State dependent arrival in bulk retrial queueing system with immediate Bernoulli feedback, multiple vacations and threshold

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State dependent arrival in bulk retrial queueing system with immediate Bernoulli feedback, multiple vacations and threshold

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Abstract. The objective of this paper is to analyse state dependent arrival in bulk retrial queueing system with immediate Bernoulli feedback, multiple vacations, threshold and constant retrial policy. Primary customers are arriving into the system in bulk with different arrival rates λ_a and λ_b. If arriving customers find the server is busy then the entire batch will join to orbit. Customer from orbit request service one by one with constant retrial rate γ . On the other hand if an arrival of customers finds the server is idle then customers will be served in batches according to general bulk service rule. After service completion, customers may request service again with probability δ as feedback or leave from the system with probability $1-\delta$. In the service completion epoch, if the orbit size is zero then the server leaves for multiple vacations. The server continues the vacation until the orbit size reaches the value ' N' (N > b). At the vacation completion, if the orbit size is ' N' then the server becomes ready to provide service for customers from the main pool or from the orbit. For the designed queueing model, probability generating function of the queue size at an arbitrary time will be obtained by using supplementary variable technique. Various performance measures will be derived with suitable numerical illustrations.

1. Introduction

Mathematical modelling of queueing system with vacations is much more flexible and useful while dealing with real time congestion problems. In vacation queueing systems the server directs to do few supplementary works at the period of its idle time, which may improve the server performance. The application of queueing system with vacations can be established from manufacturing industries, production line systems, inventory management, communication networks, etc.

Analysis of retrial queues with secondary jobs (vacations) has been taken into account by many of the researchers. Some of their works includes different models of retrial queueing systems given by Falin and Templeton [10]. A brief survey and an overview of retrial queues have been explained by Artalejo[2].Falin[9] was first introduced batch arrival retrial queueing system with the following rule: "If the server is busy at the arrival epoch, then the whole batch joins the retrial group, whereas the server is free, then one of the arriving units starts its service and the rest join the retrial group".

In order to model and analyse queueing system with retrials, the method of retrials is required. Many real time applications which exist in network systems illustrate that there are chances in queueing system with retrials such that retrial rate is independent of number of items (if any) present in the orbit, which is called constant retrial policy. Such kind of retrial policy was first constructed by Fayolle [11], who investigated "M/M/1 retrial queue, where the queue will be formed by the retrial group of customers

and request for service is possible only for customers at the head of the orbit queue after an exponentially distributed retrial time" with rate 'v'. Atencia et al. [3] have analysed bulk retrial queue with constant retrial rate and server breakdowns. Recently Jailaxmi et al. [14] examined M/G/1 retrial queue with modified vacations, collision and general retrial policy.

In all the above queueing models server is able to serve one unit at a time. But in many practical situations service is rendered in batches with different batch size. This type of models have applications in inventory systems, manufacturing industries, communication networks, etc.

In classical queueing systems, many of the researchers have contributed in the study of $M^x/G(a,b)/1$ queueing models. Bulk queue with setup times, closedown times, multiple vacations and threshold have been studied by Arumuganathan and Jeyakumar[1]. Haridass and Arumuganathan[12] analysed bulk arrival and batch service queueing model with interrupted vacation. In all the above models they derived various performance characteristics of queueing system. They extended their analysis with cost optimization. Extensive review on classical bulk arrival and batch service queueing system was given by Niranjan and Indhira[19]. State dependent service in bulk queueing system with vacation break-off was analysed by Niranjan et al [20].

Only few works have carried out in batch service queueing system with retrials. Batch service retrial queueing model with constant retrial rate has been analysed by Haridass et al. [12]. They derived some important performance measures. Cost analysis is also carried out in their work.

In the service completion, a batch of customers may seek for further service and adds to the head of the queue and this system is called queueing system with feedback. Queueing model with customer feedback will occur in many real time situations. Choi et al. [6] analysed queueing system with different types of feedback, FCFS policy and gated vacations. Krishna Kumar *et al.* [15] studied feedback queue with varying arrival rates and threshold policy. Recently Badamchi Zadeh [4] derived various performance measures of batch arrival and multi-phase queueing system with random feedback in service and single vacation policy. All the above feedback queueing models are considered as classical queueing system. Only few works are carried out in retrial queueing models with feedback. Madan et al. [18] analysed bulk queue with feedback and optional vacation policy. Krishna Kumar and Raja [16] have studied multi server queue with retrials, balking and feedback.

In all the feedback retrial queueing models, batch service is not taken into consideration. This stimulates authors to model such a queueing system called state dependent arrival in batch service retrial queueing model with active Bernoulli feedback, multiple vacations, threshold and constant retrial rate.

2. Model description

In this paper state dependent arrival in bulk queueing system with retrial, active Bernoulli feedback, multiple vacations and threshold are considered. Customers are arriving into the system in bulk with rate λ_a when the server is idle and with rate λ_b when the *server* is busy or in vacation. These type of assumptions will give motivation in analysing real time applications." An arrival of customers find the server is free then customers are served in batches with minimum of one and maximum of ' b' number of customers according to general bulk service rule". If $1 \leq \varepsilon \leq b$ then entire batch will get service, where ε is the queue length. Similarly, if $\varepsilon > b$ then service is possible only for ' b' customers, and then the remaining $\varepsilon - b$ customers will join to orbit. On the other hand if an arrival of customers finds the server is busy or on vacation then entire customers will join to orbit to get service later. "Customers from the orbit will request service one by one with constant retrial rate γ ." An orbit is a virtual queue formed by customers upon finding that the server is busy. On service completion epoch, the leaving customers may either selects for additional service as a feedback with probability δ or leave the system with a probability of $1 - \delta$. Customer who needs feedback will be taken for service immediately. After service completion, if the orbit size is empty then the server goes for vacation (secondary job). The vacation period will be continued until the orbit size reaches the threshold value ' N' (N > b). When the server finds ' N' customers waiting in the orbit during vacation completion then the server will switch on to serve customers either from orbit or from the primary pool. The pictorial representation of the proposed model is depicted below

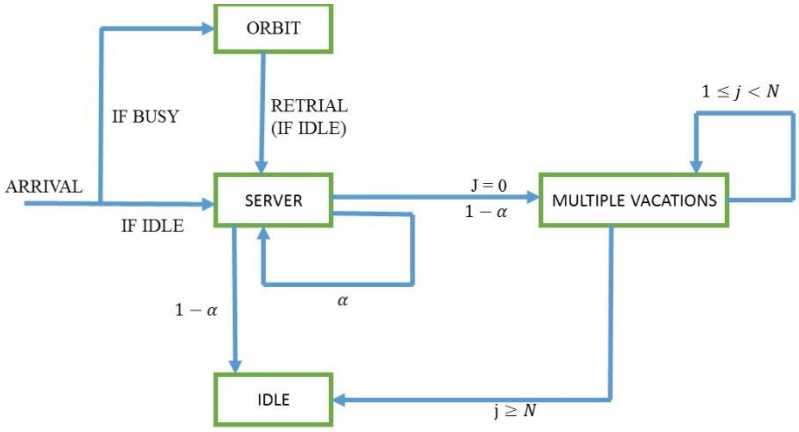

Figure 1. Pictorial representation of the queueing system: J-orbit length

2.1. Notations

Let $\lambda_a$be the Poisson arrival rate, when the server is in idle state and λ_b be the Poisson arrival rate, when the server is busy or in vacation.

Let X be the Group size random variable of the arrival, g_k be the probability that ' k' customers arrive in a batch, X (z) be the probability generating function (PGF) of X.

"N_q(t) be the number of customers waiting for service at time t".

"N_s(t) be the number of customers under the service at time t".

"$N(t)$be the number of customers in the orbit at time t".

Let'γ' be the retrial rate (constant) of the customer from the orbit.

Let A(x) (a(x)) { $\tilde{A}(\theta)$ } [A^0(x)] be the cumulative distribution function (probability density function) { Laplace-Stieltjes transform }[remaining primary service time] of service.

Let B(x) (b(x)) { ($\tilde{B}(\theta)$ } [B^0(x)] be the cumulative distribution function (probability density function){ Laplace-Stieltjes-transform }[remaining vacation time] of vacation.

Let C (t) denotes different states of the server at time t.

$$C(t)=\begin{cases}0, \text{ if the server is busy with service}\\ 1, \text{ if the server is on vacation}\\ 2, \text{ if the server is idle}\end{cases}$$

$Y(t) = j$, if the server is on j[th] vacation

The state probabilities are defined to obtain governing equations:

$$R_{ij}(x,t)dt = Pr\{N_s(t)=i, N_q(t)=j, x \le A^0(t) \le x+dt, C(t)=0\}; 1 \le i \le b, j \ge 0$$

$$S_{jn}(t)dt = Pr\{N(t)=n, x \le B^0(t) \le x+dt, C(t)=1, Y(t)=j\}, n \ge 0$$

$$I_n(t)dt = Pr\{N(t)=n, C(t)=2\},\ n \ge 0$$

3. System analysis

The following equations are obtained by using Supplementary variable technique

$$I_j(t+\Delta t) = I_j(t)(1-\lambda_a\Delta t-\gamma\Delta t)+(1-\delta)\sum_{m=1}^{b} R_{mj}(0,t)\Delta t \qquad 1 \le j \le N-1 \qquad (1)$$

$$I_j(t+\Delta t) = I_j(t)(1-\lambda_a\Delta t-\gamma\Delta t)+(1-\delta)\sum_{m=1}^{b} R_{mj}(0,t)\Delta t+\sum_{l=1}^{\infty} S_{lj}(0,t)\Delta t \qquad (2)$$
$$j \ge N$$

$$R_{1j}(x-\Delta t,t+\Delta t) = R_{ij}(x,t)(1-\lambda_b\Delta t)+\gamma I_{j+1}(t)a(x)\Delta t+I_j(t)\lambda_a g_1\, a(x)\Delta t \qquad (3)$$
$$+\delta R_{ij}(0,t)a(x)\Delta t \qquad j \ge 0$$

$$R_{i0}(x-\Delta t,t+\Delta t)=R_{i0}(x,t)(1-\lambda_b\Delta t)+I_0(t)\lambda_a g_i a(x)\Delta t+\delta R_{i0}(0,t)a(x)\Delta t \qquad 2\le i\le b \tag{4}$$

$$R_{ij}(x-\Delta t,t+\Delta t)=R_{ij}(x,t)(1-\lambda_b\Delta t)+I_j(t)\lambda_a g_i a(x)\Delta t+\delta R_{ij}(0,t)a(x)\Delta t +\sum_{k=1}^{j}R_{i\,j-k}(x,t)\,\lambda_b g_k a(x)\Delta t \qquad 2\le i\le b-1,\ j\ge 1 \tag{5}$$

$$R_{bj}(x-\Delta t,t+\Delta t)=R_{bj}(x,t)(1-\lambda_b\Delta t)+\sum_{k=1}^{j}R_{b\,j-k}(x,t)\,\lambda_b g_k\Delta t +\sum_{k=0}^{j}I_{j-k}(t)\,\lambda_a g_{b+k}s(x)\Delta t+\delta R_{bj}(0,t)a(x)\Delta t \qquad j\ge 1 \tag{6}$$

$$S_{10}(x-\Delta t,t+\Delta t)=S_{10}(x,t)(1-\lambda_b\Delta t)+(1-\alpha)R_{i0}(0,t)b(x)\Delta t \tag{7}$$

$$S_{1j}(x-\Delta t,t+\Delta t)=S_{1j}(x,t)(1-\lambda_b\Delta t)+\sum_{k=1}^{j}S_{1\,j-k}(x,t)\,\lambda_b g_k\Delta t \tag{8}$$

$$S_{l0}(x-\Delta t,t+\Delta t)=S_{l0}(x,t)(1-\lambda_b\Delta t)+S_{l-1\,0}(0,t)b(x)\,\Delta t \qquad l\ge 2 \tag{9}$$

$$S_{lj}(x-\Delta t,t+\Delta t)=S_{lj}(x,t)(1-\lambda_b\Delta t)+\sum_{k=1}^{j}S_{l\,j-k}(x,t)\,\lambda_b g_k\Delta t +S_{l-1\ j}(0,t)b(x)\,\Delta t \qquad j=1,2,\dots N-1 \tag{10}$$

$$S_{lj}(x-\Delta t,t+\Delta t)=S_{lj}(x,t)(1-\lambda_b\Delta t)+\sum_{k=1}^{j}S_{l\,j-k}(x,t)\,\lambda_b g_k\Delta t \qquad j\ge N\quad l\ge 2 \tag{11}$$

4. Steady state orbit size distribution

$$0=-(\lambda_a+\gamma)I_J+(1-\delta)\sum_{m=1}^{b}R_{mj}(0) \qquad 1\le j\le N-1 \tag{12}$$

$$0=-(\lambda_a+\gamma)I_J+(1-\delta)\sum_{m=1}^{b}R_{mj}(0)+\sum_{l=1}^{\infty}S_{lj}(0) \qquad j\ge N \tag{13}$$

$$-\frac{d}{dx}R_{1j}(x)=-\lambda_b R_{1j}(x)+\gamma I_{j+1}(t)a(x)\Delta t+I_j\lambda_a g_1 a(x)+\delta R_{ij}(0)a(x) \qquad j\ge 0 \tag{14}$$

$$-\frac{d}{dx}R_{i0}(x)=-\lambda_b R_{i0}(x)+I_0\lambda_a g_i a(x)+\delta R_{i0}(0)a(x) \qquad 2\le i\le b \tag{15}$$

$$-\frac{d}{dx}R_{ij}(x)=-\lambda_b R_{ij}(x)+\delta R_{ij}(0)a(x)+\sum_{k=1}^{j}R_{i\,j-k}(x)\,\lambda_b g_k a(x) \qquad 2\le i\le b-1,\ j\ge 1 \tag{16}$$

$$-\frac{d}{dx}R_{bj}(x)=-\lambda_b R_{bj}(x)+\sum_{k=1}^{j}R_{b\,j-k}(x)\,\lambda_b g_k+\sum_{k=0}^{j}I_{j-k}(t)\,\lambda_a g_{b+k}a(x) +\delta R_{bj}(0)a(x) \qquad j\ge 1 \tag{17}$$

$$-\frac{d}{dx}S_{10}(x)=-\lambda_b S_{10}(x)+(1-\delta)R_{i0}(0)b(x) \tag{18}$$

$$-\frac{d}{dx}S_{1j}(x)=-\lambda_b S_{1j}(x)+\sum_{k=1}^{j}S_{1\,j-k}(x)\,\lambda_b g_k \qquad j\ge 1 \tag{19}$$

$$-\frac{d}{dx}S_{l0}(x)=-\lambda_b S_{l0}(x)+S_{l-1\,0}(0)b(x) \qquad l\ge 2 \tag{20}$$

$$-\frac{d}{dx}S_{lj}(x)=-\lambda_b S_{lj}(x)+\sum_{k=1}^{j}S_{l\,j-k}(x)\,\lambda_b g_k+S_{l-1\ j}(0)b(x) \qquad j=1,2,\dots N-1 \tag{21}$$

$$-\frac{d}{dx}S_{lj}(x)=-\lambda_b S_{lj}(x)+\sum_{k=1}^{j}S_{l\,j-k}(x)\,\lambda_b g_k \qquad j\ge N\quad l\ge 2 \tag{22}$$

The Laplace – Stieltjes transform of $R_{in}(x)$ and $S_{jn}(x)$ are defined as

$$\tilde{R}_{in}(\theta) = \int_0^\infty \mathrm{e}^{-\theta \mathrm{x}}\, \mathrm{R_{in}}(\mathrm{x})dx \qquad \tilde{S}_{ln}(\theta) = \int_0^\infty \mathrm{e}^{-\theta \mathrm{x}}\, \mathrm{S_{ln}}(x)dx \tag{23}$$

$$(\theta - \lambda_b)\tilde{R}_{1j}(\theta) = R_{1\mathrm{j}}(0) - \gamma I_{j+1}\tilde{A}(\theta) - I_j\lambda_a g_1\tilde{A}(\theta) - \delta R_{ij}(0)\tilde{A}(\theta) \qquad j \geq 0 \tag{24}$$

$$(\theta - \lambda_b)\tilde{R}_{i0}(\theta) = \mathrm{R_{i0}}(0) - I_0\lambda_a g_i\tilde{A}(\theta) - \delta R_{i0}(0)\tilde{A}(\theta) \qquad 2 \leq i \leq b \tag{25}$$

$$(\theta - \lambda_b)\tilde{R}_{ij}(\theta) = \mathrm{R_{ij}}(0) - \delta R_{ij}(0)\tilde{A}(\theta) - \sum_{k=1}^{j}\tilde{R}_{i\,j-k}(\theta)\,\lambda_b g_k \quad 2 \leq i \leq b-1,\ j \geq 1 \tag{26}$$

$$\begin{aligned}(\theta - \lambda_b)\tilde{R}_{bj}(\theta) = {} & \mathrm{R_{bj}}(0) - \sum_{k=1}^{j}\tilde{R}_{b\,j-k}(\theta)\,\lambda_b g_k - \sum_{k=0}^{j} I_{j-k}\,\lambda_a g_{b+k}\tilde{A}(\theta) \\ & -\delta R_{bj}(0)\tilde{A}(\theta) \qquad j \geq 1\end{aligned} \tag{27}$$

$$(\theta - \lambda_b)\tilde{S}_{10}(\theta) = \mathrm{S_{10}}(0) - (1-\delta)R_{i0}(0)\tilde{B}(\theta) \qquad 1 \leq i \leq b \tag{28}$$

$$(\theta - \lambda_b)\tilde{S}_{1j}(\theta) = \mathrm{S_{1j}}(0) - \sum_{k=1}^{j}\tilde{S}_{1\,j-k}(\theta)\,\lambda g_k \qquad j \geq 1 \tag{29}$$

$$(\theta - \lambda_b)\tilde{S}_{l0}(\theta) = \mathrm{S_{l0}}(0) - S_{l-1\,0}(0)\tilde{B}(\theta) \qquad l \geq 2 \tag{30}$$

$$(\theta - \lambda_b)\tilde{S}_{lj}(\theta) = \mathrm{S_{lj}}(0) - \sum_{k=1}^{j}\tilde{S}_{l\,j-k}(\theta)\,\lambda g_k + S_{l-1\ j}(0)\tilde{B}(\theta) \qquad j = 1,2,\ldots N-1 \tag{31}$$

$$(\theta - \lambda_b)\tilde{S}_{lj}(\theta) = \mathrm{S_{lj}}(0) - \sum_{k=1}^{j}\tilde{S}_{l\,j-k}(\theta)\,\lambda g_k \qquad j \geq N \quad l \geq 2 \tag{32}$$

5. Probability generating function

$$\begin{aligned}&\tilde{R}_i(z,\theta) = \sum_{j=0}^{\infty}\tilde{R}_{ij}(\theta)z^j \qquad R_i(z,0) = \sum_{j=0}^{\infty}R_{ij}(0)z^j \qquad 1 \leq i \leq b \\ &\tilde{S}_j(z,\theta) = \sum_{l=1}^{\infty}\tilde{Q}_{1j}(0)z^j \qquad S_j(z,0) = \sum_{l=1}^{\infty}Q_{lj}(0)z^j \qquad j \geq 1\end{aligned} \tag{33}$$

$$\begin{aligned}(\lambda_a + \gamma)I(z) - \gamma I_0 = {} & (1-\delta)\left(\sum_{m=1}^{b}R_m(z,0) - R_{m0}(0)\right) \\ & + \sum_{l=1}^{\infty}\left(S_l(z,0) - \sum_{j=0}^{N-1}S_{lj}(0)z^j\right)\end{aligned} \tag{34}$$

$$(\theta - \lambda_b)\tilde{R}_1(z,\theta) = R_1(z,0) - \gamma\tilde{A}(\theta)\frac{1}{z}(I(z) - I_0) - \left(\lambda_a g_1 I(z) + \delta R_1(z,0)\right)\tilde{A}(\theta) \tag{35}$$

$$\left(\theta - \lambda_b + \lambda_b x(z)\right)\tilde{R}_i(z,\theta) = R_i(z,0) - \lambda_a g_i I(z)\tilde{A}(\theta) - \delta R_i(z,0) \qquad 2 \leq i \leq b-1 \tag{36}$$

$$\begin{aligned}(\theta - \lambda_b + \lambda_b x(z))\tilde{R}_b(z,\theta) = {} & R_b(z,0) - \lambda_a g_b I_0\tilde{A}(\theta) - \sum_{k=0}^{\infty}\lambda_a g_{b+k}\,z^k\,I(z)\tilde{A}(\theta) \\ & -\delta R_b(z,0)\tilde{A}(\theta)\end{aligned} \tag{37}$$

$$\left(\theta - \lambda_b + \lambda_b x(z)\right)\tilde{S}_1(z,\theta) = S_1(z,0) - \tilde{B}(\theta)(1-\delta)\,R_{i0}(0) \tag{38}$$

$$\left(\theta - \lambda_b + \lambda_b x(z)\right)\tilde{S}_l(z,\theta) = S_l(z,0) - \tilde{B}(\theta)\sum_{j=0}^{N-1}Q_{l-1\,j}(0)\,z^j \tag{39}$$

Substituting $\theta = \lambda_b$ in equation (24)

$$R_1(z,0) = \frac{\tilde{A}(\lambda_b)\left(\gamma\frac{1}{z}(I(z) - I_0) + \lambda_a g_1 I(z)\right)}{1 - \delta\tilde{A}(\lambda_b)} \tag{40}$$

Substituting $\theta = \lambda_b - \lambda_b x(z)$ in from equation (25) and (28)

$$R_i(z,0) = \frac{\tilde{A}(\lambda_b - \lambda_b x(z))\lambda_a g_i I(z)}{1 - \delta\tilde{A}(\lambda_b - \lambda_b x(z))} \qquad 2 \leq i \leq b-1 \tag{41}$$

$$R_b(z,0) = \frac{\tilde{A}(\lambda_b-\lambda_b x(z))(\lambda_a g_b I_0+\sum_{k=0}^{\infty}\lambda_a g_{b+k} z^k I(z))}{1-\delta\tilde{A}(\lambda_b-\lambda_b x(z))} \tag{42}$$

$$S_1(z,0) = \tilde{B}(\lambda_b-\lambda_b x(z))(1-\delta)R_{i0}(0) \tag{43}$$

$$S_l(z,0) = \tilde{B}(\lambda_b-\lambda_b x(z))\sum_{j=0}^{N-1} S_{l-1\,j}(0)\, z^j \tag{44}$$

By using (35) and (40)

$$\tilde{R}_1(z,\theta) = \frac{(\tilde{A}(\lambda_b)-\tilde{A}(\theta))\left(\gamma\frac{1}{z}(I(z)-I_0)+\lambda_a g_1 I(z)\right)}{(\theta-\lambda_b)(1-\delta\tilde{A}(\lambda_b))} \tag{45}$$

By using (36) and (41)

$$\tilde{R}_i(z,\theta) = \frac{\left(\tilde{A}(\lambda_b-\lambda_b x(z))-\tilde{A}(\theta)\right)\lambda_a g_i I(z)}{(\theta-\lambda_b+\lambda_b x(z))(1-\delta\tilde{A}(\lambda_b-\lambda_b x(z)))} \tag{46}$$

By using (37) and (42)

$$\tilde{R}_b(z,\theta) = \frac{\left(\tilde{A}(\lambda_b-\lambda_b x(z))-\tilde{A}(\theta)\right)(\lambda_a g_b I_0+\sum_{k=0}^{\infty}\lambda_a g_{b+k} z^k I(z))}{(\theta-\lambda_b+\lambda_b x(z))(1-\delta)\tilde{A}(\lambda_b-\lambda_b x(z))} \tag{47}$$

By using (38), (39), (43) and (44), we get

$$\sum_{l=1}^{\infty}\tilde{S}_l(z,\theta) = \frac{\left(\tilde{B}(\lambda_b-\lambda_b x(z))-\tilde{B}(\theta)\right)\left((1-\delta)R_{i0}(0)+\sum_{j=0}^{N-1} S_{l-1\,j}(0) z^j\right)}{\theta-\lambda+\lambda x(z)} \tag{48}$$

Substituting equations (40), (41), (42) and (44) in equation (34), we get

$$I(z) = \frac{\begin{pmatrix} z^{b} I_0 \begin{pmatrix} z\gamma(1-\delta\tilde{A}(\lambda_b))\left(1-\delta\tilde{A}(\lambda_b-\lambda_b x(z))\right) \\ -(1-\delta)\left(1-\delta\tilde{A}(\lambda_b-\lambda_b x(z))\right)\tilde{A}(\lambda_b) \\ +\tilde{A}(\lambda_b-\lambda_b x(z))\lambda_a g_b(1-\delta\tilde{A}(\lambda_b))z \end{pmatrix} \\ +z^{b+1}(\tilde{B}(\lambda_b-\lambda_b x(z))-1)\sum_{j=0}^{N-1} S_j z^j \end{pmatrix}}{\begin{pmatrix} z^{b+1}(\lambda_a+\gamma)(1-\delta\tilde{A}(\lambda_b))\left(1-\delta\tilde{A}(\lambda_b-\lambda_b x(z))\right)-z^b(1-\delta)\tilde{A}(\lambda_b)(\gamma+\lambda_a z g_1)\left(1-\delta\tilde{A}(\lambda_b-\lambda_b x(z))\right) \\ +\tilde{A}(\lambda_b-\lambda_b x(z))(1-\delta\tilde{A}(\lambda_b))\lambda_a\left(z^{b+1}\sum_{i=2}^{b-1} g_i - z\left(X(z)-\sum_{j=1}^{b-1} g_j z^j\right)\right) \end{pmatrix}} \tag{49}$$

The PGF of the orbit size at an arbitrary time is given by

$$P(z) = I(z) + \tilde{R}_1(z,0) + \sum_{i=2}^{b-1}\tilde{R}_i(z,0) + \tilde{R}_b(z,0) + \sum_{l=1}^{\infty}\tilde{S}_l(z,0) \tag{50}$$

Substituting $\theta = 0$ in equations from (45) to (48), then equation (50) is simplified as

$$P(z) = I_0\left[\frac{G(z)H_1+R_1K(z)}{F(z,\lambda_b)K(z)}\right] + \frac{H_1M_1+M_2S_1(-\lambda_b)(1-\delta\tilde{A}(\lambda_b))\left(1-\delta\tilde{A}(\lambda_b-\lambda_b x(z))\right)}{F(z,\lambda_b)K(z)} \tag{51}$$

where

$$R_1 = \left(\tilde{A}(\lambda_b)-1\right)\left(-\frac{\gamma}{z}\right)\left(-\lambda_b+\lambda_b x(z)\right)\left(1-\delta\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\right) + \left(\tilde{A}(\lambda_b-\lambda_b x(z))-1\right)\lambda_a g_b(-\lambda_b)\left(1-\delta\tilde{A}(\lambda_b)\right)$$

$$H_1 = F(z,\lambda_b) + \left(\tilde{A}(\lambda_b)-1\right)\left(\frac{\gamma}{z}+\lambda_a z g_1\right)\left(-\lambda_b+\lambda_b x(z)\right)\left(1-\delta\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\right) + \left(\tilde{A}(\lambda_b-\lambda_b x(z))-1\right)\left(\sum_{k=0}^{\infty}\lambda_a g_{b+k} z^k + \sum_{i=2}^{b-1}\lambda_a g_i\right)(-\lambda_b)\left(1-\delta\tilde{A}(\lambda_b)\right)$$

$$G(z)=\begin{pmatrix} z\gamma\left(1-\delta\tilde{A}(\lambda_b)\right)\left(1-\delta\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\right) \\ -(1-\delta)\left(1-\delta\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\right)\tilde{A}(\lambda_b) \\ +\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\lambda_a g_b\left(1-\delta\tilde{A}(\lambda_b)\right)z \end{pmatrix} z^b$$

$$\begin{aligned} K(z)=\ & z^{b+1}(\lambda_a+\gamma)\left(1-\delta\tilde{A}(\lambda_b)\right)\left(1-\delta\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\right) \\ & -z^b(1-\delta)\tilde{A}(\lambda_b)(\gamma+\lambda_a z g_1)\left(1-\delta\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\right) \\ & +\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\left(1-\delta\tilde{A}(\lambda_b)\right)\lambda_a\left(z^{b+1}\sum_{i=2}^{b-1}g_i-z\left(X(z)-\sum_{j=1}^{b-1}g_j z^j\right)\right) \end{aligned}$$

$$F(z,\lambda_b)=(-\lambda_b)\left(1-\delta\tilde{A}(\lambda_b)\right)\left(1-\delta\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)\right)\left(-\lambda_b+\lambda_b x(z)\right)$$

$$M_1=z^{b+1}\left(\tilde{B}(\lambda_b-\lambda_b x(z))-1\right)\sum_{j=0}^{N-1}S_j\,z^j$$

$$M_2=\left(\tilde{B}(\lambda_b-\lambda_b x(z))-1\right)\left((1-\delta)R_0+\sum_{j=0}^{N-1}S_j\,z^j\right)$$

6. Performance measures

In this section some important performance characteristics are derived from the steady-state probability distribution function given in equation (51),

6.1. Expected orbit length

$$E(Q)=\lim_{z\to 1}P'(z)$$

$$E(Q)=I_0\left[\frac{M_1K_2-M_2K_1}{2{M_1}^2}\right]+\frac{M_1L_2}{2{M_1}^2}$$

where

$S_1=E(A)\,\lambda_b E(X)$ $\quad S_2=E(A)\,\lambda_a E(X)$ $\quad S_1=E(B)\,\lambda_b E(X)$

$V_1=E(A)\lambda_b X''(1)+E(A^2)\lambda_b{}^2(E(X))^2$ $\quad V_2=E(B)\lambda_b X''(1)+E(B^2)\lambda_b{}^2(E(X))^2$

$V_3=E(A)\lambda_a X''(1)+E(A^2)\lambda_a{}^2(E(X))^2$ $\quad V_4=\left(1-\delta\tilde{A}(\lambda_b)\right)$

$K_1=G_1H_1'+R_1'U_1$ $\quad G_1=\gamma V_4(1-\delta)-(1-\delta)^2\tilde{A}(\lambda_b)+\lambda_a g_b V_4$

$H_1'=F'(z,\lambda_a)+\left(\tilde{A}(\lambda_b)-1\right)(\gamma+\lambda_a g_1)\lambda_b E(X)(1-\delta)+S_1 d_1(-\lambda_b)V_4$

$$\begin{aligned} G_2=\ & \gamma V_4-\delta\tilde{A}'(\lambda_b)-(1-\delta)\left((1-\delta)\tilde{A}'(\lambda_b)-\tilde{A}(\lambda_b)\delta S_1\right) \\ & +\lambda_a g_b\left(V_4-\delta\tilde{A}'(\lambda_b)+S_1V_4\right)+b\left(V_4-(1-\delta)^2\tilde{A}(\lambda_b)+\lambda_a g_b(1-\delta)\right) \end{aligned}$$

$R_1'=-\left(\tilde{A}(\lambda_b)-1\right)\gamma\lambda_b E(X)(1-\delta)$

$U_1=(\lambda_a+\gamma)V_4(1-\delta)-(1-\delta)^2\tilde{A}(\lambda_b)(\gamma+\lambda_a g_1)+\lambda_a\left(\sum_{i=2}^{b-1}g_i-\left(1-\sum_{j=1}^{b-1}g_j\right)\right)$

$K_2=G_1H_1''+2G_2H_1'+2R_1'U_2+R_1''U_1$

$$H_1'' = F''(z,\lambda_a) + \left(\tilde{A}(\lambda_b) - 1\right)\begin{pmatrix} 2(\gamma+\lambda_a g_1)\lambda_b E(X)(-\delta)S_1 \\ +(-\gamma+\lambda_a g_1)(\lambda_b(X''(1)+2E(X))(1-\delta)\end{pmatrix}$$
$$+2\tilde{A}'(\lambda_b)(\gamma+\lambda_a g_1)\lambda_b E(X)(1-\delta) + S_1(2d_1+2d_1')(\lambda_b)((-\delta)\tilde{A}'(\lambda_b)+V_4)$$
$$-V_4 d_1 \lambda_b V_1$$

$$U_2 = (\gamma+\lambda_a)\left((1-\delta)(-\delta)\tilde{A}'(\lambda_b) - \delta V_4 S_1\right) + (b+1)(\gamma+\lambda_a)V_4(1-\delta)$$
$$+(1-\delta)\tilde{A}(\lambda_b)(\gamma+\lambda_a g_1)\delta S_1 - (1-\delta)\tilde{A}(\lambda_b)\lambda_a g_1(1-\delta)) + S_1 V_4 d_2 \lambda_a$$
$$-(1-\delta)(\tilde{A}'(\lambda_b)+b\tilde{A}(\lambda_b))(\gamma+\lambda_a g_1)(1-\delta) + (V_4 - \delta\tilde{A}'(\lambda_b))\lambda_a(d_2' + d_2$$

$$R_1'' = \left(\tilde{A}(\lambda_b)-1\right)\gamma\left(2\delta\lambda_b E(X)S_1 + \lambda_b X''(1)(1-\delta) + 2\lambda_b E(X)(1-\delta)\right)$$
$$-2\gamma\tilde{A}'(\lambda_b)\lambda_b E(X)(1-\delta)$$

$$M_1 = -{\lambda_b}^2 E(X)(1-\delta)V_4 U_1 \qquad d_1 = \sum_{k=0}^{\infty}\lambda_a g_{b+k} + \sum_{i=2}^{b-1}\lambda_a g_i$$

$$d_1' = \sum_{k=0}^{\infty} k\lambda_a g_{b+k} \qquad d_1'' = \sum_{k=0}^{\infty} k(k-1)\lambda_a g_{b+k}$$

$$d_2' = (b+1)\sum_{i=2}^{b-1} g_i - \left(1-\sum_{j=1}^{b-1} g_j\right) - \left(E(X) - \sum_{j=1}^{b-1} j g_j\right) \quad d_2 = \sum_{i=2}^{b-1} g_i - \left(1-\sum_{j=1}^{b-1} g_j\right)$$

$$M_2 = \left(-{\lambda_b}^2\left(2E(X)\left(\delta^2(1-\delta)\tilde{A}'(\lambda_b) + V_4 S_1\right) + X''(1)(1-\delta)V_4\right)\right)U_1$$
$$+2(-{\lambda_b}^2 E(X)(1-\delta)V_4)U_2$$

$$L_2 = S_1 T_1 \sum_{j=0}^{N-1} S_j + S_1 T_2 + (1-\delta)V_4\left((1-\delta)R_0 + \sum_{j=0}^{N-1} S_j\right)(-{\lambda_b}^2 E(X)(1-\delta)V_4)$$

$$T_1 = -{\lambda_b}^2 E(X)(1-\delta)V_4 U_1 + \left(\tilde{A}(\lambda_b)-1\right)(\gamma+\lambda_a g_1)\lambda_b E(X)(1-\delta) - S_1 d_1 \lambda_b V_4$$

$$T_2 = \left((1-\delta)R_0 + \sum_{j=0}^{N-1} S_j\right)\left(-{\lambda_b}^2 E(X)(1-\delta)V_4\right)\left(-(1-\delta)\delta\tilde{A}'(\lambda_b) - V_4 S_1 \delta\right)$$

6.2. Probability that the server is busy

$$\mathrm{P(B)} = \lim_{z\to 1}\sum_{m=1}^{b}\tilde{A}_m(z,0)$$

$$\mathrm{P(B)} = \frac{(\tilde{S}(\lambda_b)-1)(\lambda_b g_1+\gamma(1-I_0))}{-\lambda_b} + E(A)I(1)\left(\sum_{i=2}^{b-1} g_i + \lambda_a g_b I_0 + \sum_{k=0}^{\infty}\lambda_b g_{b+k}\right)$$

$$where\ \ I(1) = \frac{I_0\left(\left(1-\delta\tilde{A}(\lambda_b)\right)(\gamma(1-\delta)+\lambda_a g_b) - (1-\delta)^2\tilde{A}(\lambda_b)\right)}{\left(1-\delta\tilde{A}(\lambda_b)\right)\begin{pmatrix}\frac{(\lambda_a+\gamma)(1-\delta)}{+\lambda_a\left(\sum_{i=2}^{b-1} g_i - \left(1-\sum_{j=1}^{b-1} g_j\right)\right)}\end{pmatrix} - (1-\delta)^2\tilde{A}(\lambda_b)(\gamma+\lambda_a g_1)}$$

6.3. Probability that the server is idle

$$\mathrm{P(I)} = \lim_{z\to 1} I(z)$$

$$\mathrm{P(I)} = \frac{I_0\left(\left(1-\delta\tilde{A}(\lambda_b)\right)(\gamma(1-\delta)+\lambda_a g_b) - (1-\delta)^2\tilde{A}(\lambda_b)\right)}{\left(1-\delta\tilde{A}(\lambda_b)\right)\begin{pmatrix}(\lambda_a+\gamma)(1-\delta)\\ +\lambda_a\left(\sum_{i=2}^{b-1} g_i - \left(1-\sum_{j=1}^{b-1} g_j\right)\right)\end{pmatrix} - (1-\delta)^2\tilde{A}(\lambda_b)(\gamma+\lambda_a g_1)}$$

6.4. Probability that the server is on vacation

$$\mathrm{P(V)} = \lim_{z\to 1}\sum_{l=1}^{\infty}\tilde{S}_l(z,\theta)$$

$$P(\mathrm{V}) = E(B)\lambda_b\left((1-\delta)R_0 + \sum_{j=0}^{N-1} S_j\right)$$

6.5. Steady state condition

The steady state condition for the proposed model can be obtained from the above expression

$$\lim_{z\to 1} P(z) = 1,$$

Therefore the steady state condition is derived as $\rho = \lambda_b E(A)E(X) < 1$

$$I_0 = \frac{-{\lambda_b}^2 E(X)(1-\delta)V_4 U_1}{G_1 {H_1}' + U_1 {R_1}'}$$

where

$$U_1 = (\lambda_a + \gamma)V_4(1-\delta) - (1-\delta)^2\tilde{A}(\lambda_b)(\gamma + \lambda_a g_1) + \lambda_a\left(\sum_{i=2}^{b-1} g_i - \left(1 - \sum_{j=1}^{b-1} g_j\right)\right)$$

$${H_1}' = F'(z,\lambda_a) + \left(\tilde{A}(\lambda_b) - 1\right)(\gamma + \lambda_a g_1)\lambda_b E(X)(1-\delta) + S_1 d_1(-\lambda_b)V_4$$

$${R_1}' = -\left(\tilde{A}(\lambda_b) - 1\right)\gamma\lambda_b E(X)(1-\delta) \quad G_1 = \gamma V_4(1-\delta) - (1-\delta)^2\tilde{A}(\lambda_b) + \lambda_a g_b V_4$$

Result 1

An unknown constant c_n is expressed in terms of known termD_n. "Let β_n be the probability that 'n' customers arrive into the system during an idle period", then.

$$c_n = \frac{\beta_n I_0 + \sum_{j=0}^{n-1} q_j \beta_{n-j}}{(1-\beta_0)}, \qquad \text{n=1, 2… N-1}$$

7. Special cases

The proposed model is developed with the assumption that the service time is arbitrary. However to analyse real time system, proper distribution is mandatory. This segment presents some distinct cases of the proposed system via indicating service time as "exponential distribution, hyper exponential distribution, Erlangian distribution."

7.1. Exponential bulk service time

The PDF of exponential distribution is defined as

$A(x) = e^{-\mu x}$, where μ is parameter

$$\tilde{A}\left(\lambda_b - \lambda_b x(z)\right) = \left(\frac{\mu}{\mu + (\lambda_b - \lambda_b x(z))}\right)$$

The PGF of the orbit size for exponential service time is derived by substituting the expression for $\tilde{A}\left(\lambda - \lambda x(z)\right)$ in equation (51)

$$P(z) = I_0\left[\frac{G(z)H_1 + R_1 K(z)}{F(z,\lambda_b)K(z)}\right] + \frac{H_1 M_1 + M_2 S_1(-\lambda_b)\left(1-\delta\left(\frac{\mu}{\mu+\lambda_b}\right)\right)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right)}{F(z,\lambda_b)K(z)}$$

Where

$$R_1 = \left(\left(\frac{\mu}{\mu+\lambda_b}\right) - 1\right)\left(-\frac{\gamma}{z}\right)\left(-\lambda_b + \lambda_b x(z)\right)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right)$$
$$+\left(\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right) - 1\right)\lambda_a g_b(-\lambda_b)\left(1-\delta\left(\frac{\mu}{\mu+\lambda_b}\right)\right)$$

$$H_1 = F(z,\lambda_b) + \left(\left(\frac{\mu}{\mu+\lambda_b}\right)-1\right)\left(\frac{\gamma}{z}+\lambda_a z g_1\right)\left(-\lambda_b+\lambda_b x(z)\right)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right)$$
$$+\left(\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)-1\right)\left(\sum_{k=0}^{\infty}\lambda_a g_{b+k}z^k+\sum_{i=2}^{b-1}\lambda_a g_i\right)(-\lambda_b)\left(1-\delta\left(\frac{\mu}{\mu+\lambda_b}\right)\right)$$

$$G(z) = \begin{pmatrix} z\gamma\left(1-\delta\left(\frac{\mu}{\mu+\lambda_b}\right)\right)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right) \\ -(1-\delta)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right)\left(\frac{\mu}{\mu+\lambda_b}\right) \\ +\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\lambda_a g_b\left(1-\delta\left(\frac{\mu}{\mu+\lambda_b}\right)\right)z \end{pmatrix} z^b$$

$$K(z) = z^{b+1}(\lambda_a+\gamma)\left(1-\delta\tilde{A}(\lambda_b)\right)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right)$$
$$-z^b(1-\delta)\left(\frac{\mu}{\mu+\lambda_b}\right)(\gamma+\lambda_a z g_1)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right)$$
$$+\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\left(1-\delta\left(\frac{\mu}{\mu+\lambda_b}\right)\right)\lambda_a\left(z^{b+1}\sum_{i=2}^{b-1}g_i - z\left(X(z)-\sum_{j=1}^{b-1}g_j z^j\right)\right)$$

$$F(z,\lambda_b) = (-\lambda_b)\left(1-\delta\left(\frac{\mu}{\mu+\lambda_b}\right)\right)\left(1-\delta\left(\frac{\mu}{\mu+(\lambda_b-\lambda_b x(z))}\right)\right)\left(-\lambda_b+\lambda_b x(z)\right)$$

$$M_1 = z^{b+1}\left(\tilde{B}(\lambda_b-\lambda_b x(z))-1\right)\sum_{j=0}^{N-1}S_j z^j$$

$$M_2 = \left(\tilde{B}(\lambda_b-\lambda_b x(z))-1\right)\left((1-\delta)R_0+\sum_{j=0}^{N-1}S_j z^j\right)$$

7.2. Hyper exponential bulk service time

When the service time follows hyper exponential distribution with probability density function, then $a(x) = cde^{-dx} + (1-c)\,fe^{-fx}$, where d and f are parameters, then,

$$\tilde{A}\left(\lambda_b-\lambda_b x(z)\right) = \left(\frac{dc}{d+\left(\lambda_b-\lambda_b x(z)\right)}\right)+\left(\frac{f(1-c)}{f+\left(\lambda_b-\lambda_b x(z)\right)}\right)$$

The PGF of the orbit size for hyper exponential service time is derived by substituting the expression for $\tilde{A}\left(\lambda_b-\lambda_b x(z)\right)$ in equation (51), then

$$P(z) = I_0\left[\frac{G(z)H_1+R_1K(z)}{F(z,\lambda_b)K(z)}\right]+\frac{H_1M_1+M_2S_1(-\lambda_b)M_3M_4}{F(z,\lambda_b)K(z)}$$

Where

$$R_1 = \left(\left(\frac{dc}{d+\lambda_b}+\frac{f(1-c)}{f+\lambda_b}\right)-1\right)\left(-\frac{\gamma}{z}\right)\left(-\lambda_b+\lambda_b x(z)\right)M_4$$
$$+\left(\left(\left(\frac{dc}{d+(\lambda_b-\lambda_b x(z))}\right)+\left(\frac{f(1-c)}{f+(\lambda_b-\lambda_b x(z))}\right)\right)-1\right)\lambda_a g_b(-\lambda_b)M_3$$

$$H_1 = F(z,\lambda_b) + \left(\tilde{A}(\lambda_b) - 1\right)\left(\frac{\gamma}{z} + \lambda_a z g_1\right)\left(-\lambda_b + \lambda_b x(z)\right)M_4 + \left(\left(\left(\frac{dc}{d+(\lambda_b-\lambda_b x(z))}\right) + \left(\frac{f(1-c)}{f+(\lambda_b-\lambda_b x(z))}\right)\right) - 1\right) \times \left(\sum_{k=0}^{\infty}\lambda_a g_{b+k} z^k + \sum_{i=2}^{b-1}\lambda_a g_i\right)(-\lambda_b)M_3$$

$$G(z) = \left(\begin{array}{c} z\gamma M_3 M_4 - (1-\delta)M_4\left(\frac{dc}{d+\lambda_b} + \frac{f(1-c)}{f+\lambda_b}\right) \\ \left(\left(\frac{dc}{d+(\lambda_b-\lambda_b x(z))}\right) + \left(\frac{f(1-c)}{f+(\lambda_b-\lambda_b x(z))}\right)\right)\lambda_a g_b M_3 z \end{array}\right) z^b$$

$$K(z) = z^{b+1}(\lambda_a+\gamma)M_3M_4 - z^b(1-\delta)\left(\frac{dc}{d+\lambda_b} + \frac{f(1-c)}{f+\lambda_b}\right)(\gamma+\lambda_a z g_1)M_4 + \left(\left(\frac{dc}{d+(\lambda_b-\lambda_b x(z))}\right) + \left(\frac{f(1-c)}{f+(\lambda_b-\lambda_b x(z))}\right)\right)M_3\lambda_a\left(z^{b+1}\sum_{i=2}^{b-1} g_i - z\left(X(z) - \sum_{j=1}^{b-1} g_j z^j\right)\right)$$

$$F(z,\lambda_b) = (-\lambda_b)M_3M_4\left(-\lambda_b + \lambda_b x(z)\right) \quad M_1 = z^{b+1}\left(\tilde{B}(\lambda_b - \lambda_b x(z)) - 1\right)\sum_{j=0}^{N-1} S_j\, z^j$$

$$M_3 = \left(1 - \delta\left(\frac{dc}{d+\lambda_b} + \frac{f(1-c)}{f+\lambda_b}\right)\right) \qquad M_4 = \left(1 - \delta\left(\frac{dc}{d+(\lambda_b-\lambda_b x(z))} + \frac{f(1-c)}{f+(\lambda_b-\lambda_b x(z))}\right)\right)$$

$$M_2 = \left(\tilde{B}(\lambda_b - \lambda_b x(z)) - 1\right)\left((1-\delta)R_0 + \sum_{j=0}^{N-1} S_j\, z^j\right)$$

7.3. K-Erlangian bulk service time

Let us consider that service time follows K - Erlang distribution with probability density function $a(x) = \frac{(k\mu)^k x^{k-1} e^{-(k\mu x)}}{(k-1)!}$, k > 0; where μ is the parameter, then

$$\tilde{A}\left(\lambda_b - \lambda_b x(z)\right) = \left(\frac{k\mu}{k\mu + \left(\lambda_b - \lambda_b x(z)\right)}\right)^k$$

The PGF of the orbit size K-Erlangian bulk service time is derived by substituting the expression for $\tilde{A}\left(\lambda_b - \lambda_b x(z)\right)$ in equation (51)

8. Numerical illustration

In this section, obtained theoretical results are validated with suitable numerical example. Numerical results are derived with the following assumptions

Mean arrival rate when the system is idle	λ_a
Mean arrival rate when the system is busy or on vacation	λ_b
Service time follows exponential distribution with parameter	μ
Batch size of the arrival followsgeometric distribution with mean	3
Retrial rate	γ
Vacation time follows exponential distribution with parameter	η
Maximum server capacity	b

8. 1. Effects of various parameters on performance measures

Effects of arrival rates with respect to expected orbit length is given in table 1 and table 2 with parameters $\eta = 2$, μ=3, b=4, $\delta = 0.4$ .From tables it is observed that mean orbit size increases when the arrival rate increases and mean orbit size decreases when retrial rate increases.

Table 3 and Figure 4 show the way in which the orbit size changes for different values of arrival rate λ_a (when the server is idle). Considering the service times as exponential, Erlang-2 and hyper exponential with parameters $\lambda_b = 2, \gamma = 5, \eta = 2$, μ=3, b=4, N=7, $\delta = 0.4$, it can be observed that the mean orbit size increases when the arrival rate λ_a increases.

Table 4 and Figure 5 show the way in which the orbit size changes for different values of arrival rate λ_b (when the server is busy or in vacation). Considering the service times as exponential, Erlang-2 and hyper exponential with parameters $\lambda_a = 2, \gamma = 5, \eta = 2$, μ=3, b=4, N=7, $\delta = 0.4$, it can be observed that the mean orbit size increases when the arrival rate λ_b increases.

Table 1.Retrial rate Vs Mean orbit size (Arrival rate $\lambda_b = 2$)

Retrial rate γ	**Expected orbit length E(Q)**				
	Arrival rates(λ_a)				
	2	**3**	**4**	**5**	**6**
1	3.9261	4.7963	5.8321	6.5920	8.1928
2	3.2351	4.1641	5.2193	6.0821	7.6291
3	2.7936	3.9365	4.9287	5.9768	7.4362
4	2.3261	3.7590	4.7981	5.7989	7.3190
5	2.0982	3.6963	4.6362	5.6923	7.1982
6	1.5391	3.3872	4.4902	5.4329	6.9360
7	1.1972	3.2015	4.3198	5.0763	6.2769

Table 2.Retrial rate Vs Mean orbit size (Arrival rate $\lambda_a = 2$)

Retrial rate γ	**Expected orbit length E(Q)**					
	Arrival rates(λ_b)					
	1	**2**	**3**	**4**	**5**	**6**
1	4.6321	5.7962	6.0361	6.5328	7.7912	8.3792
2	4.4982	5.5728	5.9253	6.3291	7.6324	8.0523
3	4.3026	5.3264	5.7421	6.0328	7.4561	7.8392
4	4.1982	5.1920	5.6378	5.8391	7.2793	7.5091
5	3.9751	4.9324	5.4231	5.5923	6.9261	7.2592
6	3.7782	4.8396	5.3438	5.3792	6.7938	6.9132
7	3.5329	4.6523	5.0632	5.1902	6.4982	6.7981

Table 3. Arrival rate (λ_a) Vs Expected orbit length

λ_a	**Expected orbit length**		
	Exponential	**Erlang**	**Hyper-exponential**
0.2	0.0572	0.0612	0.0644
0.4	0.0831	0.0874	0.0892
0.6	0.1357	0.2461	0.3195
0.8	0.3492	0.3964	0.4263
1.0	0.5763	0.6297	0.6921
1.2	0.8324	0.8921	0.9035
1.4	1.2497	1.3592	1.4257

Table 4. Arrival rate (λ_b) Vs Expected orbit length

λ_b	Expected orbit length		
	Exponential	**Erlang**	**Hyper-exponential**
0.2	0.0652	0.0735	0.0943
0.4	0.0839	0.0962	0.2361
0.6	0.1368	0.2437	0.4495
0.8	0.2764	0.3984	0.5327
1.0	0.4985	0.5321	0.6792
1.2	0.5362	0.7092	0.8321
1.4	0.6952	0.8361	0.9067

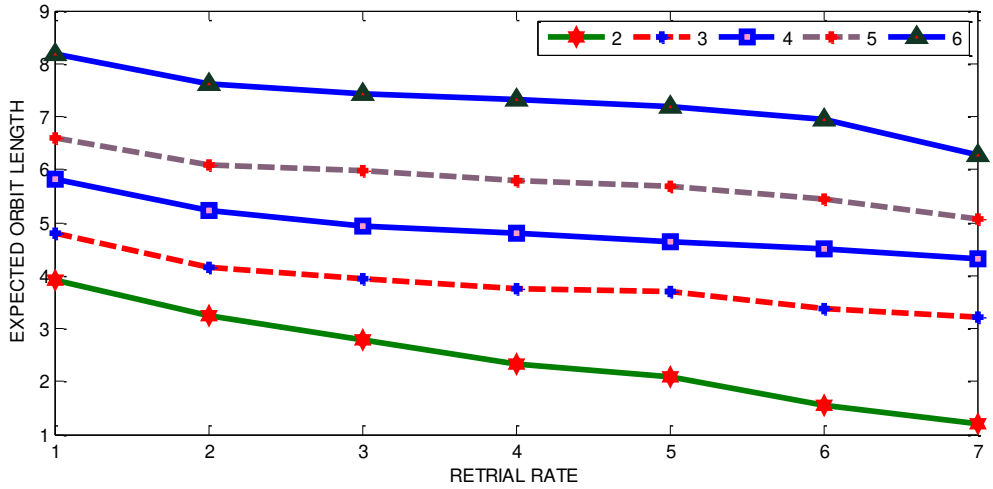

Figure 2. Retrial rate Vs Expected orbit length

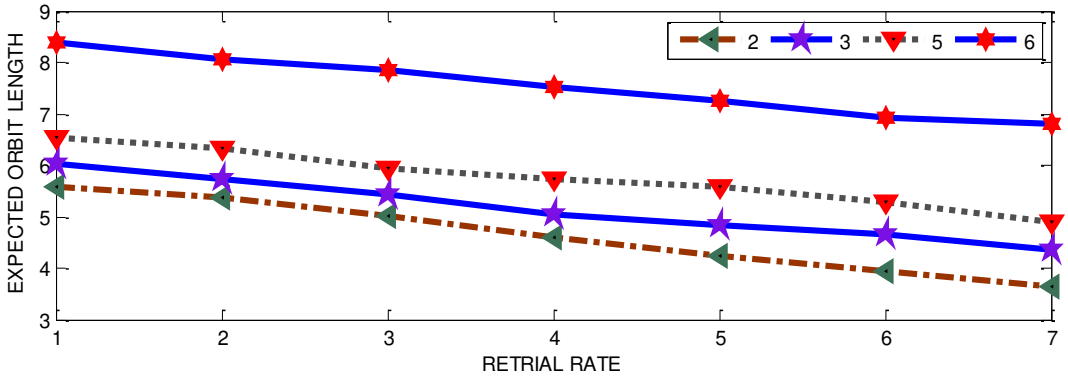

Figure 3. Retrial rate Vs Expected orbit length

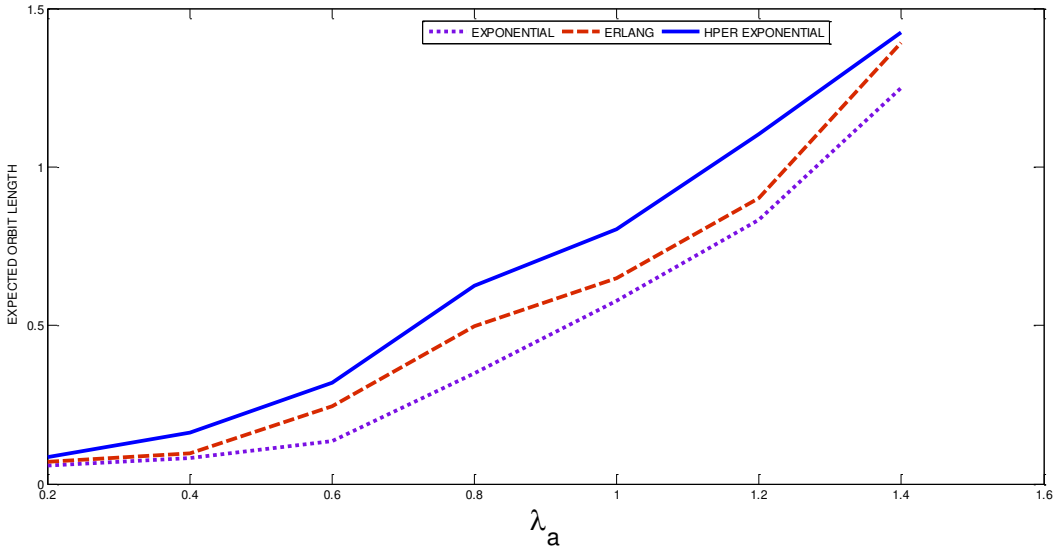

Figure 4. Arrival rate(λ_a) Vs Expected orbit length

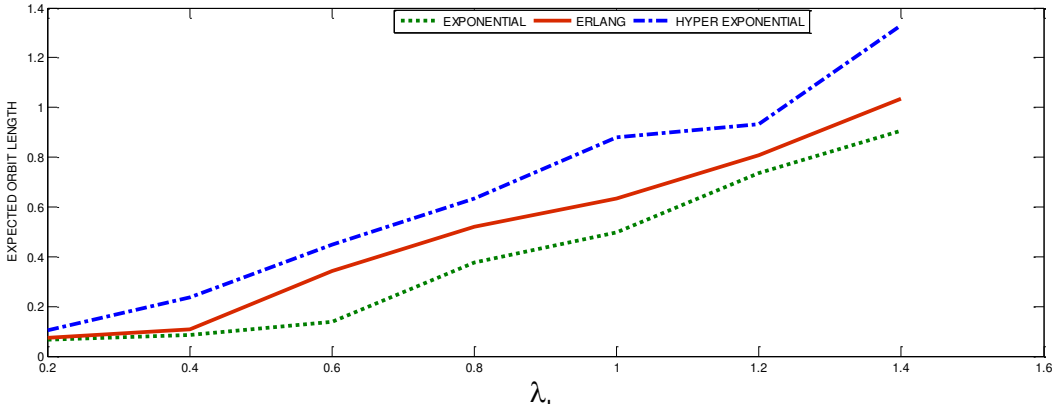

Figure 5. Arrival rate(λ_b) Vs Expected orbit length

9. Conclusions

In this paper, "state dependent arrival in batch service retrial queueing system with active Bernoulli feedback, multiple vacations and threshold" is analysed. The PGF describes the orbit size is obtained by using supplementary variable technique. Various performance characteristics are also presented with appropriate numerical illustrations.

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