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A Classical Group of Neutrosophic Triplet Groups Using $\{Z_{2p}, \times\}$

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Abstract: In this paper we study the neutrosophic triplet groups for $a \in Z_{2p}$ and prove this collection of triplets $(a, neut(a), anti(a))$ if trivial forms a semigroup under product, and semi-neutrosophic triplets are included in that collection. Otherwise, they form a group under product, and it is of order $(p-1)$, with $(p+1, p+1, p+1)$ as the multiplicative identity. The new notion of pseudo primitive element is introduced in Z_{2p} analogous to primitive elements in Z_p, where p is a prime. Open problems based on the pseudo primitive elements are proposed. Here, we restrict our study to Z_{2p} and take only the usual product modulo $2p$.

Keywords: neutrosophic triplet groups; semigroup; semi-neutrosophic triplets; classical group of neutrosophic triplets; S-semigroup of neutrosophic triplets; pseudo primitive elements

1. Introduction

Fuzzy set theory was introduced by Zadeh in [1] and was generalized to the Intuitionistic Fuzzy Set (IFS) by Atanassov [2]. Real-world, uncertain, incomplete, indeterminate, and inconsistent data were presented philosophically as a neutrosophic set by Smarandache [3], who also studied the notion of neutralities that exist in all problems. Many [4–7] have studied neutralities in neutrosophic algebraic structures. For more about this literature and its development, refer to [3–10].

It has not been feasible to relate this neutrosophic set to real-world problems and the engineering discipline. To implement such a set, Wang et al. [11] introduced a Single-Valued Neutrosophic Set (SVNS), which was further developed into a Double Valued Neutrosophic Set (DVNS) [12] and a Triple Refined Indeterminate Neutrosophic Set (TRINS) [13]. These sets are capable of dealing with the real world's indeterminate data, and fuzzy sets and IFSs are not.

Smarandache [14] presents recent developments in neutrosophic theories, including the neutrosophic triplet, the related triplet group, the neutrosophic duplet, and the duplet set. The new, innovative, and interesting notion of the neutrosophic triplet group, which is a group of three elements, was introduced by Florentin Smarandache and Ali [10]. Since then, neutrosophic triplets have been a field of interest that many researchers have worked on [15–22]. In [21], cancellable neutrosophic triplet groups were introduced, and it was proved that it coincides with the group. The paper also discusses weak neutrosophic duplets in BCI algebras. Notions such as the neutrosophic triplet coset and its connection with the classical coset, neutrosophic triplet quotient groups, and neutrosophic triplet normal subgroups were defined and studied by [20].

Using the notion of neutrosophic triplet groups introduced in [10], which is different from classical groups, several interesting structural properties are developed and defined in this paper. Here, we study the neutrosophic triplet groups using only $\{Z_{2p}, \times\}$, p is a prime and the operation $\times$ is product modulo $2p$. The properties as a neutrosophic triplet group under the inherited operation $\times$

is studied. This leads to the definition of a semi-neutrosophic triplet. However, it has been proved that semi-neutrosophic triplets form a semigroup under $\times$, but the neutrosophic triplet groups, which are nontrivial and are not semi-neutrosophic triplets, form a classical group of neutrosophic triplets under $\times$.

This paper is organized into five sections. Section 2 provides basic concepts. In Section 3, we study neutrosophic triplets in the case of Z_{2p}, where p is an odd prime. Section 4 defines the semi-neutrosophic triplet and shows several interesting properties associated with the classical group of neutrosophic triplets. The final section provides the conclusions and probable applications.

2. Basic Concepts

We recall here basic definitions from [10].

Definition 1. *Consider $(S, \times)$ to be a nonempty set with a closed binary operation. S is called a neutrosophic triplet set if for any $x \in S$ there will exist a neutral of x called neut (x), which is different from the algebraic unitary element (classical), and an opposite of x called anti (x), with both neut (x) and anti (x) belonging to S such that*

$$x * neut\,(x) = neut\,(x) * x = x$$

and

$$x * anti\,(x) = anti\,(x) * x = neut\,(x)\,.$$

The elements x, neut (x), and anti (x) are together called a neutrosophic triplet group, denoted by $(x, neut\,(x)\,, anti\,(x))$.

$neut\,(x)$ denotes the neutral of x. x is the first coordinate of a neutrosophic triplet group and not a neutrosophic triplet. y is the second component, denoted by $neut\,(x)$, of a neutrosophic triplet if there are elements x and $z \in S$ such that $x * y = y * x = x$ and $x * z = z * x = y$. Thus, (x, y, z) is the neutrosophic triplet.

We know that $(neut\,(x)\,, neut\,(x)\,, neut\,(x))$ is a neutrosophic triplet group. Let $\{S, *\}$ be the neutrosophic triplet set. If $(S, *)$ is well defined and for all $x, y \in S$, $x * y \in S$, and $(x * y) * z = x * (y * z)$ for all $x, y, z \in S$, then $\{S, *\}$ is defined as the neutrosophic triplet group. Clearly, $\{S, *\}$ is not a group in the classical sense.

In the following section, we define the notion of a semi-neutrosophic triplet, which is different from neutrosophic duplets and the classical group of neutrosophic triplets of $\{Z_{2p}, \times\}$, and derive some of its interesting properties.

3. The Classical Group of Neutrosophic Triplet Groups of $\{Z_{2p}, \times\}$ and Its Properties

Here we define the classical group of neutrosophic triplets using $\{Z_{2p}, \times\}$, where p is an odd prime. The collection of all nontrivial neutrosophic triplet groups forms a classical group under the usual product modulo $2p$, and the order of that group is $p - 1$. We also derive interesting properties of such groups.

We will first illustrate this situation with some examples.

Example 1. *Let $S = \{Z_{22}, \times\}$ be the semigroup under $\times$ modulo 22. Clearly, 11 and 12 are the only idempotents or neutral elements of Z_{22}. The idempotent $11 \in Z_{22}$ yields only a trivial neutrosophic triplet $(11, 11, 11)$ for $11 \times 21 = 11$, where 21 is a unit in Z_{22}. The other nontrivial neutrosophic triplets associated with the neutral element 12 are $H = \{(2, 12, 6)\,, (6, 12, 2)\,, (4, 12, 14)\,, (14, 12, 4)\,, (16, 12, 20)\,, (20, 12, 16)\,, (12, 12, 12)\,, (10, 12, 10)\,, (8, 12, 18)\,, (18, 12, 8)\}$. It is easily verified that $\{H, \times\}$ is a classical group of order 10 under component-wise multiplication modulo 22, with $(12, 12, 12)$ as the identity element. $(12, 12, 12) \times (12, 12, 12) = (12, 12, 12)$ product modulo 22. Likewise,*

$$(2, 12, 6) \times (2, 12, 6) = (4, 12, 14)\,,$$

$$\begin{gathered} \textit{and}\ (2,12,6)\times(4,12,14)=(8,12,18)\,; \\ (2,12,6)\times(8,12,18)=(16,12,20)\,, \\ \textit{and}\ (2,12,6)\times(16,12,20)=(10,12,10)\,; \\ (10,12,10)\times(2,12,6)=(20,12,16)\,, \\ \textit{and}\ (2,12,6)\times(20,12,16)=(18,12,8)\,; \\ (2,12,6)\times(18,12,8)=(14,12,4)\,, \\ \textit{and}\ (2,12,6)\times(14,12,4)=(6,12,2)\,; \\ (6,12,2)\times(2,12,6)=(12,12,12)\,, \\ \textit{and}\ (2,12,6)^{10}=(12,12,12)\,. \end{gathered}$$

Thus, H is a cyclic group of order 10.

Example 2. *Let* $S = \{Z_{14}, \times\}$ *be the semigroup under product modulo* 14*. The neutral elements or idempotents of* Z_{14} *are* 7 *and* 8*. The neutrosophic triplets are*

$$H = \{(2,8,4)\,,(4,8,2)\,,(6,8,6)\,,(10,8,12)\,,(12,8,10)\,,(8,8,8)\},$$

associated with the neutral element 8*. H is a classical group of order* 6*. Clearly,*

$$\begin{gathered} (10,8,12)\times(10,8,12)=(2,8,4), \\ (10,8,12)\times(2,8,4)=(6,8,6), \\ (10,8,12)\times(6,8,6)=(4,8,2), \\ (10,8,12)\times(4,8,2)=(12,8,10),\ \textit{and} \\ (10,8,12)\times(12,8,10)=(8,8,8). \end{gathered}$$

Thus, H is generated by $(10,8,12)$ *as* $(10,8,12)^6 = (8,8,8)$*, and* $(8,8,8)$ *is the multiplicative identity of the classical group of neutrosophic triplets.*

Example 3. *Let* $S = \{Z_{38}, \times\}$ *be the semigroup under product modulo* 38*.* $19, 20 \in Z_{38}$ *are the idempotents of* Z_{38}*.*

$$\begin{gathered} H = \{(2,20,10)\,,(10,20,2)\,,(4,20,24)\,,(24,20,4)\,,(20,20,20)\,,(8,20,12)\,, \\ (12,20,8)\,,(16,20,6)\,,(6,20,16)\,,(32,20,22)\,,(22,20,32)\,,(18,20,18)\,, \\ (34,20,14)\,,(14,20,34)\,,(26,20,28)\,,(28,20,26)\,,(30,2036)\,,(36,20,30)\} \end{gathered}$$

is the classical group of neutrosophic triplets with $(20,20,20)$ *as the identity element of H.*

In view of all these example, we have the following results.

Theorem 1. *Every semigroup* $\{Z_{2p}, \times\}$*, where p is an odd prime, has only two idempotents: p and* $p+1$*.*

Proof. Clearly, p is a prime of the form $2n+1$ in Z_{2p}.

$$\begin{aligned} p^2 = (2n+1)^2 &= 4n^2+4n+1 \\ &= 4n^2+2n+2n+1 \\ &= 4n^2+2n+p \\ &= 2n\,(2n+1)+p \\ &= 2np+p \\ &= p. \end{aligned}$$

Thus, p is an idempotent in Z_{2p}. Consider $p+1 \in Z_{2p}$:

$$\begin{aligned}(p+1)^2 &= p^2+2p+1\\ &= p^2+1\\ &= p+1 \quad \text{as} \quad p^2 = p.\end{aligned}$$

Thus, p and $p+1$ are the only idempotents of Z_{2p}. In fact, Z_{2p} has no other nontrivial idempotent.

Let $x \in Z_{2p}$ be an idempotent. This implies that x must be even as all odd elements other than p are units.

Let $x = 2n$ (where n is an integer), and $2 < n < p-1$ such that $x^2 = 4n^2 = x = 2n$, which implies that $2n\,(2n-1) = 0$.

This is zero only if $2n - 1 = p$ as $2n-1$ is odd. Otherwise, $2n = 0$, which is not possible, as n is even and n is not equal to 0, $x \neq 0$, so $2n-1 = p$. That is, $x = 2n = p+1$ is the only possibility. Otherwise, $x = 0$, which is a contradiction.

Thus, Z_{2p} has only two idempotents, p and $p+1$. □

Theorem 2. *Let $G = \{Z_{2p}, \times\}$, where p is an odd prime, be the semigroup under $\times$, product modulo $2p$.*

1. *If $a \in Z_{2p}$ has neut (a) and anti (a), then a is even.*
2. *The only nontrivial neutral element is $p+1$ for all a, which contributes to neutrosophic triplet groups in G.*

Proof. Let a in G be such that $a \times neut\,(a) = a$ if a is odd and $a \neq p$. Then a^{-1} exists in Z_{2p} and we have $neut\,(a) = 1$, but $neut\,(a) \neq 1$ by definition. Hence the result is true.

Further, we know $neut\,(a) \times neut\,(a) = neut\,(a)$, that is $neut\,(a)$ is an idempotent. This is possible if and only if $a = p+1$ or p.

Clearly, $a = p$ is ruled out because $ap = 0$ for all even a in Z_{2p}, hence the claim.

Thus, $neut\,(a) = p+1$ is the only neutral element for all relevant a in Z_{2p}. □

Definition 2. *Let $\{Z_{2p}, \times\}$ be the semigroup under multiplication modulo $2p$, where p is an odd prime. $H = \{(a, neut\,(a), anti\,(a))\,|a \in 2Z_{2p} \setminus \{0\}\}$. $\{H, \times\}$ is the collection of all neutrosophic triplet groups. H has the multiplicative identity $(p+1, p+1, p+1)$ under the component-wise product modulo $2p$. H is defined as the classical group of neutrosophic triplets.*

We have already given examples of them. It is important to mention this definition is valid only for Z_{2p} under the product modulo $2p$ where p is an odd prime.

Example 4. *Let $S = \{Z_{46}, \times\}$ be the semigroup under product modulo 46. Let*

$$\begin{gathered} H = \{(24,24,24), (2,24,12), (12,24,2), (4,24,6), (6,24,4), (8,24,26),\\ (26,24,8), (16,24,36), (36,24,16), (32,24,18), (18,24,32), (22,24,22),\\ (10,24,30), (14,24,28), (28,24,14), (30,24,10), (20,24,38), (38,24,20),\\ (34,24,44), (44,24,34), (40,24,42), (42,24,40)\} \end{gathered}$$

be the classical group of neutrosophic triplets, with $(24,24,24)$ as the identity under $\times$. $o\,(H) = 22$.

In view of all of this, we have to define the following for Z_{2p}.

Definition 3. *Let $\{Z_{2p}, \times\}$ be the semigroup under product modulo $2p$, where p is an odd prime. Let $K = \{2, 4, \ldots, 2p-2\}$ be the set of all even elements of Z_{2p}. For $p+1 \in K$, $x \times p+1 = x, \forall\, x \in K$. There also exists a $y \in K$ such that $y^{p-1} = p+1$. We define this y as the pseudo primitive element of $K \subseteq Z_{2p}$.*

Note: We can define pseudo primitive elements only for Z_{2p} where p is an odd prime and not for any Z_n, where n is an even integer that is analogous to primitive elements in Z_p, where p is a prime.

We will illustrate this situation with some examples.

Example 5. *Let* $\{Z_6, \times\}$ *be the modulo semigroup. For* $K = \{2, 4\}$, 2 *is the pseudo primitive element of* $K \subseteq Z_6$.

Example 6. *Let* $\{Z_{14}, \times\}$ *be the modulo semigroup under product* $\times$*, modulo* 14. *Consider* $K = \{2, 4, 6, 8, 10, 12\} \subseteq Z_{14}$. *Then* 10 *is the pseudo primitive element of* $K \subseteq Z_{14}$.

Example 7. *Let* $\{Z_{34}, \times\}$ *be the semigroup under product modulo integer* 34. 10 *is the pseudo primitive element of* $K = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32\} \subseteq Z_{34}$.

Similarly, for $\{Z_{38}, \times\}$, 10 *is the pseudo primitive element of* $K = 2Z_{38} \setminus \{0\} \subseteq Z_{38}$.

However, in the case of Z_{22}, Z_{58}, *and* Z_{26}, 2 *is the pseudo primitive element for these semigroups.*

We leave it as an open problem to find the number of such pseudo primitive elements of $K = \{2, 4, 6, \ldots, 2(p-1)\}$ of Z_{2p}.

We have the following theorem.

Theorem 3. *Let* $S = \{Z_{2p}, \times\}$ *be the semigroup under product modulo* $2p$*, where* p *is an odd prime.*

1. $K = \{2, 4, \ldots, 2p-2\} \subseteq Z_{2p}$ *has a pseudo primitive element* $x \in K$ *with* $x^{p-1} = p+1$*, where* $p+1$ *is the multiplicative identity of* K.
2. K *is a cyclic group under* $\times$ *of order* $p-1$ *generated by that* x*, and* $p+1$ *is the identity element of* K.
3. S *is a Smarandache semigroup.*

Proof. Consider Z_{2p}, where p is an odd prime. Let $K = \{2, 4, 6, \ldots, 2p-2\} \subseteq Z_{2p}$. For any $x \in K$, $(p+1)x = px + x = x$ is$px = 0(mod\ 2p)$, where x is even. Thus, $p+1$ is the identity element of Z_{2p}. There is a $x \in K$ such that $x^{p-1} = p+1$ using the principle of $2p \equiv 0$, where x is even. This x is the pseudo primitive element of K.

This $x \in K$ proves part (2) of the claim.

Since K is a group under $\times$ and $K \subseteq \{Z_{2p}, \times\}$, by the definition of Smarandache semigroup [4], S is an S-semigroup, so (3) is true. □

Next, we prove that the following theorem for our research pertains to the classical group of neutrosophic triplets and their structure.

Theorem 4. *Let* $S = \{Z_{2p}, \times\}$ *be the semigroup. Then*

$$H = \{(a, neut(a), anti(a)) \,|a \in 2Z_{2p} \setminus \{0\}\},$$

is the classical group of neutrosophic triplets, which is cyclic and of the order $p-1$.

Proof. Clearly, from the earlier theorem, $K = 2Z_{2p} \setminus \{0\}$ is a cyclic group of the order $p-1$, and $p+1$ acts as the identity element of K.

$H = \{(a, neut(a), anti(a)) \,|a \in K\}$ is a neutrosophic triplet groups collection and $neut(a) = p+1$ acts as the identity and is the unique element (neutral element) for all $a \in K$.

$(neut(a), neut(a), neut(a)) = (p+1, p+1, p+1)$ acts as the unique identity element of every neutrosophic triplet group h in H.

Since $K \subseteq Z_{2p} \setminus \{0\}$ is a cyclic group of order $p-1$ with $p+1$ as the identity element of K, we have $H = \{(a, neut\,(a)\,, anti\,(a)) \,|a \in K\}$, to be cyclic. If $x \in K$ is such that $x^{p-1} = p+1$, then that neutrosophic triplet group element $(x, p+1, anti(x))$ in H will generate H as a cyclic group of order $p-1$ as $a \times anti(a) = neut(a)$.

Hence, H is a cyclic group of order $p-1$. □

Next, we proceed to describe the semi-neutrosophic triplets in the following section.

4. Semi-Neutrosophic Triplets and Their Properties

In this section, we define the notion of semi-neutrosophic triplet groups and trivial neutrosophic triplet groups and show some interesting results.

Example 8. *Let $\{Z_{26}, \times\} = S$ be the semigroup under product modulo 26.*

We see that $13 \in Z_{26}$ is an idempotent, but $13 \times 25 = 13$, where 25 is a unit of Z_{26}. Therefore, for this 25, we cannot find anti(13), but $13 \times 13 = 13$ is an idempotent, and $(13, 13, 13)$ is a neutrosophic triplet group. We do not accept it as a neutrosophic triplet, as it cannot yield any other nontrivial triplet other than $(13, 13, 13)$.

Further, the authors of [10] defined $(0, 0, 0)$ as a trivial neutrosophic triplet group.

Definition 4. *Let $S = \{Z_{2p}, \times\}$ be the semigroup under product modulo $2p$. $p \in Z_{2p}$ is an idempotent of Z_{2p}. However, p is not a neutrosophic triplet group as $p \times (2p-1) = 2p - p = p$. Hence, $(p, neut(p), anti(p)) = (p, p, p)$ is defined as a semi-neutrosophic triplet group.*

Proposition 1. *Let $S = \{Z_{2p}, \times\}$ be the semigroup under product modulo $2p$. (p, p, p) is the semi-neutrosophic triplet group of Z_{2p}.*

Proof. This is obvious from the definition and the fact $p^2 = p$ in Z_{2p} under product modulo $2p$. □

Example 9. *Let $S = \{Z_{46}, \times\}$ be the semigroup under product modulo 46. $T = \{(23, 23, 23), (0, 0, 0)\}$ is the semi-neutrosophic triplet group and the zero neutrosophic triplet group. Clearly, T is a semigroup under $\times$, and T is defined as the semigroup of semi-neutrosophic triplet groups of order two as $(23, 23, 23) \times (23, 23, 23) = (23, 23, 23)$. $K = \{(a, neut\,(a), anti\,(a))\,|a \in 2Z_{46} \setminus \{0\} = \{2, 4, 6, 8, 10, 12, 14, 16, \ldots, 42, 44\}\}$ is a classical group of neutrosophic triplets.*

Let $P = \langle K \cup T \rangle = K \cup T$. For every $x \in K$ and for every $y \in T$, $x \times y = y \times x = (0, 0, 0)$.

Thus, P is a semigroup under product, and P is defined as the semigroup of neutrosophic triplets.

Further, we define T as the annihilating neutrosophic triplet semigroup of the classical group of neutrosophic triplets.

Definition 5. *Let $S = \{Z_{2p}, \times\}$, where p is an odd prime, be the semigroup under product modulo $2p$. Let $K = \{(a, neut\,(a), anti\,(a))\,|a \in 2Z_{2p} \setminus \{0\}, \times\}$ be the classical group of neutrosophic triplets. Let $T = \{(p, p, p), (0, 0, 0)\}$ be the semigroup of semi-neutrosophic triplets (as a minomer, we call the trivial neutrosophic triplet $(0, 0, 0)$ as a semi-neutrosophic triplet). Clearly, $\langle T \cup K \rangle = T \cup K = P$ is defined as the semigroup of neutrosophic triplets with $o\,(P) = o\,(T) + o\,(K) = p - 1 + 2 = p + 1$.*

Further, T is defined as the annihilating semigroup of the classical group of neutrosophic triplets K.

We have seen examples of classical group of neutrosophic triplets, and we have defined and studied this only for Z_{2p} under the product modulo $2p$ for every odd prime p.

In the following section, we identify open problems and probable applications of these concepts.

5. Discussions and Conclusions

This paper studies the neutrosophic triplet groups introduced by [10] only in the case of $\{Z_{2p}, \times\}$, where p is an odd prime, under product modulo $2p$. We have proved the triplets of Z_{2p} are contributed only by elements in $2Z_{2p} \setminus \{0\} = \{2, 4, \ldots, 2p-2\}$, and these triplets under product form a group of order $p-1$, defined as the classical group of neutrosophic triplets.

Further, the notion of pseudo primitive element is defined for elements $K_1 = 2Z_{2p} \setminus \{0\} = \{2, 4, 6, \ldots, 2p-2\} \subseteq Z_{2p}$. This K_1 is a cyclic group of order $p-1$ with $p+1$ as its multiplicative identity. Based on this,

$$K = \{(a, neut(a), anti(a)) \,|a \in K_1, \times\}$$

is proved to be a cyclic group of order $p-1$.

We suggest the following problems:

1. How many pseudo primitive elements are there in $\{Z_{2p}, \times\}$, where p is an odd prime?
2. Can $\{Z_n, \times\}$, where n is any composite number different from $2p$, have pseudo primitive elements? If so, which idempotent serves as the identity?

For future research, one can apply the proposed neutrosophic triplet group to SVNS and develop it for the case of DVNS or TRINS. These neutrosophic triplet groups can be applied to problems where $neut(a)$ and $anti(a)$ are fixed once a is chosen, and vice versa. It can be realized as a special case of Single Valued Neutrosophic Sets (SVNSs) where neutral is always fixed. For every a in K_1, the other factor $anti(a)$ is automatically fixed, thereby eliminating the arbitrariness in determining $anti(a)$; however, there is only one case in which $a = anti(a)$. The set $2Z_{2p} \setminus \{0\}$ can be used to model this sort of problem and thereby reduce the arbitrariness in determining $anti(a)$, which is an object of future study.

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Abbreviations

The following abbreviations are used in this manuscript:

SVNS	Single Valued Neutrosophic Set
DVNS	Double Valued Neutrosophic Set
TRINS	Triple Refined Indeterminate Neutrosophic Set
IFS	Intuitionistic Fuzzy Set

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
3. Smarandache, F. *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*; American Research Press: Rehoboth, DE, USA, 2005; ISBN 978-1-59973-080-6.
4. Vasantha, W.B. *Smarandache Semigroups*; American Research Press: Rehoboth, MA, USA, 2002; ISBN 978-1-931233-59-4.
5. Vasantha, W.B.; Smarandache, F. *Basic Neutrosophic Algebraic Structures and Their Application to Fuzzy and Neutrosophic Models*; Hexis: Phoenix, AZ, USA, 2004; ISBN 978-1-931233-87-X.
6. Vasantha, W.B.; Smarandache, F. *N-Algebraic Structures and SN-Algebraic Structures*; Hexis: Phoenix, AZ, USA, 2005; ISBN 978-1-931233-05-5.
7. Vasantha, W.B.; Smarandache, F. *Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures*; Hexis: Phoenix, AZ, USA, 2006; ISBN 978-1-931233-15-2.
8. Smarandache, F. Neutrosophic set-a generalization of the intuitionistic fuzzy set. In Proceedings of the 2006 IEEE International Conference on Granular Computing, Atlanta, GA, USA, 10–12 May 2006; pp. 38–42.
9. Smarandache, F. Operators on Single-Valued Neutrosophic Oversets, Neutrosophic Undersets, and Neutrosophic Offsets. *J. Math. Inf.* **2016**, *5*, 63–67. [CrossRef]

10. Smarandache, F.; Ali, M. Neutrosophic triplet group. *Neural Comput. Appl.* **2018**, *29*, 595–601. [CrossRef]
11. Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single valued neutrosophic sets. *Review* **2010**, *1*, 10–15.
12. Kandasamy, I. Double-Valued Neutrosophic Sets, their Minimum Spanning Trees, and Clustering Algorithm. *J. Intell. Syst.* **2018**, *27*, 163–182. [CrossRef]
13. Kandasamy, I.; Smarandache, F. Triple Refined Indeterminate Neutrosophic Sets for personality classification. In Proceedings of the 2016 IEEE Symposium Series on Computational Intelligence (SSCI), Athens, Greece, 6–9 December 2016; pp. 1–8.
14. Smarandache, F. *Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications*, 2nd ed.; Pons Publishing House: Brussels, Belgium, 2017; ISBN 978-1-59973-531-3.
15. Sahin, M.; Abdullah, K. Neutrosophic triplet normed space. *Open Phys.* **2017**, *15*, 697–704. [CrossRef]
16. Smarandache, F. Hybrid Neutrosophic Triplet Ring in Physical Structures. *Bull. Am. Phys. Soc.* **2017**, *62*, 17.
17. Smarandache, F.; Ali, M. Neutrosophic Triplet Field used in Physical Applications. In Proceedings of the 18th Annual Meeting of the APS Northwest Section, Pacific University, Forest Grove, OR, USA, 1–3 June 2017.
18. Smarandache, F.; Ali, M. Neutrosophic Triplet Ring and its Applications. In Proceedings of the 18th Annual Meeting of the APS Northwest Section, Pacific University, Forest Grove, OR, USA, 1–3 June 2017.
19. Zhang, X.H.; Smarandache, F.; Liang, X.L. Neutrosophic Duplet Semi-Group and Cancellable Neutrosophic Triplet Groups. *Symmetry* **2017**, *9*, 275–291. [CrossRef]
20. Bal, M.; Shalla, M.M.; Olgun, N. Neutrosophic Triplet Cosets and Quotient Groups. *Symmetry* **2017**, *10*, 126–139. [CrossRef]
21. Zhang, X.H.; Smarandache, F.; Ali, M.; Liang, X.L. Commutative neutrosophic triplet group and neutro-homomorphism basic theorem. *Ital. J. Pure Appl. Math.* **2017**, in press.
22. Vasantha, W.B.; Kandasamy, I.; Smarandache, F. *Neutrosophic Triplet Groups and Their Applications to Mathematical Modelling*; EuropaNova: Brussels, Belgium, 2017; ISBN 978-1-59973-533-7.