Received: 13 December 2015 | Revised: 20 January 2017 | Accepted: 24 January 2017
DOI 10.1002/dac.3301

RESEARCH ARTICLE

WILEY

Performance analysis of adaptive limited feedback coordinated zero-forcing multicell systems in time-varying channels: A practical approach

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Summary
In this paper, coordinated limited feedback zero-forcing beamforming multicell system in time-varying channel is studied. In time-varying channels, the actual rate loss arising from error in channel quantization of both interuser and intercell inferences is quantified in this work. Using the actual degradation of rate loss, a limited feedback bit allocation is proposed to reduce interuser and intercell interference simultaneously with feedback update period as one of the parameters. Furthermore, the scaling law of bits required to maintain a constant rate loss is formulated in varying channel conditions for a given feedback update period. Simulation results demonstrated the practical feedback requirement in multicell systems in the presence of both intercell and interuser interference over conventional schemes to maintain a constant rate offset. The key finding from the proposed feedback allocation is that practically much higher allocation of feedback bits and feedback scaling are required in time-varying channels to reduce throughput degradation.

KEYWORDS
coordinated zero-forcing, limited feedback, multi-antennas, throughput, time-varying channels

1 | INTRODUCTION

Wireless communication systems continuously strive to cater to the needs of ever increasing higher data rates. Designing feedback bit allocation and channel state quantization for varying channels to maximize the throughput becomes more severe when the number of antennas at the transmitter and/or at the receiver increases. The goal is practically and particularly challenging for systems that are encountering interferences, large feedback overhead, and operated in varying channel conditions. The challenge will be more critical if base stations (BS) are using coordination schemes. Conventional techniques to achieve considerable performance improvement in system parameters include mitigation of interference, automatic repeat request, sectoring, and joint transmissions. Achieving guaranteed spectral efficiency without incurring additional overhead resulted in explosion of research in theoretical and practical issues in limited feedback multiple input multiple output (MIMO) systems in time-varying channel (TVC) environments.

To quantify the achievable spectral efficiency of the MIMO systems, capacity analysis in temporal channel environments has been proposed in Godsmith et al,[1] and the capacity under rich scattering environment is studied in Telatar.[2] The capacity is tremendously increased by increasing the number of antennas at the transmitter and or at the receiver. However, transmitter and receiver channel state information (CSI) is required to nullify the effect of interferences in signal to interference plus noise ratio (SINR) degradation. The MIMO systems are mostly affected by intercell interference (ICI) and interuser interference (IUI). Because these interferences are dominating in cellular systems, the CSI of the MIMO channels is quantized and feedbacks to the BS using limited feedback backhaul. This quantized feedback is to increase the throughput, spectral efficiency, etc thereby aids in analysing the capacity of the network. The throughput degradation due to finite rate feedback in MIMO systems in zero-forcing transmission technique is analyzed in detail in Jindal.[3]

The transmission capacity of interference limited MIMO networks is studied in Hunter et al,[4] and the expression for SINR distribution and the problem of feedback resource allocation in limited feedback systems with partial CSI is also derived.[5,6] The resource allocation along with user selection using zero-forcing beamforming techniques is also proposed in Soulhli and Ohtsuki.[7] The capacity improvements with beamforming methodologies have been studied in many research works in the past. Joint beamforming method is proposed to reduce the amount of CSI sharing for interference management[8] and per stream SINR maximization in interference-limited networks.[9] In these multiantenna networks where interference (both IUI and ICI) comes out as the key capacity determining factor, the techniques, which subsidize the interference, can dramatically improve the system parameters such as throughput, back haul overhead and number of feedback bits.

To overcome the effect of intercell as well as intra-cell interferences on the performance of multiantenna systems, the CSI has been quantized adaptively in the past. Adaptive partitioning of bits for interference minimization in multiantenna systems is stuied in Bhagavatula and Heath,[10] and partitioning of bits is extended to TVCs in recent past to quantify the throughput.[11] Adaptive bit partitioning for interfering broadcast channel (IFBC) is considered in Lee and Shin.[12] The interference management in random clustering with adaptive bit partitioning is discussed in detail.[13] Closed form expressions are derived to study the system performance in adaptive bit allocation, and it is shown that the adaptive allocation of feedback bits change dramatically when the channels are closely related.[14] The sum rate in temporally correlated channels is explored, and based on the individual CSI of the channels, a 2-stage feedback scheme is also proposed.[15,16] Moreover, the bit allocation in MIMO single cell and multicell systems with coordinated limited feedback techniques is proposed by many authors to study the system performance bottleneck.

Compared to conventional noncoordination schemes, the multicell coordination schemes advocate interference nulling as a more proactive design. With coding and signal processing, the MIMO multicell coordination scheme investigates the network performance with different set of channel quality information (CQI). Multiple input multiple output coordination concepts related to scalability, coverage, and system level integration are also discussed in detail.[17,18] By combining beamforming methodology and multicell coordination, sum rate has been characterized for TVCs.[19] Further, to characterize the sum rate, threshold-based adaptive allocation has been introduced.[20] A suboptimal solution for power minimization in limited feedback coordinated beamforming approach is discussed in Seongjin et al,[21] and the comparison of adaptive ICI cancellation between single cell coordinated transmission and multicell coordinated transmission is explored in Zhang et al.[22] To enhance the performance of multiantenna systems under study, consequently, the coordination between BS and mobile stations are used to mitigate both the IUI and ICI effects on the achievable throughput of the system. However, the capacity of backhaul link challenges the coordination range.

The expressions derived for bit partitioning in Lee and Shin[12] are not applicable to channels that are time varying in nature. In Kim et al,[19] closed form expressions are derived for TVCs without incorporating IUI. Further, in Lee and Shin,[12] feedback update policy is not considered, and Kim et al[19] does not provide closed form solutions for feedback scaling to maintain a constant rate loss with respect to perfect CSI feedback. Moreover, expressions for feedback scaling with channel feedback update period in TVCs that are unavailable and suitable expressions must be arrived. These shortcomings motivate us to derive expressions for feedback scaling and propose a closed form solutions for required feedback in limited feedback systems. Nevertheless, the intuition is to propose a feedback allocation scheme for multiantenna system in interference limited regime to quantify the practical throughput degradation in TVC conditions, ie, the proposed actual model quantifies the practical system performance by simultaneously considering IUI, ICI, and channel temporal correlation. The contributions are summarized as follows.

First, using Gauss Markov model and random vector quantization, the expression for rate loss arising from both IUI and ICI is derived by using coordinated zero-forcing beamforming (CZFB). From the rate loss, optimization problem is formulated to derive the number of feedback bits required to reduce quantization error. After optimizing the number of feedback bits, the rate loss is characterized against feedback update duration. Then, with the given feedback duration, a closed form solution is proposed for the number of bits to be scaled to maintain a constant rate offset. By considering IUI, ICI, and channel temporal correlations simultaneously, the proposed actual with IUI and ICI estimates the practical degradation of the system performance. To compensate the performance losses arising from IUI, ICI, and temporal correlation, proposed extra with IUI and ICI investigates additional requirements to minimize the degradation. Finally, we analyzed the cell average sum rate variations by simultaneously varying the feedback update duration and number of feedback bits via proposed actual and proposed extra methods. The key finding from the proposed models is to illustrate the actual and extra feedback requirements to achieve the practical throughput in TVCs when its performance is degraded by both IUI and ICI. Moreover, it is also demonstrated how frequent the feedback to be updated to meet the required cell average sum rate.

The rest of the paper is organized as follows. Section 2 deals with the system model considered in the work; Section 3 describes the proposed bit allocation and feedback scaling. In Section 4, the feedback scaling required to meet the guaranteed throughput is numerically simulated, and the results are corroborated with analytical expressions. Finally, concluding remarks are given in Section 5.

2 | SYSTEM MODEL

The K-cell multiantenna downlink finite-rate feedback system is considered in this work. One such a model of the system with L users in each cell and BS arrangement are shown in the Figure 1. There are K BS and one BS in each cell. The BS consists of M antennas and offers service to L users. The power of each BS is shared between L users in a given time, and each user has single antenna receivers. The user l in i-th cell is represented as (l, i)

The m-th channel use between desired user l and the serving i-th base station BS_i is $\boldsymbol{h}_{l,i,i}[m] \in C^M$. The subscript l,i,i indicates that the user l is in i-th cell and receives the signal from i-th BS, and similarly, l,i,j represents the signal from j-th BS to the user l in i-th cell. The time index, ie, number of channel uses is represented as m. The received signal of the user l in i-th cell is given by

$$y_{l,i} = \sqrt{P_{l,i,i}}\boldsymbol{h}^H_{l,i,i}[m]\boldsymbol{x}_i[m] + \sum_{j=1,j\neq i}^{K}\sqrt{P_{l,i,j}}\boldsymbol{h}^H_{l,i,j}[m]\boldsymbol{x}_j[m] + g_{l,i}, \tag{1}$$

where $P_{l,i,i}$,$P_{l,i,j}$ represent the signal power and interference power at the user (l,i) from respective BS. The parameter $\boldsymbol{h}^H_{l,i,j}[m]$ is the interference vector coefficient from j-th BS of a size $MX1$. The signal vectors from the BS are given as $x_i[m]$, $x_j[m]$ with a size of $MX1$. The expected value of the desired signal vector is $\mathbb{E}\left(Tr\left(\boldsymbol{x}_i[m]\boldsymbol{x}_i^h[m]\right)\right) = E^t$. Each channel vectors are drawn from independent identically distributed complex random distribution having zero mean and unit variance. The additive white Gaussian noise $g_{l,i}$ at each user (l,i) is a scalar, and it is having a variance of σ^2. The path loss of the desired signal to the user from the i-th BS is given by $(1+d_i)^{-\alpha}$, where d_i is the distance of the user (l,i) from the desired i-th BS, and α represents path loss exponent.

2.1 | Time-varying channel feedback model

Because of the time-varying behaviour of the interfering and desired channel, the time varying behaviour is modelled from well-known Gauss Markov model.[10] Using the Gauss Markov model, the TVC m-th use between the user and the BS of interest is

$$\boldsymbol{h}_{l,i,i}[m] = \eta_{l,i,i}\boldsymbol{h}_{l,i,i}[m-1] + \sqrt{1-\eta^2_{l,i,i}}\boldsymbol{w}_{l,i}[m]. \tag{2}$$

The interfering channel between the l-th user and BS $(i\neq j, j=1,2,\ldots,K)$ is

$$\boldsymbol{h}_{l,i,j}[m] = \eta_{l,i,j}\boldsymbol{h}_{l,i,j}[m-1] + \sqrt{1-\eta^2_{l,i,j}}\boldsymbol{w}_{l,j}[m], \tag{3}$$

where $\eta_{l,i,i} = J_0\left(2\pi f_d^{lii}T_s\right)$ and $\eta_{l,i,j} = J_0\left(2\pi f_d^{lij}T_s\right)$ are the correlation coefficients to model the delay associated with channel time-varying behaviour. In the correlation coefficients, J_0 is zeroth order Bessel's coefficient. The Doppler frequency between the respective BS and the user (l,i) is represented in the correlation coefficients as f_d^{lii} , f_d^{lij} . The parameter T_s is the frame duration. $\boldsymbol{w}_{l,i}[m]$ and $\boldsymbol{w}_{l,j}[m]$ represent the beamforming

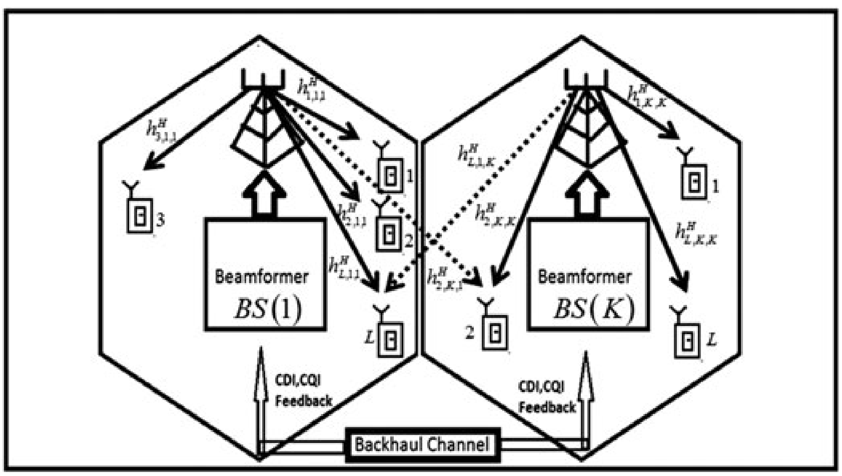


FIGURE 1 System model of multicell limited feedback. BS, base station

vectors for the user (l, i) having a size $MX1$. The beamforming vectors are normalized to unit value, ie, $\|\boldsymbol{w}_{l,i}[m]\| = 1$.

The user (l, i) where $l = 1, 2, \ldots, L$ and $i = 1, 2, \ldots, K$ in each of the K-cells has perfect knowledge of CSI of the desired channels, ie, $\boldsymbol{h}_{l,i,j}$ by using the reference orthogonal signals. Once the CSI is known at the user, the users transmits back the serving BS, ie, BS_i, the required CQI, and channel direction information (CDI) of all available channel links. To send back the CSI, the received channel vectors directions are quantized, ie, $\widetilde{\boldsymbol{h}}_{l,i,j}[m] = \frac{\boldsymbol{h}_{l,i,j}[m]}{\|\boldsymbol{h}_{l,i,j}[m]\|}$ for $j = \{1, 2, \ldots, K\}$ by using different quantization codebooks. From the codebook size $C_{l,i,j} = \left\{C^1_{l,i,j}, C^2_{l,i,j}, \ldots C^{Z_j}_{l,i,j}\right\}, \left\{Z_j = 2^{B_{l,i,j}}\right\}$ and using the principle of minimum chordal distance metric, channel distribution information indices of interfering BS channel vectors[3] are found to be

$$\widehat{\boldsymbol{h}}_{l,i,j} = \boldsymbol{c}^{z_j}_{l,i,j} \text{ and } z_j = \arg \max_{1 \le m \le 2^{B_{l,i,j}}} \left| \boldsymbol{c}^{m\ H}_{l,i,j} \widetilde{\boldsymbol{h}}_{l,i,j} \right|. \tag{4}$$

To find the indices of CDI, we apply random vector quantization (RVQ) method. Once the CDI is quantized using RVQ,[23] users inform the indices of CDI z_j to the serving BS through backhaul link, which is considered as error free in this paper. In the CZBF feedback analysis, the feedback for CQI is not included in total feedback budget. Let us assume that each user uses B_T bits to feedback during a sub-frame or a total frame t_F and $B_{l,i,i}$ bits are required to quantize $\boldsymbol{h}_{l,i,i}[m]$ if feedback occurs. Similarly $B_{l,i,j}$ bits are required for $\boldsymbol{h}_{l,i,j}[m]$. Let $\tau_{l,i,i}, \tau_{l,i,j}$ are respective feedback update periods and $N_{l,i,i}$, $N_{l,i,j}$ are the feedback update volumes during a frame period i.e. $N_{l,i,i} = \frac{t_F}{\tau_{l,i,i}}, N_{l,i,j} = \frac{t_F}{\tau_{l,i,j}}$. The above assumption concludes that $\tau_{l,i,i}, \tau_{l,i,j}$ are integers and $(\tau_{l,i,i}, \tau_{l,i,j}) \ge 1$.

Since the objective is to reduce the rate loss with minimum total feedback budget, we consider a more general scenario with less frequent feedback updation. Hence to reduce frequent feedback updation, we incorporate a minimal update policy by considering $(\tau_{l,i,i}, \tau_{l,i,j}) \ge 2$. From the above analysis $\sum_{l=1}^{L-1} N_{l,i,i} B_{l,i,i} + \sum_{j=1, j \ne i}^{K} N_{l,i,j} B_{l,i,j} = B_T$. Interestingly $\boldsymbol{h}_{l,i,i}[m]$ is feedback if and only if m is an integer multiple of $\tau_{l,i,i}$ i.e. $m = p\tau_{l,i,i}$ where p is $1, 2, \ldots, N_{l,i,i}$. Suppose for $\{m | p\tau_{l,i,i} < m < (p+1)\tau_{l,i,i}\}$ if the BS is not receiving any channel distribution information, then BS has to use $\boldsymbol{h}_{l,i,i}[p\tau_{l,i,i}]$ which is the only most recent CDI. The channel vector $h_{l,i,j}[m]$ is also feedback in the manner defined above. Because of the delay in the limited feedback and the channels are updated according to the policy defined above i.e. the BS uses pervious channel estimates of all the desired and interfering channels, and hence the rate loss tends to be maximum.

2.2 | Time varying channel CZBF rate

To study the rate and amount of feedback required to scale the feedback to reach the perfect CSI sum-rate, the ICI and IUI to be nullified. In order to mitigate both these interferences, the coordinated zero-forcing is used.[24] In this section, we describe the CZFB rate analysis. After applying the RVQ and indices of CDI sent to the BS, the signal received at the user[12] is from Equation 1 is written as

$$\begin{aligned} y_{l,i} = {} & \sqrt{P_{l,i,i}} \boldsymbol{h}^H_{l,i,i}[m] \boldsymbol{w}_{l,i}[m] s_{l,i}[m] \\ & + \underbrace{\textstyle\sum_{u=1, u \ne l}^{L} \sqrt{P_{l,i,i}} \boldsymbol{h}^H_{l,i,i}[m] \boldsymbol{w}_{u,i}[m] s_{u,i}[m]}_{IUI} \\ & + \underbrace{\textstyle\sum_{j=1, j \ne i}^{K} \sqrt{P_{l,i,m}} \boldsymbol{h}^H_{l,i,j}[m] \sum_{l=1}^{L} \boldsymbol{w}_{l,j}[m] s_{l,j}[m]}_{ICI} \quad + g_{l,i}, \end{aligned} \tag{5}$$

where the parameter $s_{l,i}$ denotes the data symbol to the designated user (l, i) and $\mathbb{E}\left(|s_{l,i}[m]^2|\right) = 1$. After sharing the quantized CDI of serving and interfering users, each BS in the system calculates the vector $\boldsymbol{w}_{l,i}$ in such a manner that the IUI and ICI reduce to zero. To be more precise, the vectors are constructed so that $\begin{bmatrix} \overline{\widehat{\boldsymbol{H}}}_{i,i}[m] \\ \underbrace{\widehat{\boldsymbol{H}}_{S_{i,j}}[m]}_{(L-1+Q)XM} \end{bmatrix} \boldsymbol{w}_{l,i}[m] = 0$. In this, $\overline{\widehat{\boldsymbol{H}}}_{i,i}[m]$ represents the intracell network channel matrix for nullifying IUI and is equal to $\overline{\widehat{\boldsymbol{H}}}_{i,i}[m] = \left[\widehat{\boldsymbol{h}}_{1,i,i}[m], \ldots, \widehat{\boldsymbol{h}}_{i-1,i,i}[m], \widehat{\boldsymbol{h}}_{l+1,i,i}[m], \ldots, \widehat{\boldsymbol{h}}_{L,i,k}[m]\right]^H$. The matrix $\widehat{\boldsymbol{H}}_{S_{l,i}}[m] = \left[\widehat{\boldsymbol{h}}_{p,k,i}[m], \ldots, \widehat{\boldsymbol{h}}_{m,j,i}[m]\right]^H$ stands for network channel to mitigate ICI, and s_l is a set of users present in other $K-1$ cells, which receives ICI from BS_i transmission. The function of s_l is to find the size of the rows of $\widehat{\boldsymbol{H}}_{S_{l,i}}$, which in turn determine the dimension of the matrix of $\widehat{\boldsymbol{H}}_{S_{l,i}}[m]$. This dimension will have maximum value in situations where all other $K-1$ cell users are in the boundary of the i-th cell, and i-th cell should have KL antennas to completely nullify the IUI and ICI generated by this $K-1$ cell users.

Because of the quantization of CDI of serving and interfering BS channels, there always a residual ICI and IUI present in a limited feedback system. Because of this residual interference, the rate supported by the system is written as

$$R^{FB}_{l,i}[m] = \log_2\left(1 + \frac{E^t_{l,i,i}(1+d_i)^{-\alpha} \|\boldsymbol{h}_{l,i,i}[m]\|^2 |\widetilde{\boldsymbol{h}}^H_{l,i,i}[m] \boldsymbol{w}_{l,i}[m]|^2}{I_{IUI} + I_{ICI} + 1}\right), \tag{6}$$

where the term $\widetilde{I}_{IUI} = E^t_{l,i,i}(1+d_i)^{-\alpha} \|\boldsymbol{h}_{l,i,i}[m]\|^2 \sum_{u=1, u \ne l}^{L} |\widetilde{\boldsymbol{h}}^H_{u,i,i}[m] \boldsymbol{w}_{u,i}[m]|^2$ represents IUI, and ICI is represented as

$\widetilde{I}_{ICI} = \sum_{j=1,j\neq i}^{K}\left(1+d_j\right)^{-\alpha}E_{l,i,j}^{t}\|\boldsymbol{h}_{l,i,j}[m]\|^2\sum_{l=1}^{L}|\widetilde{\boldsymbol{h}}_{l,i,j}^{H}[m]\boldsymbol{w}_{l,j}[m]|^2$. The received signal power of l-th user in i-th cell $P_{l,i,i}$ is written with expected value of transmit signal power $E_{l,i,i}^{t}$ as $P_{l,i,i} = \frac{E_{l,i,i}^{t}}{(1+d_i)^{\alpha}}$ and correspondingly the signal from the j-th interfering station as $P_{l,i,j} = \frac{E_{l,i,j}^{t}}{\left(1+d_j\right)^{\alpha}}$. To estimate the effect of residual interference on achievable rate of limited feedback system, the channel $\widetilde{\boldsymbol{h}}_{l,i,j}[m]$ is split into 2 components. The orthogonal basis function, which gives raise to the quantized CDI, is $\widetilde{\boldsymbol{h}}_{l,i,i}[m] = \widehat{\boldsymbol{h}}_{l,i,j}[m]\left(\cos\theta_{l,i,j} + \boldsymbol{q}_{l,i,j}[m]\left(\sin\theta_{l,i,j}\right)\right)$, where $\theta_{l,i,j}$ is the angle between quantized and original desired channel vectors. The quantity $\boldsymbol{q}_{l,i,j}$ is the components of the error vector due to channel quantization effect. The rate in Equation 6 because of quantization error in CDI is

$$R_{l,i}^{FB} = \log_2\left(1+\frac{E_{l,i,i}^{t}(1+d_i)^{-\alpha}\|\boldsymbol{h}_{l,i,i}[m]\|^2|\widetilde{\boldsymbol{h}}_{l,i,i}^{H}[m]\boldsymbol{w}_{l,i}[m]|^2}{1+\widetilde{I}_{IUI}+\widetilde{I}_{ICI}}\right), \quad (7)$$

where IUI is $\widetilde{I}_{IUI} = E_{l,i,i}^{t}(1+d_i)^{-\alpha}\|\boldsymbol{h}_{l,i,i}[m]\|^2\sin^2\theta_{l,i,i}\sum_{u=1,u\neq l}^{L}|\boldsymbol{q}_{u,i,i}^{H}[m]\boldsymbol{w}_{u,i}[m]|^2$ and ICI is termed as $\widetilde{I}_{ICI} = \sum_{j=1,j\neq i}^{K}\left(1+d_j\right)^{-\alpha}E_{l,i,j}^{t}\|\boldsymbol{h}_{l,i,j}[m]\|^2\sum_{l=1}^{L}\sin^2\theta_{l,i,j}|\boldsymbol{q}_{l,i,j}^{H}[m]\boldsymbol{w}_{l,j}[m]|^2$. Now, we describe the rate loss of the proposed model with respect to perfect CSI system.

2.3 | Rate loss characterization

The rate loss $\Delta R_{l,i}[m]$ is the amount of difference between the achievable rate of the user in a limited feedback system with residual interference to the system where perfect CSI is available, ie, $\Delta R_{l,i}[m] = \mathbb{E}\left(R_{l,i}[m] - R_{l,i}^{FB}[m]\right)$ where $R_{l,i}[m]$ denotes the rate of the user in a perfect CSI system.

Theorem 1. *In a finite rate K-cell TVC feedback system, per user rate loss is given by*

$$\Delta R_{l,i}[m] \leq \log_2\left(\begin{array}{c} 1+(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)2^{B_{l,i,i}}\beta\left(2^{B_{l,i,i}},\frac{M}{M-1}\right)+1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)+ \\ (L)\sum_{j=1,j\neq i}^{K}\left(1+d_j\right)^{-\alpha}E_{l,i,j}^{t}\left(\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\left(\frac{M}{M-1}\right)2^{B_{l,i,j}}\beta\left(2^{B_{l,i,j}},\frac{M}{M-1}\right)+1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\right)\end{array}\right). \quad (8)$$

Proof: *See Appendix A*

The rate loss of Equation 8 is now a function of number of limited feedback bits and channel temporal correlations, which is a parameter of channel update period. To minimize the loss, the quantification of number of bits for interferences is necessary. Now, we derive the closed form expression for number of bits.

3 | PROPOSED BIT ALLOCATION AND SCALING

In the preceding section, we derived the rate loss due to the residual IUI and ICI. This residual interference comes from the fact that the quantization of interfering channels and causes performance degradation. To mitigate this performance degradation, optimal number of bits should be allocated between IUI and ICI. When the total number of feedback bits per user is fixed, the number of bits to IUI and ICI is allocated as

$$\min_{B_{l,i,i}\in(0,\mathbb{Z}^{+})}\left(\left\{\left(2^{B_{l,i,i}}\beta\left(2^{B_{l,i,i}},\frac{M}{M-1}\right)\right)A_i+\left((L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\right)\right\}\right.$$
$$\left.+\left\{\sum_{j=1,j\neq i}^{K}\left(\left[2^{B_{l,i,j}}\beta\left(2^{B_{l,i,j}},\frac{M}{M-1}\right)\right]A_j+\left[(L)\left(1+d_j\right)^{-\alpha}E_{l,i,j}^{t}\left(1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\right)\right]\right)\right\}\right), \textit{subject to } \sum_{i=1}^{K}B_{l,i,j}\leq B_T. \quad (9)$$

In the above Equation 9 for notational brevity, we have used $A_i = (L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)$ and $A_j = (L)\left(1+d_j\right)^{-\alpha}E_{l,i,j}^{t}\left(\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\left(\frac{M}{M-1}\right)\right)$ to represent the IUI and ICI. The solution for bit allocation in the above equation requires exhaustive search and involves high computational complexity when each individual user's bits $B_{l,i,i}$ becomes very high. Therefore, by using the upper bounds of quantization error, ie, $\mathbb{E}\left(\sin^2\theta_{l,i,j}\right)<2^{-\frac{B_{l,i,j}}{M-1}}$, a suboptimal closed form solution is proposed for the bit allocation.

Theorem 2. *The actual number of feedback bits required at the i-th user, which minimizes the Equation 9 is derived as*

$$B_i = \min\left(B_T,\ \max\left\{0, (M-1)log_2\left(\frac{\left\{(L-1)E^t_{l,i,i}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\}\ln(2)}{\left(\left\{(L-1)E^t_{l,i,i}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\}\prod_{j=1,j\neq i}^{K}A_j\ln(2)\right)^{\frac{1}{K}}}\right)+\frac{B_T}{K}\right\}\right). \tag{10}$$

Proof. *See Appendix B.*

The results in Equation 10 show that the number of bits allocated for IUI is proportional to (1) ratio of the total IUI power multiplied by channel temporal correlation coefficient and (2) the mean of geometric IUI and ICI power multiplied with channel temporal correlation coefficient. The channel temporal correlation coefficient in-turn affected by feedback update period, and this suggests that channel variations of IUI and ICI determine the feedback allocation. This B_i gives rise to the practical (actual) feedback requirement and actual achievable throughput in temporally correlated channels with inclusion of both ICI and IUI.

After deriving actual feedback bits for IUI, we focus on scaling of bits. To show how to maintain constant rate loss when the channel is time varying and modelled using channel temporal correlation coefficient, the required scaling of feedbacks to maintain the rate loss is given in the following corollary.

Corollary 1. *Given the number of feedback bits for IUI derived in Theorem 2, the number of feedback bits per user to be scaled to maintain a constant rate loss of* $log_2(\sigma)$ *bps/Hz is*

$$B_T = K(M-1)\left\{log_2\left(1/\left(\sigma-\left(1+K(L-1)E^t_{l,i,i}(1+d_i)^{-\alpha}\left(1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\right)\right)\right)\right. \tag{11}$$
$$\left.+\log_2\left(\left(\left\{K(L-1)E^t_{l,i,i}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\}\prod_{j=1,j\neq i}^{K}A_j\right)^{\frac{1}{K}}\right)\right\}.$$

Proof. *See Appendix C*

The scaling law of bits in Equation 11 is now a function of fading correlation coefficient, ie, to maintain a constant rate loss, the total power received including IUI Power and ICI power from all cells to be scaled appropriately. In the numerical simulation section, we illustrate the actual throughput performance with increasing number of bits.

4 | SIMULATION RESULTS

In this section, we demonstrated the performance of proposed scheme (proposed actual with IUI and ICI) to visualize results derived in the previous sections. The numerical calculation of the throughput of the proposed method (actual with ICI and IUI) with bit allocations between the serving BS and interfering base stations is plotted. Further, the scaling of bits to obtain the required sum-rate in both channel models (IFBC and TVC) is illustrated. Moreover, proposed extra with IUI and ICI is included in the plots to realize the extra feedback bits required compared to the proposed actual with IUI and ICI to reach to the performance as that of the schemes, which consider only the ICI.

The parameters of simulation are as follows. A simple 3-cell model shown in Figure 2 is considered, and each cell radius is set to be 500 m. Throughout the simulation, the number of antenna in each BS is 6 ($M=6$), the path loss exponent $\alpha = 3.8$, and in each cell, 2 users are simultaneously active, ie, L = 2, since 3 adjacent cells are taken, hence K = 3. The distance between serving BS_i and user (l, i) d_i varies from 0 to 500 m, and d_j varies from 500 to 1000 m. The maximum received signal and interference power at the user (l,i) $P_{l,i,i}$,$P_{l,i,j}$ in the simulation is set to be 20 dB at a

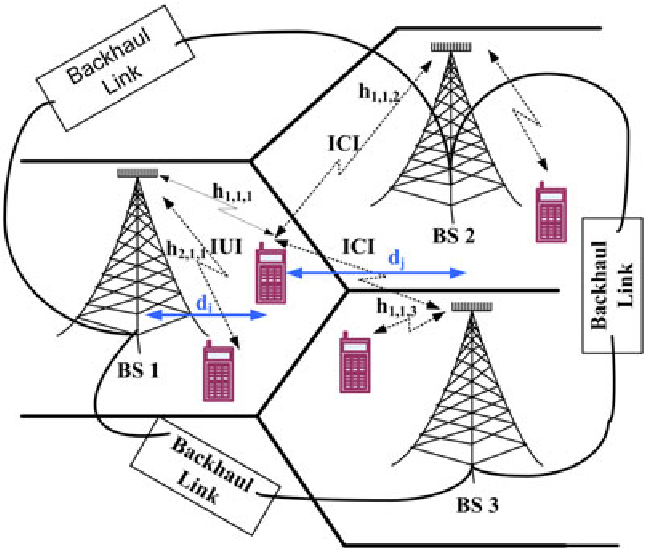


FIGURE 2 Simple one dimensional simulation model. BS, base station; ICI, intercell interference

distance of 500 m. The $\eta_{l,i,i}$,$\eta_{l,i,j}$ is calculated when the carrier frequency is 2 GHz and the frame duration T_s is 5 m. The mobile speed between user and the tagged BS is assumed to be varying between 0 and 10 km/h.

In Figure 3, the adaptive bit partitioning between the desired (serving) BS and interfering BS with respect to user distance from the serving BS when $B_T = 15$ is illustrated. From the plot, it is very clear that the proposed method starts allocating bits at little earlier distance compared to the previously reported allocation in Lee and Shin.[12] Because, in the proposed method, the channel temporal correlations are also taken into account while allocating bits, the proposed allocation starts allocating based on the actual strengths of IUI and ICI. Moreover, inside Figure 3, the bits allocation of 2 interfering BS are also plotted. On the basis of the proportion of bits between interfering and serving BS, the level of cooperation among BS to eliminate stronger interference (ICI and or IUI) can easily be determined. Also, one can easily infer from the plot is that, if the user is nearer to the serving BS, the IUI is dominating, and hence, the allocated bits of IUI are higher compared to the IUI. As the user moving away from the desired BS, the value of IUI is stats-reducing, but at the same time, the ICI is increasing and the interfering base stations will get more bits.

Figure 4 depicts the rate loss and subframe time. The rate loss is periodic with the least common multiple of subframe time among the BS. It is very important to measure that the proposed actual rate loss of system is 1.8 bps/Hz compared to 1.5 bps/Hz of TVC reported in Kim et al.[19] This higher loss is mainly because of significant IUI whose effects are not taken into account while determining the rate loss of TVC in Kim et al.[19] If one compare with the loss with respect to recently reported loss of IFBC in Lee and Shin,[12] the proposed actual loss is still higher. This is mainly due to the negligence of channel temporal correlation effect in IFBC, and the proposed actual rate loss includes both the omissions of other 2 methods. To demonstrate the effect feedback bits on rate loss with feedback update period, the rate loss is projected with 2 different combinations of bits. If the allocated bits are higher, one can observe from Figure 4, the rate loss tends to decrease and trying to approach TVC of Kim et al.[19] To compensate the loss (proposed extra with IUI and ICI), 26 additional bits are required to achieve the loss as that of TVC for $B_T = 10$ ($B_{l,1,1} = B_{l,1,2} = 4; B_{l,1,3} = 2$). Similarly, 18 additional bits are required if the total number of bits among the base stations is comparatively higher when $B_T = 18(B_{l,1,1} = B_{l,1,2} = 7; B_{l,1,3} = 4)$ to reach to the TVC level.

Continuing in the same manner, if the feedback update duration to quantize the CSI is higher, the expected loss tends to increase. The loss gap tends to be minimum compared to the TVC case for lower update duration. This necessitates that

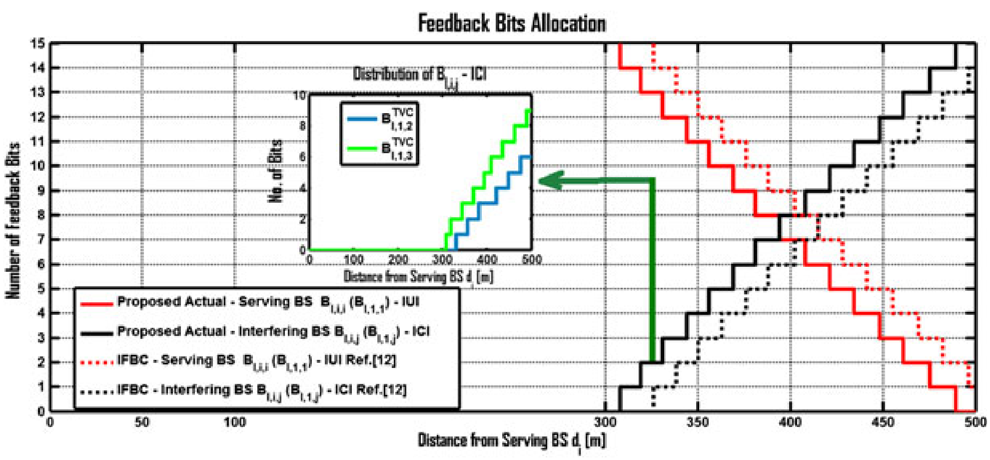


FIGURE 3 Feedback allocation strategy between interuser interference (IUI) and intercell interference (ICI) when $B_T = 15$; $\nu_{1,1,1} = 8km/h$; $\nu_{1,1,2} = 6km/h$; $\nu_{1,1,3} = 5km/h$; and $T_s = 5ms$. BS, base station

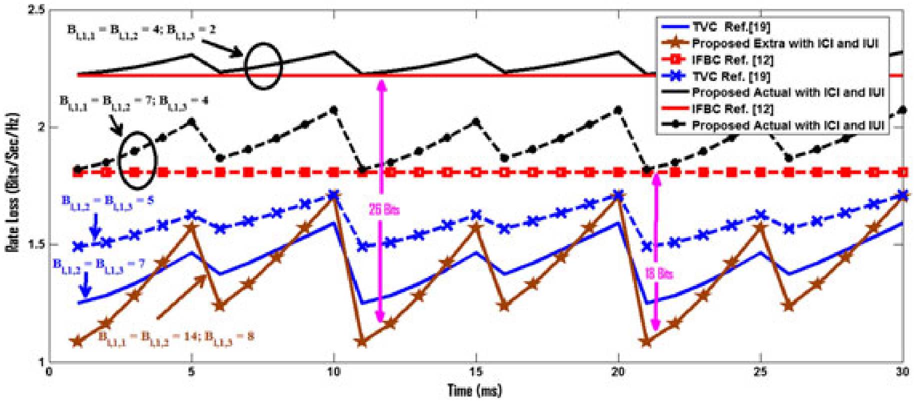


FIGURE 4 Rate loss versus subframe index (T_s) when $\nu_{1,1,1} = 8km/h$; $\nu_{1,1,2}$=$6km/h$; $\nu_{1,1,3}$=$5km/h$; $P_{2,1,1} = P_{1,1,2} = 3dB$; $P_{1,1,3} = 0dB$; $\tau_{1,1,1} = \tau_{1,1,2} = 5$; $\tau_{1,1,3} = 10$; and $t_F = 30$. BS, base station; ICI, intercell interference; IFBC, interfering broadcast channel; IUI, interuser interference; TVC, time-varying channel

to minimize the throughput loss, the CSI has to be updated frequently. The longer the update duration, the higher will be the loss, and the corresponding losses for different schemes are shown in Figure 5. In IFBC, channel temporal correlations are not considered, and hence, the expected loss is constant.

To illustrate the effect of feedback bits on achievable throughput, the following table (Table 1) lists the actual cell average sum rate with increasing total feedback bits B_T. One can easily understand that proposed scheme achieves the throughput as that of TVC and IFBC if the SINR commonly here as signal-to-noise ratio (SNR) is increased suitably or the limited feedback bits are scaled to the required extent. The increase in feedback bits and or SNR is due to additional losses arising from channel temporal correlations and interference. For example, to achieve a throughput of 1.8 bps, the proposed scheme requires 29 feedback bits at SNR of 0 dB. But at the same time, if the SNR is increased to 20 dB, it requires only 15 feedback bits to achieve the same throughput.

By the same token, the impact of proposed feedback allocation with cell edge SNR on achievable throughput is studied in Figure 6 ie, by fixing the total feedback bits and analysing the throughput with respect to SNR. When the total bits B_T ($B_{l,1,1} = B_{l,1,2} = 14$, $B_{l,1,3} = 8$ in IFBC or $B_{l,1,2}$=$B_{l,1,3} = 18$ in TVC) is fixed, the proposed actual allocation experiences a throughput of around 2.5 bps, which is lesser by mere 8% compared to IFBC of Lee and Shin[12] and around 40% with respect to TVC of Kim et al.[19] The decrease in throughput in the proposed scheme mainly because aforementioned effects such as inclusion of channel temporal correlations and IUI. To compensate the throughput degradation, if the total bits are increased (ie, if $B_T = 108$, the increase of 72 bits with respect to proposed actual scheme), one can easily infer from the plot that the proposed method with extra bits achieve comparatively higher throughput compared to the other schemes. It is of about 50% gain in throughput with respect to IFBC of Lee and Shin,[12] and improvement of around 10% against TVC of Kim et al.[19]

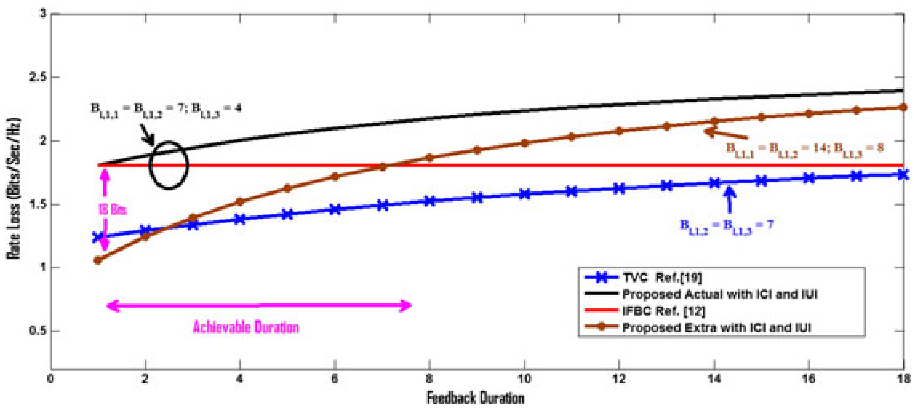


FIGURE 5 Rate loss against feedback duration $\tau_{1,1,1}$ when $\nu_{1,1,1}$= $8km/h$; $\nu_{1,1,2} = 6km/h$; $\nu_{1,1,3} = 5km/h$; $P_{2,1,1} = P_{1,1,2} = 3dB$; $P_{1,1,3}$=$0dB$; $\tau_{1,1,1} = \tau_{1,1,2} = \tau_{1,1,3}$; and $T_s = 5ms$. ICI, intercell interference; IFBC, interfering broadcast channel; IUI, interuser interference; TVC, time-varying channel

TABLE 1 Achievable throughput with increasing B_T when $\nu_{1,1,1} = 8km/h$; $\nu_{1,1,2} = 6km/h$; $\nu_{1,1,3}$=5km/h, $P_{2,1,1}$=$P_{1,1,2}$=3dB; $P_{1,1,3}$=0dB; $\tau_{1,1,1}$=$\tau_{1,1,2} = \tau_{1,1,3}$=2; and $T_s = 5ms$

Total Bits B_T	Throughput, bits/sec/Hz								
	SNR = 0 dB			SNR = 10 dB			SNR = 20 dB		
	TVC	IFBC	Proposed Actual	TVC	IFBC	Proposed Actual	TVC	IFBC	Proposed Actual
15	1.83342	1.72273	1.57354	2.38257	2.0114	1.78796	2.43214	2.04657	1.81309
17	1.83342	1.7801	1.61599	2.51064	2.15177	1.88601	2.57603	2.19933	1.9188
19	1.83342	1.83366	1.6565	2.6371	2.29449	1.98415	2.72014	2.35653	2.02537
21	1.83342	1.88337	1.69501	2.76124	2.43893	2.08195	2.86381	2.51778	2.13239
23	1.83342	1.92922	1.73147	2.88244	2.58444	2.17898	3.00636	2.68265	2.23941
25	1.83342	1.97127	1.76587	3.00008	2.73037	2.27484	3.14716	2.85077	2.34601
27	1.83342	2.00963	1.79822	3.11361	2.87607	2.36913	3.28557	3.02172	2.45178
29	1.83342	2.04447	1.82853	3.22255	3.02089	2.46147	3.42096	3.19514	2.5563
31	1.83342	2.07597	1.85685	3.32648	3.16418	2.55152	3.55277	3.37066	2.65919
33	1.83342	2.10433	1.88323	3.42506	3.3053	2.63896	3.68045	3.54791	2.76006
35	1.83342	2.12977	1.90773	3.51804	3.44365	2.72351	3.80349	3.72656	2.85856

Abbreviations: IFBC, interfering broadcast channel; SNR, signal-to-noise ration; TVC, time-varying channel.

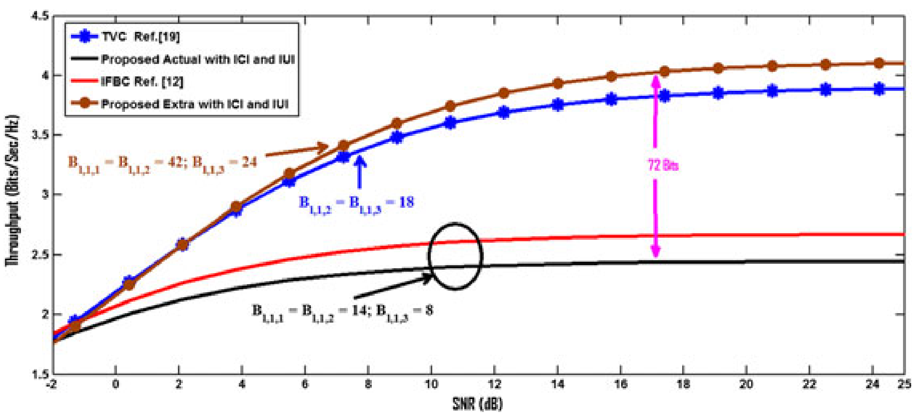


FIGURE 6 Cell edge sum rate performance when $\nu_{1,1,1}$= 8km/h; $\nu_{1,1,2}$=6km/h ; $\nu_{1,1,3}$= 5km/h; $P_{1,1,1}$= $P_{2,1,1}$= $P_{1,1,2}$; $P_{1,1,3} = P_{1,1,1} - 1dB$; $\tau_{1,1,1}$=$\tau_{1,1,2}$=$\tau_{1,1,3}$ = 2; and $T_s = 5ms$. ICI, intercell interference; IFBC, interfering broadcast channel; IUI, interuser interference; TVC, time-varying channel

The scaling of feedback bits of the proposed actual with IUI and ICI scheme derived in Equation 11 is verified in Figure 7 against TVC and IFBC. One can easily estimate that the proposed scheme maintains a rate loss within $LLog_2(\sigma)$. More clearly, if $\sigma = 2 + \left(K(L{-}1)E^t_{l,i,i}(1 + d_i)^{-\alpha}\left(1{-}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)\right)$ and $E^t_{l,i,i}(1 + d_i)^{-\alpha}\left(1{-}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)$ at a distance of 500 m is −5 dB, the proposed actual and extra methods maintains a rate offset of close to 3.10 bps with respect to perfect CSI in larger portion of received SNR. At the same time, if $E^t_{l,i,i}(1 + d_i)^{-\alpha}\left(1{-}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)$ at a distance of 500 m is maintained at 0 dB, the proposed methods fall within 4.64 bps throughput loss against perfect CSI. These results one can easily verify from the plot of Figure 7.

Figure 8 depicts the network performance with increasing number of bits when both the feedback update duration and target SNR vary simultaneously. In non-TVC conditions, though feedback duration increases, the achievable throughput is not affected. This shows that the increasing feedback update duration does not affect the number of feedback bits which in turn not affecting the performance. But for a TVC conditions, both the SNR and feedback update duration change the requirement of number of feedback bits, and if the feedback update duration increases, the throughput decreases for a given value of SNR. This proves that in a

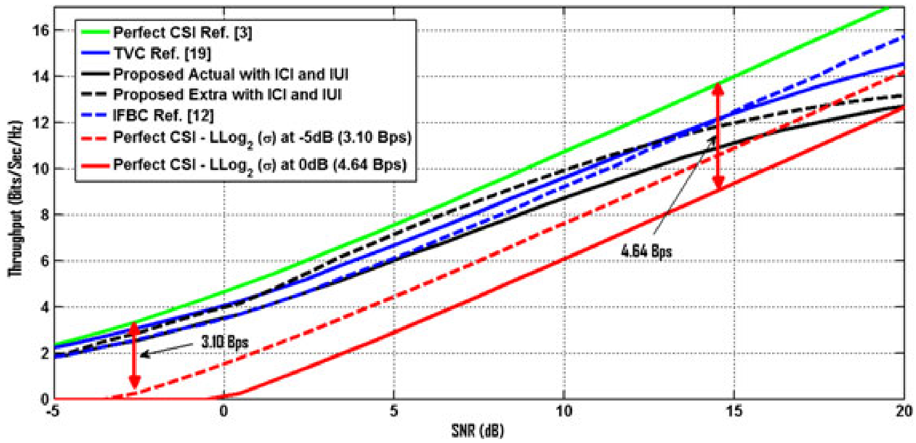
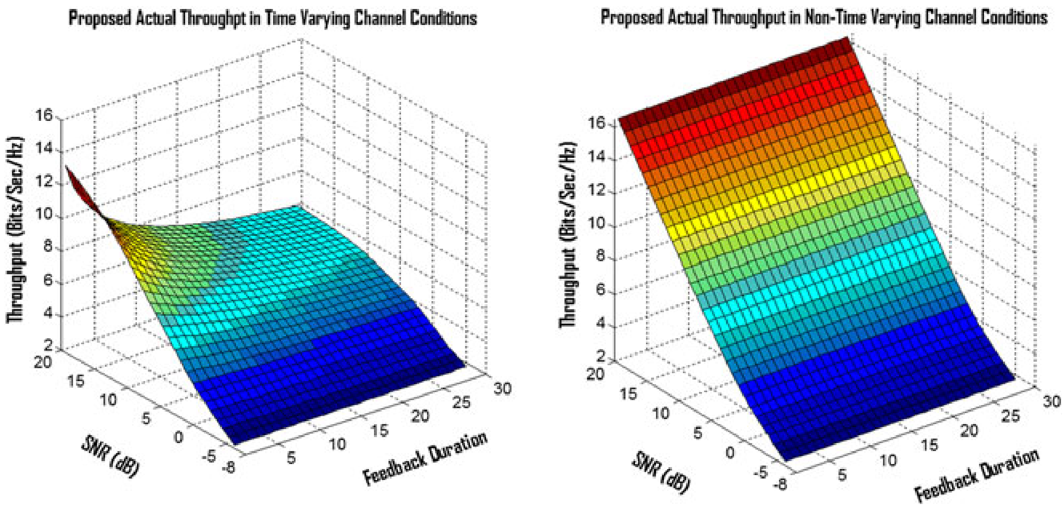


FIGURE 7 Cell edge sum rate performance for increasing B_T when $\nu_{1,1,1}=4km/h$; $\nu_{1,1,2}=6km/h$; $\nu_{1,1,3}=5km/h$; $P_{1,1,1}=P_{2,1,1}$; $P_{1,1,2}=P_{1,1,3}=P_{1,1,1}-3dB$; $\tau_{1,1,1}=\tau_{1,1,2}=\tau_{1,1,3}=2$; and $T_s=5ms$. CSI, channel state information; ICI, intercell interference; IFBC, interfering broadcast channel; IUI, interuser interference; SNR, signal-to-noise ratio; TVC, time-varying channel



(A) (B)

FIGURE 8 Cell edge sum rate performance with simultaneous variations of signal-to-noise ratio and feedback duration. A, $\nu_{1,1,1}=4km/h$; $\nu_{1,1,2}=6km/h$; $\nu_{1,1,3}=5km/h$; $P_{1,1,1}=P_{2,1,1}$; $P_{1,1,2}=P_{1,1,3}=P_{1,1,1}-3dB$; $T_s=5ms$. B, $P_{1,1,1}=P_{2,1,1}$; $P_{1,1,2}=P_{1,1,3}=P_{1,1,1}-3dB$; and $\eta_{1,1,1}=\eta_{1,1,2}=\eta_{1,1,3}=1$

TVC, frequent update of channel information is required to achieve the guaranteed average sum rate.

By evaluating the cell average sum-rate with increasing number of feedback bits and feedback update duration, it is proved from the above results that the practical feedback requirement requires extra allocation to meet the guaranteed throughput in MIMO systems with IUI and ICI in TVC conditions.

5 | CONCLUSION

In this paper, practical achievable performance of coordinated zero-forcing multicell network is characterized in TVC. Adaptive feedback allocation scheme for IUI and ICI channels is proposed to reduce the throughput degradation. The proposed scheme estimates the actual cell average sum-rate by optimally allocating per user feedback and maintaining total feedback budget constant. It is also demonstrated from numerical simulation and derived expressions; how many extra bits required per user in TVC conditions to compensate the loss emanating from channel temporal correlations and interferences. Finally, we have proved that to maintain a specific rate offset with a predetermined throughput, the total feedback bit per user is weighed linearly with SNR. The feedback scaling is proportional to channel temporal correlation, number of antennas M, and number of cells K besides IUI and ICI powers. The results of the work can also be extended to study performance of multicell networks by jointly optimizing the feedback bits and feedback update period.

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How to cite this article: Balaji S, Mallick P S. Performance analysis of adaptive limited feedback coordinated zero-forcing multicell systems in time-varying channels: A practical approach. *Int J Commun Syst*. 2017;e3301. https://doi.org/10.1002/dac.3301

APPENDIX A

The rate of the user in a perfect feedback system is given by[3]

$$R_{l,i}[m] = \log_2\left(1 + E^t_{l,i,i}(1+d_i)^{-\alpha}|\boldsymbol{h}^H_{l,i,i}[m]\boldsymbol{w}^P_{l,i}[m]|^2\right), \quad (12)$$

where $\boldsymbol{w}^P_{l,i}[m]$ is the perfect CSI beamforming vector at BS_i. Since this CSI is perfect, it is assumed that the beamforming vector completely cancels both ICI and IUI perfectly. After defining the rate in perfect CSI system and coordinated zero-forcing system, the rate loss is the difference between Equation 7 and Equation 12. The expected rate loss is

$$\Delta R_{l,i}[m] = \mathbb{E}\left\{log_2\left(1+E^t_{l,i,i}(1+d_i)^{-\alpha}\left|\boldsymbol{h}^H_{l,i,i}[m]\boldsymbol{w}^P_{l,i}[m]\right|^2\right)\right\} -\mathbb{E}\left\{log_2\left(1+\frac{E^t_{l,i,i}(1+d_i)^{-\alpha}\|\boldsymbol{h}_{l,i,i}[m]\|^2\left|\widetilde{\boldsymbol{h}}^H_{l,i,i}[m]\boldsymbol{w}_{l,i}[m]\right|^2}{1+\widetilde{I}_{IUI}+\widetilde{I}_{ICI}}\right)\right\}. \quad (13)$$

After logarithmic manipulation, the above Equation 13 becomes

$$\Delta R_{l,i}[m] = \mathbb{E}\left\{log_2\left(1+E^t_{l,i,i}(1+d_i)^{-\alpha}\left|\boldsymbol{h}^H_{l,i,i}[m]\boldsymbol{w}^P_{l,i}[m]\right|^2\right)\right\} + \mathbb{E}\left\{log_2\left\{\widetilde{I}_{IUI}+\widetilde{I}_{ICI}+1\right\}\right\} -\mathbb{E}\left\{log_2\left(1+\widetilde{I}_{IUI}+\widetilde{I}_{ICI}+\left(E^t_{l,i,i}(1+d_i)^{-\alpha}\|\boldsymbol{h}_{l,i,i}[m]\|^2\left|\widetilde{\boldsymbol{h}}^H_{l,i,i}[m]\boldsymbol{w}_{l,i}[m]\right|^2\right)\right)\right\}. \quad (14)$$

The log function is monotonically increasing function, and due to the fact that $\boldsymbol{w}^P_{l,i}[m]$ and $\boldsymbol{w}_{l,i}[m]$ distributions are identical and isotropic to $\boldsymbol{h}^H_{l,i,i}[m]$, the expected rate loss reduces to the residual value of the interference coming from ICI and IUI as

$$\Delta R_{l,i}[m] = \mathbb{E}\left\{\log_2\left(1+E^t_{l,i,i}(1+d_i)^{-\alpha}\left|\boldsymbol{h}^H_{l,i,i}[m]\boldsymbol{w}^P_{l,i}[m]\right|^2\right)\right\} -\mathbb{E}\left\{\log_2\left(1+E^t_{l,i,i}(1+d_i)^{-\alpha}\|\boldsymbol{h}_{l,i,i}[m]\|^2\left|\widetilde{\boldsymbol{h}}^H_{l,i,i}[m]\boldsymbol{w}_{l,i}[m]\right|^2\right)\right\} +\mathbb{E}\left\{log_2\left(\widetilde{I}_{IUI}+\widetilde{I}_{ICI}+1\right)\right\}. \quad (15)$$

Now, the residual value of the interference is

$$\Delta R_{l,i}[m] = \mathbb{E}\left\{log_2\left(\widetilde{I}_{IUI}+\widetilde{I}_{ICI}+1\right)\right\}. \quad (16)$$

Now, applying to Jensen's inequality to the last equation and taking the expectation to inside the logarithm

$$\Delta R_{l,i}[m] = \log_2\left(\mathbb{E}\left(\widetilde{I}_{IUI}\right)+\mathbb{E}\left(\widetilde{I}_{ICI}\right)+1\right). \quad (17)$$

The expected value of IUI and ICI needs to be calculated to estimate $\Delta R_{l,i}[m]$. The expected rate loss $\Delta R_{l,i}[m]$ defined in Equation 17 reaches to its maximum value if IUI and ICI go to maximum. To characterize the rate loss, the IUI and ICI defined in Equation 7 is now rewritten with previous channel update as

$$\mathbb{E}\left(\widetilde{I}_{IUI}\right) = E^t_{l,i,i}(1+d_i)^{-\alpha}\sum_{u=1,u\neq l}^{L}\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,i}\left[\tau_{l,i,i}-1\right]\boldsymbol{w}_{l,i}\left[\tau_{l,i,i}\left\lfloor\frac{\tau_{l,i,i}-1}{\tau_{l,i,i}}\right\rfloor\right]\right|^2\right]$$
$$\mathbb{E}\left(\widetilde{I}_{ICI}\right) = \sum_{j=1,j\neq i}^{K}\left(1+d_j\right)^{-\alpha}E^t_{l,i,j}\sum_{l=1}^{L}\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,j}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}\left[\tau_{l,i,i}\left\lfloor\frac{\tau_{l,i,i}-1}{\tau_{l,i,i}}\right\rfloor\right]\right|^2\right]. \quad (18)$$

Since the TVC condition is the first order Gauss Markov model, the characteristic of $\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,i}[m-1]\boldsymbol{w}_{u,i}[m]\right|^2\right]$ does not depend on time index m, but it is a function of time difference based on the properties of Gauss Markov Model.[19] After incorporating the above property, Equation 18 is reduced to

$$\mathbb{E}\left(\widetilde{I}_{IUI}\right) = E^t_{l,i,i}(1+d_i)^{-\alpha}\sum_{u=1,u\neq l}^{L}\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,i}\left[\tau_{l,i,i}-1\right]\boldsymbol{w}_{l,i}[0]\right|^2\right] \quad (19)$$
$$\mathbb{E}\left(\widetilde{I}_{ICI}\right) = \sum_{j=1,j\neq i}^{K}\left(1+d_j\right)^{-\alpha}E^t_{l,i,j}\sum_{l=1}^{L}\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,j}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^2\right].$$

Substituting the expected value of IUI and ICI in Equation 17

$$\Delta R_{l,i}[m] = \log_2\left\{1+\mathbb{E}\left(\widetilde{I}_{IUI}\right)+\mathbb{E}\left(\widetilde{I}_{ICI}\right)\right\} = \log_2\left(1+E^t_{l,i,i}(1+d_i)^{-\alpha}\sum_{u=1,u\neq l}^{L}\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,i}\left[\tau_{l,i,i}-1\right]\boldsymbol{w}_{l,i}[0]\right|^2\right] +\sum_{j=1,j\neq i}^{K}\left(1+d_j\right)^{-\alpha}E^t_{l,i,j}\sum_{l=1}^{L}\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,j}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^2\right]\right). \quad (20)$$

The expected value of ICI for a general case where $\{\tau_{l,i,i},\tau_{l,i,j}\}\geq 2$ from Kim et al[19] as

$$\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,j}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^2\right] = \eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)} \mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,j}[0]\boldsymbol{w}_{l,j}[0]\right|^2\right] + 1-\eta^2_{l,i,j}\sum_{i=0}^{\tau_{l,i,j}-2}\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}, \quad (21)$$

$$\mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,j}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^2\right] = \eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)} \mathbb{E}\left[\left|\boldsymbol{h}^H_{l,i,j}[0]\boldsymbol{w}_{l,j}[0]\right|^2\right] + 1-\eta^2_{l,i,j}\left(\frac{1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}}{1-\eta^2_{l,i,j}}\right), \quad (22)$$

$$\mathbb{E}\left[\left|\boldsymbol{h}_{l,i,j}^{H}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^{2}\right]=\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\mathbb{E}\left[\left|\|\boldsymbol{h}_{l,i,j}[0]\|\right|^{2}\right]$$

$$\mathbb{E}\left[\left|\widetilde{\boldsymbol{h}}_{l,i,j}^{H}[0]\boldsymbol{w}_{l,j}[0]\right|^{2}\right]+1-\eta_{l,i,j}^{2}\left(\frac{1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}}{1-\eta_{l,i,j}^{2}}\right). \quad (23)$$

By substituting the value of $\widetilde{\boldsymbol{h}}_{l,i,j}^{H}[0]$ from the orthogonal basis function defined earlier, the expected value becomes

$$\mathbb{E}\left[\left|\boldsymbol{h}_{l,i,j}^{H}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^{2}\right]=\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\mathbb{E}\left[\left\|\boldsymbol{h}_{l,i,j}[0]\right\|^{2}\right]$$

$$\mathbb{E}\left[\left|\left(\widehat{\boldsymbol{h}}_{l,i,j}[0]\left(\cos\theta_{l,i,j}\right)+\ \boldsymbol{q}_{l,i,j}[0]\left(\sin\theta_{l,i,j}\right)\right)^{H}\boldsymbol{w}_{l,j}[0]\right|^{2}\right]$$

$$+1-\eta_{l,i,j}^{2}\left(\frac{1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}}{1-\eta_{l,i,j}^{2}}\right), \quad (24)$$

$$\mathbb{E}\left[\left|\boldsymbol{h}_{l,i,j}^{H}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^{2}\right]=\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\mathbb{E}\left[\left\|\boldsymbol{h}_{l,i,j}[0]\right\|^{2}\right]$$

$$\mathbb{E}\left[\left|\left(\boldsymbol{q}_{l,i,j}[0]\right)^{H}\boldsymbol{w}_{l,j}[0]\right|^{2}\right]\times\left(\mathbb{E}\left[\left(sin^{2}\theta_{l,i,j}\right)\right]\right)$$

$$+1-\eta_{l,i,j}^{2}\left(\frac{1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}}{1-\eta_{l,i,j}^{2}}\right). \quad (25)$$

To obtain the closed form, we use the fact that $\mathbb{E}\left[\left\|\boldsymbol{h}_{l,i,j}[0]\right\|^{2}\right]=M$ and $|(\boldsymbol{q}_{l,i,j}[0])^{H}\boldsymbol{w}_{l,j}[0]|^{2}$ is a beta distribution with parameters $\beta(1,M-2)$,because both $(\boldsymbol{q}_{l,i,j}[0])^{H}$ and $\boldsymbol{w}_{l,j}[0]$ are independent and isotropically distributed in $M-1$ dimensional space. The value of $\mathbb{E}\left[\left(\sin^{2}\theta_{l,i,j}\right)\right]$ is upper bounded as $\mathbb{E}\left(\sin^{2}\theta_{l,i,j}\right)<2^{B_{l,i,j}}\beta\left(2^{B_{l,i,j}},\frac{M}{M-1}\right)<2^{-\frac{B_{l,i,j}}{M-1}}$. Hence, the above Equation 25 reduces to

$$\mathbb{E}\left[\left|\boldsymbol{h}_{l,i,j}^{H}\left[\tau_{l,i,j}-1\right]\boldsymbol{w}_{l,j}[0]\right|^{2}\right]=\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\left(\frac{M}{M-1}\right)2^{B_{l,i,j}}$$

$$\beta\left(2^{B_{l,i,j}},\frac{M}{M-1}\right)+1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}. \quad (26)$$

The expected value of IUI is also calculated in the same way defined (From Equation 21 to Equation 26) for ICI. After substituting IUI and ICI in the Equation 20, the rate loss reduces to

$$\Delta R_{l,i}[m]=\log_{2}\left\{\begin{array}{l}1+E_{l,i,i}^{t}(1+d_{i})^{-\alpha}\sum_{u=1,u\neq l}^{L}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)2^{B_{l,i,i}}\beta\left(2^{B_{l,i,i}},\frac{M}{M-1}\right)+1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\\+\sum_{j=1,j\neq i}^{K}(1+d_{j})^{-\alpha}E_{l,i,j}^{t}\sum_{l=1}^{L}\left(\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\left(\frac{M}{M-1}\right)2^{B_{l,i,j}}\beta\left(2^{B_{l,i,j}},\frac{M}{M-1}\right)+1-\eta_{l,i,j}^{2\left(\tau_{l,i,j}-1\right)}\right)\end{array}\right\}. \quad (27)$$

Since there are L users, after simple algebraic manipulations, desired result is obtained.

APPENDIX B

First, we relax the integer constraint on $B_{l,i,i}$, and using the upper bounds on quantization error, the bit allocation in Equation 9 is now reformulated generally as

$$\begin{array}{ll}\min\limits_{B_{l,i,i}} & \sum_{i=1}^{K}P2^{-\frac{B_i}{M-1}}\\ subject\ \text{to} & \sum_{i=1}^{K}B_i\leq B_T.\end{array} \quad (28)$$

In the above Equation 28, for notational simplicity, we dropped the index (l,i,i) on B and assign simply i, ie, $B_{l,i,i}$ is B_i. The objective function in the bit allocation problem is now convex, and by applying convex optimization from the previous studies,[12,25] the Lagrangian of the problem is

$$L(B_i,\lambda)=\sum_{i=1}^{K}\left\{\left(2^{-\frac{B_i}{M-1}}\right)A_i+\left((L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\right)\right\}+\lambda\left(\sum_{i=1}^{K}B_i-B_T\right) \quad (29)$$

The derivatives with respect to B_i and λ of the above Lagrangian problem are

$$\frac{\partial L(B_i,\lambda)}{\partial B_i}=-\frac{A_i\ln(2)}{(M-1)}2^{\frac{-B_i}{(M-1)}}+\lambda, \quad (30)$$

$$\frac{\partial L(B_i,\lambda)}{\partial\lambda}=\sum_{i=1}^{K}B_i-B_T. \quad (31)$$

The solution to $\frac{\partial L(B_i,\lambda)}{\partial B_i}=0$ is given by

$$B_i=(M-1)\log_2\left(\frac{A_i\ln(2)}{(M-1)\lambda}\right). \quad (32)$$

Substituting the above Equation 32 in the Equation 31 produces

$$\sum_{i=1}^{K}B_i-B_T=\sum_{i=1}^{K}(M-1)\log_2\left(\frac{A_i\ln(2)}{(M-1)\lambda}\right)-B_T, \quad (33)$$

$$\sum_{i=1}^{K}B_i-B_T=(M-1)\log_2\left(\prod_{i=1}^{K}\left(\frac{A_i\ln(2)}{(M-1)\lambda}\right)\right)-B_T. \quad (34)$$

The solution for λ satisfying the above constraint $\frac{\partial L(B_i,\lambda)}{\partial\lambda}=0$ is given by

$$\lambda=\left\{2^{\frac{-B_T}{(M-1)}}\prod_{i=1}^{K}\left(\frac{A_i\ln(2)}{(M-1)}\right)\right\}^{\frac{1}{K}}. \quad (35)$$

Hence, by substituting λ in the Equation 32 and finding B_i results in

$$B_i = (M-1)\log_2\left(\frac{A_i \ln(2)}{\left(\prod_{i=1}^{K} A_i \ln(2)\right)^{\frac{1}{K}}}\right) + \frac{B_T}{K}. \tag{36}$$

Substitute A_i from Equation 9, then B_i becomes

$$B_i = (M-1)\log_2\left(\frac{\left\{(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\}\ln(2)}{\left(\left\{(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\}\prod_{j=1,j\neq i}^{K} A_j \ln(2)\right)^{\frac{1}{K}}}\right) + \frac{B_T}{K}. \tag{37}$$

The optimal solution of B_i is the maximum value of B_i , ie, $B_i \geq 0$ and B_T.

APPENDIX C

To scale the feedback, we use the previously derived IUI feedback bits (Theorem 2), which minimize the residual interference. Using upper bound of quantization error, ie, $\mathbb{E}\left(\sin^2\theta_{l,i,j}\right) < 2^{-\frac{B_{l,i,j}}{M-1}}$, the upper bound of the rate loss from Equation 8 is rewritten as

$$\Delta R_{l,i}[m] \leq \log_2\left(1 + \sum_{i=1}^{K}\left\{\left(2^{-\frac{B_i}{M-1}}\right)A_i + \left((L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\right)1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right\}\right). \tag{38}$$

If the user wants to maintain the rate loss under $\log_2(\sigma)$, the upper bounds of rate loss is now written as

$$\log_2(\sigma) \leq \log_2\left(1 + \sum_{i=1}^{K}\left\{\left(2^{-\frac{B_i}{M-1}}\right)A_i + \left((L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\right)\right\}\right). \tag{39}$$

By substituting the feedback bit allocation from Equation 10 into the above Equation 39, the rate loss is

$$\sigma - 1 - \sum_{i=1}^{K}\left((L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\right) \geq 2^{\frac{-B_T}{K(M-1)}}\sum_{i=1}^{K}\left\{\left(\frac{\left\{(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\}\ln(2)}{\left(\left\{(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\}\prod_{j=1,j\neq i}^{K} A_j \ln(2)\right)^{\frac{1}{K}}}\right)^{-1} A_i\right\}. \tag{40}$$

Substituting the value of A_i and after algebraic manipulations, the upper bound of rate loss becomes

$$\sigma - 1 - \sum_{i=1}^{K}\left((L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\right) \geq 2^{\frac{-B_T}{K(M-1)}} \sum_{i=1}^{K}\left\{\left(\left[(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha} \times \left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right]\prod_{j=1,j\neq i}^{K} A_j\right)^{\frac{1}{K}}\right\}. \tag{41}$$

Since the term on the right side of the above Equation 41 simply summed up by K times, the above Equation 41 can now be rewritten as

$$\sigma - 1 - \left(K(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(1-\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\right)\right) \geq 2^{\frac{-B_T}{K(M-1)}} K\left(\left\{(L-1)E_{l,i,i}^{t}(1+d_i)^{-\alpha}\left(\eta_{l,i,i}^{2\left(\tau_{l,i,i}-1\right)}\left(\frac{M}{M-1}\right)\right)\right\} \times \prod_{j=1,j\neq i}^{K} A_j\right)^{\frac{1}{K}}. \tag{42}$$

On simplification, the total number of bits to be scaled is reduced to Equation 11 and the proof is complete.