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Unsteady MHD free convective boundary layer flow of a nanofluid past a moving vertical plate

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Abstract. An investigation on hydromagnetic boundary layer flow over a past an exponentially accelerated vertical plate of a nanofluid in presence of the transverse magnetic field. The fluid considered here is a non-scattering medium and the Boussinesq's approximation is used to describe pressure gradient and radiative heat flux in the energy equation. We have considered three types of nanofluids containing Cu- water, TiO_2- water and Al_2O_3 - water. The governing partial differential equations are transformed to dimensionless with suitable dimensional quantities and then employed Laplace transform technique in order to a closed form solution for velocity, temperature and concentration fields without any restriction. The numerical values of nanofluids on the temperature, transverse velocity, skin-friction, and Nusselt number are studies through graphs.

1. Introduction

Nanomaterials are being applied in more and more fields with tremendous applications in the areas of medicine, electronics, and material sciences. Nanoparticles used as a part of nanofluids are normally formed metals (Al, Cu), oxides (Al_2O_3, TiO_2), nitrides (AlN, SiN), and carbides (SiC). Base liquid is normally a conductive liquid; for example, H_2O, engine oil, and $C_2H_6O_2$. Effective thermal conductivity of ethylene glycol is stretched out by up to 40% for a nanofluid including $C_2H_6O_2$ containing around 0.3% vol. Copper nanoparticles of mean width<10nm, analyzed by [3-8]. Examined a two to four fold rise in thermal conductivity development for nanofluid containing TiO_2-water or Al_2O_3-water nanoparticles over a small temperature vary from 21^o-51^oC. An improvement in the thermal conductivity relies on the shape, estimate and thermal attributes of nanoparticles; see [11] and [12]. The existing literature was shown that ensures the enlargement of nanoparticles in the base fluid may accomplish an essentially reducing in the heat transfer; for a comprehensive review, see [16, 17, and 21].

The investigation of Magnetohydrodynamics with heat transfer in the presence of thermal radiation effects have an interest in the consideration of a huge number of researchers because of varying applications in nuclear reactor, for the designs of the fins, steel rolling, manufacturing engineering and various propulsion devices for aircraft, in cooling of reactors in geophysics and astrophysics, it is associated with focus on the stellar and solar structures, radio spread by the ionosphere etc. It is concerned with the interaction of electromagnetic fields and electrically conducting fluids. When a conducting fluid moves through a magnetic field, an electric field and therefore a current may be started, and in this manner, the current interacts with the magnetic field to make a body force on the fluid. Such interactions occur both in nature and in new man-made devices. In the research center, numerous new devices have been made which use the magnetohydrodynamic interaction directly, such as impetus units and power generators or which include liquid electromagnetic field interactions, for example, electrical discharges, MHD pumps, electron beam dynamics, MHD bearing, and traveling wave tubes. The various effects of an unsteady MHD natural convective boundary layer flow of nanofluids past a vertical permeable plate have been studied analytically, for extensive review; see [10, 13, 14, 19, and 24]. From the innovative point of view, magnetohydrodynamic convective flow problems are very significant in the fields of chemical engineering, planetary and stellar magnetospheres and aeronautics [25, 26, and 28].

The objective of the present study is to investigate the hydro-magnetic boundary-layer flow in nanofluids past an exponentially accelerated vertical plate in the transverse magnetic field when the magnetic field fixed with respect to the fluid or to the moving plate despite the fact that this circumstance includes in many engineering application. In this paper, it is considering various types of nanofluids containing Cu-water, TiO_2-water, and Al_2O_3-water. The non-dimensional governing coupled linear PDEs are solved analytically.

2. Mathematical analysis

Consider the unsteady boundary-layer flow past an exponentially accelerated vertical flat plate of a nanofluid within the sight of thermal radiation affected by the magnetic field, when the magnetic field fixed with respect to the fluid or to the plate. The x-axis is taken the plate in the vertically upward direction of the movement, whereas y-axis is taken normal to it. Initially, at the time $t \le 0$, both the surface and adjacent fluid have the same temperature in the static condition for every one of the focuses in whole stream area $y \ge 0$. At the time $t \ge 0$, the plate starts to move in its own plane with the velocity $u_0 f(t')$ in the vertically upward direction against to the gravitational field. At the same time a constant temperature T_w. A magnetic field of uniform strength B_0 is considered to be devoted normal to the direction of the fluid flow. It is considered various types of nanofluids containing Cu-water, TiO_2-water, and Al_2O_3-water. Fluid phase and the suspended nanoparticles are assumed to be in thermal equilibrium. The energy equation the dissipation of viscosity is ignored.

Table 1. Thermophysical properties of water and nanoparticles [20]

Physical properties	Pure water/ base fluid	Cu	Al_2O_3	TiO_2
ρ (kg/m^3)	997.1	8933	3970	4250
c_p (J/kg. K)	4179	385	765	686.2
k (W/m. K)	0.613	401	40	8.9538
$\beta \times 10^{-5}$(K^{-1})	21	1.67	0.85	0.90
$\sigma\,(\Omega\,\text{m})^{-1}$	0.05	5.96×10^{7}	1×10^{-10}	1×10^{-12}

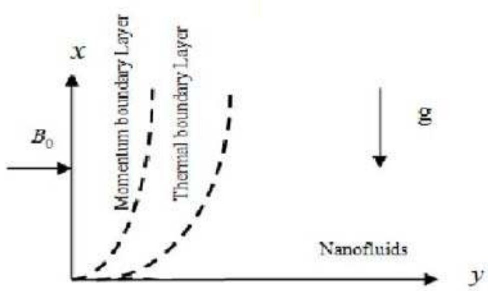

Figure 1. Physical coordinate system

Based on these assumptions, the fluid flow can be represented by the following system of governing equations:

$$\rho_{nf}\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\sigma_{nf}B_0^2u-\frac{\mu_{nf}}{K_1}u \tag{1}$$

$$(\rho c_p)_{nf}\frac{\partial T}{\partial t}=k_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{2}$$

Equation (1) is replaced by (see [1, 22, 27])

$$\rho_{nf}\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\sigma_{nf}B_0^2\left[u-u_0f\left(t'\right)\right]-\frac{\mu_{nf}}{K_1}u \tag{3}$$

Combining (1) and (3)

$$\rho_{nf}\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\sigma_{nf}B_0^2\left[u-K\,u_0f\left(t'\right)\right]-\frac{\mu_{nf}}{K_1}u \tag{4}$$

$$K=\begin{cases}0 & \text{if } B_0 \text{ is fixed relative to the fluid}\\ 1 & \text{if } B_0 \text{ is fixed relative to the plate}\end{cases}$$

where

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\quad \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\ (\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s,\ (\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\phi(\rho c_p)_s,$$

$$\sigma_{nf}=\sigma_f\left[1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right] \tag{5}$$

The effective thermal conductivity of the nanofluid given by [15] and [20] is given by

$$k_{nf}=k_f\left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right] \tag{6}$$

In view Eqs. (1) to (6), respectively.

The initial and boundary conditions are as follows:

$$t\le 0: u=0,\quad T=T_\infty \quad \text{for all } y\ge 0$$

$$t>0: u=u_0f(t'),\quad T=T_w,\quad \text{at } y=0$$

$$u\to 0,\quad T\to T_\infty,\quad \text{as } y\to\infty$$

The radiative heat flux for an optically thick fluid can be found from [23] approximation and its formula is derived from the diffusion concept of radiative heat transfer in the following way

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

$$T^4\cong 4TT_\infty^3-3T_\infty^4 \tag{8}$$

In view Eqs. (7) and (8), Eq. (2) becomes

$$\frac{\partial T}{\partial t}=\frac{1}{(\rho c_p)_{nf}}\left(k_{nf}+\frac{16\sigma^*T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} \tag{9}$$

Introducing non-dimensional variables

$$\eta=\frac{u_0y}{v_f},\ \tau=\frac{u_0^2t}{v_f},\ u_1=\frac{u}{u_0},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ Sc=\frac{v}{D},\ t=\frac{t'u_0^2}{v},,\ a_0=\frac{a'v}{u_0^2},\ k=\frac{vk_l}{u_0^2},\ Gr=\frac{g\beta v(T_w-T_\infty)}{u_0^3} \tag{10}$$

We get the subsequent governing equations which are non-dimensional.

$$\frac{\partial u_1}{\partial \tau} = a_1 \frac{\partial^2 u_1}{\partial \eta^2} + Gra_2\theta - \frac{a_1}{P} - M^2 a_3 (u_0 - Ke^{a_0 t}) \tag{11}$$

$$\frac{\partial \theta}{\partial \tau} = a_4 \frac{\partial^2 \theta}{\partial \eta^2} \tag{12}$$

where $x_1 = \left[(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right]$, $x_2 = \left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]$, $x_3 = \left[(1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]$, $x_4 = \left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right]$,

$$x_5 = \left[1+\frac{3(\frac{\sigma_s}{\sigma_f}-1)\phi}{(\frac{\sigma_s}{\sigma_f}+2)-(\frac{\sigma_s}{\sigma_f}-1)\phi}\right],\quad a_1 = \frac{1}{(1-\phi)^{2.5}x_1},\ a_2 = \frac{x_2}{x_1},\ a_3 = \frac{x_5}{x_1},\ a_4 = \frac{1}{x_3 \Pr}(x_4+Nr) \tag{13}$$

and $M^2 = \frac{\sigma_f B_0^2 v_f}{\rho_f u_0^2}$, is magnetic parameter, $Gr = \frac{g\beta_f v_f (T_w - T_\infty)}{u_0^3}$ the thermal Grashof number, $\Pr = \frac{\mu_f c_p}{k_f}$ is Prandtl number and $Nr = \frac{16\sigma^* T_\infty^3}{3k_f k^*}$ the radiation parameter.

The initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned} &t \le 0: u_1 = 0, \quad \theta = 0, \quad \text{for all } \eta \ge 0 \\ &t > 0: u_1 = \exp(a_0 t), \quad \theta = 1, \quad \text{at } \eta = 0 \\ &u_1 \to 0, \quad \theta \to 0 \quad \text{at } \eta \to 0 \end{aligned} \tag{14}$$

Equations from (11) and (12), subject to the boundary conditions (14) are solved by usual integral transform technique and the solutions are expressed in terms of exponential and complementary error functions.

$$\theta(\eta,\tau) = erfc\left(\frac{\eta}{2\sqrt{a_4\tau}}\right) \tag{15}$$

$$\begin{aligned} u_1(\eta,\tau) &= \frac{(1-a_9)\exp(a_0\tau)}{2}\left[\exp(y\sqrt{b_0})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_0a_1\tau}\right)+\exp(-y\sqrt{b_0})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_0a_1\tau}\right)\right] \\ &+\frac{a_8}{2}\left[\exp(y\sqrt{b_1})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_1a_1\tau}\right)+\exp(-y\sqrt{b_1})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_1a_1\tau}\right)\right] \\ &-\frac{a_8\exp(a_7\tau)}{2}\left[\exp(y\sqrt{b_2})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_2a_1\tau}\right)+\exp(-y\sqrt{b_2})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_2a_1\tau}\right)\right]+a_9\exp(-a_5\tau)erfc(\frac{\eta}{2\sqrt{a_1\tau}}) \\ &-a_8 erfc(\frac{\eta}{2\sqrt{a_4\tau}})+a_9\exp(a_0\tau)+a_9\exp(-a_5\tau) \\ &+\frac{a_8\exp(a_7\tau)}{2}\left[\exp(y\sqrt{b_3})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}+\sqrt{b_3a_1\tau}\right)+\exp(-y\sqrt{b_3})erfc\left(\frac{\eta}{2\sqrt{a_1\tau}}-\sqrt{b_3a_1\tau}\right)\right] \end{aligned} \tag{16}$$

Where

$$a_5 = a_3M^2,\ a_6 = \frac{-Gra_2a_4}{a_1-a_4},\ a_7 = \frac{a_5a_4}{a_1-a_4},\ a_8 = \frac{a_6}{a_7},\ a_9 = \frac{a_5K}{a_0+a_5},\ b_0 = \frac{a_0+a_5}{a_1},\ b_1 = \frac{a_5}{a_1},\ b_2 = \frac{a_5+a_7}{a_1},\ b_3 = \frac{a_7}{a_4}$$

2.1. Skin-friction

From Equation (16), we get skin-friction as follows

$$\tau_x = -\frac{1}{(1-\phi)^{2.5}}\left[\frac{\partial u_1}{\partial \eta}\right]_{\eta=0} = (1-a_9)\exp(a_0\tau)\left(\sqrt{b_0}\,erf\left(\sqrt{b_0a_1\tau}\right)+\frac{e^{-a_1b_0\tau}}{\sqrt{\pi a_1\tau}}\right)$$

$$+a_8\left(\sqrt{b_1}erf\left(\sqrt{b_1a_1\tau}\right)+\frac{e^{-a_1b_1\tau}}{\sqrt{\pi a_1\tau}}\right)-a_8\left(\sqrt{b_2}erf\left(\sqrt{b_2a_1\tau}\right)+\frac{e^{-a_1b_2\tau}}{\sqrt{\pi a_1\tau}}\right)-a_9\frac{e^{-a_5\tau}}{\sqrt{\pi a_1\tau}}+a_8\frac{1}{\sqrt{\pi a_4\tau}}$$
$$+a_8\exp(a_7\tau)\left(\sqrt{b_3}erf\left(\sqrt{b_3a_4\tau}\right)+\frac{\exp(-a_4b_3\tau)}{\sqrt{\pi a_4\tau}}\right) \quad (17)$$

2.2. Nusselt Number
From Equation (15), we get Nusselt number as follows

$$Nu=-\frac{k_{nf}}{k_f}\left[\frac{\partial\theta}{\partial\eta}\right]_{\eta=0}=-x_4\left[\frac{\partial\theta}{\partial\eta}\right]_{\eta=0}=x_4\left[\frac{1}{\sqrt{a_4\pi\tau}}\right] \quad (18)$$

3. Results and Discussion
Equations (11)-(12) with the boundary condition (14) were solved analytically using the Laplace transform technique. To verify the exactness in the obtained solution, we have compared the case with the results when the magnetic field is being fixed to the fluid (K=0) as those reported by [10] are very good in agreement. A part of this study, we have also investigated the case when the magnetic field is being fixed to the plate (K=1). The impacts of various governing dimensionless parameters are examined, namely the magnetic parameter (M^2), radiation parameter (Nr), volume fraction parameter (ϕ), time(τ) and thermal Grashof number (Gr) in transit of flow field, velocity, temperature, skin friction, and Nusselt number are studied graphically shown Figures 2-13.

The behavior of fluid velocity by the influence of nanoparticles, magnetic parameter, time, accelerated parameter, radiation parameter, and volume fraction are shown in Figures 2-7. From Figure 2 it is observed that the velocity distribution for TiO_2–water and Al_2O_3-water are same as their densities, but due to the high density of Cu*,* Cu- water the dynamic viscosity increases more leads to thinner boundary layer than other particles. Figure 3 depicts that the velocity various magnetic parameter. It is observed that the momentum boundary layer thickness increases with increasing value of the magnetic parameter in case of moving plate. Figure 4 displays the effect of ϕ nanoparticles on the fluid velocity, it decreases due to the absence of surface tension forces and hence, the momentum boundary layer thickness decrease. From Figure 5 the fluid velocity increases with the increasing values of radiation parameter Nr. It is noted that the momentum of boundary layer thickness increases when Nr tends to increase inside a boundary layer region and consequently it accelerates the viscosity of the nanofluid. The effects of τ on the velocity field are shown in Figure 6 shows that the velocity of the nanofluid increases with increasing values of τ. Figure 7 reveals that the velocity increases with the increasing values of a_0.

The behavior of fluid temperature by the influence of radiation parameter, Prandtl number and volume fraction are illustrated in Figures 8-10. From Figures 8 and 9 it is seen that the effect of volume fraction ϕ of nanoparticles and radiation parameter Nr on the temperature distribution. Metallic nanoparticles have much higher heat conductivity than common liquids. It also observed that with increasing the volume fraction ϕ of the nanoparticles the thermal boundary layer is increased. It is noticed that the fluid temperature increases as Nr increments due to the fact the conduction effect of the nanofluid increases in the presence of thermal radiation. Figure 10 depicts that the temperature decreases with the increasing values of Pr.

From Figure 11 reveals that the Nusselt number increases with an increasing the values of ϕ. The skin-friction at the plate is presented in Figures 12 - 13. From Figure 12 it is seen that the density of Cu –water nanofluid is higher than the Al_2O_3 and TiO_2 – water nanofluids, the shear stress for Cu – water nanofluid is found to be lower. From Figure 13 it is identified that skin-friction increases with increasing values of ϕ and moreover the momentum boundary layer thickness decreases.

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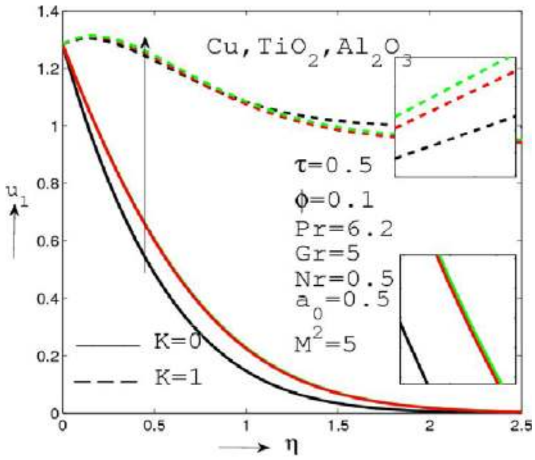

Figure 2. Influence of nanofluids on velocity

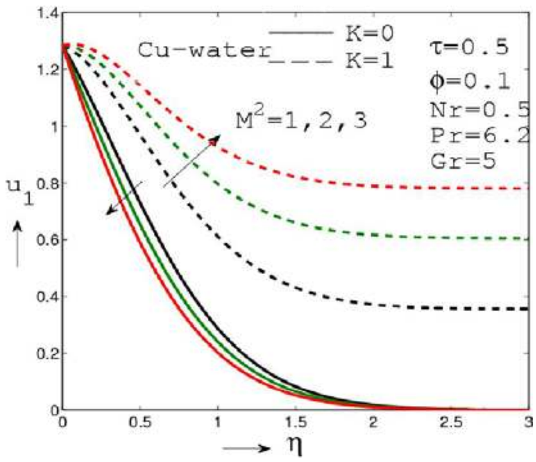

Figure 3. Influence of M^2 on velocity

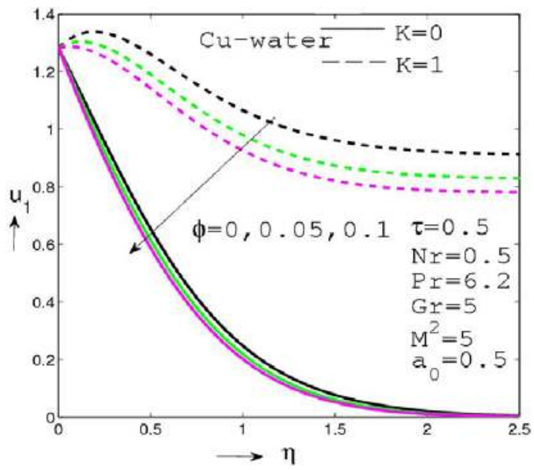

Figure 4. Influence of ϕ on velocity

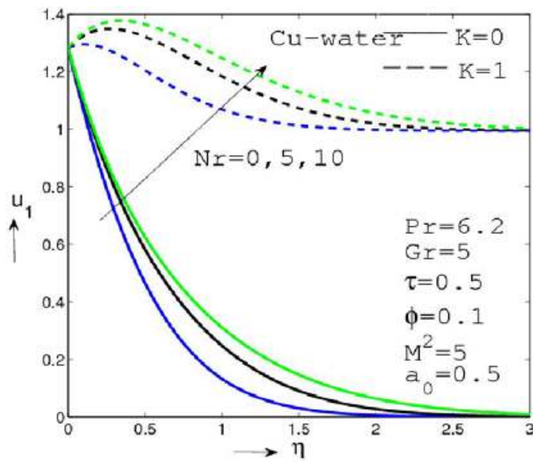

Figure 5. Influence of *Nr* on velocity

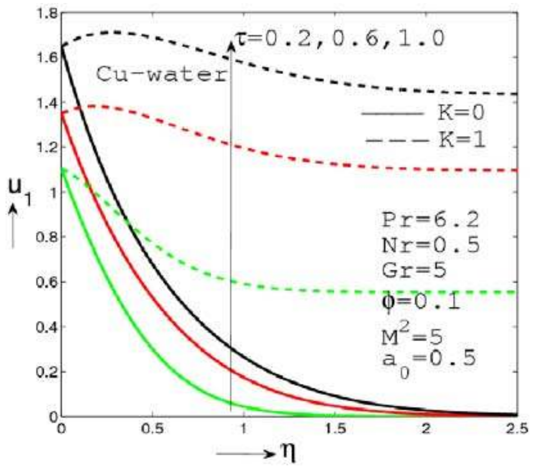

Figure 6. Influence of τ on velocity

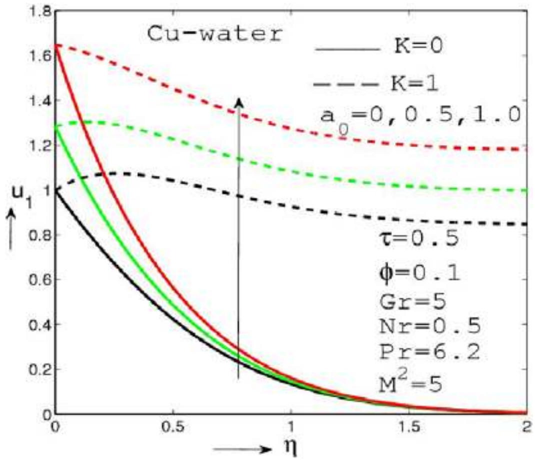

Figure 7. Influence of a_0 on velocity

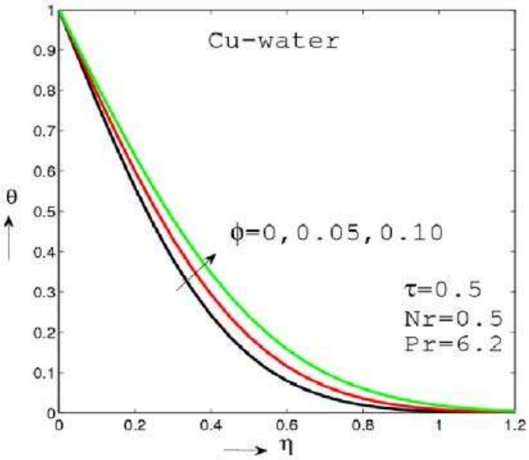

Figure 8. Influence of ϕ on temperature

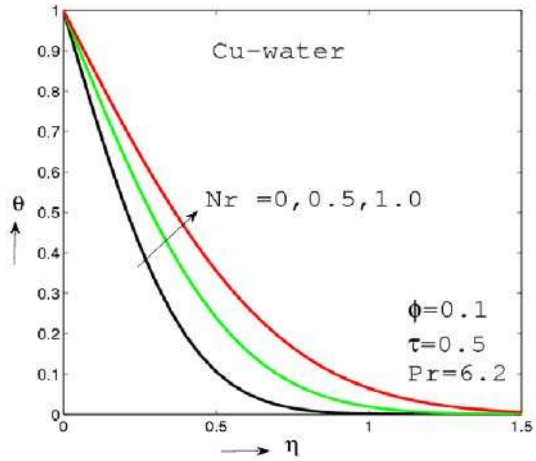

Figure 9. Influence of *Nr* on Temperature

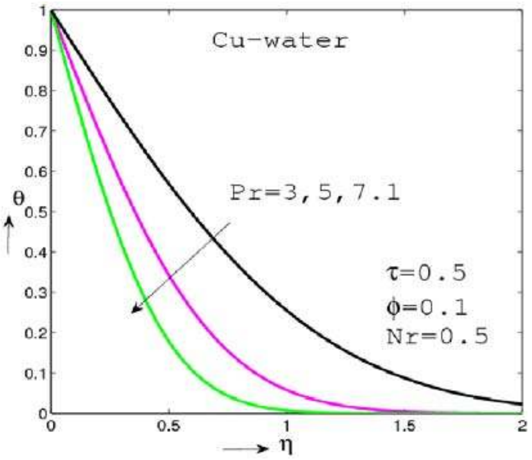

Figure 10. Influence of *Pr* on temperature

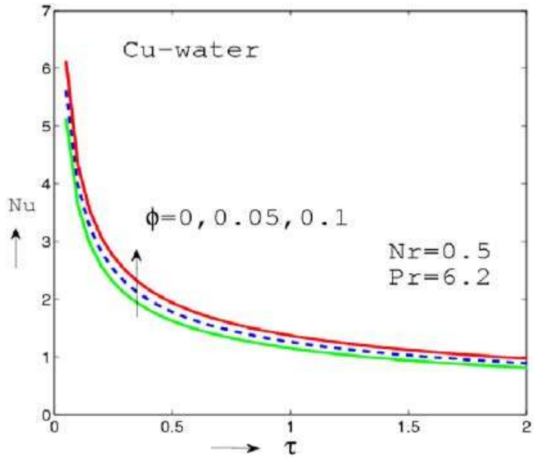

Figure 11. Influence of ϕ on Nusselt number

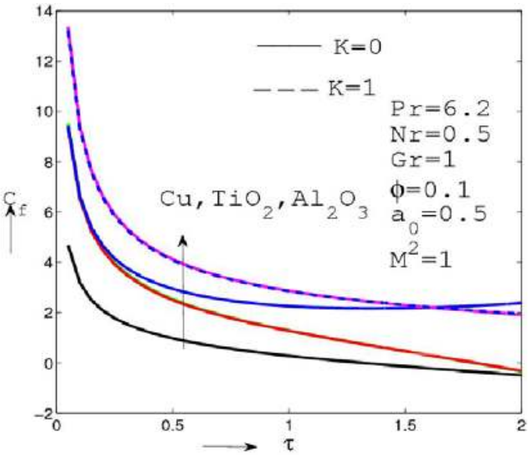

Figure 12. Influence of nanofluids on Skin-friction

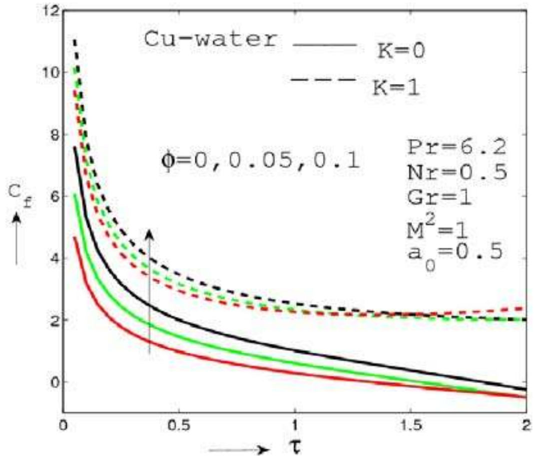

Figure 13. Influence of ϕ on Skin-friction

4. Conclusions

The unsteady natural convective hydromagnetic boundary layer flow over a past an exponentially accelerated vertical plate of a nanofluid in the sight of the transverse magnetic field. The result reveals that the flow characteristics of the concentration, temperature, velocity, the rate of heat transfer and the shear stress at the plate are also discussed. And the results are good in agreement with those studied by [10]. With the above results the following conclusions are made:

- The temperature of nanofluid increases with increasing the values of ϕ .
- The temperature of the nanofluid decreases with increasing *Pr*.
- The skin-friction at the surface of the wall for Cu- water nanofluid is found to be lesser than other particles.

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