{A}(n)$ is said to be in the class $\mathcal{R}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta)$ if there exists a function $g \in \mathcal{R}_n(\gamma, \lambda, \alpha, \mu, \beta)$ such that the inequality (3.1) holds true.

Theorem 3. *If* $g \in \mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$ *and*

$$b = 1 - \frac{\binom{\lambda+n}{n} \delta\rho}{(n+1)\left[\binom{\lambda+n}{n} \rho - \beta|\gamma|\right]}, \tag{3.2}$$

then

$$N_{n,\delta}(g) \subset \mathcal{S}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta), \tag{3.3}$$

where ρ *is given already by* (2.8).

Proof. Assuming that $f \in N_{n,\delta}(g)$, we find from the definition (1.2) that

$$\sum_{k=n+1}^{\infty} k|a_k - b_k| \leqq \delta, \tag{3.4}$$

which readily implies the following coefficient inequality:

$$\sum_{k=n+1}^{\infty} |a_k - b_k| \leqq \frac{\delta}{n+1} \quad (n \in \mathbb{N}). \tag{3.5}$$

Since $g \in \mathcal{S}_n(\gamma, \lambda, \alpha, \mu, \beta)$, we have [*cf.* Eq. (2.9)].

$$\sum_{k=n+1}^{\infty} b_k = \frac{\beta|\gamma|}{\binom{\lambda+n}{n} \rho}, \tag{3.6}$$

so that

$$\left| \frac{f(z)}{g(z)} - 1 \right| < \frac{\sum\limits_{k=n+1}^{\infty} |a_k - b_k|}{1 - \sum\limits_{k=n+1}^{\infty} b_k}$$

$$\leqq \frac{\delta}{n+1} \cdot \frac{\binom{\lambda+n}{n} \delta\rho}{\left[\binom{\lambda+n}{n} \rho - \beta|\gamma|\right]} =: 1 - b, \tag{3.7}$$

provided that b is given precisely by (3.2). Thus, by Definition 1, we conclude that

$$f \in \mathcal{S}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta)$$

for b given by (3.2). This evidently completes the proof of Theorem 3. □

The proof of Theorem 4 below is much akin to that of Theorem 3, and so the details involved are being omitted here.

Theorem 4. *If* $g \in \mathcal{R}_n(\gamma, \lambda, \alpha, \mu, \beta)$ *and*

$$b = 1 - \frac{\binom{\lambda+n}{n}\delta\left[\alpha\mu(n+1)^3 + (\alpha-\mu-\alpha\mu)(n+1)^2 + (1-\alpha+\mu)(n+1)\right]}{(n+1)\left[\binom{\lambda+n}{n}\left[\alpha\mu(n+1)^3 + (\alpha-\mu-\alpha\mu)(n+1)^2 + (1-\alpha+\mu)(n+1)\right] - \beta|\gamma|\right]}, \tag{3.8}$$

then

$$N_{n,\delta}(g) \subset \mathcal{R}_n^{(b)}(\gamma, \lambda, \alpha, \mu, \beta). \tag{3.9}$$

Remark 2. A special case of Theorem 3 when $\alpha = \mu = 0$ was proven recently by Murugusundaramoorthy and Srivastava [16, p. 6, Theorem 3].

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