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A comparative study on passive vibration damping in thin carbon nanotube based hybrid composite spherical shell panel

Ashirbad Swain, Tarapada Roy*, Bijoy Kumar Nanda

Department of Mechanical Engineering, National Institute of Technology Rourkela, Rourkela, India, Pin: 769008

Abstract

The present article deals with the vibration problem associated with thin carbon fiber reinforced composite (CFRC) shell structures that can be mitigated by addition of carbon nanotubes (CNTs) in the polymer matrix phase of such CFRC structures. Mori-Tanaka in conjunction with strength of material method has been employed to obtain the elastic properties of such hybrid composite. Shell structure is discretized by eight noded shell element having five degree of freedom at each node where stress resultant-type Koiter's shell theory is applied. Impulse response using Duhamel integral and frequency response analysis have been carried out to analyze the damping phenomena of hybrid composite structure considering Raleigh damping. Results revealed that there are profound influence of carbon nanotube content on the dynamic response of the system. It is also found that the both the damping and stiffness of the structure is increased by the inclusion of carbon nanotubes.

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Keywords: Conventional CFRC structures; CNTs; Mori – Tanaka Method; Finite element; Settling time; Frequency Response;

1. Introduction

Due to the extraordinary mechanical property of carbon nanotube, it can be considered as an effective reinforcement agent for composites. Studies by Thostenson and Chow [1] and Odegard et al. [2] have revealed the effective property of CNT reinforced composite. They showed that vibration in either first mode or multiple modes can be used to determine the elastic properties and damping ratios. Modal testing was done by impulse excitation methods. Kyriazoglou and Guild [3] found damping ratio using experimental methods and by FEM. Hybrid composites contain more than one fiber or one matrix system in a laminate. The first step here is to determine the properties of

*Tarapada Roy. Tel.: +91-661-2462507
E-mail address: tarapada@nitrkl.ac.in

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nanocomposite which is done by using Mori – Tanaka method. Assuming perfect bonding between carbon fibers and nanocomposite matrix, the effective properties of the hybrid composite has been evaluated using mechanics of materials approach. An eight node shell element has been used for the finite element analysis of hybrid composite spherical shell structure. A sixteen layered laminate with stacking sequence [0 -45 45 90]2S has been considered for vibration analysis of simply supported structure.

2. Modelling for material properties

A mathematical model has been applied to obtain the mechanical properties of this hybrid nanocomposite based on the strength of material approach along with the Mori-Tanaka method. Figure 1 shows the various constituents of CNT based hybrid composite.

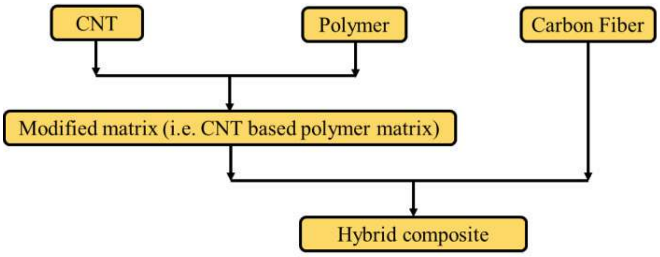

Fig. 1. Constituents of hybrid composite

In first the effective properties of modified matrix (PMNC) have been got by implementing the Mori-Tanaka method then effective properties of the hybrid composite which is a combination of CNT based polymer matrix and carbon fiber is determined using strength of material method. Detailed formulations are presented in the following subsections.

2.1. Modelling for CNT based polymer matrix

The procedure to determine the isotropic properties of randomly oriented straight MWCNTs dispersed in polymer matrix is presented in this subsection. When CNTs are completely randomly oriented in the polymer, the CNT based polymer matrix is then isotropic, and its bulk (K_{nc}) modulus shear modulus (G_{nc}) and the Young's modulus (E_{nc}) are expressed as [4]

$$\left.\begin{aligned} K_{nc} &= K_m + \frac{v_{cnt}^{PMNC}(\delta - 3K_m\alpha)}{3\left(v_m^{PMNC} + v_{cnt}^{PMNC}\alpha\right)} \\ G_{nc} &= G_m + \frac{v_{cnt}^{PMNC}(\eta - 2G_m\beta)}{2\left(v_m^{PMNC} + v_{cnt}^{PMNC}\beta\right)} \\ E_{nc} &= \frac{9K_{nc}G_{nc}}{3K_{nc} + G_{nc}} \\ v_m^{PMNC} &+ v_{cnt}^{PMNC} = 1 \end{aligned}\right\} \tag{1}$$

Where Km and Gm are bulk and shear moduli of polymer, v_m^{PMNC} and v_{cnt}^{PMNC} are the volume fractions of polymer and CNT respectively and the other related parameters can be found from ref. 4.

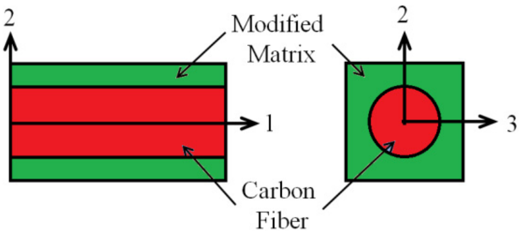

Fig. 2. Representative of hybrid composite

2.2. Modelling for hybrid composite

Figure 2 shows a hexagonal RVE of hybrid composite consisting of carbon fibers oriented in CNT based polymer matrix (modified matrix). Assuming perfect bonding between carbon fiber and modified matrix, the normal strains in hybrid composite, carbon fiber and modified matrix are equal along the fiber direction and the transverse stresses in the same phase are equal along the transverse direction to the fiber from isofield conditions. Based on the principal material co-ordinate (1-2-3) axes as shown in Figure 2, the constitutive relations for the constituent phases of the proposed hybrid composite can be written as

$$\sigma^r = \left[C^r\right] \varepsilon^r \ ; \ r = f, NC \tag{2}$$

Using rule of mixture, the longitudinal and transverse strains, and stresses can be expressed in terms of volume fractions of modified matrix and carbon fiber.

$$v_f + v_{NC} = 1 \tag{3}$$

Where v_f and v_{NC} are volume fractions of carbon fiber and modified matrix respectively in the hybrid composite. Imposing iso-field conditions and rules of mixture for satisfying the perfect bonding conditions between the carbon fiber and modified matrix can be expressed as [5 - 7]

$$\begin{aligned} & \varepsilon_1^f \quad \sigma_2^f \quad \sigma_3^f \quad \sigma_{23}^f \quad \sigma_{13}^f \quad \sigma_{12}^f \quad {}^T = \varepsilon_1^{NC} \quad \sigma_2^{NC} \quad \sigma_3^{NC} \quad \sigma_{23}^{NC} \quad \sigma_{13}^{NC} \quad \sigma_{12}^{NC} \quad {}^T \\ & = \varepsilon_1^{HC} \quad \sigma_2^{HC} \quad \sigma_3^{HC} \quad \sigma_{23}^{HC} \quad \sigma_{13}^{HC} \quad \sigma_{12}^{HC} \quad {}^T \end{aligned} \tag{4}$$

Where NC and HC represents nanocomposite and hybrid composite respectively.

$$\begin{aligned} & v_f \ \sigma_1^f \quad \varepsilon_2^f \quad \varepsilon_3^f \quad \varepsilon_{23}^f \quad \varepsilon_{13}^f \quad \varepsilon_{12}^f \quad {}^T + v_{NC} \ \sigma_1^{NC} \quad \varepsilon_2^{NC} \quad \varepsilon_3^{NC} \quad \varepsilon_{23}^{NC} \quad \varepsilon_{13}^{NC} \quad \varepsilon_{12}^{NC} \quad {}^T \\ & = \sigma_1^{HC} \quad \varepsilon_2^{HC} \quad \varepsilon_3^{HC} \quad \varepsilon_{23}^{HC} \quad \varepsilon_{13}^{HC} \quad \varepsilon_{12}^{HC} \quad {}^T \end{aligned} \tag{5}$$

Relation of stress and strain in the hybrid composite lamina can be found by using the Eqs. 1- 5. The constitutive relations for the proposed Hybrid composite can be expressed as follows.

$$\sigma^{HC} = \left[C^{HC}\right] \varepsilon^{HC} \tag{6}$$

Where $\left[C^{HC}\right]$ is the effective elastic matrix of the proposed hybrid composite and is given by,

$$\left[C^{HC}\right] = C_1\ V_3^{\ -1} + C_2\ V_4^{\ -1} \tag{7}$$

3. Finite element (FE) formulation and analyses

The stress-resultant type Koiter's shell theory [8] has been considered in the present finite element formulation of the hybrid composite shell structures. The effect of shear deformation to the Koiter Shell theory based on the Midlin's hypothesis [9, 10] has also been considered in the present FE formulation.

3.1. Strain displacement relations

Neglecting normal strain component in the thickness direction, the five strain components of a doubly curved shell may be express as

$$\left[\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{xz}\right]^T = \left[\varepsilon_{xx}^0 \quad \varepsilon_{yy}^0 \quad \gamma_{xy}^0 \quad \gamma_{yz}^0 \quad \gamma_{xz}^0\right]^T + z\left[k_{xx} \quad k_{yy} \quad k_{xy} \quad 0 \quad 0\right]^T \tag{8}$$

Where $\varepsilon_{xx}^0, \varepsilon_{yy}^0$ and γ_{xy}^0 is the in-plane strains of the midsurface in the cartesian coordinate system and k_{xx}, k_{yy} and k_{xy} are the bending strains (curvatures) of the midsurface in the cartesian coordinates system. After incorporating the effect of transverse stain in Koiter's shell theory [10], in plane and transverse strain-displacement relations may be expressed as described in the following subsections.

3.2. In-plane/bending strain-displacement matrix

The strain components on the midsurface of shell element are

$$\varepsilon = \left[\varepsilon_{xx}^0 \quad \varepsilon_{yy}^0 \quad \gamma_{xy}^0 \quad k_{xx} \quad k_{yy} \quad k_{xy}\right]^T \tag{9}$$

By using isoparametric 8-noded shell element, the displacement component on the shell midsurface at any point within an element can be expressed as

$$u_0 \quad v_0 \quad w \quad \theta_1 \quad \theta_2 \ ^T = N \quad d^e \tag{10}$$

The mid-surface strains and curvatures from the Koiter's shell theory are

$$\begin{aligned}
\varepsilon_{xx}^0 &= \frac{1}{A_1}\frac{\partial u}{\partial \alpha_1} + \frac{v}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{w}{R_1};\ \varepsilon_{yy}^0 = \frac{1}{A_2}\frac{\partial v}{\partial \alpha_2} + \frac{u}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{w}{R_2} \\
\gamma_{xy}^0 &= \frac{1}{A_1}\frac{\partial v}{\partial \alpha_1} + \frac{1}{A_2}\frac{\partial u}{\partial \alpha_2} - \frac{u}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} - \frac{v}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{2w}{R_{12}} \\
k_{xx} &= \frac{1}{A_1}\frac{\partial \theta_1}{\partial \alpha_1} + \frac{\theta_2}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{1}{2R_{12}}\left(\frac{1}{A_1}\frac{\partial v}{\partial \alpha_1} - \frac{1}{A_2}\frac{\partial u}{\partial \alpha_2} - \frac{u}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{v}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}\right) \\
k_{yy} &= \frac{1}{A_2}\frac{\partial \theta_2}{\partial \alpha_2} + \frac{\theta_1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} - \frac{1}{2R_{12}}\left(\frac{1}{A_1}\frac{\partial v}{\partial \alpha_1} - \frac{1}{A_2}\frac{\partial u}{\partial \alpha_2} - \frac{u}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{v}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}\right) \\
k_{xy} &= \frac{1}{A_1}\frac{\partial \theta_2}{\partial \alpha_1} + \frac{1}{A_2}\frac{\partial \theta_1}{\partial \alpha_2} - \frac{\theta_1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} - \frac{\theta_2}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} - C_0\left(\frac{1}{A_1}\frac{\partial v}{\partial \alpha_1} - \frac{1}{A_2}\frac{\partial u}{\partial \alpha_2} - \frac{u}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{v}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}\right)
\end{aligned} \tag{11}$$

Where $C_0 = 0.5\left[\ 1/R_1\ -\ 1/R_2\ \right]$ and by using 8-noded isoparametric shape functions, the strain components at any point on the shell midsurface can be expressed

$$\varepsilon\ = \left[B_b^e\right]\ d^e \tag{12}$$

3.3. Transverse strain displacement matrix

According to the FSDT, the transverse shear strain vector of a doubly curved shell element may be expressed as

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \begin{Bmatrix}\theta_2 + \dfrac{1}{A_2}\dfrac{\partial w}{\partial\alpha_2} - \dfrac{u}{R_{12}} - \dfrac{v}{R_2}\\ \theta_1 + \dfrac{1}{A_1}\dfrac{\partial w}{\partial\alpha_1} - \dfrac{u}{R_1} - \dfrac{v}{R_{12}}\end{Bmatrix} \tag{13}$$

And hence the transverse shear strain at any point on the shell mid surface can be expressed

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \left[B_s^e\right]\ d^e \tag{14}$$

3.4. Equation of motion

The dynamic finite element formulation has been derived by using the Hamilton's principle as follows

$$\int_{t_1}^{t_2}[\delta(T-U)+\delta W]dt = 0 \tag{15}$$

where t_1 and t_2 defines the time interval. T is the kinetic energy of the system, U is the elastic strain energy and W is the external work done by the force on the structure. After putting the energy expressions in Eq. 15 and taking first variation, the equation of motion can be written as

$$\left[M_{uu}^e\right]\ \ddot{d}^e\ + \left[K_{uu}^e\right]\ d^e\ =\ F^e \tag{16}$$

Mass matrix in its final form can be expressed as

$$\left[M_{uu}^e\right] = \int_{-1}^{1}\int_{-1}^{1}\int_{-h/2}^{h/2} \rho\ N^T\ N\ dz\left|J\right| d\xi d\eta \tag{17}$$

$$F^e\ = \int_A N^T\ \ f_s^e\ \ x, y\ \ dA \tag{18}$$

3.5. Free vibration or modal analysis

After assembling the elemental matrices, for free vibration analysis the equation of motion can be written as

$$M \ddot{d} + K d = 0 \tag{19}$$

The Eq. (19) can be further deduced to an eigenvalue problem as

$$K - \omega^2 M \quad X = 0 \tag{20}$$

Where ω^2 is the eigenvalue and X is the eigenvector of the system.

3.6. Modelling for damping properties of hybrid composite structures

The damping in large systems can be modeled using the Rayleigh damping or proportional damping model. Calculating Rayleigh damping coefficients for large degree of freedom system has been provided with detail in Chowdhury and Dasgupta [11]. Here the C matrix is found such that

$$X^T C X = \begin{bmatrix} \alpha+\beta\omega_1^2 & 0 & \cdots & 0 \\ 0 & \alpha+\beta\omega_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha+\beta\omega_N^2 \end{bmatrix} = \begin{bmatrix} 2\xi_1\omega_1 & 0 & \cdots & 0 \\ 0 & 2\xi_2\omega_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 2\xi_N\omega_N \end{bmatrix} \tag{21}$$

Where X is the eigenvector of the system. α and β are the coefficients to be determined for N simultaneous equations.

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \tag{22}$$

ω_i and ξ_i are i^{th} modal natural frequency and damping ratio respectively. As for any linear system under vibration, only few modes are enough to study its overall dynamics. In the present study first 3% of the total modes are taken as significant mode and 2.5 times of the chosen modes (and beyond the significant modes) are used. The damping ratio of the first and last significant mode is taken as 0.01 and 0.03 respectively after which all the damping ratios either interpolated or extrapolated depending on their sequences for linear interpolation. Then three set of data $\alpha^I, \beta^I, \xi_i^I$, $\alpha^{II}, \beta^I, \xi_i^{II}$ and α, β, ξ_i are found as shown in Figure 5, to get final set of data . After assembly of elemental matrices, the equation of motion of whole structure can be represented as

$$M \ddot{d} + C \dot{d} + K d = F \tag{23}$$

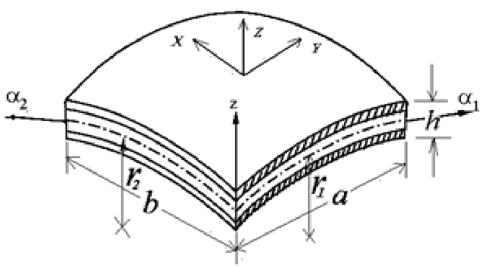

Fig. 3. Geometry of layered composite shell panel in Cartesian coordinate system

4. Results and discussion

The Transient response states that with the increase in carbon nanotube content the settling time decrease. Secondly it came clear that from the frequency response, with increase in carbon nanotube content, the absolute amplitude is decreasing and the peaks are also shifting towards the right.

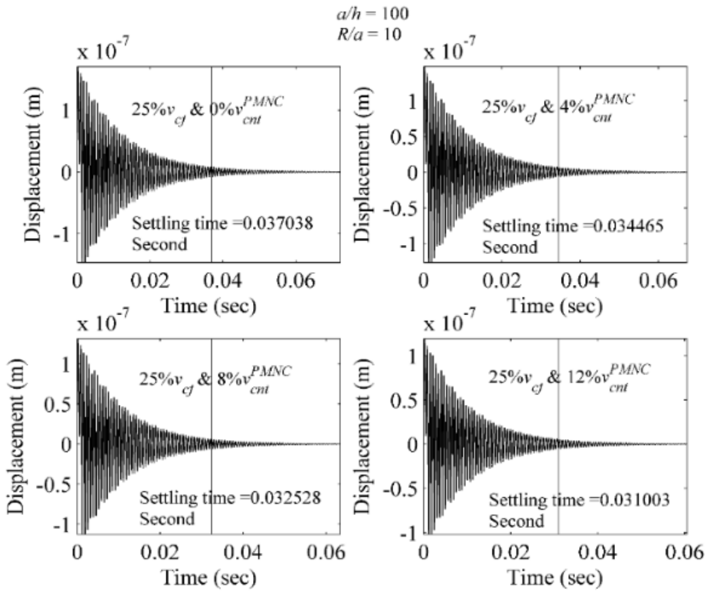

Fig. 4. Transient responses of the composite shell panel for *R/a* = 10 and *a/h* = 100 for a particular v_{cf} with variation of CNTs

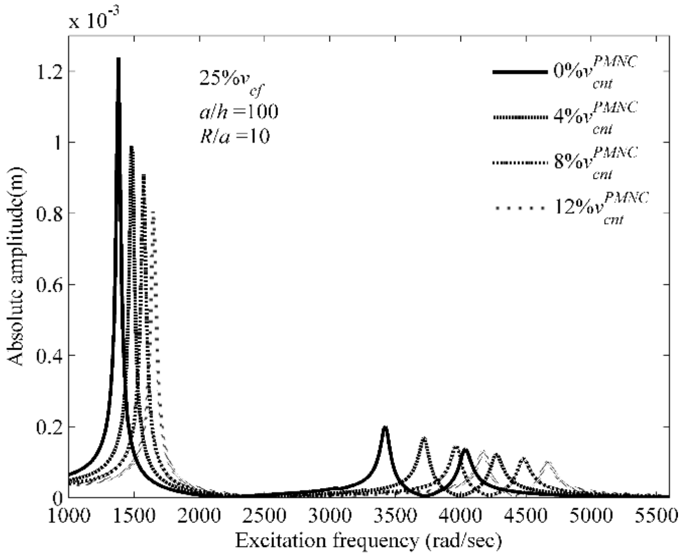

Fig. 5. Frequency responses of the composite shell panel for *R*/*a* = 10 and *a*/*h* = 100 for a particular v_{cf} with variation of CNTs

5. Conclusion

The article presents mitigation of the vibration problem associated with thin CFRC shell structures by addition of CNTs in the polymer matrix phase of such CFRC structures. The effective material properties of the hybrid composite structures have been determined by Mori – Tanaka and strength of material method. From the transient response analysis, it is found that the settling time decreases with the CNTs content. It has also been observed from the frequency response that increases in CNTs content shifts the peaks of amplitude to higher frequency level where as the absolute amplitude drops making the structures operable at higher frequencies than that of conventional CFRC spherical shell structures. Hence it can be concluded that there is a profound effect of CNTs content on the dynamic responses of such shell structures.

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