A Small Signal Analysis of DC-DC Boost Converter

M. Sai Krishna Reddy*, Ch. Kalyani, M. Uthra and D. Elangovan

Power Electronics and Drives, M. Tech, VIT University, Vellore, Tamil Nadu, 632014, India;
saikrishna.mule@gmail.com;

Abstract

This paper presents simulation and small signal analysis of dc-dc boost converter with closed loop control. Small signal model of the boost converter used to analyze the small deviations around the steady-state operating point which will help in modeling the closed loop converter parameters. Complete state-space analysis is done to obtain output voltage to duty ratio transfer-functions for both ideal and Non-ideal boost converter. PI controller is designed using root locus plots for both ideal and Non-ideal cases. The model of the converter is designed and simulated for both the cases with closed loop voltage mode control providing load disturbance using MATLAB. Results are observed and compared for both ideal and Non-ideal cases.

Keywords: DC-DC Boost Converter, PI Controller, Small Signal Model (SSM), Voltage Mode Control

1. Introduction

All Switching DC-DC converters have been widely used to supply a stable DC output voltage for the industrial products. The conventional open loop control strategy used for DC-DC converter with no load and line regulation has several disadvantages compared to closed loop control of switching converter[1,3]. Closed control more-over helps in reducing ripple in voltages too. It is very important to know load changes time to time and responding to that changes is quickly is one of the essential requirement for a reliable converter products. For the closed loop control of any switching DC-DC converter one need to model the system for modeling to obtain Small Signal Model (SSM) of the converter is essential. Using small signal analysis we can achieve complete transfer functions of the given DC-DC converter[4,7]. More-over small signal analysis helps in identifying the deviation around steady state operating point which helps in ripple reduction and achieving more stable and regulated output voltage. Small signal analysis for ideal case DC-DC boost converter has given in literature but for Non-ideal case including all the resistance and diode drops is taken into consideration and modeled the system transfer functions. Comparison of duty variation for both ideal and Non-ideal cases is shown using theoretical expressions.

In this paper, both ideal and Non-ideal DC-DC boost converter are taken with certain assumptions. Transfer functions are obtained for both ideal and Non-ideal converters using small signal model (SSM) of the converters. Closed loop PI controller is designed for both the cases using Root locus technique[6]. Finally script code is written for obtaining root locus for the given transfer functions using MATLAB and simulations are done for different cases using MATLAB Simulink to validate the controller response for step changes in load.

---

* Author for correspondence

2. Small Signal Analysis of the Ideal Boost Converter

The required circuits to analyze the ideal case boost converter are formed as ideal boost converter circuit along with its ON state and OFF state equivalents which are shown in Figures 1(a), 1(b), 1(c).
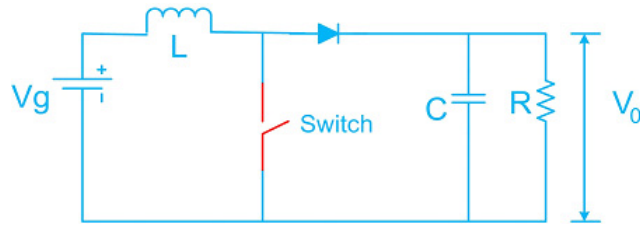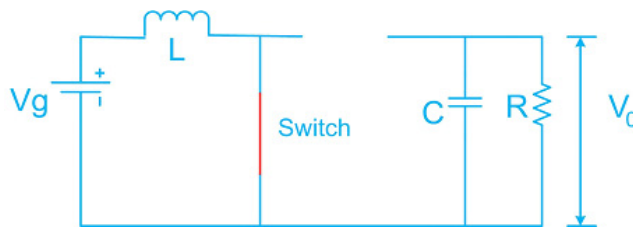

Figure 1(a). Ideal boost converter circuit diagram.

Figure 1(b). Ideal boost converter switch ON-state equivalent circuit diagram.

Figure 1(c). Ideal boost converter switch OFF-state equivalent circuit diagram.

Following assumptions are taken into consideration for Ideal case boost converter SSM.

- Inductance L is large enough to consider current through it is constant.
- All components are ideal.
- The switching period is T, and the switch is closed for time DT and open for (1-D) T.
- The capacitor is very large, and the output voltage is held constant.
-

$$\begin{bmatrix} L\frac{di_l(t)}{dt} \\ C\frac{dV(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R} \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} V_g(t) \\ 0 \end{bmatrix} \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} L\frac{di_l(t)}{dt} \\ C\frac{dV(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -\frac{1}{R} \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} V_g(t) \\ 0 \end{bmatrix} \tag{3}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} \tag{4}$$

Both ON-state and OFF-state equations can be averaged by using state-space averaging technique. Generally state space representation of state variables is shown below

$$\dot{X} = Ax + Bu \quad \dot{X} = Ax + Bu \tag{5}$$

$$y = Cx + Du \quad y = Cx + Du \tag{6}$$

Where in most of the analysis we assume D is almost equal to zero. Using above representation we have A1, B1, C1 matrices from ON state dynamic equations and A2, B2, C2 matrices from OFF state dynamic equations. Now state-space averaging is done using following steps.

$$A = A1 * D + A2 * D'$$

$$B = B1 * D + B2 * D'$$

$$C = C1 * D + C2 * D'$$

After performing above steps A, B, C matrices and complete dynamic equations in state space representation obtained as follows

$$A = \begin{bmatrix} 0 & -D' \\ D' & -\frac{1}{R} \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; c = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} L\frac{di_l(t)}{dt} \\ C\frac{dV(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -D' \\ D' & -\frac{1}{R} \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}V_g(t) \tag{7}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} \tag{8}$$

We can obtain the steady state solution by putting $\dot{X} = 0$ and using relation $X = A^{-1} \ast B \ast U$, then equation (7) relation between output voltage in terms of duty is expressed as

$$V = \frac{V_g}{1 - D} \tag{9}$$

Now To obtain linear model which is easier to analyze a small signal model has been constructed and that is linearised about steady state operating point to get required transfer functions.
Linearization is done by using the below expression.

$$\hat{X} = A\hat{x} + B\hat{u} + \{(A_1 - A_2)X + (B_1 - B_2)U\}\hat{d}$$

$$A1 - A2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \quad B_1 - B_2 = 0$$

$$\begin{bmatrix} L\frac{d\hat{i}_l(t)}{dt} \\ C\frac{d\hat{V}(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -D' \\ D' & -\frac{1}{R} \end{bmatrix}\begin{bmatrix} \hat{i}_l(t) \\ \hat{V}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} \widetilde{V_g}(t) \\ 0 \end{bmatrix}\begin{bmatrix} V \\ -I \end{bmatrix}\hat{d} \tag{10}$$

On solving above equation (10) using Laplace and inverse Laplace transforms and making $\widehat{v_g}(s)$ is equal to zero one can get the transfer function between control input 'd' to output voltage 'V' is given as equation (11)

$$\frac{\hat{v}(s)}{\hat{d}(s)} = \frac{-\frac{I}{C}\left[s - \frac{D' \ast V}{I \ast L}\right]}{\left[s^2 + \frac{s}{c \ast R} + \frac{D'^2}{L \ast C}\right]} \tag{11}$$

3. Small Signal Analysis of the Non-Ideal Boost Converter

Following assumptions are taken into consideration for Non-ideal case boost converter SSM.

- Inductance L is large enough to consider current through it is constant.
- For Input Source inductance resistance (R), Switch resistance (R_{on}) and diode drop (V_{DD}) has taken into consideration.
- The switching period is T, and the switch is closed for

time DT and open for (1-D) T.
- The capacitor is very large, and the output voltage is held constant.

The required circuits to analyze the Non-ideal case boost converter are formed as Non-ideal boost converter circuit along with its ON state and OFF state equivalents which are shown in Figues 23(), 23(), 23(c).
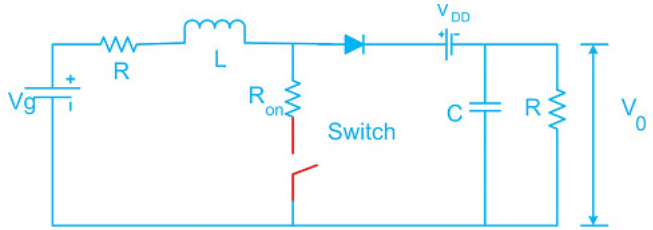ON state dynamic equations:

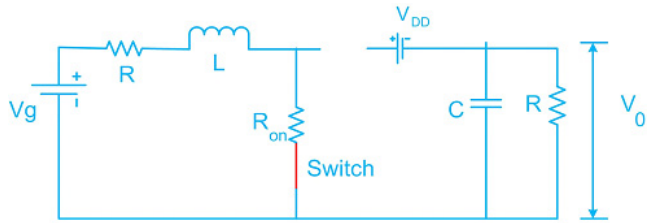**Figure 2(a).** Non-ideal boost converter circuit diagram.

Figure 2(b). Non-ideal boost converter ON-state equivalent circuit diagram.
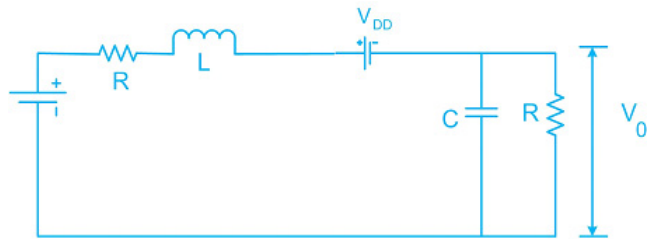

Figure 2(c). Non-ideal boost converter OFF-state equivalent circuit diagram.

$$\begin{bmatrix} L\frac{di_l(t)}{dt} \\ C\frac{dV(t)}{dt} \end{bmatrix} = \begin{bmatrix} (-R - R_{on}) & 0 \\ 0 & -\frac{1}{R} \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_g(t) \\ V_{DD} \end{bmatrix} \tag{12}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} \tag{13}$$

OFF-state dynamic equations:

$$\begin{bmatrix} L\dfrac{di_l(t)}{dt} \\ C\dfrac{dV(t)}{dt} \end{bmatrix} = \begin{bmatrix} -R & -1 \\ 1 & -\dfrac{1}{R} \end{bmatrix} \begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_g(t) \\ V_{DD} \end{bmatrix} \tag{14}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} \tag{15}$$

Using standard state space representation we have A1, B1, C1 matrices from ON state dynamic equations and A2, B2, C2 matrices from OFF state dynamic equations. Now state-space averaging is done using following steps.

$$A = A1 * D + A2 * D'$$

$$B = B1 * D + B2 * D'$$

$$C = C1 * D + C2 * D'$$

After performing above steps A, B, C matrices and complete dynamic equations in state space representation obtained as follows,

$$A = \begin{bmatrix} -(R + Ron*D) & -D' \\ D' & -\dfrac{1}{R} \end{bmatrix}; B = \begin{bmatrix} 1 & -D' \\ 0 & 0 \end{bmatrix}; c = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} L\dfrac{di_l^{\square}(t)}{dt} \\ C\dfrac{dV^{\square}(t)}{dt} \end{bmatrix} = \begin{bmatrix} -R - Ron*D & -D' \\ D' & \dfrac{1}{R} \end{bmatrix} \begin{bmatrix} i_l^{\square}(t) \\ V^{\square}(t) \end{bmatrix} + \begin{bmatrix} 1 & -D' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_g^{\square}(t) \\ \widehat{V_{DD}(t)} \end{bmatrix} + \begin{bmatrix} -Ron*I + V + V_{DD} \\ -I \end{bmatrix} d^{\square \square} \tag{19}$$

On solving above equation (19) using Laplace and inverse Laplace transforms and making $\widehat{v_g}(s)$ $\widehat{v_g}(s)$ is equal to zero one can get the transfer

$$\begin{bmatrix} L\dfrac{di_l(t)}{dt} \\ C\dfrac{dV(t)}{dt} \end{bmatrix} = \begin{bmatrix} -(R + Ron*D) & -D' \\ D' & -\dfrac{1}{R} \end{bmatrix} \begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} + \begin{bmatrix} 1 & -D' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Vg(t) \\ V_{DD} \end{bmatrix} \tag{16}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} i_l(t) \\ V(t) \end{bmatrix} \tag{17}$$

We can obtain the steady state solution by putting $\dot{x} = 0$ and using relation $X = A^{-1} * B * U$, then equation (16) relation between output voltages in terms of duty is expressed as,

$$V = \dfrac{R * \left[-D' * \left(V_g(t) - D' * V_{DD} \right) \right]}{R + R_{on} * D + D'^2} \tag{18}$$

Now to obtain linear model which is easier to analyze a small signal model has been constructed and that is linearised about steady state operating point to get required transfer functions.

Linearization is done by using the below expression.

$$\widehat{X} = A\widehat{x} + B\widehat{u} + \{(A_1 - A_2)X + (B_1 - B_2)U\}\widehat{d}$$

$$A1 - A2 = \begin{bmatrix} -R_{on} & 1 \\ -1 & 0 \end{bmatrix}$$

$$B_1 - B_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

function between control input 'd' to output voltage 'V' is given as equation (20).

$$\dfrac{\widehat{v}(s)}{\widehat{d}(s)} = \dfrac{(V + V_{DD} - I * R_{on}) * D' - I * (L * s + R + R_{on} * D)}{C * s(L * s + R + R_{on} * D) + D'^2 + \dfrac{[(R]_{on} * D + R + L * s)}{R}} \tag{20}$$

The graph between duty cycle 'D' Vs Gain (V/V$_g$) is shown in Fig 2(d)3.4 from which one can observe that ideal converter output voltage curve tends to infinity

when duty cycle value tends towards unity where as in non-ideal case output voltage curve tends to zero this is because of D' component in steady state expression.
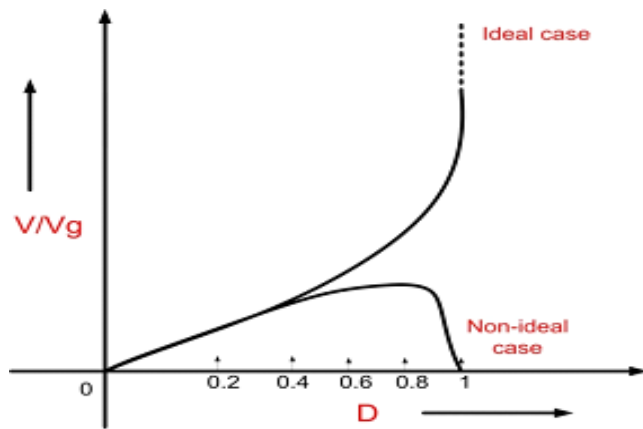
Figure 2(d). Duty cycle Vs gain graph for both ideal and non-ideal cases.

4. PI Controller Design using Root Locus

PI controller parameters are designed using root locus plots, in MATLAB editor window script was written to produce root locus plot based on SSM and from obtained root locus plot with specific iterations closed loop pole points are located and corresponding gain of control structure was calculated. Obtained root locus plot is shown in igure 34(a) below.
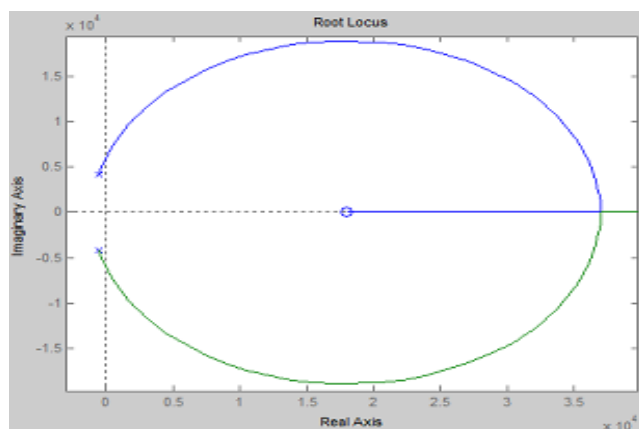

Figure 3(a). Root locus plot of the boost converter.

Continuous looping statement is used to make iterations. And for different step responses controller gain are calculated. After arriving best step response with less steady state error corresponding gains are taken into consideration for controller design.

5. Simulation Results for Different Cases

The boost converter with both ideal and non-ideal cases is simulated with closed loop PI controller action. Control logic is modeled based on duty ratio formulae and it is used to generate pulses for MOSFET. To maintain load regulation and to observe the controller response disturbance in load is created using step input. Ideal boost converter simulink schematic output voltage and current responses are shown in Fgures 45a) to 45(c), also Non-ideal boost converter simulink schematic output voltage and current responses are shown in Fgures 45d) to 45(f).

From the simulation we can observe that there is a voltage response change in both cases after certain time when disturbance created and it is quickly settling to steady state value. Also for Non-ideal case there is a dip in output voltage compared to ideal case response due to resistance and diode drops. Both ideal and Non-ideal cases are simulated for same parameters.

Also we can observe there is increment in Load current during disturbance and it is also quickly settled for the given PI Controller design. Current waveforms are shown in Fgures 45() and 45(f). For Non-ideal case capacitor Equivalent Series Resistor (ESR) is not included for less complexity.
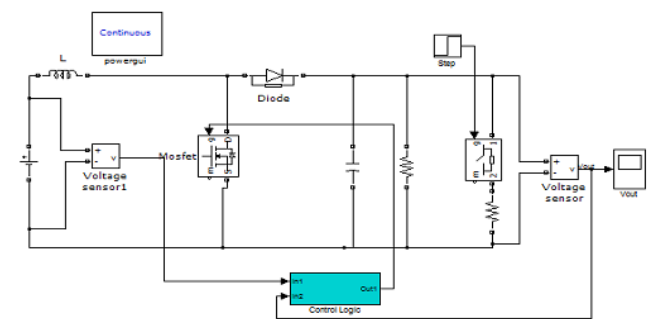

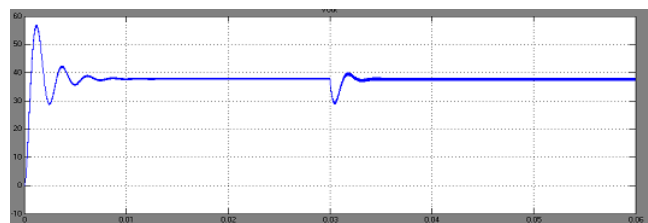**Figure 4(a).** Simulink schematic of ideal case boost converter.

Figure 4(b). Output voltage response of ideal case boost converter.
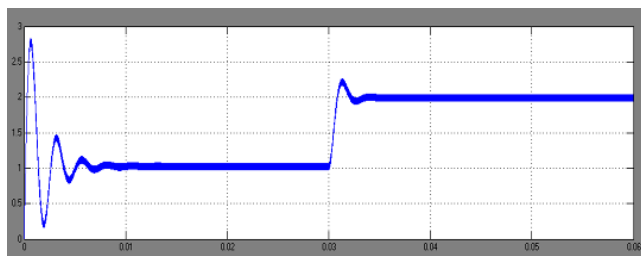
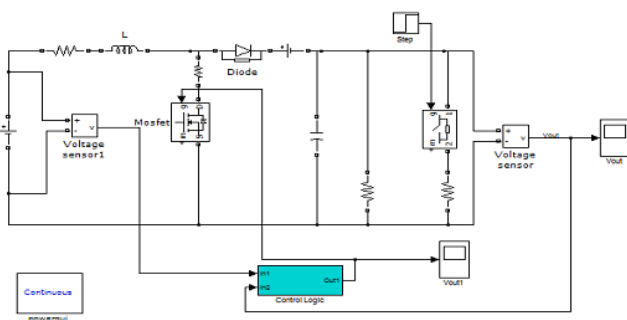**Figure 4(c).** Output current response of ideal case boost converter.

Figure 4(d). Simulink schematic of Non-ideal case boost converter.
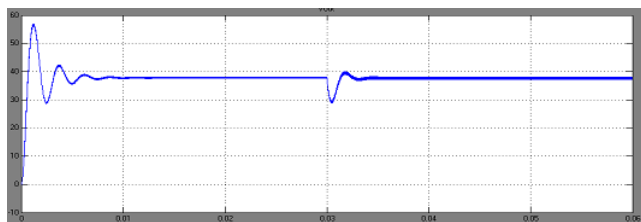

Figure 4(e). Output voltage response of Non-ideal case boost converter.
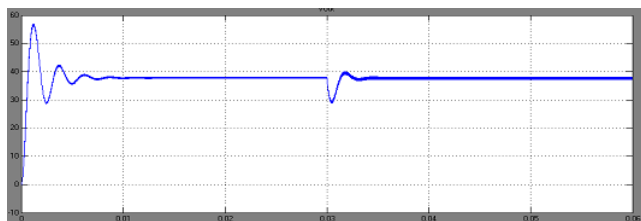

Figure 4(f). Output current response of Non - ideal case boost converter.

6. Conclusion

In Boost converter Small Signal Model (SSM), complete analysis is done for both ideal and non-ideal case. From the SSM of boost converter, transfer function of the both converters derived. Based on small signal output to controlled input (d), transfer functions root locus are plotted using MATLAB editor and script. Loop iterations are done to check the best step response and to obtain proper gain values of the PI control structure. After designing the controller parameters both ideal and Non-ideal cases are simulated using MATLAB simulink and voltage response for both cases are compared.

7. References

1. Chen Z, Gao W, Hu J, Ye X. Closed-loop analysis and cascade control of a non-minimum phase boost converter. IEEE Transactions on power electronics. 2011 Apr; 26(4):1237–52.
2. Radosevich A. Root locus based, integrated circuit design of a switch mode, boost current regulator 2010 Dec 17.
3. Guo L. Design and implementation of digital controllers for buck and boost converters using linear and nonlinear control methods [Ph.D thesis] 2006 Aug 7.
4. Dave MR, Dave KC. Analysis of boost converter using pi control algorithms. Int J Eng Trends Tech. 2012; 3(2).
5. Zhou C, Jovanovic M. Design trade-offs in continuous current-mode controlled boost power-factor correction circuit. 1992 May. Virginia.
6. Copeland BR. The design of PID controllers using Ziegler Nichols Tuning. 2008 Mar.
7. Mohan N. First course on power electronics. 2007 ed. Minneapolis: MNPERE; 2007.
8. Erickson RW. Fundamentals of power electronics, Norwell, Massachusetts: Kluwer Academic Publishers; 1997.
9. Brown M. Power supply cookbook. Newton, Massachusetts: Butterworth-Heinemann; 1994.
10. Ramirez A, Perez E. Stability of current-mode control of DC-DC power converters. Syst Contr Lett. 2002 May; 45:113–9.
11. Erickson RW, Maksimovic D. A primer on simulation, modeling, and design of the control loops of switching regulators. IEEE APEC Tutorial Seminar; 2003.
12. Current Sense Circuit Collection. Linear Technology Application Note 105. 2005.