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ORIGINAL RESEARCH

Neutrosophic AHP-Delphi Group decision making model based on trapezoidal neutrosophic numbers

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Abstract The main objective of this research is to study the integration of Analytic Hierarchy Process (AHP) into Delphi framework in neutrosophic environment and present a new technique for checking consistency and calculating consensus degree of expert's opinions. In some pragmatism bearings, the experts might be not able to assign deterministic evaluation values to the comparison judgments due to his/her confined knowledge or the differences of individual judgments in group decision making. To overcome these challenges, we have used neutrosophic set theory to handle the integration of AHP into Delphi framework, where each pairwise comparison judgment is symbolized as a trapezoidal neutrosophic number. The power of AHP is enhanced by adding Delphi technique, since it can reduce noise which result from focusing on group and/or individual interests rather than concentriciting on problem disband and it also increase consensus degree about ideas. Obtaining a consistent trapezoidal neutrosophic preference relation is very difficult in decision making process because in creating a consistent trapezoidal preference relation; each expert should make $\frac{n\times(n-1)}{2}$ consistent judgments for n alternatives. When the number of alternatives is increasing, the workload of giving judgments for the experts is heavy and it makes them tired and leads to inconsistent judgments. In the proposed model,experts will focus only on $(n-1)$ restricted judgments and this also enhances the performance of AHP over the traditional version that is proposed by Saaty. A real life example is developed based on expert opinions about evaluation process of many international search engines. The problem is solved to show the validation of the suggested method in neutrosophic path.

Keywords Analytic hierarchy process (AHP) · Delphi technique · Neutrosophic set theory · Trapezoidal neutrosophic numbers · Consistency · Consensus degree

1 Introduction

Multi-Criteria Decision Making (MCDM) is a formal and structured decision making methodology for dealing with complex problems (Daneshvar Rouyendegh 2011). Saaty (2008) founded the Analytic Hierarchy Process (AHP) in the late 1970s. Today it is the common and the most widely used procedure for dealing with Multi-Criteria Decision Making. Analytic Hierarchy Process solves complex problems by decomposing it into sub problems, consist of criteria and alternatives. Then, a series of pairwise comparisons matrices between criteria and alternatives are made. Furthermore, each expert should make $\frac{n\times(n-1)}{2}$ consistent judgments for n alternatives and this makes experts tired and leads to inconsistent judgments by increasing number of alternatives. And in this research, we treated this drawback by making experts focus only on $(n-1)$ restricted judgments. The analysis of AHP require applying a scale system for pair-wise comparisons matrix and this scale play an important role in transforming qualitative analysis to quantitative analysis (Lv 2001). The traditional 1–9 scale of analytic hierarchy

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process was used by most of the previous research. Lv et al. (2003) determined serious of mathematical shortages of Saaty's scale such as:

- Large hole between ranking results and human judgments.
- Conflicting between ruling matrix and human intellect.

Then, in this research we proposed anew scale from 0 to 1 to overcome previous drawbacks. Although analytic Hierarchy Process allows the use of qualitative, as well as quantitative criteria in estimation but in real life problems, the experts may be unable to allocate preference values to the objects considered due to vague knowledge and/or differences of individual or group interests (Jain et al. 2016; Samuel et al. 2017; Sangaiah and Thangavelu 2013; Sangaiah et al. 2015, 2017; Sangaiah and Jain 2016). And to overcome these drawbacks, we integrate AHP with Delphi mechanism. Helmer and Rescher (1959) advanced the Delphi mechanism at the Rand Corporation in the 1950s; it is a very important and excessively used method for obtaining consensus of opinions from experts about real world topics. Delphi mechanism was built on a logical basis that "*n* heads are better than one" (Miller 2006). Delphi is a procedure for collecting data from defendants according to their areas of experience. Although most of surveys seek to distinguish "what is", the Delphi technique tries to achieve "what could/should be" (Miller 2006). Delphi technique used in many fields such as program planning, resource utilization, policy judgment and needs assessment. A Delphi technique has the following advantages:

1. Deal with complex problems effectively.
2. Able to define and modify a wide range of alternatives.
3. Create different judgments on the same topic and use feedback about individual's judgments to let individuals revise their views.
4. Achieve a high degree of consensus.
5. Increase consistency by decreasing noise which result from focusing on group and/or individual interests rather than concentriciting on problem disbanding.

Tavana et al. (1993) integrated AHP with Delphi and applied it to the conflict resolution in hiring decisions. The analytic hierarchy process also used with Delphi method, to evaluate Chinese search engines (Lewandowski et al. 2011). Kim et al. (2013) applied Delphi-AHP methods to select the ranking of waste electrical and electronic equipment. The fuzzy Delphi method and fuzzy analytical hierarchy process applied to managerial competence of multinational corporation executives (Liu 2013). To rank effective material selection criteria, fuzzy Delphi-AHP technique was used (Kazemi et al. 2015).

Since neutrosophic set is a popularization of crisp sets, fuzzy sets and intuitionistic fuzzy sets to exemplify ambiguous, conflicting, and incomplete information about real world. Then, this research represents AHP-Delphi in neutrosophic surroundings. The organization of the research as it is summed up:

Section 2 gives an insight into some basic definitions on neutrosophic sets. Section 3 explains the proposed methodology of neutrosophic AHP-Delphi group decision making model. Section 4 introduces numerical example. Finally Sect. 5 concludes the research with future work.

2 Preliminaries

In this section, the essential definitions involving neutrosophic set, single valued neutrosophic sets, trapezoidal neutrosophic numbers and operations on trapezoidal neutrosophic numbers are defined.

Definition 1 (*Saaty and Vargas* 2006; *Hezam et al.* 2015) Let X be a space of points and $x \in X$. A neutrosophic set A in X is definite by a truth-membership function $T_A(x)$ an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$.$T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $[-0, 1^{+}]$. That is $T_A(x)$ (x): $X \to [-0, 1^{+}]$, $I_A(x)$ $X \to [-0, 1^{+}]$ and $F_A(x)$ $X \to [-0, 1^{+}]$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ so $0- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$.

Definition 2 (*El-Hefenawy et al.* 2016; *Saaty and Vargas* 2006; *Hezam et al.* 2015) Let X be a universe of discourse. A single valued neutrosophic set A over X is an object taking the form $A = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \right\} : x \in X\}$, where $T_A(x)$: $X \to [0,1]$, $I_A(x) X \to [0,1]$ and $F_A(x)$: $X \to [0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x)$ 3 for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. For convenience, a SVN number is represented by $A = (a$, b, c), where a, b, c ∈ [0, 1] and $a + b + c \leq 3$.

Definition 3 (*Abdel-Baset et al.* 2016; *Mahdi et al.* 2002) suppose that $\tilde{a}, \tilde{a}, \tilde{a} \in [0, 1]$ and $a_1, a_2, a_3, a_4 \in R$ where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then a single valued trapezoidal neutrosophic number, $\tilde{a} = \left\langle \left(a_1, a_2, a_3, a_4 \right); \tilde{a}, \tilde{a}, \tilde{a} \right\rangle$ is a special neutrosophic set on the real line set R, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as:

$$T_{\tilde{a}}(x)=\begin{cases}\alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right) & (a_1\le x\le a_2)\\ \alpha_{\tilde{a}} & (a_2\le x\le a_3)\\ \alpha_{\tilde{a}}\left(\frac{a_4-x}{a_4-a_3}\right) & (a_3\le x\le a_4)\\ 0 & otherwise\end{cases}, \tag{1}$$

$$I_{\tilde{a}}(x)=\begin{cases}\frac{(a_2-x+\theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1\le x\le a_2)\\ \theta_{\tilde{a}} & (a_2\le x\le a_3)\\ \frac{(x-a_3+\theta_{\tilde{a}}(a_4-x))}{(a_4-a_3)} & (a_3\le x\le a_4)\\ 0 & otherwise\end{cases}, \tag{2}$$

$$F_{\tilde{a}}(x)=\begin{cases}\frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1\le x\le a_2)\\ \beta_{\tilde{a}} & (a_2\le x\le a_3)\\ \frac{(x-a_3+\beta_{\tilde{a}}(a_4-x))}{(a_4-a_3)} & (a_3\le x\le a_4)\\ 1 & otherwise\end{cases}, \tag{3}$$

where $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, and $\beta_{\tilde{a}}$ represent the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively. A single valued trapezoidal neutrosophic number $\tilde{a}=\langle(a_1,a_2,a_3,a_4);\tilde{a},\tilde{a},\tilde{a}\rangle$ may express an ill-defined quantity of the range, which is approximately equal to the interval $[a_2,a_3]$.

Definition 4 (*Abdel-Baset et al. 2016; Hezam et al. 2015*) Let $\tilde{a}=\langle(a_1,a_2,a_3,a_4);\alpha_{\tilde{a}},\theta_{\tilde{a}},\beta_{\tilde{a}}\rangle$ and $\tilde{b}=\langle(b_1,b_2,b_3,b_4);\alpha_{\tilde{b}},\theta_{\tilde{b}},\beta_{\tilde{b}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $\gamma\neq 0$ be any real number. Then,

1. Addition of two trapezoidal neutrosophic numbers

$$\tilde{a}+\tilde{b}=\langle(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4);\ \alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle$$

2. Subtraction of two trapezoidal neutrosophic numbers

$$\tilde{a}-\tilde{b}=\langle(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1);\ \alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle$$

3. Inverse of trapezoidal neutrosophic number

$$\tilde{a}^{-1}=\left\langle\left(\frac{1}{a_4},\frac{1}{a_3},\frac{1}{a_2},\frac{1}{a_1}\right);\alpha_{\tilde{a}},\theta_{\tilde{a}},\beta_{\tilde{a}}\right\rangle \quad where\ \tilde{a}\neq 0$$

4. Multiplication of trapezoidal neutrosophic number by constant value

$$\gamma\tilde{a}=\begin{cases}\langle(\gamma\tilde{a}_1,\gamma\tilde{a}_2,\gamma\tilde{a}_3,\gamma\tilde{a}_4);\alpha_{\tilde{a}},\theta_{\tilde{a}},\beta_{\tilde{a}}\rangle & if\ (\gamma>0)\\ \langle(\gamma\tilde{a}_4,\gamma\tilde{a}_3,\gamma\tilde{a}_2,\gamma\tilde{a}_4);\alpha_{\tilde{a}},\theta_{\tilde{a}},\beta_{\tilde{a}}\rangle & if\ (\gamma<0)\end{cases}$$

5. Division of two trapezoidal neutrosophic numbers

$$\frac{\tilde{a}}{\tilde{b}}=\begin{cases}\left\langle\left(\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\right\rangle & if\ a_4>0,b_4>0\\ \left\langle\left(\frac{a_4}{b_4},\frac{a_3}{b_3},\frac{a_2}{b_2},\frac{a_1}{b_1}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\right\rangle & if\ a_4<0,b_4>0\\ \left\langle\left(\frac{a_4}{b_1},\frac{a_3}{b_2},\frac{a_2}{b_3},\frac{a_1}{b_4}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\right\rangle & if\ a_4<0,b_4>0\end{cases}$$

6. Multiplication of trapezoidal neutrosophic number

$$\tilde{a}\tilde{b}=\begin{cases}\langle(a_1b_1,a_2b_2,a_3b_3,a_4b_4);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & if\ a_4>0,b_4>0\\ \langle(a_1b_4,a_2b_3,a_3b_2,a_4b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & if\ a_4<0,b_4>0\\ \langle(a_4b_4,a_3b_3,a_2b_2,a_1b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\theta_{\tilde{a}}\vee\theta_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}}\rangle & if\ a_4<0,b_4<0\end{cases}$$

3 Methodology

In this section, we present the steps of the proposed model.

Although Delphi technique designed to query a group of experts about specific topic for achieving consensus of ideas and opinions, it also adopted in the construction process of criteria (Mahdi et al. 2002; Wen et al. 2015, 2016). The Delphi survey including, questionnaire design, recuperation, feedback and overtures collection from experts. In the proposed model we used Delphi technique to identify criteria, evaluating them and experts also evaluate their judgments by using trapezoidal neutrosophic numbers. Since previous researches noted that, AHP scale (1–9) shows many drawbacks as illustrated by Lv et al. (2003); then, in our model we proposed a new scale from 0 to 1 to overcome previous drawbacks. Also the consensus degree on the obtained decisions should be calculated. Next, use $(n-1)$ judgments instead of $\frac{n\times(n-1)}{2}$ to obtain consistent trapezoidal neutrosophic preference relations. Finally, AHP is used for selecting the priority alternatives.

The steps of our proposed model can be concluded in the following steps:

Step 1 Prepare Delphi technique for constructing the criteria and alternatives.

To prepare Delphi survey, we should do the following:

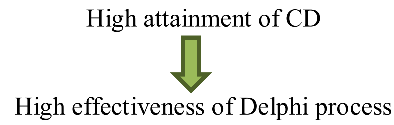
1. Select experts:
   In our Delphi survey, four experts who have a strong background about the problem domain were invited to participate.
2. Design questionnaire:

Rounds of questionnaire range from five to three depending on consensus degree of experts. Here, we used only four rounds to obtain maximum consensus degree of experts.

- The first questionnaire designed from literature review about problem domain because the adjusted Delphi process is appropriate if basic information concerning the target issue is available and applicable according to Kerlinger opinion (Kerlinger and Lee 1999). The first questionnaire will send to each expert through his/her own e-mail. Each expert in first round questionnaire will be asked to answer the following questions: "what are criteria, sub criteria of the problem according to literature review", "what other criteria should be added according to your opinion", "what is the rank of criteria according to your opinion", "work on criteria and write your suggestions", "please construct the hierarchy of problem according to your opinion" and "resend the answer sheet to research team".
- In the second questionnaire, the feedback of experts and first questionnaire was sent to each expert and requiring him/her to rehearsal updated criteria. The research team will check all experts' feedback on the second tour questionnaire and might note that there was little variation in each expert's opinion from the first tour.
- In the third tour questionnaire, each expert will receive the information and judgments from previous questionnaire and will be asked to revise his/her judgments and make a reconstruction of problem hierarchy according to updated criteria. In case of keeping expert on his/her old opinion, he/she should give reasons for staying on his/her opinion.
- In the final questionnaire, the list of criteria, its ranking, the final judgments are obtained and for the last time the experts that are not consensus on ideas will be asked to revise his/her judgments.

Step 2 Calculate the consensus degree (CD) according to Delphi survey as follows:

$CD = \frac{NE}{N} \times 100\%$, where NE is the number of experts that have the same opinion and N is the total numbers of experts in Delphi process.



Consensus degree should be greater than 50%; otherwise we should increase number of rounds of questionnaires.

Step 3 Form the problem hierarchically at various levels according to final Delphi survey from all experts.

The first level of hierarch represents the overall goal, the second level represents the decision criteria and sub-criteria that have obtained from Delphi technique and third level is composed of all possible alternatives.

Step 4 Let experts construct a pairwise comparisons matrix of criteria and alternatives with regard to each criteria by focusing only on $(n-1)$ consensus judgments instead of $\frac{n\times(n-1)}{2}$.

Step 5 Check consistency of criteria and alternatives with respect to criteria.

When the experts evaluate their judgments by using fuzzy numbers, Liu et al. (2016) shows that the preference relations with fuzzy numbers are inconsistent in nature and for this reason, we focus here on additive approximation-consistency of trapezoidal neutrosophic additive reciprocal matrices and its properties. Prior to give the definition of trapezoidal neutrosophic additive reciprocal preference relations let us firstly assume that the scale system 0–1 is applied by all experts. The following trapezoidal neutrosophic additive reciprocal preference relation is given in as follows:

$$\tilde{R} = (\check{r})n\times n = \begin{bmatrix} (0.5,0.5,0.5,0.5) & \left(l_{12}, m_{12_L}, m_{12_U}, u_{12}\right) & \cdots & \left(l_{1n}, m_{1n_L}, m_{12_U}, u_{1n}\right) \\ \left(l_{21}, m_{21_L}, m_{21_U}, u_{21}\right) & (0.5,0.5,0.5,0.5) & \cdots & \left(l_{2n}, m_{2n_L}, m_{2n_U}, u_{2n}\right) \\ \vdots & \vdots & & \vdots \\ \left(l_{n1}, m_{n1_L}, m_{n1_U}, u_{n1}\right) & \left(l_{n2}, m_{n2_L}, m_{n2_U}, u_{n2}\right) & \cdots & (0.5,0.5,0.5,0.5) \end{bmatrix} \quad (4)$$

where $\check{r}_{ij}$ is explained as the neutrosophic number degree of the alternative (criterion) x_i over x_j. $l_{ij}\left(m_{ij_L}, m_{ij_u}\right)$ and u_{ij} indicate the lower, median and upper bounds of the trapezoidal neutrosophic number r_{ij}. $l_{ij}, m_{ij_L}, m_{ij_u}$ and u_{ij} are non-negative real numbers with $0 \le l_{ij} \le m_{ij_L} \le m_{ij_u} \le u_{ij} \le 1$, and $l_{ij} + u_{ij} = m_{ij_L} + m_{ij_L} = m_{ij_U} + m_{ij_U} = u_{ij} + l_{ij} = 1$, for all $i,j = 1,2,\ldots,n$. The preference relation may be adequately expressed as the matrix $\tilde{R} = \left(\check{r}_{ij}\right)n\times n$ where $\check{r}_{ij} = T_{\check{r}}\left(x_i, x_j\right), I_{\check{r}}\left(x_i, x_j\right)$ and $F_{\check{r}}\left(x_i, x_j\right)$ it is explained as the preference ratio of the alternative x_i over x_j, for all $i,j = 1,2,\ldots,n$ For example, $\check{r}_{ij} = \frac{1}{2}$ indicates that there is no difference between x_i and x_j, $\check{r}_{ij} = 1$ means that x_i is absolutely preferred to x_j, $\frac{1}{2} < \check{r}_{ij} < 1$ implies that x_i is very strongly preferred to x_j and $0 < \check{r}_{ij} < \frac{1}{2}$ implies that x_i is preferred to x_j.

Definition 5 The consistency of trapezoidal neutrosophic reciprocal preference relations

$\tilde{R} = (\check{r}_{ij})n \times n$ can be expressed as:

$$\check{r}_{ij} = \check{r}_{ik} + \check{r}_{kj} - (0.5, 0.5, 0.5, 0.5) \tag{5}$$

where $i, j, k = 1, 2, \dots, n$. The reason is based on the following analysis:

Making use of the operation laws of trapezoidal neutrosophic numbers, Eq. (5) is also rewritten as

$$l_{ij} = l_{ik} + l_{kj} - (0.5, 0.5, 0.5, 0.5),\ m_{ij_L} = m_{ik_L} + m_{kj_L} - (0.5, 0.5, 0.5, 0.5),$$
$$m_{ij_U} = m_{ik_U} + m_{kj\,U} - (0.5, 0.5, 0.5, 0.5), u_{ij} = u_{ik} + u_{kj} - (0.5, 0.5, 0.5, 0.5),$$

where $i, j, k = 1, 2, \dots, n$. One can find from Definition 5 that two following matrices will be consistence:

$$\begin{bmatrix} 0.5 & l_{12} & \cdots & l_{1n} \\ l_{21} & 0.5 & \cdots & l_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \mathrm{l}_{n1} & \mathrm{l}_{n2} & \cdots & 0.5 \end{bmatrix}, \begin{bmatrix} 0.5 & u_{12} & \cdots & u_{1n} \\ u_{21} & 0.5 & \cdots & u_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ u_{n1} & u_{n2} & \cdots & 0.5 \end{bmatrix} \tag{6}$$

It is convenient to construct four preference relations from a trapezoidal neutrosophic preference relation $\tilde{R} = (\check{r}_{ij})n \times n = \left(l_{ij}, m_{ij_L}, m_{ij_U}, u_{ij}\right)n \times n$ as follows:

$$\check{r}^{l}_{ij} = \begin{cases} l_{ij}, & i < j \\ 0.5, & i = j \\ u_{ij}, & i > j \end{cases} \tag{7}$$

$$\check{r}^{l}_{ij} = \begin{cases} u_{ij}, & i < j \\ 0.5, & i = j\,, \\ l_{ij}, & i > j \end{cases} \tag{8}$$

$$\check{r}^{ml}_{ij} = m_{ij_L},\ \text{for every } i,j = 1, 2, \dots, n. \text{ And } \check{r}^{mu}_{ij} = m_{ij_U},\ \text{for every } i,j = 1, 2, \dots, n \tag{9}$$

For a limited set of alternatives $X = \{x_1, x_2, \dots, x_n\}$ and $(n \geq 2)$ there are $n!$ possible comparison matrices corresponding to the permutation of alternatives. Hence, we define the following function:

$$P{:}k \to P(k),\ K = 1, 2, \dots, n, \tag{10}$$

where the function P denotes a permutation of $(1, 2, \dots, n)$, $P(k_1) \neq P(k_2)$ when

$k_1 \neq k_2, \left(k_1, k_2 \in \{1, 2, \dots, n\}\right)$

From Eq. (10) the trapezoidal neutrosophic reciprocal matrix with a permutation P can be expressed as $\tilde{R}_P = \left(\check{r}_{p(i)_{p(j)}}\right)n \times n$ where $\check{r}_{p(i)_{p(j)}} = \left(l_{p(i)_{p(j)}}, m^{l}_{p(i)_{p(j)}}, m^{u}_{p(i)_{p(j)}}, u_{p(i)_{p(j)}}\right)$. Similarly, four preference relations can be written as follows:

$$\check{r}^{l}_{p(i)p(j)} = \begin{cases} l_{p(i)p(j)}, & i < j \\ 0.5, & i = j \\ u_{p(i)p(j)}, & i > j \end{cases} \tag{11}$$

$$\check{r}^{u}_{p(i)p(j)} = \begin{cases} u_{p(i)p(j)}, & i < j \\ 0.5, & i = j \\ l_{p(i)p(j)}, & i > j \end{cases} \tag{12}$$

$$\check{r}^{ml}_{p(i)p(j)} = m_{Lp(i)p(j)},\ \text{for every } i,\ j = 1, 2, \dots, n. \quad \text{And } \check{r}^{mu}_{p(i)p(j)} = m_{Up(i)p(j)} \text{ for every } i,\ j = 1, 2, \dots, n. \tag{13}$$

Then, we conclude a new definition of trapezoidal neutrosophic additive reciprocal preference relation:

Definition 6 In order to check whether a trapezoidal neutrosophic reciprocal preference relation $\tilde{R}$ is of additive approximation-consistency or not, one should check the consistency of $\check{r}^{l}_{(p(i)p(j))}, \check{r}^{u}_{(p(i)p(j))}, \check{r}^{ml}_{(p(i)p(j))}$ *and* $\check{r}^{mu}_{(p(i)p(j))}$. In other words, if $\tilde{R}$ is not of additive approximation-consistency, there is at least one of $\check{r}^{l}_{(p(i)p(j))}, \check{r}^{u}_{(p(i)p(j))}, \check{r}^{ml}_{(p(i)p(j))}$ *and* $\check{r}^{mu}_{(p(i)p(j))}$ without additive approximation-consistency for any permutation of alternatives.

The characterization of additive consistency of additive reciprocal matrices as follows:

Proposition 1 *For an additive reciprocal preference relation $R = (\check{r}_{ij})n \times n$, the following statements are equal*:

- $\check{r}_{ik} + \check{r}_{kj} + \check{r}_{ji} = \frac{1}{2}$, for every i, j, k
- $\check{r}_{ik} + \check{r}_{kj} + \check{r}_{ji} = \frac{1}{2}$, for every $i < j < k$.

Proposition 2 *For an additive reciprocal preference relation* $R = \left(\check{r}_{ij}\right)n \times n$, *the following statements are equal*:

- $\check{r}_{ik} + \check{r}_{kj} + \check{r}_{ji} = \frac{1}{2}$, for every i, j, k
- $b_{i(i+1)} + b_{i(i+1)(i+2)} + \cdots + b_{i(j-1)j} + b_{ji} = \frac{j-i+1}{2}$, *for every* $i < j$.

Proposition 3 *For an additive reciprocal preference relation* $R = \left(\check{r}_{ij}\right)n \times n$, *the following statement is true*:

$\check{r}_{ik} = 1 - \check{r}_{kj}$

Then, the characterization of additive approximation-consistency of trapezoidal neutrosophic additive reciprocal preference relations as follows:

Theorem 1 *A trapezoidal neutrosophic additive reciprocal preference relations* $\tilde{R}$ *is of additive approximation-consistency if and only if there is a permutation P such that*

$$\begin{cases} l_{p(i)p(k)} + l_{p(k)p(j)} + u_{p(j)p(i)} = 2 \\ {m^l}_{p(i)p(k)} + {m^l}_{p(k)p(j)} + {m^l}_{p(j)p(i)} = 2 \\ {m^u}_{p(i)p(k)} + {m^u}_{p(k)p(j)} + {m^u}_{p(j)p(i)} = 2 \\ u_{p(i)p(k)} + u_{p(k)p(j)} + l_{p(j)p(i)} = 2 \end{cases} \tag{14}$$

For every $i \le k \le j$

Proof It is seen that $\tilde{R}$ has additive approximation-consistency, if and only if there is a permutation P such that four additive reciprocal preference relations $\check{r}^l_{(p(i)p(j))}$, $\check{r}^u_{(p(i)p(j))}$, $\check{r}^{ml}_{(p(i)p(j))}$ *and* $\check{r}^{mu}_{(p(i)p(j))}$ are additively consistent. Otherwise, $\tilde{R}$ is said to be not of additive approximation consistency. Making use of Proposition 1, Eq. (14) is satisfied. Inversely, if Eq. (14) is satisfied, $\check{r}^l_{(p(i)p(j))}$, $\check{r}^u_{(p(i)p(j))}$, $\check{r}^{ml}_{(p(i)p(j))}$ *and* $\check{r}^{mu}_{(p(i)p(j))}$ additively consistent for the permutation P. So we have confirmed the Theorem.

Theorem 2 *For a trapezoidal neutrosophic additive reciprocal preference relation* $\tilde{R}$.

$$\begin{cases} l_{p(i)p(k)} + l_{p(k)p(j)} + u_{p(j)p(i)} = 2 \\ {m^l}_{p(i)p(k)} + {m^l}_{p(k)p(j)} + {m^l}_{p(j)p(i)} = 2 \\ {m^u}_{p(i)p(k)} + {m^u}_{p(k)p(j)} + {m^u}_{p(j)p(i)} = 2 \\ u_{p(i)p(k)} + u_{p(k)p(j)} + l_{p(j)p(i)} = 2 \end{cases} \tag{I}$$

For every $i \le k \le j$

$$\begin{cases} l_{p(i)p(i+k)} + l_{p(i+1)p(i+2)} + \ldots + l_{p(j-1)p(j)} + u_{p(j)p(i)} = \frac{i-i+1}{2} \\ {m^l}_{p(i)p(i+1)} + {m^l}_{p(i+1)p(i+2)} + \ldots + {m^l}_{p(j-1)p(j)} + {m^l}_{p(i)p(i+1)} = \frac{i-i+1}{2} \\ {m^u}_{p(i)p(i+1)} + {m^u}_{p(i+1)p(i+2)} + \ldots + {m^u}_{p(j-1)p(j)} + {m^u}_{p(j)p(i)} = \frac{i-i+1}{2} \\ u_{p(i)p(i+1)} + u_{p(i+1)p(i+2)} + \ldots + u_{p(j-1)p(i)} + l_{p(j)p(i)} = \frac{i-i+1}{2} \end{cases}$$

For $i < j$.

(II)

Proof Since $\check{r}^l_{(p(i)p(j))}, \check{r}^u_{(p(i)p(j))}, \check{r}^{ml}_{(p(i)p(j))}$ *and* $\check{r}^{mu}_{(p(i)p(j))}$ are constructed from $\tilde{R}$ by using Eqs. (11), (12) and (13), it is seen from Proposition 2 that two statements (I) and (II) are equivalent.

If the preference relation $\check{R}$ not be a trapezoidal neutrosophic additive reciprocal matrix for $u_{ij} > 1$ or $l_{ij} < 0$ then,

Do the following adjustment to obtain the acceptable preference relation. $R = (\check{r}'_{ij})n \times n$, when $u_{ij} > 1$ then:

$$\breve{r}'_{ij} = \frac{\breve{r}_{ij} + c_x}{1 + 2c_x} \tag{15}$$

where, $C_x = \max\{u_{ij} - 1, 0 - l_{ij}\}$ For every $i, j = 1, 2 \ldots, n\}$.

It is easy to see that

$$0 \le l_{ij} \le m'_{ij} \le u'_{ij} \le 1$$

Proof Application of Theorems 1 or 2 yields the following:

$$\check{r}'_{ij} = \check{r}'_{ik} + \check{r}'_{kj} - \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right),$$

And because the scale of our system is 0–1, then the maximum range of $\check{r}'_{ij}$ is 2 and by applying Eq. (15), we ensure that, the obtained $\check{r}'_{ij}$ does not exceed the specified range.

When $l^x_{ij} < 0$, apply Eq. (16), to ensure that, the obtained $\check{r}'_{ij}$ is not less than the specified range.

$$\check{r}'_{ij} = \frac{-\check{r}'_{ij} + c_x}{1 + 2c_x} \tag{16}$$

Let us consider the case of $l_{ij} > m_{ij}$ or $m_{ij} > u_{ij}$. We propose an adjustment method for making $l_{ij} > m_{ij}$ to $l_{ij} \le m_{ij}$. Because the estimation value $\check{r}_{ij}$ is obtained by using the additive consistency from $\check{r}_{ik}$ and $\check{r}_{kj}$. It is further seen that $\check{r}_{ik}$ and $\check{r}_{kj}$ are given directly by the expert or one of them is from the expert and another is indirectly obtained. When $\check{r}_{ik}$ and $\check{r}_{kj}$ are given by the expert the adjustment process has the following possible cases:

The first case is, $k < i < j$or $j < i < k$. By using additive approximation consistency given in Definition 6, we have the following:

$$l_{ij} = u_{ik} + l_{kj} - 0.5,$$

$$m_{ij} = u_{ik} + l_{kj} - 0.5,$$

Let

$$u'_{ik} = u_{ij} - v^u_{ik}, l'_{kj} = l_{kj} - v^l_{kj}, m'_{kj} = m_{kj} + v^m_{kj}, m'_{ik} = m_{ik} + v^m_{ik}$$

where

$$u_{ik} + v_{kj}^{l} + v_{ik}^{m} + v_{kj}^{m} \geq l_{ij} - m_{ij} = \Delta > 0$$

for $v_{ik}^{u} \geq 0, v_{kj}^{l} \geq 0, v_{ik}^{m} \geq 0$ and $v_{kj}^{m} \geq 0$.

Then, $l_{ij} = u_{ik} + l_{kj} - v_{ik}^{u} - v_{kj}^{l} - 0.5, m'_{ij} = m_{ik} + m_{kj} + v_{ik}^{m} + v_{kj}^{m} - 0.5.$

Then, $l_{ij}^{\}} \leq m_{ij}$.

The second case is,

$i < k < j$or $j < k < i$. By using additive approximation consistency given in Definition 6, we have the following:

$l_{ij} = u_{ik} + l_{kj} - 0.5,$

$m_{ij} = u_{ik} + l_{kj} - 0.5,$

Then, we can get the following:

$$l'_{ij} = l_{ik} + u_{kj} - v_{ik}^{l} - v_{kj}^{u} - 0.5,\ m'_{ij} = m_{ik} + m_{kj} + v_{ik}^{m} + v_{kj}^{m} - 0.5,$$

where $v_{ik}^{m} + v_{kj}^{m} + v_{ik}^{l} + v_{kj}^{u} \geq \Delta$ for $v_{kj}^{m} \geq 0, v_{ik}^{l} \geq 0, v_{ik}^{l} \geq 0, v_{kj}^{u} \geq 0$.

Then, $l'_{ij} \leq m'_{ij}$,

From the previous, we can conclude that:

In order to keep the original information as much as possible, we usually make.

$m'_{ij} - l'_{ij} = \Delta$. When $m_{ij} > u_{ij}$, the method is similar to the previous and we keep the original information as much as possible by using the following equation:

$$u_{ij} - m_{ij} = \Delta, \tag{17}$$

We can transform additive reciprocal preference relation to multiplicative preference relation using the following equation:

$$l_{ij}^{new} = \frac{l_{ij}}{1 - l_{ij}}, u_{ij}^{new} = \frac{u_{ij}}{1 - u_{ij}},\ m^{l}ij^{new} = \frac{m^{l}ij}{1 - m^{l}ij}, \quad m^{u}ij^{new} = \frac{m^{u}ij}{1 - m^{u}ij} \quad For,\ j = 1, 2 \ldots n \tag{18}$$

After transforming additive reciprocal preference relation to multiplicative preference relation, the scale of system will transform from 0 to 1 to Saaty's scale.

The following trapezoidal neutrosophic multiplicative reciprocal preference relation is given in as follows:

where $\check{r}_{ij}$ is explained as the neutrosophic number degree of the alternative (criteria).

x_i over x_j. $l_{ij}, (m_{ij_L}, m_{ij_U})$ and u_{ij} indicate the lower, median and upper bounds of the trapezoidal neutrosophic number $\check{r}_{ij}$ $l_{ij}, (m_{ij_L}, m_{ij_U})$ and u_{ij} are non-negative real numbers with $\frac{1}{9} \leq l_{ij} \leq m_{ij_L} \leq m_{ij_U} \leq u_{ij} \leq 9$, and $l_{ij}u_{ij} = m_{ij_L}m_{ij_L} = m_{ij_U}m_{ij_U} = u_{ij}l_{ij} = 1$, for all $i, j = 1, 2, \ldots, n$. The preference relation may be conveniently expressed as the matrix $\tilde{R} = (\check{r}_{ij})n \times n$, where $\check{r}_{ij} = T_{\check{r}}(x_i, x_j), I_{\check{r}}(x_i, x_j)$ and $F_{\check{r}}(x_i, x_j)$ it is interpreted as the preference ratio of the alternative x_i over x_j, for all $i, j = 1, 2, ..., n$.

Definition 7 The consistency of trapezoidal multiplicative preference relations $\tilde{R} = (\check{r}_{ij})n \times n$ can be expressed as:

$$\check{r}_{ij}\check{r}_{ki}\check{r}_{ik} = \check{r}_{ik}\check{r}_{kj}\check{r}_{ji} \quad \text{For} \quad i, j, k = 1, 2, ..., n \tag{20}$$

Step 6 Calculate weight of criteria and alternatives.

To determine weight of each criterion and alternative from corresponding neutrosophic pairwise comparison matrix, we first transformneutrosophic pairwise comparison matrix to deterministic pairwise comparison matrix, using the following equations:

Let $\tilde{a}_{ij} = \langle (a_1, b_1, c_1, d_1)\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ be a single valued trapezoidal neutrosophic number, then

$$\mathrm{S}(\tilde{a}_{\mathrm{ij}}) = \frac{1}{16}[\mathrm{a}_1 + \mathrm{b}_1 + \mathrm{c}_1 + \mathrm{d}_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}}) \tag{21}$$

And

$$\mathrm{A}(\tilde{a}_{\mathrm{ij}}) = \frac{1}{16}[\mathrm{a}_1 + \mathrm{b}_1 + \mathrm{c}_1 + \mathrm{d}_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}}) \tag{22}$$

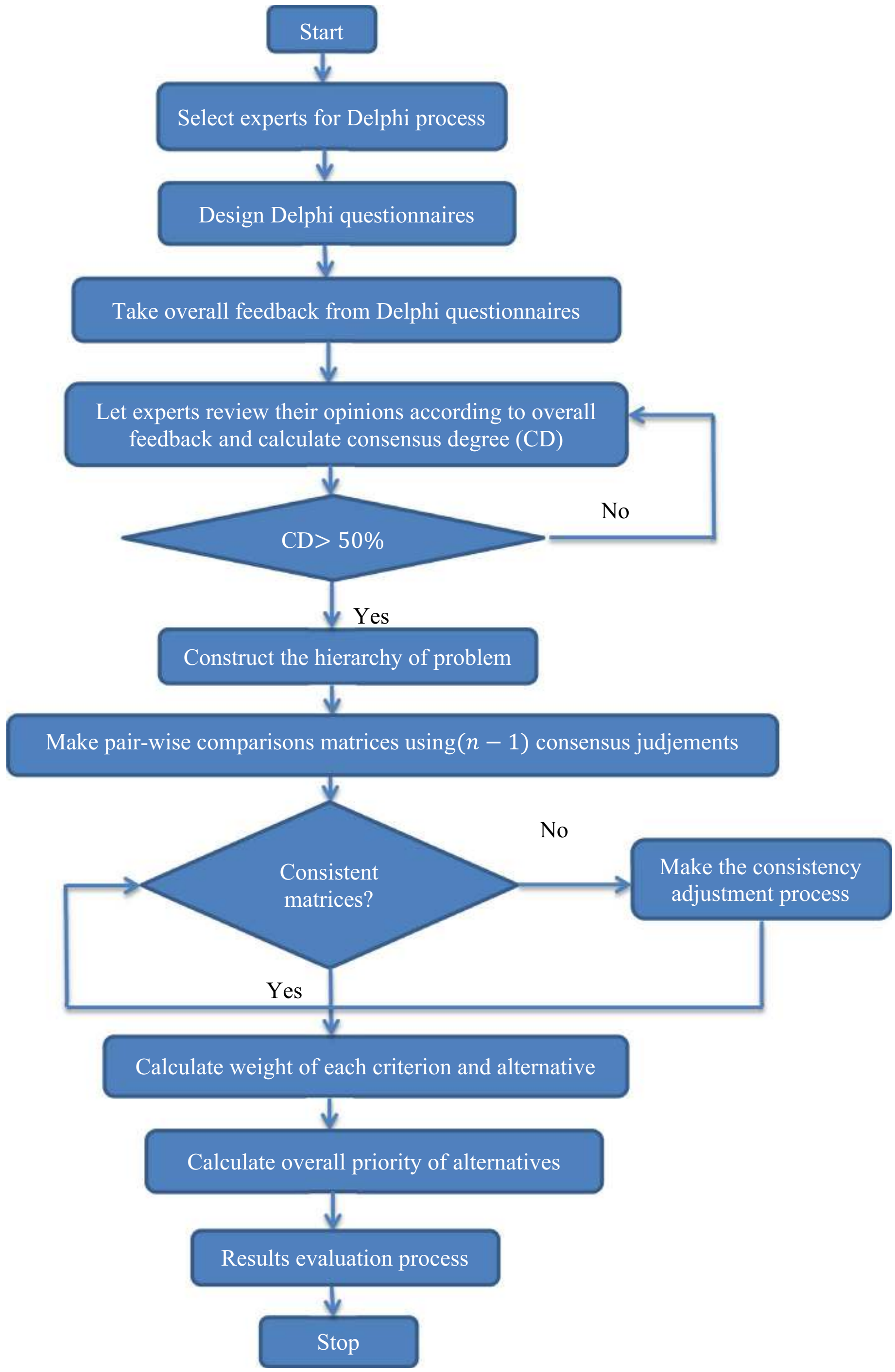
is called the score and accuracy degrees of $\tilde{a}_{ij}$ respectively.

From the deterministic matrix we can easily find ranking of priorities, namely the Eigen Vector X as follows;

1. Equalize the column entries by dividing each entry by the sum of the column.
2. Take the overall row averages.

$$\tilde{R} = (\check{r}_{ij})n \times n = \begin{bmatrix} (1,1,1,1) & (l_{12}, m_{12_L}, m_{12_U}, u_{12}) & (l_{1n}, m_{1n_L}, m_{12_U}, u_{1n}) \\ (l_{21}, m_{21_L}, m_{21_U}, u_{21}) & (1,1,1,1) & (l_{2n}, m_{2n_L}, m_{2n_U}, u_{2n}) \\ \vdots & \vdots & \vdots \\ (l_{n1}, m_{n1_L}, m_{n1_U}, u_{n1}) & (l_{n2}, m_{n2_L}, m_{n2_U}, u_{n2}) & (1,1,1,1) \end{bmatrix} \tag{19}$$

Fig. 1 Schematic diagram of neutrosophic AHP-Delphi group decision making model



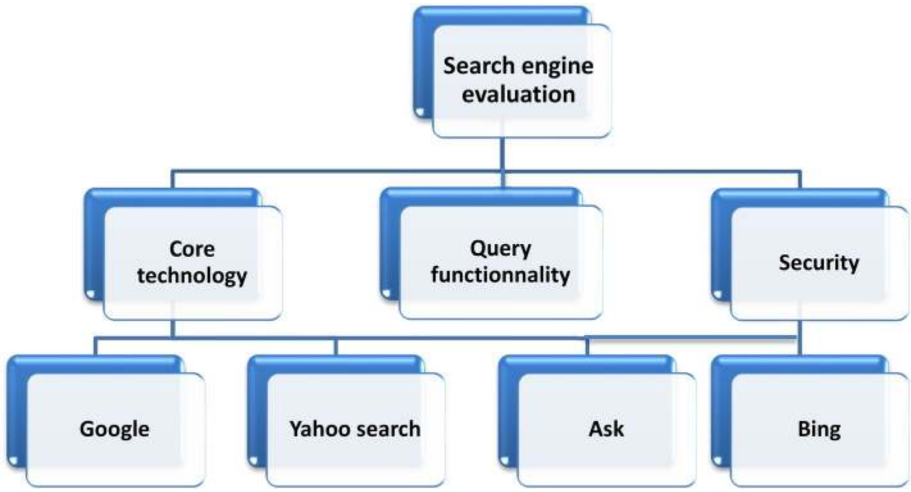


Fig. 2 Hierarchy structure of search engines according to first expert opinion

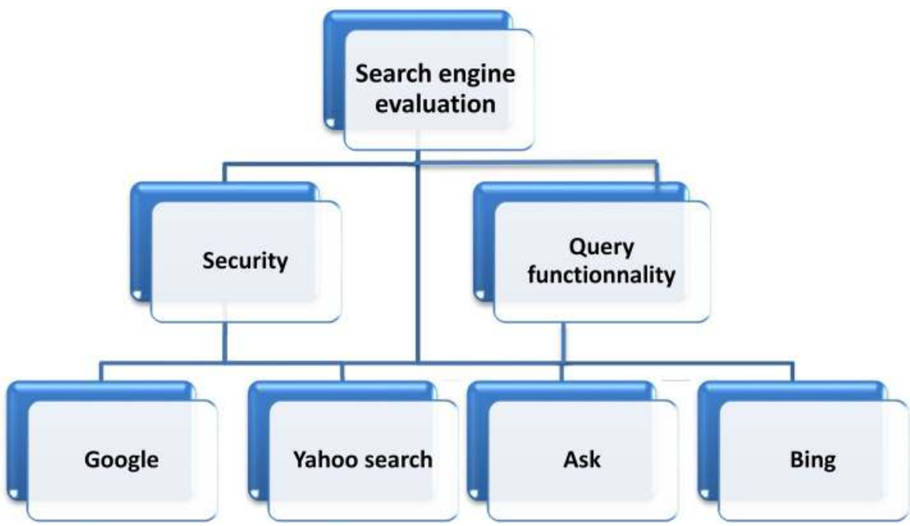


Fig. 3 Hierarchy structure of search engines according to second expert opinion

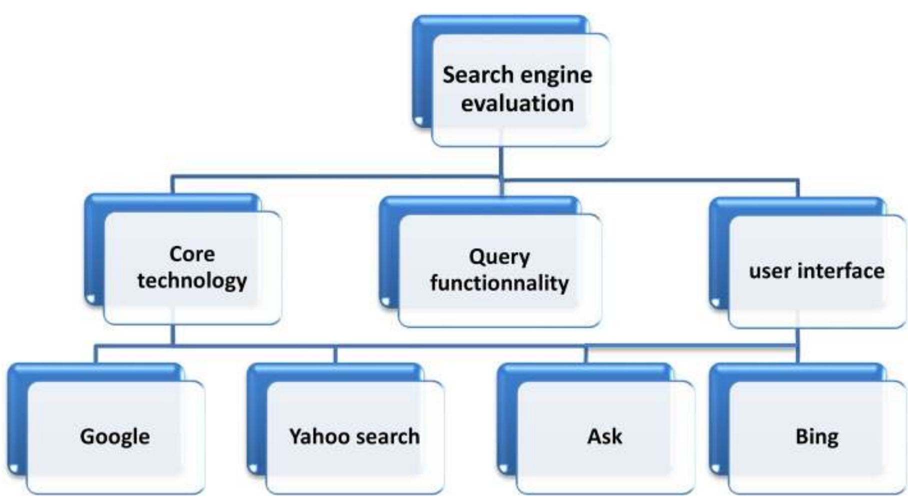


Fig. 4 Hierarchy structure of search engines according to third expert opinion

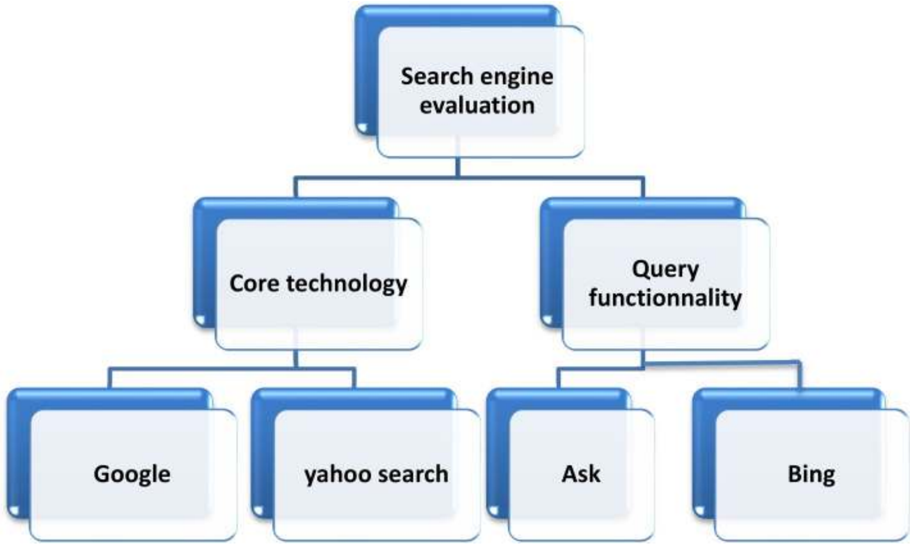


Fig. 5 Hierarchy structure of search engines according to fourth expert opinion

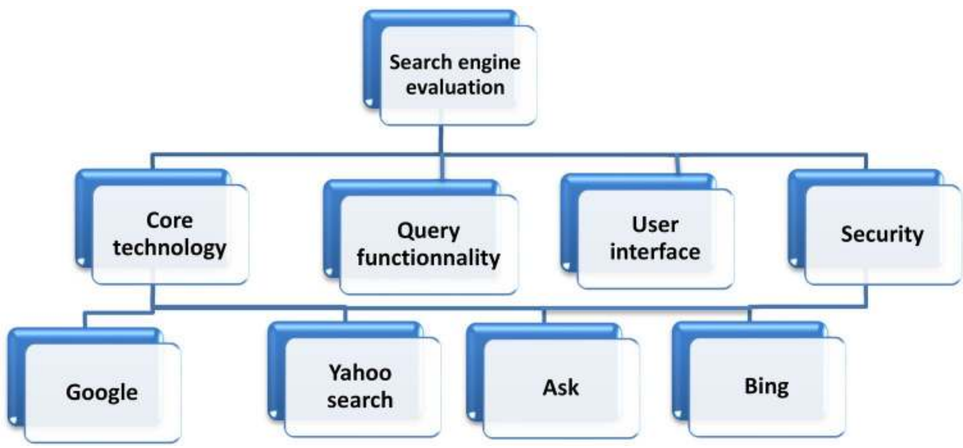


Fig. 6 Hierarchy structure of search engines

Step 7 Calculate overall priority of each alternative and determine final ranking of all alternatives using AHP program.

Step 8 Evaluate results.

The previous steps can be concluded in Fig. 1.

4 Illustrative example and comparison analysis

In this section, to illustrate the efficiency and applicability of the proposed algorithm, we consider a search engine evaluation problem and make a comparison analysis of the proposed algorithm with other existing algorithms.

4.1 Illustrative example

This example illustrates the evaluation process of the most-popular search engines.

Step 1 Prepare Delphi technique for constructing the criteria.

In our Delphi survey, four experts who have a strong experience and academic background in the domain of search engines were invited to participate.

According to first expert's opinion, the most important criteria for evaluating search engines presented in Fig. 2.

According to second expert's opinion, the most important criteria for evaluating search engines presented in Fig. 3.

According to third expert's opinion, the most important criteria for evaluating search engines presented in Fig. 4.

According to fourth expert's opinion, the most important criteria for evaluating search engines presented in Fig. 5.

After preparing rounds of Delphi questionnaires and taking overall feedback from all experts, making reviews and update of criteria, research team noted that three experts from the total number are consensuses on the following criteria for evaluating search engines;

- Core technology
- Query functionality
- Security
- User interface

$\tilde{R}_{13} = \tilde{r}_{12} + \tilde{r}_{23} - (0.5, 0.5, 0.5, 0.5) = (0.3, 0.5, 0.65, 0.9),$
$\tilde{R}_{31} = 1 - \tilde{R}_{13} = 1 - (0.3, 0.5, 0.65, 0.9) = (0.1, 0.35, 0.5, 0.7),$
$\tilde{R}_{32} = \tilde{r}_{31} + \tilde{r}_{12} - (0.5, 0.5, 0.5, 0.5) = (-0.2, 0.15, 0.4, 0.7),$
$\tilde{R}_{21} = 1 - \tilde{R}_{12} = 1 - (0.2, 0.3, 0.4, 0.5) = (0.5, 0.6, 0.7, 0.8),$
$\tilde{R}_{14} = \tilde{r}_{13} + \tilde{r}_{34} - (0.5, 0.5, 0.5, 0.5) = (0.1, 0.4, 0.65, 1.2),$
$\tilde{R}_{24} = \tilde{r}_{21} + \tilde{r}_{14} - (0.5, 0.5, 0.5, 0.5) = (0.1, 0.5, 0.85, 1.5),$
$\tilde{R}_{41} = 1 - \tilde{R}_{14} = 1 - (0.1, 0.4, 0.65, 1.2) = (-0.2, 0.35, 0.6, 0.9),$
$\tilde{R}_{42} = 1 - \tilde{R}_{24} = 1 - (0.1, 0.5, 0.85, 1.5) = (-0.5, 0.15, 0.5, 0.9),$
$\tilde{R}_{43} = 1 - \tilde{R}_{34} = 1 - (0.3, 0.4, 0.5, 0.8) = (0.2, 0.5, 0.6, 0.7),$

Then, the pairwise comparison matrix will be as follows:

$$\tilde{R} = \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.2,0.3,0.4,0.5) & (0.3,0.5,0.65,0.9) & (0.1,0.4,0.65,1.2) \\ (0.5,0.6,0.7,0.8) & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.75,0.9) & (0.1,0.5,0.85,1.5) \\ (0.1,0.35,0.5,0.7) & (-0.2,0.15,0.4,0.7) & (0.5,0.5,0.5,0.5) & (0.3,0.4,0.5,0.8) \\ (-0.2,0.35,0.6,0.9) & (-0.5,0.15,0.5,0.9) & (0.2,0.5,0.6,0.7) & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

Step 2 Calculate the consensus degree (CD) according to Delphi survey as follows:

$CD = \frac{NE}{N} \times 100\% = \frac{3}{4} \times 100\% = 75\%$, where NE is the number of experts that have the same opinion and N is the total numbers of experts in Delphi process.

According to Definition 6, one can see that $\tilde{R}$ is not trapezoidal neutrosophic additive reciprocal preference relations.

By using Eqs. (15), (16) and (17) one can obtain the following:

$$\tilde{R} = \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.2,0.3,0.4,0.5) & (0.3,0.5,0.65,0.9) & (0.1,0.4,0.65,1.2) \\ (0.5,0.6,0.7,0.8) & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.75,0.9) & (0.1,0.5,0.85,1.5) \\ (0.1,0.35,0.5,0.7) & (0.2,0.2,0.4,0.7) & (0.5,0.5,0.5,0.5) & (0.3,0.4,0.5,0.8) \\ (0.2,0.35,0.6,0.9) & (0.5,0.5,0.5,0.9) & (0.2,0.5,0.6,0.7) & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

Step 3 Form the final hierarchical structure of problem as in Fig. 6.

Step 4 Checking consistency.

The experts were consensual on the following pairwise comparison of criteria according to previously Delphi process:

Then the previous matrix appears consistent according to Definition 6. And then by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, experts should determine the maximum

$$\tilde{R} = \begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.2,0.3,0.4,0.5) & x & x \\ x & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.75,0.9) & x \\ x & x & (0.5,0.5,0.5,0.5) & (0.3,0.4,0.5,0.8) \\ x & x & x & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

where, x indicates preference values that are not agreed by experts, and then we can calculate these values and make it consistent with their judgments.

We can complete the previous matrix as follows:

By applying Theorems 1 or 2 we can obtain the following:

truth-membership degree (α), minimum indeterminacy-membership degree (θ) and minimum falsity-membership degree (β) of single valued neutrosophic numbers as in Definition 3. Then,

$$\tilde{R} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.5; 0.7, 0.2, 0.5) & (0.3, 0.5, 0.65, 0.9; 0.5, 0.2, 0.1) & (0.1, 0.4, 0.65, 1; 0.5, 0.2, 0.1) \\ (0.5, 0.6, 0.7, 0.8; 0.7, 0.2, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.75, 0.9; 0.5, 0.2, 0.1) & (0.1, 0.5, 0.85, 1.5; 0.4, 0.5, 0.6) \\ (0.1, 0.35, 0.5, 0.7; 0.8, 0.2, 0.1) & (0.2, 0.2, 0.4, 0.7; 0.3, 0.1, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.3, 0.4, 0.5, 0.8; 0.7, 0.2, 0.5) \\ (0.2, 0.35, 0.6, 0.9; 0.6, 0.2, 0.3) & (0.5, 0.5, 0.5, 0.9; 0.6, 0.2, 0.3) & (0.2, 0.5, 0.6, 0.7; 0.9, 0.4, 0.6) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

Step 5 Calculate weight of criteria as follows:

To determine weight of each criterion from the previous neutrosophic pairwise comparison matrix, we first transformneutrosophic pairwise comparison matrix to deterministic pairwise comparison matrix, using the following equations:

Let $\tilde{a}_{ij} = \left\langle \left(a_1, b_1, c_1, d_1\right)\tilde{a}, \tilde{a}, \tilde{a} \right\rangle$ be a single valued trapezoidal neutrosophic number, then

$$S\left(\tilde{a}_{ij}\right) = \frac{1}{16}\left[a_1 + b_1 + c_1 + d_1\right] \times 2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}} \tag{21}$$

And

$$A\left(\tilde{a}_{ij}\right) = \frac{1}{16}\left[a_1 + b_1 + c_1 + d_1\right] \times 2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}} \tag{22}$$

is called the score and accuracy degrees of $\tilde{a}_{ij}$ respectively.

From the deterministic matrix we can easily find ranking of priorities, namely the Eigen Vector X as follows;

1. Equalize the column entries by dividing each entry by the sum of the column.
2. Take the overall row averages.

By using Eq. 21, we obtain the following deterministic matrix:

$$\tilde{R} = \begin{bmatrix} 0.5 & 0.262 & 0.323 & 0.295 \\ 0.325 & 0.5 & 0.44 & 0.199 \\ 0.299 & 0.159 & 0.5 & 0.25 \\ 0.269 & 0.315 & 0.237 & 0.5 \end{bmatrix}$$

By making normalization of matrix by dividing each entry by the sum of the column, we obtain the following:

$$\tilde{R} = \begin{bmatrix} 0.36 & 0.21 & 0.21 & 0.24 \\ 0.23 & 0.40 & 0.29 & 0.16 \\ 0.21 & 0.13 & 0.33 & 0.2 \\ 0.19 & 0.25 & 0.16 & 0.4 \end{bmatrix}$$

By taking overall raw averages, then the weights of criteria are;

$$X = \begin{bmatrix} 0.25 \\ 0.27 \\ 0.22 \\ 0.25 \end{bmatrix}$$

Step 6 Pairwise comparison matrix of alternatives with respect to each criterion according to Delphi survey.

The pairwise comparison matrix of alternatives with respect to core technology criterion as follows:

$$\tilde{A}_{ct} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.5, 0.6, 0.7, 0.8) & x & x \\ x & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.6, 0.7) & x \\ x & x & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.5) \\ x & x & x & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

We can complete the previous matrix by applying Theorems 1 or 2 as follows:

$$\tilde{A}_{ct} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.5, 0.6, 0.7, 0.8) & (0.4, 0.6, 0.8, 1) & (0.1, 0.4, 0.7, 1) \\ (0.2, 0.3, 0.4, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.6, 0.7) & (0.1, 0.3, 0.5, 0.7) \\ (0, 0.2, 0.4, 0.6) & (0, 0.3, 0.6, 0.9) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.5) \\ (0, 0.3, 0.6, 0.9) & (0.3, 0.5, 0.7, 0.9) & (0.5, 0.6, 0.7, 0.8) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

Then the previous matrix appears consistent according to Definition 6. And then by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, experts should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ) and minimum falsity-membership degree (β) of single valued neutrosophic numbers as in Definition 3. Then,

$$\tilde{A}_{ct} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.5, 0.6, 0.7, 0.8; 0.7, 0.2, 0.5) & (0.4, 0.6, 0.81, 0.4; 0.5, 0, 0.6) & (0.1, 0.4, 0.7, 1; 0.6, 0.2, 0.3) \\ (0.2, 0.3, 0.4, 0.5; 0.8, 0.2, 0.6) & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.6, 0.7; 0.5, 0.2, 0.1) & (0.1, 0.3, 0.5, 0.7; 0.5, 0.2, 0.1) \\ (0, 0.3, 0.6, 0.9; 0.5, 0.3, 0.4) & (0, 0.3, 0.6, 0.9; 0.8, 0.5, 0.3) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.5; 0.6, 0.4, 0.2) \\ (0, 0.3, 0.6, 0.9; 0.5, 0.2, 0.1) & (0.3, 0.5, 0.7, 0.9; 0.3, 0.1, 0.5) & (0.5, 0.6, 0.7, 0.8; 0.3, 0.1, 0.5) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

By using Eq. 21, we can determine the deterministic values of previous neutrosophic matrix as follows:

$$\tilde{A}_{ct} = \begin{bmatrix} 0.5 & 0.32 & 0.23 & 0.29 \\ 0.17 & 0.5 & 0.30 & 0.22 \\ 0.13 & 0.22 & 0.5 & 0.17 \\ 0.25 & 0.25 & 0.28 & 0.5 \end{bmatrix}$$

According to Definition 6, one can see that $\tilde{A}_{qf}$ is not trapezoidal neutrosophic additive reciprocal preference relations.

By using Eqs. 15, 16 and 17 one can obtain the following:

$$\tilde{A}_{qf} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0, 0.5, 0.8, 1) & (0.4, 0.5, 0.7, 0.9) & (0.3, 0.5, 0.9, 1) \\ (0.5, 0.5, 0.5, 1) & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.6, 0.7, 1) & (0.1, 0.5, 0.6, 0.8) \\ (0.1, 0.3, 0.5, 0.6) & (0.4, 0.4, 0.8, 1) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8) \\ (0.8, 0.8, 0.8, 1) & (0.2, 0.4, 0.5, 0.8) & (0.2, 0.4, 0.5, 0.8) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

By making normalization of matrix we obtain the following:

$$\tilde{A}_{ct} = \begin{bmatrix} 0.47 & 0.25 & 0.17 & 0.24 \\ 0.16 & 0.39 & 0.23 & 0.19 \\ 0.12 & 0.17 & 0.38 & 0.14 \\ 0.24 & 0.19 & 0.21 & 0.42 \end{bmatrix}$$

Then the previous matrix appears consistent according to Definition 6. And then by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, experts should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ) and minimum falsity-membership degree (β) single valued neutrosophic numbers as in Definition 3. Then,

$$\tilde{A}_{qf} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0, 0.5, 0.8, 1; 0.7, 0.2, 0.5) & (0.4, 0.5, 0.7, 0.9; 0.5, 0.2, 0.1) & (0.3, 0.5, 0.9, 1; 0.6, 0.2, 0.3) \\ (0.5, 0.5, 0.5, 1; 0.8, 0.2, 0.6) & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.6, 0.7, 1; 0.5, 0.2, 0.1) & (0.1, 0.5, 0.6, 0.8; 0.5, 0.2, 0.1) \\ (0.1, 0.3, 0.5, 0.6; 0.5, 0.3, 0.4) & (0.4, 0.4, 0.8, 1; 0.8, 0.5, 0.3) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8; 0.6, 0.4, 0.2) \\ (0.8, 0.8, 0.8, 1; 0.5, 0.2, 0.1) & (0.2, 0.4, 0.5, 0.8; 0.3, 0.1, 0.5) & (0.2, 0.4, 0.5, 0.8; 0.3, 0.1, 0.5) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

By taking overall raw averages, then the weights are as follows;

$$X = \begin{bmatrix} 0.28 \\ 0.24 \\ 0.20 \\ 0.26 \end{bmatrix}$$

The pairwise comparison matrix of alternatives with respect to query functionality criterion as follows:

By using Eq. 21, we can determine the deterministic values of previous neutrosophic matrix as follows:

$$\tilde{A}_{qf} = \begin{bmatrix} 0.5 & 0.29 & 0.31 & 0.25 \\ 0.31 & 0.5 & 0.40 & 0.27 \\ 0.17 & 0.325 & 0.5 & 0.26 \\ 0.47 & 0.28 & 0.30 & 0.5 \end{bmatrix}$$

By making normalization of matrix we obtain the following:

$$\tilde{A}_{qf} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & x & (0.4, 0.5, 0.7, 9) & x \\ x & (0.5, 0.5, 0.5, 0.5) & x & x \\ x & x & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8) \\ x & (0.2, 0.4, 0.5, 0.8) & x & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

We can complete the previous matrix by applying Theorems 1 or 2 as follows:

$$\tilde{A}_{qf} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0, 0.5, 0.8, 1.5) & (0.4, 0.5, 0.7, 9) & (-0.3, 0.5, 0.9, 1.8) \\ (-0.5, 0.2, 0.5, 1) & (0.5, 0.5, 0.5, 0.5) & (-0.6, 0.2, 0.7, 1.4) & (0.1, 0.5, 0.6, 0.8) \\ (0.1, 0.3, 0.5, 0.6) & (-0.4, 0.3, 0.8, 1.6) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8) \\ (-0.8, 0.1, 0.5, 1.3) & (0.2, 0.4, 0.5, 0.8) & (0.2, 0.4, 0.5, 0.8) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

$$\tilde{A}_{qf} = \begin{bmatrix} 0.34 & 0.20 & 0.21 & 0.19 \\ 0.21 & 0.36 & 0.15 & 0.21 \\ 0.11 & 0.23 & 0.33 & 0.20 \\ 0.47 & 0.2 & 0.2 & 0.39 \end{bmatrix}$$

By taking overall raw averages, then the weights are as follows;

$$X = \begin{bmatrix} 0.23 \\ 0.23 \\ 0.21 \\ 0.28 \end{bmatrix}$$

The pairwise comparison matrix of alternatives with respect to user interface criterion as follows:

$$\tilde{A}_{ui} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.9, 1) & x & x \\ x & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.8, 0.9) & x \\ x & x & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8) \\ x & x & x & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

We can complete the previous matrix by applying Theorems 1 or 2, then;

$$\tilde{A}_{ui} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.9, 1) & (0.7, 0.9, 1.2, 1.4) & (0.4, 0.7, 1.3, 1.7) \\ (0, 0.1, 0.3, 0.4) & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.8, 0.9) & (0.3, 0.5, 0.9, 1.2) \\ (-0.4, -0.2, 0.1, 0.3) & (-0.3, 0, 0.5, 0.8) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8) \\ (-0.7, -0.3, 0.3, 0.6) & (-0.6, -0.1, 0.7, 1.1) & (0.2, 0.4, 0.5, 0.8) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

According to Definition 6, one can see that $\tilde{A}_{ui}$ is not trapezoidal neutrosophic additive reciprocal preference relations.

By using Eqs. 15, 16 and 17 one can obtain the following:

$$\tilde{A}_{ui} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.9, 1) & (0.7, 0.9, 1, 1) & (0.4, 0.7, 1, 1) \\ (0, 0.1, 0.3, 0.4) & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.8, 0.9) & (0.3, 0.5, 0.9, 1) \\ (0.4, 0.4, 0.4, 0.4) & (0.3, 0.3, 0.5, 0.8) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8) \\ (0.7, 0.7, 0.7, 0.7) & (0.6, 0.6, 0.7, 1) & (0.2, 0.4, 0.5, 0.8) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

The previous matrix appears consistent according to Definition 6. And then by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, experts should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ) and minimum falsity-membership degree (β) of single valued neutrosophic numbers as in Definition 3. Then,

$$\tilde{A}_{ui} = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.9, 1; 0.7, 0.2, 0.5) & (0.7, 0.9, 1, 1; 0.5, 0.2, 0.1) & (0.4, 0.7, 1, 1; 0.5, 0.2, 0.3) \\ (0, 0.1, 0.3, 0.4; 0.8, 0.2, 0.6) & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.8, 0.9; 0.5, 0.2, 0.1) & (0.3, 0.5, 0.9, 1; 0.5, 0.2, 0.1) \\ (0.4, 0.4, 0.4, 0.4; 0.5, 0.3, 0.4) & (0.3, 0.3, 0.5, 0.8; 0.8, 0.5, 0.3) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8; 0.6, 0.4, 0.2) \\ (0.7, 0.7, 0.7, 0.7; 0.5, 0.2, 0.1) & (0.6, 0.6, 0.7, 1; 0.3, 0.1, 0.5) & (0.2, 0.4, 0.5, 0.8; 0.3, 0.1, 0.5) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

By using Eq. 21, we can determine the deterministic values of previous neutrosophic matrix as follows:

$$\tilde{A}_{ui} = \begin{bmatrix} 0.5 & 0.4 & 0.49 & 0.41 \\ 0.1 & 0.5 & 0.41 & 0.37 \\ 0.18 & 0.24 & 0.5 & 0.56 \\ 0.38 & 0.30 & 0.20 & 0.5 \end{bmatrix}.$$

By making normalization of matrix we obtain the following:

$$\tilde{A}_{ui} = \begin{bmatrix} 0.43 & 0.27 & 0.30 & 0.22 \\ 0.08 & 0.35 & 0.26 & 0.20 \\ 0.15 & 0.16 & 0.31 & 0.30 \\ 0.33 & 0.21 & 0.12 & 0.27 \end{bmatrix}$$

By taking overall raw averages, then the weights are as follows;

$$X = \begin{bmatrix} 0.30 \\ 0.22 \\ 0.23 \\ 0.23 \end{bmatrix}$$

The pairwise comparison matrix of alternatives with respect to security criterion as follows:

$$\tilde{A}_S = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.5, 0.6, 0.7, 0.8) & x & x \\ x & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.6, 0.7) & x \\ x & x & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.5) \\ x & x & x & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

We can complete the previous matrix by using Theorems 1 or 2:

$$\tilde{A}_S = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.5, 0.6, 0.7, 0.8) & (0.4, 0.6, 0.8, 1) & (0.1, 0.4, 0.7, 1) \\ (0.2, 0.3, 0.4, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.6, 0.7) & (0.1, 0.3, 0.5, 0.7) \\ (0, 0.2, 0.4, 0.6) & (0, 0.3, 0.6, 0.9) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.5) \\ (0, 0.3, 0.6, 0.9) & (0.3, 0.5, 0.7, 0.9) & (0.5, 0.6, 0.7, 0.8) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

Then the previous matrix appears consistent according to Definition 6. And then by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, experts should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ) and minimum falsity-membership degree (β) of single valued neutrosophic numbers as in Definition 3. Then,

$$\tilde{A}_S = \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.5, 0.6, 0.7, 0.8; 0.7, 0.2, 0.5) & (0.4, 0.6, 0.8, 1; 0.4, 0.5, 0.6) & (0.1, 0.4, 0.7, 1; 0.6, 0.2, 0.3) \\ (0.2, 0.3, 0.4, 0.5; 0.8, 0.2, 0.6) & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.6, 0.7; 0.5.0.2, 0.1) & (0.1, 0.3, 0.5, 0.7; 0.5, 0.2, 0.1) \\ (0, 0.2, 0.4, 0.6; 0.5, 0.3, 0.4) & (0, 0.3, 0.6, 0.9; 0.8, 0.5, 0.3) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.5; 0.6, 0.4, 0.2) \\ (0, 0.3, 0.6, 0.9; 0.5, 0.2, 0.1) & (0.3, 0.5, 0.7, 0.9; 0.3, 0.1, 0.5) & (0.5, 0.6, 0.7, 0.8; 0.3, 0.1, 0.5) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

By using Eq. 21, we can determine the deterministic values of previous neutrosophic matrix as follows:

$$\tilde{A}_S = \begin{bmatrix} 0.5 & 0.32 & 0.23 & 0.29 \\ 0.17 & 0.5 & 0.30 & 0.22 \\ 0.13 & 0.22 & 0.5 & 0.17 \\ 0.25 & 0.25 & 0.28 & 0.5 \end{bmatrix}$$

By making normalization of matrix we obtain the following:

$$\tilde{A}_S = \begin{bmatrix} 0.47 & 0.25 & 0.17 & 0.24 \\ 0.16 & 0.39 & 0.23 & 0.19 \\ 0.12 & 0.17 & 0.38 & 0.14 \\ 0.24 & 0.19 & 0.21 & 0.42 \end{bmatrix}$$

By taking overall raw averages, then the weights are as follows:

$$\mathrm{X} = \begin{bmatrix} 0.28 \\ 0.24 \\ 0.20 \\ 0.26 \end{bmatrix}$$

Step 7 Calculate overall priority of alternatives.

$$\begin{bmatrix} 0.28 & 0.23 & 0.30 & 0.28 \\ 0.24 & 0.23 & 0.22 & 0.24 \\ 0.20 & 0.21 & 0.23 & 0.20 \\ 0.26 & 0.28 & 0.23 & 0.26 \end{bmatrix} \times \begin{bmatrix} 0.25 \\ 0.27 \\ 0.22 \\ 0.25 \end{bmatrix} \times \begin{bmatrix} 0.27 \\ 0.23 \\ 0.20 \\ 0.25 \end{bmatrix}$$

Priority matrix

Criteria weights

The AHP ranking of decision alternatives are showed in Fig. 7:

It's obvious from Fig. 7 that, Google is the best search engine followed by yahoo, Ask and finally Bing search engine.

Search engine	Priority
Google	0.27
Yahoo	0.25
Ask	0.23
Bing	0.20

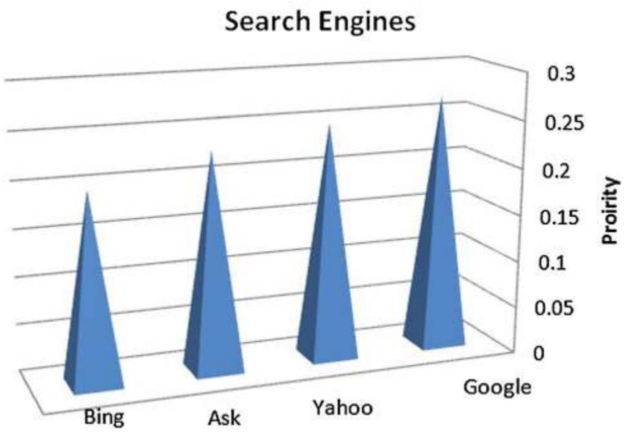


Fig. 7 The AHP ranking of decision alternatives

Step 8 Evaluation process.

A total of 20 graduated students from computer science department at Zagazig University in Egypt were picked randomly to estimate the interpretation of search engines according to this research. The graduated students were 13 men and seven women. The research team sends an estimation questionnaire to each entrant by e-mail. The entrants were asked to measure performance of four search engines with respect to our system criteria and ranking them according to their opinions. The answer sheet of each participant was resend to research team. The research team noted that 80% of participants were agreed with research results.

4.2 Comparative analysis

The proposed Delphi-AHP group decision making model compared with other existing approaches in this subsection.

1. Tavana et al. (1993) integrated the analytic hierarchy process (AHP) into Delphi framework, the proposed model did not introduce any method to calculate consensus degree on decisions and if the pairwise comparison matrix is not consistent they did not provide any technique to make it consistent. Lewandowski et al. (2011) did not consider consistency and consensus degree. But in our research we considered these points.
2. Almost previous studies, used Saaty's scale in the pairwise comparison judgments, but in our research we proposed anew scale to treat Saaty's scale drawbacks which illustrated by Lv et al. (2003).
3. (Kazemi et al. 2015; Liu 2013; Saaty 2008; Tavana et al. 1993) used $\frac{n\times(n-1)}{2}$ pairwise comparison judgments and it is a time consuming, but in our research we used only $(n-1)$ pairwise comparison judgments.
4. Because fuzzy set has only single valued function used to express evidence of acceptance and rejection at the same time in many practical situations as in Kazemi et al. 2015; Liu 2013, then it cannot represent the problem domain effectively. But in our research we integrate Delphi into AHP framework by using neutrosophic, which has three valued functions.
5. Because the problem domain should has a precise knowledge, otherwise the people have some uncertainty in assigning the preference evaluation values and this makes the decision making process appear the characteristics of confirmation, refusal and indeterminacy. So neutrosophic is very important and efficient in dealing with uncertainty and vagueness rather than other methods.

5 Conclusions and future works

Neutrosophic set includes classical set, fuzzy set and intuitionistic fuzzy set as it does not mean only truth-membership and falsity-membership but also considers indeterminacy function which is very obvious in real life situations. Since the precise judgments are not logical in simulating uncertainty associated with vagueness of decision making process, then in this research we have considered parameters of AHP-Delphi comparison matrices as trapezoidal neutrosophic numbers. The power of AHP is enhanced by adding Delphi technique, since it reduced noise which result from focusing on group and/or individual interests rather than concentriciting on problem disband and it also increased consensus dessssssgree about ideas. Each expert in traditional AHP should make $\frac{n\times(n-1)}{2}$ consistent judgments for n alternatives and this makes experts tired and leads to inconsistent judgments by increasing number of alternatives. We treated this drawback by making experts focus only on $(n-1)$ consensus judgments. In the future, we will apply the proposed model in different practical problems which require verdicts about qualitative features from a number of experts. We also will apply trapezoidal neutrosophic multiplicative reciprocal preference relation to AHP-Delphi technique, and present a new method to convert consistent trapezoidal neutrosophic additive reciprocal preference relation to consistent multiplicative preference relation. We also will develop the proposed example to include different types of search engines such as business, academic and governmental search engines.

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Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interests regarding the publication of this paper.

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