Spatial modulation and physical layer network coding based bidirectional relay network with transmit antenna selection over Nakagami-\(m\) fading channels

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Summary
In this paper, a high data rate bidirectional relay network is proposed by combining the merits of spatial modulation (SM) and physical layer network coding. All nodes in the network are equipped with multiple antennas. Spatial modulation technique is used to reduce hardware complexity and interchannel interference by activating only one antenna at any time during transmission. In the proposed bidirectional relay network, transmit antennas are selected at the source nodes and relay node on the basis of the order statistics of channel power. It increases received signal power and provides a significant improvement in the outage performance. Also, the data rate of the proposed network is improved by physical layer network coding at the relay node. A closed form analytical expression for the outage probability of the network over Nakagami-\(m\) fading channel is derived and validated by Monte Carlo simulations. In addition, asymptotic analysis is investigated at high signal-to-noise ratio region. The outage performance of the proposed network is compared with SM and physical layer network coding bidirectional relay network without transmit antenna selection and point-to-point SM. With approximate \(\text{SNR} \approx 1\) dB difference between the two networks, the same data rate is achieved.

KEYWORDS
decode and forward, Nakagami-\(m\) fading channel, physical layer network coding, spatial modulation, transmit antenna selection

1 | INTRODUCTION
The demand for high data rate and better quality of service is of the utmost importance in next-generation wireless communication systems. The recently proposed spatial modulation (SM) has attracted a great deal of attention in the past few years in 5G wireless networks.\(^1\) In SM, the information bits are partitioned into two streams of which one is mapped to the symbol and the other is used to activate a particular antenna out of all transmit antennas.\(^2\) Therefore, information is carried not only by the symbol but also by the antenna index. When the information bits are transmitted solely through the transmit antenna index, SM is reduced to space shift keying (SSK).\(^3\) Spatial modulation eliminates the synchronization between the transmit antennas and interchannel interference because only one antenna is active during transmission. Hence, only one radio frequency (RF) chain is required in SM because of single antenna...
transmission at any transmission time slot. Besides these advantages, SM\textsuperscript{4} may be flexibly configured for diverse transmit and receive antenna constellations. In multiple-input multiple-output (MIMO) systems, the error performance is directly related to interantenna interference, synchronization, and a significant amount of power consumed due to the number of transmitting RF chains. Studies on SM\textsuperscript{5,6} have shown that it is capable of outperforming MIMO transmission techniques with practical implementation constraints. Spatial modulation in previous literatures\textsuperscript{7-9} has been proved as an efficient low complexity and energy efficient transmission technique to improve the spectral efficiency and bandwidth.

An optimal maximum likelihood (ML) detection algorithm\textsuperscript{10} was proposed for SM receiver in which the detector carries out an exhaustive search for the global optimum in the entire signal space. To increase the robustness of SM-MIMO system under the limited feedback, Link Adaptation (LA) techniques have been proposed in Jeganathan et al\textsuperscript{11} and Wang et al.\textsuperscript{12} The SM-MIMO link adaptation schemes including transmit antenna selection (TAS), the optimal criterion is to design the link by maximizing the minimum Euclidean distance (ED) among the legitimate transmit vectors. The ED-based algorithms result in larger complexity because of exhaustive search. Therefore, to reduce the complexity of ED-based algorithms, TAS criteria were developed such as in previous studies.\textsuperscript{13-16} In Xiao et al\textsuperscript{17} and Wei et al,\textsuperscript{18} a low complexity TAS for the SM was proposed to provide a more sophisticated trade-off between the computational complexity and system performance. In Sun et al,\textsuperscript{19} a novel Euclidean distance optimized antenna selection equivalent criterion, relaying on matrix dimension reduction, is proposed to reduce complexity reduction and large-scale MIMO applications. In Kumbhani and Kshetrimayum,\textsuperscript{20} analytical frameworks evaluating the outage probability for SM-MIMO were developed on the basis of the TAS with capacity optimized antenna selection. In Chen et al,\textsuperscript{21} exact outage probability and error performances based on TAS and maximal ratio combining are analyzed for a MIMO scheme over Nakagami-$m$ fading channels with an arbitrary value of $m$. In Di Renzo and Haas,\textsuperscript{22,23} the error performance of SM over fading channels has been analyzed for various channel models and MIMO system models using moment generating functions. In Di Renzo and Haas,\textsuperscript{24} the moment generating function framework is extended to analyze bit error rate (BER) performance of SM-MIMO systems over generalized fading channels. Recently, in Peppas et al,\textsuperscript{25} a new unified analytical framework was proposed to analyze BER performance of multiple-input single-output and MIMO SSK over generalized fading channels. In Peppas et al,\textsuperscript{26} the analytical framework for the computation of the diversity and coding gains of SM over generalized channels was proposed.

Physical layer network coding (PLNC) is a new paradigm for wireless relay networks introduced in Li et al\textsuperscript{27} and Zhang et al\textsuperscript{28} to improve the data rate and outage performance or spectral efficiency. In PLNC, the relay node receives the signal from the source nodes and uses bitwise exclusive-OR (XOR) to combine the signals and then broadcasts the encoded PLNC mapped signal to the source nodes. It provides a significant improvement in the achievable data rates compared with the traditional relay network. In previous studies,\textsuperscript{29-31} information theoretic studies on achievable rates of PLNC-based relay network have been discussed. The deployment of SM in a point-to-point wireless network is now extended to relay-aided and cooperative communication systems with different kinds of diversity protocols. Cooperative communication using relays is an effective means to combat fading and shadowing and to improve coverage, capacity, and transmit power.

In Xie et al,\textsuperscript{32} SM for a two-way network coded channel has been studied and BER performance analysis is carried out when the end nodes use SSK. Som and Chockalingam\textsuperscript{33} had analyzed the BER performance of cooperative relay network with SSK and decode and forward (DF) protocol, considering multiple relays with threshold-based best relay selection, and selection combining of direct and relayed path at the destination. In Namboodiri et al,\textsuperscript{34} the PLNC bidirectional relay network design modulation schemes have been studied. In Unnikrishnan and Rajan,\textsuperscript{35} the BER performance of the spatial modulated PLNC for bidirectional relay networks is improved by using coordinate interleaved orthogonal design. In Laneman et al,\textsuperscript{36} the concept of SM in cooperative communication with different kinds of diversity protocols forms the virtual MIMO to combat fading and to increase the coverage. In previous studies,\textsuperscript{37-41} SM technique is applied to various relay systems for achieving high throughput based on relay selection information. In Mesleh and Ikki,\textsuperscript{42} SM network with multiple DF relay nodes is proposed in which the relays detect the source signal and forward the decoded signal to the destination in predetermined orthogonal channels. However, the performance of SM is severely affected by the achieved diversity gain because of the small number of receive antennas.

In this paper, the merits of SM and PLNC are combined in bidirectional half-duplex (HD) relay network with TAS and are proposed by choosing a set of transmit antennas based on the order statistics of the channel power. From the chosen set, transmit antenna is selected to transmit information of the modulated symbols. The major contributions of this paper are as follows:
• The advantage of SM and PLNC has been combined in the newly proposed bidirectional network with DF relay to improve spectral efficiency. In addition, order statistics for TAS is used at both the source nodes and relay node of the bidirectional relay network.

• A closed form analytical expression for outage probability of bidirectional relay network with SM and PLNC is derived in Nakagami-\(m\) fading environment. The Nakagami-\(m\) fading model represents a wide variety of realistic line of sight and non-line of sight fading channels encountered in practice. Also, the asymptotic analysis is evaluated at high signal-to-noise ratio (SNR) region. Hence, the derived analytical expression of outage probability is used to investigate the outage characteristics under different fading severity conditions of the proposed bidirectional relay network.

• The outage performance of the proposed network is compared with the conventional SM and PLNC bidirectional relay network without TAS and point-to-point SM network.

The rest of the paper is organized as follows: Section 2 describes the system model for the proposed SM and PLNC-based bidirectional wireless relay network. In Section 3, analytical expression for the outage performance of the proposed network is derived. Numerical and simulation results on the outage performance of the proposed network are discussed in Section 4. Section 5 concludes the paper.

2 | SYSTEM MODEL

Consider SM and PLNC–based bidirectional HD relay network shown in Figure 1. Each source node is equipped \(N_i\) antennas and the relay node \(R\) with \(N_r\) antennas. In the proposed network, on the basis of the channel impulse response between the source nodes and relay node, only \(U \leq N_i\) antennas are chosen. Both the source nodes exchange \((G)\) bits of information in two time slots with the help of DF relay node. The \((G)\) bits of information are split into two groups with length \(b_1 = \log_2 U \) and \(b_2 = G - b_1\). The first group of \(b_1\) bits are used for TAS from the best \(U = 2^{b_1}\) antennas among \(N_i\) transmit antennas, and the second group of \(b_2\) bits are used for actual data transmission by the selected transmit antenna. Now, the total number of bits \((G)\) that can be transmitted by the source nodes in each time slot is given by \(G = b_1 + b_2\).

In the first time slot, on the basis of channel impulse response between \(j\)th source nodes \(S_j, j = 1, 2\), and relay node \(R\), source nodes \(S_j, j = 1, 2\), select \(U\) antennas from \(N_i\) transmit antennas to maximize the received SNR at the relay node \(R\). Let \(h_{i,k}^{S,R}\), \(i = 1, 2, \ldots, N_r\), and \(k = 1, 2, \ldots, N_i\), be the channel impulse response coefficients between \(j\)th source nodes \(S_j, j = 1, 2\), and relay node \(R\). The channel coefficients are modeled as independent and identically distributed Nakagami-\(m\) random variables with shape parameter \(m\) and scale parameter \(\Omega\). The scale and shape parameters are defined as \(\Omega = \mathbb{E}\left[h_{i,k}^{S,R}\right]^2\) and \(m = \mathbb{E}\left[(X^2 - \Omega^2)^2\right]\), respectively. The probability density function (PDF) of Nakagami-\(m\) distribution\(^{43}\) is given by

\[
f_{h_{i,k}^{S,R}}(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right) m^{m - 1} e^{-x^2 m \left(\frac{m}{\Omega}\right)^2}, \quad j = 1, 2, \tag{1}
\]

where \(\Gamma(.)\) is the Gamma function. Let \(z^{S_1}\) and \(z^{S_2}\) be the \(N_r \times 1\) vectors that contain the sum of the squared magnitude of the channel coefficients between the links \(S_1\) and \(R\), and \(S_2\) and \(R\), respectively. The elements \(z^{S_1}\) and \(z^{S_2}\) are defined as

![FIGURE 1 System model for spatial modulation and physical layer network coding–based bidirectional half-duplex relay network with transmit antenna selection](image-url)

The antennas correspond to the index of the chosen antennas at different sources. The PDF of \( z_{k}^{j} \), \( j = 1, 2 \), is given by

\[
f_{z_{k}^{j}}(x) = \frac{1}{2^{mN_{r}}(mN_{r})!(mN_{r})^{-1}} e^{-mN_{r}} x^{mN_{r}-1}, \quad x \geq 0, \quad j = 1, 2, \quad k = 1, 2, \ldots, N_{r}. \tag{4}\]

The cumulative distribution function (CDF) of \( z_{k}^{j} \), \( j = 1, 2 \), is given by

\[
F_{z_{k}^{j}}(x) = 1 - e^{-mN_{r}} \sum_{c=0}^{N_{r}} \frac{(mN_{r})^{c}}{c!}, \quad x \geq 0, \quad j = 1, 2, \quad k = 1, 2, \ldots, N_{r}. \tag{5}\]

The antennas correspond to the index of the \( U \) largest values in vectors \( z^{1} \) and \( z^{2} \) are chosen for transmission at source nodes \( S_1 \) and \( S_2 \), respectively. By arranging the elements of \( z_{k}^{j} \), \( j = 1, 2 \), between the \( N_{r} \) largest values in vectors \( z^{1} \) and \( z^{2} \), \( j = 1, 2 \), are chosen for transmission at source nodes \( S_1 \) and \( S_2 \), respectively. By arranging the elements of \( z_{k}^{j} \), \( j = 1, 2 \), between the \( N_{r} \) largest values in vectors \( z^{1} \) and \( z^{2} \), according to the order statistics, the PDF of \( U \times 1 \) vector \( z_{(c)}^{j} \) is written as

\[
f_{z_{(c)}^{j}}(x) = \frac{1}{B(c, N_{r}-c+1)} \left\{ F_{z_{k}^{j}}(x) \right\}^{c-1} \left\{ 1 - F_{z_{k}^{j}}(x) \right\}^{N_{r}-c} f_{z_{k}^{j}}(x), \tag{6}\]

where \( c = N_{r} - U + 1 \) and \( B(., .) \) is the beta function. In the first time slot, on the basis of the first \( b_{1} \) bits of data with the length of \( G \) bits, antenna \( q_{j} \) is selected from the \( U \) chosen antennas at \( j \)th source node. The symbol \( x_{j} \) with the length of \( b_{2} \) bits is transmitted by the selected antenna \( q_{j} \). This phase is known as multiple access channel phase. The \( N_{r} \times 1 \) receive signal vector at the relay node \( R \) is given by

\[
y_{R} = h_{S_{1}, R}^{q_{1}} x_{1} + h_{S_{2}, R}^{q_{2}} x_{2} + w_{R}, \tag{7}\]

where \( h_{S_{j}, R}^{q_{j}}, \ j = 1, 2 \), is the column corresponding to the antenna index of the selected antennas in the \( N_{r} \times U \) channel matrix \( H_{S_{j}, R}^{U} \), \( j = 1, 2 \), between the \( j \)th source node and relay node \( R \). \( w_{R} \) is modeled as circularly symmetric complex Gaussian noise vector at the relay node \( R \) with zero mean and variance \( N_{0}I_{N_{r}} \). The PDF of the received SNR obtained from the signals of the selected transmit antennas is given by

\[
f_{y_{S_{j}, R}}(x) = \frac{1}{N_{r}^{-c} + 1} \sum_{i=1}^{N_{r}} \frac{1}{B(i, N_{r}-i+1)} \left\{ F_{z_{k}^{j}}(x) \right\}^{i-1} \left\{ 1 - F_{z_{k}^{j}}(x) \right\}^{N_{r}-i} f_{z_{k}^{j}}(x), \quad j = 1, 2. \tag{8}\]

As the concept of PLNC is used at relay node, the average received SNR at relay node is the minimum of \( y_{S_{j}, R} = \frac{\| h_{S_{j}, R} \|^{2}}{N_{0}I_{N_{r}}} \) and \( y_{S_{j}, R} = \frac{\| h_{S_{j}, R} \|^{2}}{N_{0}I_{N_{r}}} \) for the derivation of lower bound outage probability. By substituting Equations 5 and 6 in Equation 8, the PDFs of the received SNR at the source nodes \( S_{j}, \ j = 1, 2 \), can be written as

\[
f_{y_{S_{j}, R}}(x) = \frac{1}{(N_{r}^{-c} + 1)(mN_{r})} \sum_{i=1}^{N_{r}} \frac{1}{B(i, N_{r}-i+1)} \sum_{b=0}^{i-1} \binom{i-1}{b} (-1)^{b} \times \sum_{l=0}^{M_{l}} C_{l}(b, mN_{r}) x^{mN_{r}+l-1} e^{-mx(N_{r}-i+b+1)}, \quad j = 1, 2. \tag{9}\]
where $M_r = (mN_r - 1)(N_r - i + b)$ and $C_i(b, mN_i)$ is the coefficient of $x^i$ in the expansion of $\left(\sum_{t=0}^{mN_r - 1} \frac{1}{t!} \right)_{N_r - i + b}$. Given the channel state information at the DF relay node $R$, the antenna indices and symbols $q_j$, $x_j$, $j = 1, 2$, are detected using the ML method as given by

$$ (\hat{q}_1, \hat{q}_2, \hat{x}_1, \hat{x}_2) = \arg \min_{q_j, x_j} \left| y_{R} - \sum_{j=1}^{2} h_{R}^x x_j \right|^2. \quad (10) $$

In Equation 10, $\hat{q}_1$ and $\hat{q}_2$ are the detected symbols corresponding to the antenna indices of source nodes $S_1$ and $S_2$, respectively. Similarly, $\hat{x}_1$ and $\hat{x}_2$ are the detected symbols corresponding to the data transmitted from the selected transmit antenna indices of source nodes $S_1$ and $S_2$, respectively. Let the bitwise XOR operation $q_j = \hat{q}_1 \oplus \hat{q}_2$ be based on the concept of PLNC to be used to select the antenna indices at relay node $R$ and the corresponding data bits $x_r = \hat{x}_1 \oplus \hat{x}_2$. In the second time slot, the number of bits transmitted by relay node $R$ is given by $G = \log_2 U + b_2$. Let $z^R$ be $N_r \times 1$ vectors that contain the sum of the squared magnitude of the channel coefficients between the links $R$ and $S_j$, $j = 1, 2$. The elements of the vector $z^R$ are defined as

$$ z^R_k = \sum_{i=1}^{N_t} \left| h_{RS}^k \right|^2, \quad k = 1, 2, \ldots, N_r, j = 1, 2 \quad (11) $$

In the second time slot, antennas that correspond to the index of the $U$ largest values in vectors $z^R$ are chosen for transmission at the relay node $R$. The $N_r \times 1$ receive signal vectors at the source nodes $S_j$, $j = 1, 2$, are given by

$$ y_{S} = h_{RS}^S x^r + w_{S}, \quad j = 1, 2, \quad (12) $$

where $h_{RS}^S$ is the column corresponding to the antenna index of the selected antenna in the $N_r \times U$ channel matrix $H_{RS}^S$, $j = 1, 2$, between the relay node $R$ and $j$th source node. $w_{S}$, $j = 1, 2$, is modeled as a circularly symmetric complex Gaussian noise vector at the source nodes $S_j$, $j = 1, 2$, with zero mean and variance $N_0 I_{N_r}$. Given channel state information at the source nodes, the parameters $q_r x_r$ are detected using ML method given by

$$ (\hat{q}_r, \hat{x}_r) = \arg \min_{q_r, x_r} \left| y_{S} - h_{RS}^S x^r \right|^2, \quad j = 1, 2. \quad (13) $$

On the basis of the concept of PLNC, the $b_1$ bits corresponding to $\hat{q}_r$ and $q_j$ and $b_2$ bits of the detected data symbols $\hat{x}_r$ and $x_r$ are bitwise XORed in the source node to detect the $(G)$ bits of information from the other source node. According to order statistics, the PDF of the instantaneous received SNR at relay node obtained from the signals of selected transmit antennas is given by

$$ f_{y_{R},S}^\prime(x) = \frac{1}{(N_r - r + 1) \Gamma(mN_t)} \sum_{i=1}^{N_t} \frac{1}{B(i, N_r - i + 1)} \sum_{b=0}^{i-1} \frac{(i-1)}{b} \left( -1 \right)^b \times \prod_{t=0}^{M_t} C_i(b, mN_t) x^{mN_t-i-1} e^{-mx(N_r-i+b+1)}, \quad j = 1, 2, \quad (14) $$

where $M_t = (mN_t-1)(N_r - i + b)$, $r = N_r - U + 1$, and $C_i(b, mN_t)$ is the coefficient of $x^i$ in the expansion of

$$ \left( \sum_{t=0}^{mN_r - 1} \frac{1}{t!} \right)_{N_r - i + b}, \quad y_{R,S} = \frac{\left| h_{RS}^S \right|^2}{N_0 I_{N_r}} \text{ is the average receive SNR at the source nodes } S_j, j = 1, 2. $$

3 | OUTAGE PERFORMANCE

In this section, analytical expression for outage probability is derived for the proposed SM and PLNC based bidirectional HD relay network over Nakagami-$m$ fading channel.
### 3.1 | Outage probability at the relay node in the first time slot

The relay node $R$ receives information from both source nodes $S_1$ and $S_2$ in the first time slot; the outage probability at the relay node $R$ for a data rate of $R_d$ bits/s/Hz is written as

$$P_{\text{out}}^R(R_d) = \Pr \left[ \log_2 \left( 1 + \min \left( \gamma_{S_1,R}^j, \gamma_{S_2,R}^j \right) \right) < R_d \right].$$  \hspace{1cm} (15)

Let $\chi_R = \min \left( \gamma_{S_1,R}^j, \gamma_{S_2,R}^j \right)$ and $\gamma_{th} = \frac{2R_d - 1}{\gamma_{S,R}}$, and then the outage probability can be rewritten as

$$P_{\text{out}}^R(\gamma_{th}) = \Pr [\chi_R \leq \gamma_{th}].$$ \hspace{1cm} (16)

The outage probability at relay node $R$ is written in terms of CDF denoted by $F_{\chi_R}(\gamma_{th})$ given by

$$F_{\chi_R}(\gamma_{th}) = 1 - \left[ \left( 1 - F_{\gamma_{S_1,R}^j}^R(\gamma_{th}) \right) \left( 1 - F_{\gamma_{S_2,R}^j}^R(\gamma_{th}) \right) \right].$$ \hspace{1cm} (17)

The CDF at the source nodes $S_j, j = \{1, 2\}$, is defined as

$$F_{\gamma_{S_j,R}^j}(\gamma_{th}) = \Pr \left[ \gamma_{S_j,R}^j \leq \gamma_{th} \right], j = 1, 2.$$ \hspace{1cm} (18)

By using Equation 9, the CDF $F_{\gamma_{S_j,R}^j}(\gamma_{th})$ can be rewritten as

$$F_{\gamma_{S_j,R}^j}(\gamma_{th}) = \frac{1}{N_t - c + 1} \sum_{i=c}^{N_t} \frac{1}{B(i, N_t - i + 1)} \times \prod_{b=0}^{i-1} \left( \frac{1}{b} \right) \left( -1 \right)^b \sum_{t=0}^{M_t} C_t(b, mN_t) \times \int_0^{\gamma_{th}} \chi^{mN_t + t - 1} e^{-\chi(N_t - i + b + 1)} dx, j = 1, 2.$$ \hspace{1cm} (19)

Using table of integrals, the CDF $F_{\gamma_{S_j,R}^j}(\gamma_{th})$ expression is derived as

$$F_{\gamma_{S_j,R}^j}(\gamma_{th}) = \frac{1}{(N_t - c + 1) \Gamma(mN_t)} \sum_{i=c}^{N_t} \frac{1}{B(i, N_t - i + 1)} \times \prod_{b=0}^{i-1} \left( \frac{1}{b} \right) \left( -1 \right)^b \sum_{t=0}^{M_t} C_t(b, mN_t) \times \gamma^{mN_t + t, m\gamma_{th}(N_t - i + b + 1)} (N_t - i + b + 1)^{mN_t + t}, j = 1, 2.$$ \hspace{1cm} (20)

Substituting Equation 20 in Equation 17, the outage probability proposed SM and PLNC–based bidirectional HD relay network at the relay node $R$ is written as

$$P_{\text{out}}^R(R_d) = 1 - \prod_{j=1}^{2} \left[ 1 - \frac{1}{\Gamma(mN_t)} \sum_{i=c}^{N_t} \frac{1}{B(i, N_t - i + 1)} \times \prod_{b=0}^{i-1} \left( \frac{1}{b} \right) \left( -1 \right)^b \sum_{t=0}^{M_t} C_t(b, mN_t) \times \gamma^{mN_t + t, m\gamma_{th}(N_t - i + b + 1)} (N_t - i + b + 1)^{mN_t + t} \right].$$ \hspace{1cm} (21)
3.2 | Outage probability at the source nodes in the second time slot

The source nodes $S_j, j = 1, 2$, receive information from relay node $R$ in the second time slot; the outage probability at source nodes for a data rate of $R_d$ bits/s/Hz is defined as

$$P_{\text{out}}^{S_j}(R_d) = \text{Pr}\left[\log_2 \left(1 + \gamma_{R,S_j} \right) < R_d\right], j = 1, 2. \quad (22)$$

The outage probability at source nodes $S_j, j = 1, 2$, can be rewritten as

$$P_{\text{out}}^{S_j}(R_d) = \text{Pr}\left[\frac{1}{R_{S_j}} < \gamma_{th}\right], j = 1, 2. \quad (23)$$

By using Equation 14, the outage probability at the source nodes can be expressed as

$$P_{\text{out}}^{S_j}(R_d) = \frac{1}{N_r-r+1\Gamma(mN_r)} \sum_{i=0}^{N_r} \frac{1}{B(i,N_r-i+1)} \times \sum_{b=0}^{i-1} \left(\frac{i-1}{b}\right) \left(-1\right)^b \sum_{t=0}^{M_r} C_t(b,mN_r) \times \gamma(mN_r+t,m\gamma_{th}(N_r-i+b+1)) \times (N_r-i+b+1)^{mN_r+t}, j = 1, 2. \quad (24)$$

The end-to-end outage probability of the proposed SM and PLNC–based bidirectional HD relay network is obtained by

$$P_{\text{out}}^{\text{SM\&PLNC}} = P_{\text{out}}^{R}(R_d) + \left[1-P_{\text{out}}^{S_j}(R_d)\right] \times \left(P_{\text{out}}^{S_j}(R_d)\right). \quad (25)$$

Substituting Equations 21 and 24 in Equation 25, the end-to-end outage probability of the proposed SM and PLNC–based bidirectional relay network with TAS is expressed as

$$P_{\text{out}}^{\text{SM\&PLNC}} = 1 - \prod_{j=1}^{2} \left[1 - \left(\frac{1}{N_r-c+1\Gamma(mN_r)} \sum_{i=0}^{N_r} \frac{1}{B(i,N_r-i+1)} \times \sum_{b=0}^{i-1} \left(\frac{i-1}{b}\right) \left(-1\right)^b \sum_{t=0}^{M_r} C_t(b,mN_r) \times \gamma(mN_r+t,m\gamma_{th}(N_r-i+b+1)) \times (N_r-i+b+1)^{mN_r+t}\right) \right] \times \frac{1}{N_r-r+1\Gamma(mN_r)} \sum_{i=0}^{N_r} \frac{1}{B(i,N_r-i+1)} \times \sum_{b=0}^{i-1} \left(\frac{i-1}{b}\right) \left(-1\right)^b \sum_{t=0}^{M_r} C_t(b,mN_r) \times \gamma(mN_r+t,m\gamma_{th}(N_r-i+b+1)) \times (N_r-i+b+1)^{mN_r+t}, j = 1, 2. \quad (26)$$

To evaluate the asymptotic outage probability of the proposed bidirectional HD relay network at high SNR, let $i = c, b = 0$, and $t = 0$ for $P_{\text{out}}^{R}(R_d)$ and let $i = r, b = 0$, and $t = 0$ for $P_{\text{out}}^{S_j}(R_d)$ in Equation 25. After simple manipulations, the outage probability of SM and PLNC Equation 26 is similar to outage expression derived in previous studies. The asymptotic outage probability is expressed as
The asymptotic outage probability $P_{\text{out}}^{\text{Asymp}}$ in the closed form is expressed as

$$
P_{\text{out}}^{\text{Asymp}} \approx 1 - \prod_{j=1}^{2} \left[ 1 - \left( \frac{(2^j-1)^{cmN_t}}{(N_t-c+1)B(c,N_t-c+1)c} \left( \frac{1}{\Gamma(N_t+1)} \right)^c \gamma_{\text{th}}^{cmN_t} + \mathcal{O}\left( \frac{1}{\left( \gamma_{\text{th}}^{cmN_t} \right)^{c}} \right) \right) \right] \times \frac{\gamma_{\text{th}}^{cmN_t}}{(N_t-r+1)B(r,N_t-c+1)r} \left( \frac{1}{\Gamma(N_t+1)} \right)^c , j = 1, 2.
$$

### 4 | NUMERICAL RESULTS AND DISCUSSIONS

In this section, the outage performance of the proposed SM and PLNC–based bidirectional relay network over Nakagami-$m$ fading channel is analyzed using the analytical expressions and validated by Monte Carlo simulation. The parameters for the outage analysis are listed in Table 1.

In Figure 2, the proposed network model was simulated for various number of transmit antennas at the source nodes $N_t = 4, 6, 8$, at relay node $N_r = 4$, set of antennas selected for transmission is $U = 2$, shape parameter $m = 1$ (Rayleigh fading environment) with binary phase shift keying modulation at a fixed data rate of $R_d = 3$ bps/Hz. When $N_t = 6$ and $N_r = 4$, the minimum SNR requirement to achieve at $10^{-2}$ outage probability is approximately $4$ dB, whereas in SM and PLNC–based bidirectional relay network without TAS, the SNR required is $10$ dB. A significant improvement in the outage probability is observed for the proposed system over conventional SM and PLNC–based bidirectional relay network without TAS by increasing the number of antennas at the source nodes and hence provides transmit diversity. In the proposed network, the number of transmit antenna at the source nodes is increased by the factor of $2$ without changing the number of active antennas. This in turn implies the number of RF chains remains the same and hence is energy efficient. When compared with a single source to destination communication, the proposed bidirectional relay network achieve same data rate with SNR compensation of approximately $1$ dB.

In Figure 3, the outage performance of the proposed SM and PLNC–based bidirectional relay network with TAS for different values of $m$ in Nakagami-$m$ fading channel is shown. It is observed that outage performance of the proposed network works well as we decrease the fading severity of the channel. Even for high fading environment $m = 0.5$, the SNR requirement to achieve $10^{-1}$ outage is approximately $6$ dB in the proposed network, whereas the network without TAS requires $13$ dB.

**TABLE 1** List of parameters

<table>
<thead>
<tr>
<th>S. No</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Data rate ($R_d$)</td>
<td>1-6 bps/Hz</td>
</tr>
<tr>
<td>2</td>
<td>Number of transmit antennas at source nodes ($N_t$)</td>
<td>4, 6, 8</td>
</tr>
<tr>
<td>3</td>
<td>Number of transmit antennas at relay nodes ($N_r$)</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Number of transmit antennas selection at source nodes ($U$)</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>SNR at source nodes</td>
<td>0-20 dB</td>
</tr>
</tbody>
</table>
In Figure 4, the outage performance of the proposed SM and PLNC–based bidirectional relay network with antenna selection is plotted by varying data rate $R_d$ from 1 to 6 bps/Hz. The number of antennas at the source nodes is assumed $N_t=4,6,8$ , at relay node $N_r=4$, and set of antennas selected is $U=2$ for a fixed SNR of 5 dB. It is observed for $N_t=6$ and $N_r=4$ the data rate supported by the proposed network is 3.1 bps/Hz for an outage probability of $10^{-2}$ whereas SM without TAS can support data rate of 1.7 bps/Hz at the same outage probability. When the number of antennas at the source nodes is increased to $N_t=8$, the data rate supported by the proposed network is increased to 3.5 bps/Hz at the same outage probability. It is observed that the increase in the number of antennas at the source nodes increases the data rate significantly for the proposed system compared with SM without TAS. Figure 5 illustrates the asymptotic outage analysis of the proposed bidirectional relay network. It is observed that the asymptotic expression in Equation 28 approaches to the exact end-to-end outage probability given in Equation 26 at high SNR.

**FIGURE 2** Outage performance of the proposed SM and physical layer network coding–based bidirectional relay network over Nakagami-$m$ fading channel with $R_d = 3$ bps/Hz, $N_t=4,6,8, U=2$, and $N_r=4$. SM, spatial modulation; SNR, signal-to-noise ratio; TAS, transmit antenna selection

**FIGURE 3** Outage performance of the proposed SM and physical layer network coding–based bidirectional relay network with TAS for different values of $m$ in Nakagami-$m$ fading channel $R_d = 4$ bps/Hz, $N_t=4$, $U=2$, and $N_r=4$. SM, spatial modulation; SNR, signal-to-noise ratio; TAS, transmit antenna selection
5 | CONCLUSION

In this paper, the outage performance of SM and PLNC–based bidirectional HD relay network with TAS over Nakagami-\(m\) fading is analyzed. Set of transmit antennas at source nodes and relay node is selected on the basis of the order statistics of the channel power. A closed form analytical expression is derived for the end-to-end outage probability of the proposed bidirectional relay network in Nakagami-\(m\) fading channel environment. In addition, asymptotic analysis is evaluated at the high SNR region. The outage performance of the proposed system model shows a significant improvement over the conventional SM without TAS at the rate of increase in number of antennas at the source nodes. It has been proved that the proposed bidirectional relay network with SM and PLNC provides high data rate when compared with point-to-point network using SM. The analytical results are validated by using Monte Carlo simulation.

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