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2-Power Domination in Certain Interconnection Networks

R. Sundara Rajan[a], J. Anitha[a,*], Indra Rajasingh[a]

[a]*School of Advanced Sciences, VIT University, Chennai-600127, India*

Abstract

The k-power domination problem generalizes domination and power domination problems. The k-power domination problem is to determine a minimum size vertex set $S \subseteq V(G)$ such that after setting $X = N[S]$ and iteratively adding to X vertices x that have a neighbour v in X such that at most k neighbours of v are not yet in X till we get $X = V(G)$. The least cardinality of such set is called the k-power domination number of G and is denoted by $\gamma_{p,k}(G)$. In this paper, we restrict our discussion to $k = 2$, referred to as 2-power domination. We compute 2-power domination number for certain interconnection networks such as hypertree, sibling tree, X-tree, Christmas tree, mesh, honeycomb mesh, hexagonal mesh, cylinder, generalized Petersen graph and subdivision of graphs.

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Keywords: 2-power domination; hypertree; hexagonal mesh; honeycomb mesh; cylinder; Christmas tree.

1. Introduction

Electric power companies need to continually monitor the state of their systems as in the case of voltage magnitude at loads and machine phase angle at generators. One method of monitoring these variables is to place Phase Measurement Units, called PMUs, at selected locations in the system. The problem is to locate a smallest set of PMUs to monitor the entire system. In electric power system, a vertex represents an electric node and an edge represents a transmission line joining two electrical nodes[1].

A set $S \subseteq V$ is a dominating set in a graph $G(V, E)$ if every vertex in $V \backslash S$ has at least one neighbour in S, that is $N[S] = V$. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of dominating sets of G. The power domination problem is considered as a variation of the dominating set problem. We define a set S to be a power dominating set (PDS) if every vertex in G is observed by S. The k-power domination is a generalization of domination and power domination problems. The k-power domination of G, denoted by $\gamma_{p,k}(G)$, is the minimum cardinality of a k-power dominating set of G. For any graph G, $1 \leq \gamma_{p,k}(G) \leq \gamma_p(G) \leq \gamma(G)$.

Generalized power domination number of any connected graph G of order n, satisfies $\gamma_{p,k}(G) \leq \frac{n}{k+2}$. Also for any claw-free $(k + 2)$-regular graph of order n, $\gamma_{p,k}(G) \leq \frac{n}{k+3}$[2]. Generalized power domination has been well studied for regular graphs[3], Sierpinski graphs[4], trees[1], interval graphs, circular-arc graphs[5], grid[6], claw-free[7], block

* J. Anitha.
E-mail address: anithaharish78@gmail.com

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graphs[8], product graphs[9], cylinder, torus and generalized Petersen graph[10]. Moreover, the power domination number $\gamma_p(G) \leq n/3$ for any graph G of order $n \geq 3$.

Definition 1.1. [1] *For $v \in V(G)$, the open neighbourhood of v, denoted as $N_G(v)$, is the set of vertices adjacent to v; and the closed neighbourhood of v, denoted by $N_G[v]$, is $N_G(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the open neighbourhood of S is defined as $N_G(S) = \bigcup_{v \in S} N_G(v) \backslash S$ and the closed neighbourhood of S is defined as $N_G[S] = N_G(S) \cup S$. For brevity, we denote $N_G[S]$ by $N[S]$.*

Definition 1.2. [1] *For a graph $G(V, E)$, $S \subseteq V$ is a dominating set of G if every vertex in $V \backslash S$ has at least one neighbour in S. The domination number of G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set of G.*

Definition 1.3. [2] *Let $G(V, E)$ be a graph and let $S \subseteq V(G)$. For $k \geq 0$, we define the sets $M^i(S)$ of vertices monitored by S at level i, $i \geq 0$, inductively as follows:*

1. $M^0(S) = N[S]$.
2. $M^{i+1}(S) = \cup\{N[v] : v \in M^i(S)$ *such that* $|N[v] \backslash M^i(S)| \leq k\}$.

Note that $M^i(S) \subseteq M^{i+1}(S) \subseteq V(G)$ for any i. Moreover, every time a vertex of the set $M^i(S)$ has at most k neighbours outside the set, we add its neighbours to the next generation $M^{i+1}(S)$. If $M^{i_0}(S) = M^{i_0+1}(S)$ for some i_0, then $M^j(S) = M^{i_0}(S)$ for any $j \geq i_0$. We thus define $M^\infty(S) = M^{i_0}(S)$.

Definition 1.4. [2] *Let $G = (V, E)$ be a graph, let $S \subseteq V(G)$, and let $k \geq 0$ be an integer. If $M^\infty(S) = V(G)$, then the set S is called a k-power dominating set of G, abbreviated kPD-set. The minimum cardinality of a kPD-set in G is called the k-power domination number of G written $\gamma_{p,k}(G)$.*

In this paper, we restrict our discussion to $k = 2$. In general, the k-power domination problem is NP-complete[2]. In fact, the problem has been shown to be NP-complete even when restricted to bipartite graphs and chordal graphs[1].

2. Main Results

In this section, we solve 2-power domination problem for certain interconnection networks.

A tree is a connected graph that contains no cycles. The most common type of tree is the binary tree. It is so named because each node can have at most two descendants. A binary tree is said to be a complete binary tree if each internal node has exactly two descendants. These descendants are described as left and right children of the parent node. Binary tree are widely used in data structures because they are easily stored, easily manipulated, and easily retrieved. Also many operations such as searching and storing can be easily performed on tree data structures. Furthermore, binary trees appear in communication pattern of divide-and-conquer type algorithms, functional and logic programming, and graph algorithms. A rooted tree represents a data structure with a hierarchical relationship among its various elements.

Definition 2.1. *Let T be the tree formed from a star $K_{1,n}$ and identifying each of its pendant vertices with binary trees; that is T has at most one vertex of degree 4 or more. We call such a tree T an extended spider tree and denote it by T_n^*. The vertex of degree n in $K_{1,n}$ is said to be at level 0 in the extended spider. Its neighbours which are n is number are at level 1. Their descendents are in level 2 and so on.*

Theorem 2.2. *Let G be a tree, then $\gamma_{p,2}(G) = 1$ if and only if G is an extended spider or a binary tree.*

Proof. *Let us assume that G is an extended spider tree T^*. Consider $S = \{v : deg(v) > 3\}$. Then v is the root vertex and $|S| = 1$. Now $M^0(S) = N[S] = \{v, x_i / x_i \text{ is the root of a binary tree}, 1 \leq i \leq n\}$. In other words, $M^0(S)$ contains all the vertices at level 0 and level 1 of the extended spider. Since every vertex at level 1 has exactly two children, $M^1(S)$ includes all the vertices at level 2 and no more. Thus at every step, $M^i(S)$ includes all the vertices in level $i + 1$, $1 \leq i \leq r - 1$ where r is the maximum height of the binary trees identified with vertices of $K_{1,n}$. Thus $M^{r-1}(S) = V(G)$. Therefore, $\gamma_{p,k}(G) = 1$. Further if G is a binary tree, then by Theorem 2.5, $\gamma_{p,k}(G) = 1$.*

Conversely, let us assume that $\gamma_{p,2}(G) = 1$. Let $S = \{v\}$ be a 2-power dominating set of G. Consider $M^0(S) = N[v]$.

For every $u \in M^0(S)$, $|N[u] \backslash M^0(S)| \leq 2$. In other words, every vertex in $N(v)$ has at most 2 descendents. Similarly for $i \geq 1$, for every $u \in M^i(S)$, $|N[u] \backslash M^i(S)| \leq 2$. This implies that G is either a binary tree or an extended spider.

Theorem 2.3. *Let G be a graph and $\gamma_{p,2}(G) = 1$. Then G contains an extended spider T_n^*.*

Proof. *Let $v \in S \subseteq V(G)$. Let $S = \{v\}$ be a 2-power dominating set of G, and $M^0(S) = N[v]$. Then there exists a vertex $u \in M^i(S)$, $i \geq 1$ such that each node in $M^i(S)$, $i \geq 1$ has at most two children and is dominated by $M^i(S)$, $i \geq 1$. Therefore, G contains an extended spider T_n^*.*

Remark 2.4. *The converse of Theorem 2.3 is not necessarily true.*
The graph in Figure 1(a) contains an extended spider tree. But $\gamma_{p,2}(G) \neq 1$ shown in figure 1(b).

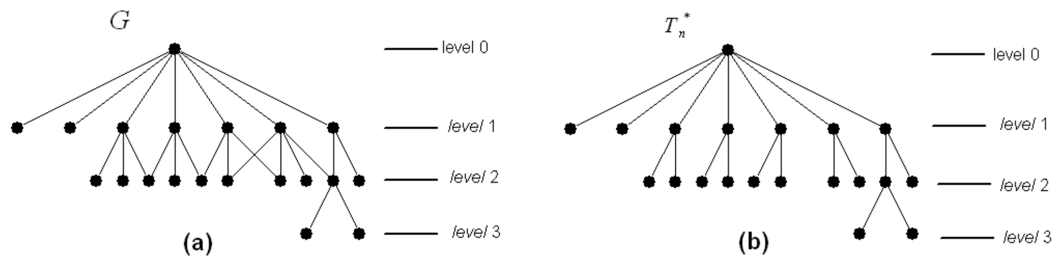

Fig. 1. (a) Graph G (b) Extended spider T_n^*

2.1. Tree Architectures

In 2012, Chang et al.[2] obtained the following results for a connected graph G.

Theorem 2.5. [2] *If G is connected and $\Delta(G) \leq k+1$, then $\gamma_{p,k}(G) = 1$.*

Remark 2.6. *The converse of Theorem 2.5 is not true. That is, $\gamma_{p,k}(G) = 1$ does not necessarily imply that $\Delta(G) \leq k+1$. For example See Figure 1(b).*

Theorem 2.7. *Let G be a binary tree or an extended spider T_n^*. Let G^* be a graph obtained from G by adding new edges joining vertices of the same level of G. Then $\gamma_{p,2}(G^*) = 1$ with the 2-power dominating set S as the singleton set containing the root at level 0 of the extended spider.*

Proof. *The vertex at level 0 of G is a 2-power dominating set of G. As the new edges join vertices at the same level of G, for any vertex u in $M^i(S)$ it is still live that $|N[u] \backslash M^i(S)| \leq 2$. Hence $\gamma_{p,2}(G^*) = 1$.*

Theorem 2.7 generates a number of connected graphs with $\gamma_{p,2}(G) = 1$. Some of the interconnection networks belong to this category. We now heighlight a few such architectures with $\gamma_{p,2}(G) = 1$.

2.1.1. Hypertree

A hypertree is an interconnection topology for incrementally expansible multicomputer systems, which combines the easy expandability of tree structures with the compactness of the hypercube; that is, it combines the best features of the binary tree and the hypercube. These two properties make this topology particularly attractive for implementation of multiprocessor networks of future, where a complete computer with a substantial amount of memory can fit on a single VLSI chip.

Definition 2.8. [11] *The basic skeleton of a hypertree is a complete binary tree T_r. Here the nodes of the tree as numbered as follows: The root node has label 1. The root is said to be at level 0. Labels of left and right children are formed by appending 0 and 1, respectively to the labels of the parent node. Here the children of the nodes x are labeled as $2x$ and $2x+1$. Additional links in a hypertree are horizontal and two nodes in the same level of the tree are joined if their label difference is 2^{i-2}. We denote an r-level hypertree as $HT(r)$. It has $2^{r+1} - 1$ vertices and $3(2^r - 1)$ edges. See Figure 2(a).*

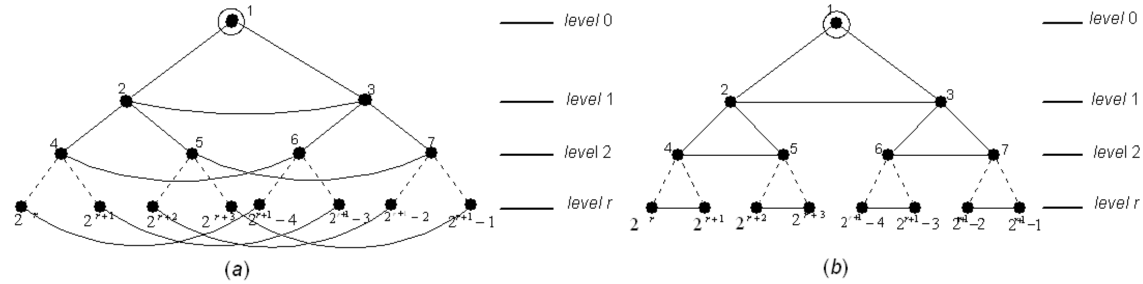

Fig. 2. (a) r-level hypertree $HT(r)$ (b)1-rooted sibiling tree ST_r^1

Theorem 2.9. *Let G be a hypertree $HT(r)$, $r \geq 3$. Then $\gamma_{p,2}(G) = 1$.*

2.1.2. 1- rooted sibiling tree

Definition 2.10. [12] *1-rooted sibling tree ST_r^1 is obtained from the 1-rooted complete binary tree T_r^1 by adding edges (sibiling edges) between left and right children of the same parent node. See Figure 2(b).*

Theorem 2.11. *Let G be 1-rooted sibiling tree ST_r^1, $r \geq 2$. Then $\gamma_{p,2}(G) = 1$.*

2.1.3. X-tree

Definition 2.12. *An X-tree XT_r is obtained from complete binary tree on $2^{r+1} - 1$ vertices of height $2^i - 1$, and adding paths P_i left to right through all the vertices at level i, $1 \leq i \leq r$. See Figure 3(a).*

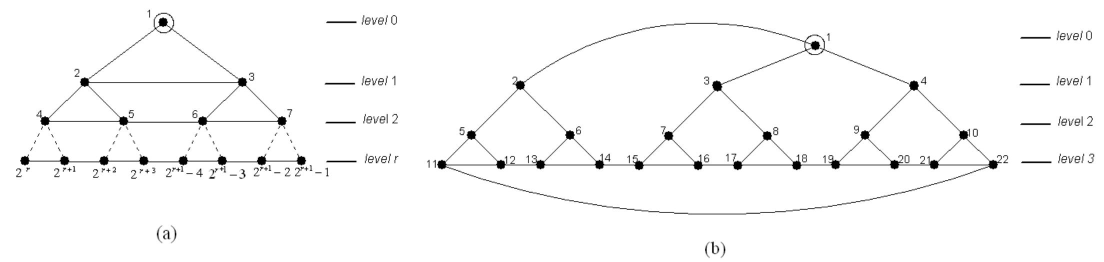

Fig. 3. (a) X-tree XT_r (b) Christmas tree $CT(3)$

Theorem 2.13. *Let G be X-tree XT_r, $r \geq 2$. Then $\gamma_{p,2}(XT_r) = 1$.*

2.1.4. Christmas tree

Definition 2.14. [13] *An s^{th} slim tree $ST(s)$ as $ST(s) = (V, E, u, l, r)$, where V is the node set, E is the edge set, $u \in V$ is the root node, $l \in V$ is the left node,$r \in V$, and $s \geq 2$ is an integer. The sth slim tree $ST(s)$ is recursively define as follows:*

1. *$ST(2)$ is the complete graph K_3 with its nodes labeled with u, l and r .*
2. *The sth slim tree $ST(s)$, with $s \geq 3$ is composed of a root node u and two disjoint copies of $(s-1)$th slim trees as the left sub tree and right sub tree, denoted by $ST^l(s-1) = (V_1, E_1, u_1, l_1, r_1)$ and $ST^r(s-1) = (V_2, E_2, U_2, l_2, r_2)$, respectively, where in particular $u \notin V_1 \cup V_2$. To be specific, $ST(s) = (V, E, u, l, r)$ is given by $V = V_1 \cup V_2 \cup \{u\}$, $E = E_1 \cup E_2 \cup \{(u, u_1), (u, u_2), (r_1, l_2)\}$,$l = l_1$, $r = r_2$.*

Definition 2.15. *The Christmas tree $CT(s)$ is composed of an sth slim tree $ST^l(s) = (V_1, E_1, u_1, l_1, r_1)$ and an $(s+1)$th slim tree $ST^r(s) = (V_2, E_2, u_2, l_2, r_2)$.The node set of $CT(s)$ is $V_1 \cup V_2$ and the edge set of $CT(s)$ is $E = E_1 \cup E_2 \cup \{(u_1, u_2); (l_1, r_2), (l_2, r_1)\}$. See Figure 3(b).*

Theorem 2.16. *Let G be a Christmas tree $CT(s)$, $s \geq 2$. Then $\gamma_{p,2}(G) = 1$.*

2.1.5. Mesh network

One of the most popular architecture is mesh-connected computer, in which processors are being connected by a communication link to its neighbours in up to four directions. It is well known that there are three possible tessellations of a plane with regular polygons of the same kind: square, triangle and hexagonal. They are basis for the designs of direct interconnection networks with highly competitive overall performance [16]. Regular square mesh is applied in military communications, medical monitoring, public service communications, security systems and so on. Hexagonal and honeycomb networks are applied in cellular phone station placement, the representation of benzenoid hydrocarbons, computer graphics and image processing, cylinder network applied in dynamic system and probability [17]

Definition 2.17. [14] *The cartesian product of two graphs G and H, denoted by $G \times H$, is the graph with vertex set $V(G) \times V(H)$. Two vertices (g, h) and (g', h') are adjacent in $G \times H$ if they are equal in one coordinate and adjacent in the other. The graph $P_m \times P_n$ is called $m \times n$ mesh graph, and is denoted by $M(m, n)$, $m, n \geq 2$. See Figure 4.*

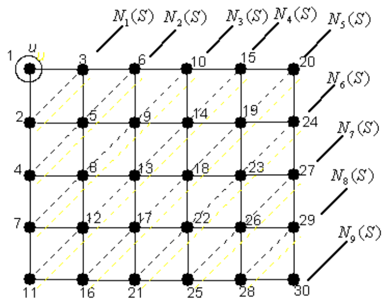

Fig. 4. Mesh network $M(5, 6)$

Theorem 2.18. *Let G be an $M(m, n)$, $m, n \geq 2$ mesh network. Then $\gamma_{p,2}(G) = 1$.*

Proof. *Consider the left most top vertex u and label it 1 as shown in Figure 4. Label the vertex set $N_i(u)$, $1 \leq i \leq m + n - 2$, consecutively from 2 to mn beginning with vertex in the hieghest row to the vertex with the lowest row. Initiate labeling with $i = 1$. The breadth first search tree rooted at a vertex of degree 2, say vertex labelled 1 yields a tree isomorphic to a comb with spine length m and hair length n. Drawn as a rooted tree at 1 we find that vertices on the dotted lines shown in Figure 4 are all at the same level. Vertices of degree 2 in the comb which are degree 4 in the mesh have one edge joined to a vertex one level above and another edge joined to one level below. This implies that, for every $u \in M^i(S)$, $|N[u] \backslash M^i(S)| \leq 2$. Thus $S = \{1\}$ is a 2-power dominating set.*

2.1.6. Hexagonal network

Definition 2.19. *For any n, the triangular grid T_n, is the graph whose vertices are ordered triples (i, j, k) of non-negative integers summing to n, and two vertices are joined by an edge if they agree in one co-ordinate and differ by one in the other two.*

Definition 2.20. [15] *The higher dimensional hexagonal network is generalization of a triangular network. Nodes in a n-dimensional hexagonal network are placed at the vertices of a n-triangular tessalation, so that each node has up to $2k + 2$ neighbours and it is denoted by $HX(n)$. See Figure 5.*

Theorem 2.21. *Let G be an n-dimensional hexagonal network $HX(n)$, $n \geq 2$. Then $\gamma_{p,2}(G) = 1$.*

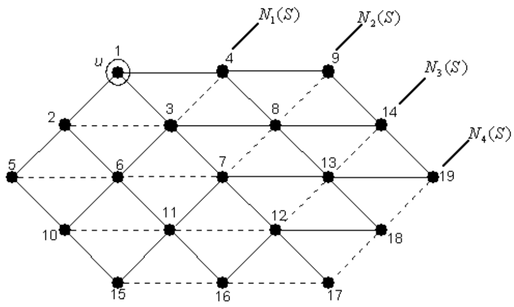

Fig. 5. Hexagonal network $HX(2)$

Proof. *Consider the left most top vertex u and label it* 1 *as shown in Figure* 5*. Label the vertex set* $N_i[S],\ 1 \leq i \leq 2^n$*, consecutively from* 2 *to mn beginning with vertex in the hieghest row to the vertex with the lowest row. Initiate labeling with* $i = 1$*. The breadth first search tree rooted at a vertex of degree* 3*, say vertex labelled* 1 *yields a tree isomorphic to an extended spider with length* $2^n + 1$ *from the root vertex labelled as* 1*. Drawn as a rooted tree at* 1 *we find that vertices on the dotted lines shown in Figure* 5 *are all at the same level. This implies that, for every* $u \in M^i(S),\ |N[u] \backslash M^i(S)| \leq 2$*. Thus* $S = \{1\}$ *is a 2-power dominating set.*

2.1.7. Honeycomb network

Definition 2.22. [16] *For given positive integers m, n such that* $m < n$, $[m, n] = \{m, m+1, ..., n-1, n\}$*. The hexagonal honeycomb mesh of dimension* $n \geq 1$, $n \in Z$, $HC(n)$ *has vertex set* $V(HC_n) = \{(x, y, z) \backslash x, y, z \in [-n+1, n]$ *and* $1 \leq x+y+z \leq 2\}$ *and two vertices* (x_1, y_1, z_1) *and* (x_2, y_2, z_2) *are adjacent if and only if* $|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| = 1$*. See Figure* 6(*a*).

Theorem 2.23. *Let G be an n-dimensional honeycomb network* $HC(n)$, $n \geq 2$*. Then* $\gamma_{p,2}(G) = 1$.

Proof. *Label the vertices of* $HC(n)$, $n \geq 2$*, consecutively from 1 to* $6n^2$ *from left to right beginning with left most vertex labelled as* 1*. The breadth first search tree rooted at a vertex of degree* 2*, say vertex labelled* 1 *yields a tree isomorphic to a binary tree with length* $3n + 1$ *from the root vertex labelled as* 1*. This implies that, for every* $u \in M^i(S),\ |N[u] \backslash M^i(S)| \leq 2$*. Thus* $S = \{1\}$ *is a 2-power dominating set.*

2.1.8. **Cylinder**

Definition 2.24. [17] *The cylinder* $C_m \times P_n$*, where* $m, n \geq 3$ *is a* $P_m \times P_n$ *grid with wraparound edge in each row. It is clear that the vertex set of* $P_m \times P_n$ *is* $V = \{x_1x_2 : 0 \leq x_i \leq d_i - 1, i = 1, 2\}$ *and two vertices* $x = x_1x_2$ *and* $y = y_1y_2$ *are linked by an edge, if* $|x_1 - y_1| + |x_2 - y_2| = 1$*. See Figure* 6(*b*).

Theorem 2.25. *Let G be a cylinder network* $C(m, n)$, $m, n \geq 3$*. Then* $\gamma_{p,2}(G) = 1$.

3. Generating graphs with $\gamma_{p,2}(G) = 1$

Definition 3.1. [19] *The ordinary subdivision graph* $S(G)$ *of the graph G is obtained from G by inserting a new vertex of degree* 2 *on each edge of G. For* $k \geq 1$*, the kth subdivision graph* $S_k(G)$ *is obtained from G by inserting k new vertices of degree* 2 *on each edge of G. Thus,* $S_0(G) \cong G$ *and* $S_1(G) \cong S(G)$*. For* $k \geq 1$, $S_k(G_1 \cup G_2) = S_k(G_1) \cup S_k(G_2)$.

Theorem 3.2. *Let G be a subdivision of graph H . Then* $\gamma_{p,2}(G) = 1$*, where* $\Delta(H) \leq 3$ *or H is isomorphic to* $M(m, n)$ *or H is isomorphic to* $C(m, n)$*.*

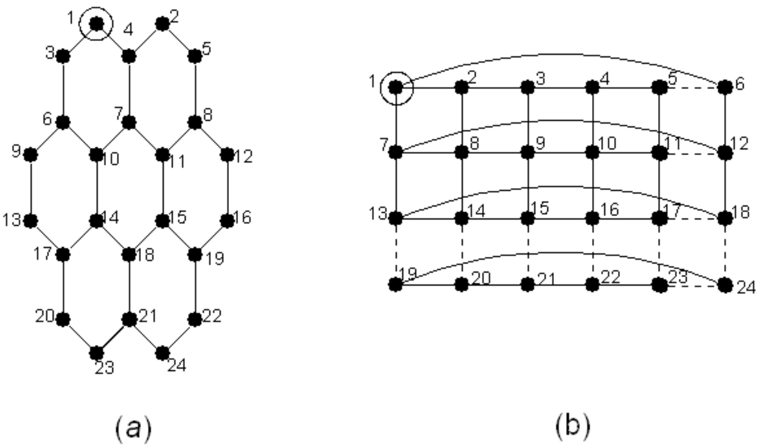

Fig. 6. (a) Honeycomb network $HC(2)$ (b) Cylinder $C(4,6)$

4. Conclusion

In this paper, we have computed 2-power domination number is one for certain interconnection networks.

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