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ScienceDirect
Procedia Engineering 127 (2015) 1087 – 1094
Procedia Engineering
www.elsevier.com/locate/procedia

International Conference on Computational Heat and Mass Transfer-2015

The Influence of Slip, Wall Properties on the Peristaltic Transport of a Conducting Bingham Fluid with Heat Transfer

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Abstract

In this paper, we investigate the effects of wall slip conditions, elasticity wall properties and heat transfer on the peristaltic transport of conducting Bingham fluid in a non-uniform channel under the assumptions of long wavelength and low-Reynolds number. The expressions for velocity, stream function, temperature and heat transfer coefficient are obtained. The effects of the physical parameters on the velocity and temperature fields are presented through graphs. The effect of increase in the two $(E_1 \& E_2)$ of the three wall property parameters is to enhance the velocity and temperature in the channel whereas opposite behaviour is noticed regarding the other parameter E_3. Also the size of the trapped bolus gets reduced due to the increase in the magnetic parameter M .

Keywords: Peristaltic transport; MHD Bingham fluid; Slip conditions; Wall properties; Trapping phenomena.

1. Introduction

Peristalsis is a mechanism of fluid transport that occurs due to the propagation of sinusoidal waves across the walls of channel. This phenomenon widely occurs in several industrial and biomedical applications including swallowing of food through esophagus, chyme motion in the gastrointestinal tract, blood circulation in small blood

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doi:10.1016/j.proeng.2015.11.469

vessels, sanitary fluid transport of corrosive fluids, transport of toxic liquid in nuclear industry etc. Many modern mechanical devices are designed through peristaltic pumping for fluid transport without internal moving parts. Most of the physiological fluids (blood, chyme etc.) are observed to be non-Newtonian in nature. Several investigations on the peristaltic transport of non-Newtonian fluids under the assumptions of long wavelength and low Reynolds number have been reported in the literature. Peristaltic flows with heat transfer and MHD have potential applications in the biomedical sciences. Therefore extensive work is reported by several authors such as Srinivasacharya et al. [1], Hayat et al. [2], Mekheimer and Elmaboud [3], Vajravelu et al. [4-7].

Peristaltic flows with heat transfer have important applications in the biomedical sciences. Bio-heat transfer processes include thermoregulation, metabolic heat generation, evaporation, convection, perfusion of blood flow, skin burning, fever and hypothermia in mammals. The magnetohydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain problems of the movement of conductive physiological fluids, such as blood flow in small blood vessels and blood pump machines.

The no-slip boundary condition is widely used for flows involving non-Newtonian fluids past rigid boundaries. However it has been observed that several polymeric fluids slip or stick-slip on solid boundaries. In such cases the flow behaviour corresponds to a slip flow and the fluid presents a loss of adhesion at the wetted wall making the fluid slide along the wall. When the molecular mean free-path length of the fluid is comparable to the distance between the plates (nanochannels or micro channels), the fluid exhibits non-continuum effects such as slip flow. Flows with slip would be useful for the study of problems in chemical engineering, for example, flows through pipes in which chemical reactions occur at the walls, two-phase flows in porous slider bearings.

Consideration of wall properties such as wall stiffness, wall rigidity, wall tension etc. is very important in peristalsis. In particular the increased intensity of such effects can significantly influence the blood pressure in human body. Peristaltic motion in a channel with complaint walls has been discussed by a few researchers previously [8 - 11]. In the present study the effects of both wall slip conditions and heat transfer on peristaltic flow of a conducting Bingham fluid in a non-uniform channel with elastic wall properties have been investigated under the assumptions of long wavelength and low-Reynolds number. The expressions for velocity, temperature, stream function and heat transfer coefficient are obtained. The effects of various physical parameters on velocity and temperature are analyzed through graphs. Further the trapping phenomenon is discussed in detail

2. Mathematical Formulation

We consider the peristaltic flow of a Bingham fluid through a two-dimensional non-uniform channel bounded by flexible walls. The flow is considered to be induced by sinusoidal wave trains propagating along the channel walls with a constant speed c. The wall deformation is given by

$$\bar{\eta}(\bar{x},\bar{t}) = d(x) - a\,Sin\frac{2\pi}{\lambda}\left(\bar{x}-c\bar{t}\right) \quad \text{where} \quad d(x) = d + \bar{m}x,\ \bar{m} << 1 \tag{1}$$

The governing equations which describe the present study are given by

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2}$$

$$\rho\left[\frac{\partial}{\partial \bar{t}} + \bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right]\bar{u} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} - \frac{\partial}{\partial \bar{y}}(\tau_0 - \mu\frac{\partial \bar{u}}{\partial \bar{y}}) - \sigma B_0^2 \bar{u} \tag{3}$$

$$\rho\left[\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right]\bar{v}=-\frac{\partial \bar{p}}{\partial \bar{y}}+\mu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \quad (4)$$

$$\xi\left[\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right]T=\frac{k}{\rho}\left(\frac{\partial^2 T}{\partial \bar{x}^2}+\frac{\partial^2 T}{\partial \bar{y}^2}\right)+\upsilon\left\{2\left[\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2+\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2\right]+\left(\frac{\partial \bar{v}}{\partial \bar{x}}+\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2\right\} \quad (5)$$

where $\bar{u}, \bar{v}, \rho, \mu, p, d, a, \lambda, c, \bar{m}, \xi, \upsilon, k, T, \sigma, B_0$ and τ_0 are the axial velocity, transverse velocity, fluid density, viscosity of the fluid, pressure, mean width of the channel, amplitude, wavelength, wave speed, dimensional non-uniformity of the channel, specific heat at constant volume, kinematic viscosity, thermal conductivity of the fluid, temperature, electrical conductivity of the fluid, applied magnetic field and yield stress.

The equation of motion of the flexible wall is expressed as

$$L^*(\bar{\eta})=\bar{p}-\bar{p_0} \quad (6)$$

where L^* is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

$$L^*=-\tau\frac{\partial^2}{\partial \bar{x}^2}+m_1\frac{\partial^2}{\partial \bar{t}^2}+C\frac{\partial}{\partial \bar{t}} \quad (7)$$

where τ is the elastic tension in the membrane, m_1 is the mass per unit area, C is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to the tension in the muscles and *h* is the dimensional slip parameter. We assumed $p_0=0$.

Continuity of stress at $\bar{y}=\bar{\eta}$ and using *x*- momentum equation, yield

$$\frac{\partial}{\partial \bar{x}}L^*(\bar{\eta})=\frac{\partial \bar{p}}{\partial \bar{x}}=\mu\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}-\frac{\partial}{\partial \bar{y}}\left(\tau_0-\mu\frac{\partial \bar{u}}{\partial \bar{y}}\right)-\sigma B_0^2\bar{u}-\rho\left[\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right]\bar{u} \quad (8)$$

$$\bar{u}=-h\frac{\partial \bar{u}}{\partial \bar{y}} \text{ at } \bar{y}=\bar{\eta}=[d+\bar{m}\bar{x}+a\sin\frac{2\pi}{\lambda}(\bar{x}-c\bar{t})] \quad (9)$$

$$\frac{\partial T}{\partial \bar{y}}=0 \text{ on } \bar{y}=\bar{y}_0,\quad T=T_1 \text{ on } \bar{y}=\bar{\eta} \quad (10)$$

Introducing ψ such that

$$\bar{u}=\frac{\partial \bar{\psi}}{\partial \bar{y}} \text{ and } \bar{v}=-\frac{\partial \bar{\psi}}{\partial \bar{x}}$$

and the following non-dimensional parameters are given by

$$\left.\begin{aligned}&x=\frac{\bar{x}}{\lambda},\ y=\frac{\bar{y}}{d},\ \psi=\frac{\bar{\psi}}{c\,d},\ p=\frac{d^2\bar{p}}{\mu c\lambda},\ t=\frac{c\bar{t}}{\lambda},K=\frac{k}{d^2},\ m=\frac{\lambda\bar{m}}{d},\delta=\frac{d}{\lambda},\varepsilon=\frac{a}{d}\\&y_0=\frac{\bar{y}_0}{d},\eta=\frac{\bar{\eta}}{d}=1+mx+\varepsilon\sin 2\pi(x-t),\mathrm{R}=\frac{\rho cd}{\mu},\theta=\frac{(T-T_0)}{(T_1-T_0)},M=\sqrt{\frac{\sigma}{\mu}}B_0,\\&\mathrm{Pr}=\frac{\rho\upsilon\xi}{k},Ec=\frac{c^2}{\xi(T_1-T_0)},E_1=\frac{-\tau d^3}{\lambda^3\mu c},E_2=\frac{m_1cd^3}{\lambda^3\mu},E_3=\frac{cd^3}{\lambda^2\mu},\beta=\frac{h}{d}\end{aligned}\right\}\tag{11}$$

where R is the Reynolds number, δ and ε are the dimensionless geometric parameters, M is the Hartman number, Pr is the Prandtl number, Ec is the Eckert number, E_1, E_2 *and* E_3 are the dimensionless elasticity parameters, m is the non-uniform parameter and β is the Knudsen number (slip parameter) . Equation (9) describes the fluid slip at the flexible elastic wall.

3. Solution of the problem

Using non-dimensional quantities and applying the long wavelength and low Reynolds number approximation, the basic equations (1) - (10) reduce to

$$0=-\frac{\partial p}{\partial x}-\frac{\partial}{\partial y}\left(\tau_0-\frac{\partial^2\psi}{\partial y^2}\right)-M^2\frac{\partial\psi}{\partial y}\tag{12}$$

$$0=\frac{\partial p}{\partial y}\tag{13}$$

$$0=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}+E\left(\frac{\partial^2\psi}{\partial y^2}\right)^2\tag{14}$$

$$\frac{\partial}{\partial y}\left(-\tau_0+\frac{\partial^2\psi}{\partial y^2}\right)-M^2\frac{\partial\psi}{\partial y}=\left[E_1\frac{\partial^3\eta}{\partial x^3}+E_2\frac{\partial^3\eta}{\partial x\partial t^2}+E_3\frac{\partial^2\eta}{\partial x\partial t}\right]\tag{15}$$

$$\frac{\partial\psi}{\partial y}=-\beta\frac{\partial^2\psi}{\partial y^2}\ \text{ at }\ y=\eta=[1+mx+\varepsilon\sin 2\pi(x-t)]\tag{16}$$

Further, it is assumed that the streamline takes zero value at the line $y = 0$, i.e.

$$\begin{aligned}&\psi_p(0)=0,\\&\psi_{yy}(0)=\tau_0\ \text{ at }\ y=0\\&\psi=\psi_p\ \text{ at }\ y=y_0\end{aligned}\tag{17}$$

$$\frac{\partial\theta}{\partial y}=0\ \text{ at }\ y=y_0,\ \ \theta=1\ \text{ at }\ y=\eta\tag{18}$$

By differentiating equation (12) with respect to y we obtain

$$\frac{\partial^2}{\partial y^2}\left(-\tau_0+\frac{\partial^2\psi}{\partial y^2}\right)-M^2\frac{\partial^2\psi}{\partial y^2}=0\tag{19}$$

By solving equation (19) with boundary conditions (15), (16) and (17) we obtain
The stream function in the plug flow region $(0 \le y \le y_0)$ as

$$\psi_p = \left[\frac{\tau_0}{M} \left(\sinh My_0 - L_1 \cosh My_0 \right) - \frac{A}{M^2} \right] y \tag{20}$$

and corresponding plug flow velocity is given by

$$u_p = \frac{\tau_0}{M} \left(\sinh My_0 - L_1 \cosh My_0 \right) - \frac{A}{M^2} \tag{21}$$

where $y_0 = \frac{1}{M} \tanh^{-1} \left(1/L_1 \right)$, $A = -8 \varepsilon \pi^3 \left[(E_1 + E_2) \cos 2\pi(x-t) - \frac{E_3}{2\pi} \sin 2\pi(x-t) \right]$

and $L_1 = \dfrac{\sinh M\eta + \beta M \cosh M\eta - \dfrac{A}{M^2}}{\cosh M\eta + \beta M \sinh M\eta}$

The stream function in the non-plug flow region $(y_0 \le y \le \eta)$ is given by

$$\psi = \frac{\tau_0}{M^2} \left(\cosh My - L_1 \sinh My \right) - \frac{\left(Ay + B + \tau_0 \right)}{M^2} \tag{22}$$

where $B = \tau_0 \left((My_0 L_1 - 1) \cosh My_0 - (My_0 - L_1) \sinh My_0 - 1 \right)$

The corresponding velocity in the non-plug flow region is given by

$$u = \frac{\tau_0}{M} \left(\sinh My - L_1 \cosh My \right) - \frac{A}{M^2} \tag{23}$$

Using equation (22) in equation (14) subject to the condition (18) we obtain
the temperature as

$$\theta = -Br\tau_0^{\,2} \left[\frac{Cosh2My}{4M^2}(1+L_1^2) + \frac{y^2}{4}(1-L_1^2) - L_1 \frac{Sinh2My}{4M^2} \right] + C_1 y + C_2 \tag{24}$$

where $C_1 = Br\tau_0^{\,2} \left[\frac{Sinh2My_0}{4M}(1+L_1^2) + \frac{y_0}{2}(1-L_1^2) - L_1 \frac{Cosh2My_0}{2M} \right]$

$$C_2 = 1 + Br\tau_0^{\,2} \left[\frac{Cosh2M\eta}{8M^2}(1+L_1^2) + \frac{\eta^2}{4}(1-L_1^2) - L_1 \frac{Sinh2M\eta}{4M^2} \right] - C_1 \eta$$

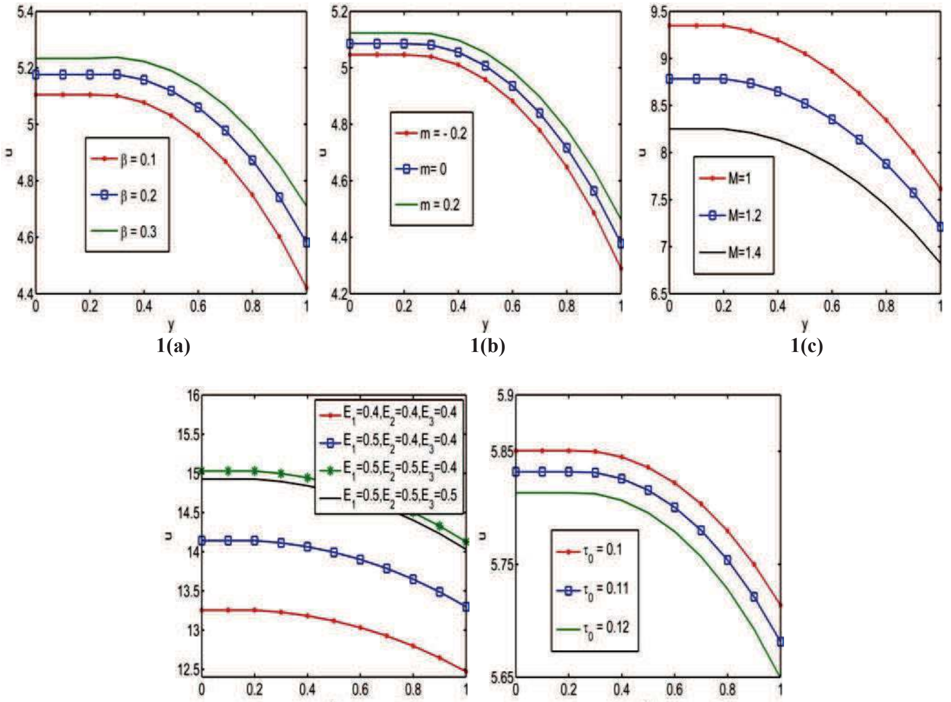
and $Br = Ec\,\Pr$ is the Brinkman number

The coefficient of heat transfer at the wall is given by

$$Nu = -\left(\theta_y \right)_{at\, y=\eta} \tag{25}$$

4. Results and discussions

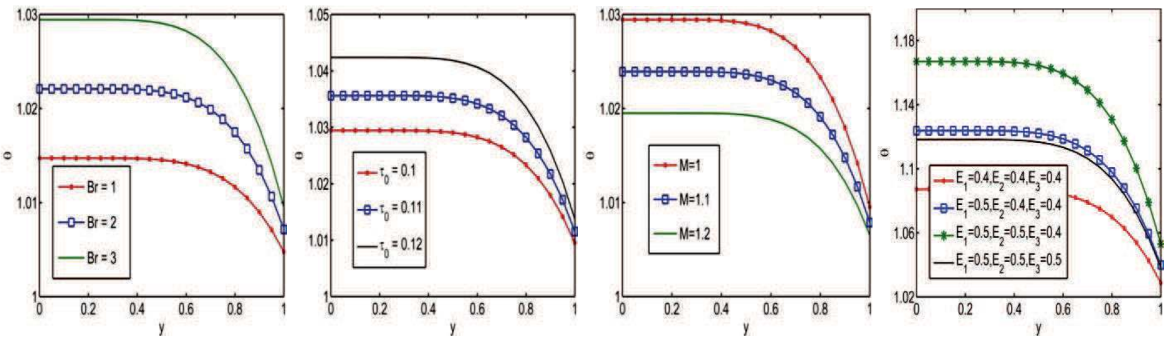
Equation (23) gives the expression for velocity as a function of y. Velocity profiles are plotted in Fig.1 to study the effects of different parameters such as slip parameter β, non-uniform parameter m, magnetic parameter M and yield stress τ_0 etc. on the velocity distribution. We notice that the velocity profiles are parabolic and the velocity increases with increasing β where as it decreases with increasing yield stress τ_0 and magnetic parameter M. Also we observe that the velocity for a divergent channel (m > 0) is higher compared with uniform channel (m = 0) and convergent (m < 0) channels. Further it is noticed that the velocity increases with increasing E_1 and E_2 while it decreases with increasing E_3.



1(a) **1(b)** **1(c)**

1(d) **1(e)**

Fig.1. Velocity profiles for : $x = 0.2,\ t = 0.1, m = 0.1,\ \varepsilon = 0.3, \beta = 0.1, \tau_0 = 0.4, M = 1.5, E_1 = 0.4, E_2 = 0.3, E_3 = 0.2.$

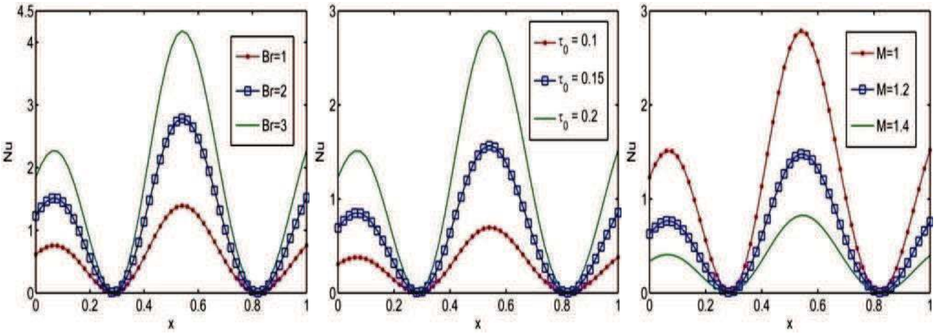


2(a) **2(b)** **2(c)** **2(d)**

Fig.2. Temperature profiles for : $x = 0.2,\ t = 0.1, m = 0.1,\ \varepsilon = 0.3, \beta = 0.1, \tau_0 = 0.4, M = 1.5, Br = 2, E_1 = 0.4, E_2 = 0.3, E_3 = 0.2.$

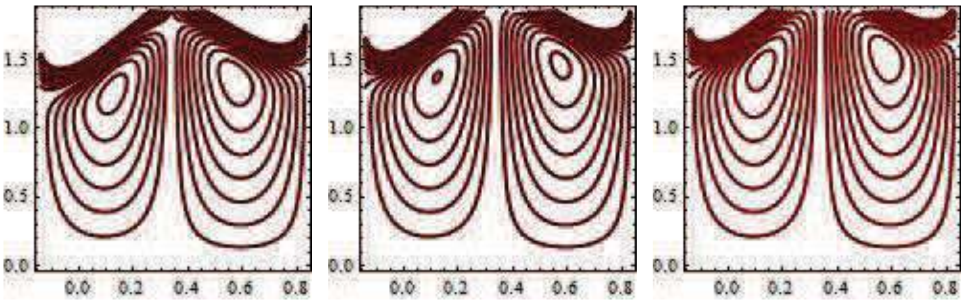
The expression for the temperature in terms of y is given by equation (24). The effect of heat transfer on peristalsis is illustrated in Fig.2. It is observed that the temperature field decreases with the increase of magnetic parameter M. Also it is noticed that the temperature increases with increasing Brinkman number Br and yield stress τ_0. Further we find that the temperature increases with increasing E_1 and E_2 whereas it decreases with increasing E_3.

The variation in coefficient of heat transfer Nu for various values of the interest parameters can be analyzed through Fig.3. It is observed that due to peristalsis, the heat transfer coefficient is in oscillatory behaviour. The absolute value of heat transfer coefficient increases with increase of Br and τ_0 while it decreases with increasing M.



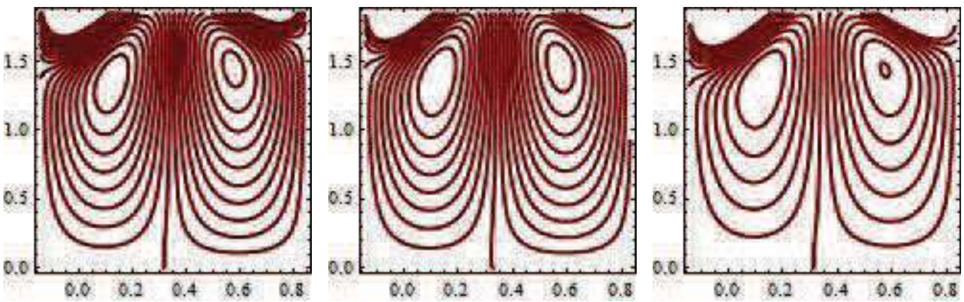
3(a) **3(b)** **3(c)**

Fig.3. The variation of Nusselt number for : $x=0.2$, $t=0.1, m=0.1$, $\varepsilon=0.5, \beta=0.1, \tau_0=0.4, M=1.5, Br=2, E_1=0.4, E_2=0.3, E_3=0.2$.



4(a) 4(b) 4(c)

Fig.4 Stream lines for : $t=1, \varepsilon=0.3, m=0.2, \tau_0=0.8, M=6, E1=0.4, E2=0.2, E3=0.1$, (a) $\beta=0.05$, (b) $\beta=0.1$, (c) $\beta=0.15$



5(a) 5(b) 5(c)

Fig.5 Stream lines for : $t=0.1, \varepsilon=0.3, m=0.2, \tau_0=0.8, \beta=0.05, E1=0.3, E2=0.2, E3=0.1$, (a) $M=5$, (b) $M=6$, (c) $M=7$

5. Trapping phenomenon

Trapping is an interesting phenomenon which refers to closed circulating streamlines that exist at every high flow rates and when occlusions are very large. Streamlines are plotted to study the effect of slip parameter β and magnetic parameter M in figures 4 and 5. We observe that the size of trapped bolus increases with increasing slip parameter where as it decreases with increasing magnetic parameter M.

6. Conclusions

In this work, we study the influence of wall slip, wall properties, yield stress and heat transfer on the peristaltic transport of conducting Bingham fluid in a non-uniform channel under long wavelength and low-Reynolds number approximations. When the flow of polymer melts occurs due to an applied pressure gradient, there is a sudden increase in the throughput at a critical pressure gradient and this phenomenon is known as "spurt". The slip phenomenon has been attributed as the cause of spurt. In view of this, the study of slip on certain non-Newtonian fluid flows is important and hence the present results may have applications in analysis of polymer melts. Some of the interesting observations are summarized as follows:

- The velocity increases with increasing slip parameter β whereas it decreases with increasing yield stress τ_0 and magnetic parameter M.
- We notice that the velocity for a divergent channel is higher compared with uniform channel and convergent channels. Further the velocity increases with increasing E_1 and E_2 while it decreases with increasing E_3.
- It is observed that the temperature field decreases with the increase of magnetic parameter M and it increases with increasing Brinkman number Br. Further the temperature increases with increasing E_1 and E_2 while it decreases with increasing E_3.
- The absolute value of heat transfer coefficient increases with increase of Brinkman number and yield stress while it decreases with increasing Magnetic parameter.
- The size of trapped bolus increases with increasing slip parameter where as it decreases with increasing magnetic parameter.

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