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A survey on fuzzy transportation problems

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A survey on fuzzy transportation problems

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Abstract. An uncertain transportation problem is a transportation problem in which the parameters are fuzzy numbers. This paper presents a survey on single objective fuzzy transportation problem (SOFTP) and multi-objective fuzzy transportation problem (MOFTP) with its mathematical models.

1. Introduction
The transportation problem (TP) is a minimum-cost planning problem for transporting a commodity from factories to warehouses with the shipping cost from one point to another point. The basic TP was originally developed by Hitchcock [36]. TP can be a single objective problem or multi objective problem. The aim of the TP is to find the transportation schedule that minimizes the total transportation cost. The unit costs of transportation, supply and demand quantities are the parameters of the TP. This paper is organized as follows: Section 2 projects the membership functions. Section 3 presents SOFTP with its mathematical formulation and MOFTP with its mathematical formulation in section 4 and the last section presents the conclusion.

2. Membership functions and ranking technique
One of the main assumptions in solving fuzzy mathematical programming problems involves the use of non-linear (exponential and hyperbolic) membership functions and linear membership function for all fuzzy sets involved in a decision making process.

2.1. Triangular Membership Function
Triangular membership function can be stated as follows.

$$\mu_B(y) \;=\; \begin{cases} 0, & y<b_1 \\[4pt] \dfrac{y-b_1}{b_2-b_1} & b_1 \le y \le b_2 \\[8pt] \dfrac{b_3-y}{b_3-b_2} & b_2 \le y \le b_3 \\[8pt] 0 & y \ge b_3 \end{cases} \tag{1}$$

2.2. Trapezoidal Membership Function
Trapezoidal membership function can be defined as follows.

$$\mu_B(y) = \begin{cases} 0 & \text{for } y \le b_1 \\ \dfrac{(y\text{-}b_1)}{(b_2\text{-}b_1)} & \text{for } b_1 \le y \le b_2 \\ 1 & \text{for } b_2 \le y \le b_3 \\ \dfrac{(b_4\text{-}y)}{(b_4\text{-}b_3)} & \text{for } b_3 \le y \le b_4 \\ 0 & \text{for } y \ge b_4 \end{cases} \tag{2}$$

2.3. Linear Membership Function

A linear membership function can be defined as follows.

$$\mu_m(z_m(y)) = \begin{cases} 1 & \text{if } z_m \le L_m, \\ 1 - \dfrac{z_m - L_m}{U_m - L_m} & \text{if } L_m < z_m < U_m, \\ 0 & \text{if } z_m \ge U_m. \end{cases} \tag{3}$$

Where L_m is the aspiration level of achievement and U_m is the highest acceptable level of achievement for the mth objective function.

2.4. Exponential Membership Function

An exponential membership function can be defined as follows.

$$\mu^E{}_m(z_m(y)) = \begin{cases} 1 & \text{if } z_m \le L_m, \\ \dfrac{e^{-s\psi_m(y)} - e^{-s}}{1 - e^{-s}} & \text{if } L_m < z_m < U_m, \\ 0 & \text{if } z_m \ge U_m. \end{cases} \tag{4}$$

where $\psi_m(y) = (z_m - L_m)/(U_m - L_m)$ $m = 1, 2, ..., M$ and s is a non-zero parameter recommended by the decision maker.

2.5. Hyperbolic Membership Function

A hyperbolic membership function can be defined as follows.

$$\mu_m^H(z_m(y)) = \begin{cases} 1 & \text{if } z_m \le L_m, \\ \dfrac{1}{2} + \dfrac{1}{2} \dfrac{e^{\left\{\frac{U_m + L_m}{2} - z_m\right\}\alpha_m} - e^{-\left\{\frac{U_m + L_m}{2} - z_m\right\}\alpha_m}}{e^{\left\{\frac{U_m + L_m}{2} - z_m\right\}\alpha_m} + e^{-\left\{\frac{U_m + L_m}{2} - z_m\right\}\alpha_m}} & \text{if } L_m < z_m < U_m, \\ 0 & \text{if } z_m \ge U_m. \end{cases} \tag{5}$$

where $\alpha_m = \dfrac{6}{U_m\text{-}L_m}$.

Hyperbolic membership function has the following properties.

(1) $\mu_m^H(z_m(y))$ is strictly monotonously decreasing function with respect to $Z_m(y)$;

(2) $\mu_m^H(z_m(y)) = \frac{1}{2} \Leftrightarrow z_m(y) = \frac{1}{2}(U_m + L_m);$

(3) $\mu_m^H(z_m(y))$ strictly   convex   for $z_m(y) \geq \frac{1}{2}(U_m + L_m)$ and   strictly   concave   for

$z_m(y) \leq \frac{1}{2}(U_m + L_m);$

(4) $\mu_m^H(z_m(y))$ satisfies $0 < \mu_m^H(z_m(y)) < 1$ for $L_m < z_m(y) < U_m$ and   approaches

asymptotically $\mu_m^H(z_m(y)) = 0$ and $\mu_m^H(z_m(y)) = 1$ as $Z_m(y) \to \infty$ and $-\infty$, respectively.

2.6.Ranking Technique
Ranking of fuzzy data is an essential part of the decision process in various applications. Fuzzy data must be ranked before an action is taken by a decision maker. In [41,42], Jain introduced a method using the notion of maximizing set to order the fuzzy numbers. In [96], Yager presented a robust's ranking method to convert the FTP into TP.

*2.6.1.Robust'sRanking Technique.*The Robust's ranking is defined as $R(\tilde{c}) = \int_0^1 0.5(c_\alpha^L, c_\alpha^U) d\alpha$, where

(c_α^L, c_α^U) is the α-level cut of the fuzzy number $\tilde{c}$ .Robust's ranking approach satisfies the property of compensation, linearity and additive.

3. Single objective fuzzy transportation problem (SOFTP)
In many real life situations, transportation problems are modelled and solved as single objective problems. The occurrence of randomness and imprecision in the real life is unavoidable due to some unexpected circumstances. To deal quantitatively with uncertain data in making decisions, Bellman and Zadeh [3] and Zadeh [97] proposed the concept of fuzziness. In practice, due to some uncontrollable factors cost coefficients, the supply and demand amount of a TP may be imprecise. Such TP is known as FTP. The objective of the FTP is to find the transportation schedule that minimizes the total fuzzy transportation cost. O'heigeartaigh [35] proposed a heuristic approach for the solution of TP where the availability and requirements are fuzzy numbers with linear triangular membership functions. Using the parametric programming approach in terms of the Bellman-Zadeh criterion, Chanas et al. [11] discussed TP with fuzzy parameters. Their method solved the solution which simultaneously satisfied the constraints and the objective to a maximum level. Lio and Hwang [61] developed transportation model which solved the problem when supply and demand are fuzzy and costs are crisp. Chanas et al. [10] formulated the FTP in three different cases and presented an algorithm for solving the formulated FTP. Chanas and Kuchta [12] discussed the concept of the solution of the LPP with the objective functions as interval coefficients. Chanas and Kuchta [13] discussed an algorithm for finding an optimal solution to fuzzy integer TP by considering the parameters that are availability and requirement as L − R type fuzzy numbers. Parra et al. [75] presented a new procedure for solving FTP and then to obtain the possibility distribution of the objective value of the TP. For finding an optimal solution to interval TP, Sengupta and Pal [81] applied a technique of fuzzy programming and they also considered the midpoint and width of the interval in the objective function. Nagoor Gani and Abdul Razak [69] solved the TP by considering the constraints as uncertain parameters and they discussed the utilization of Kuhn-Tucker conditions related to the parametric problem. For finding an optimal solution to a TP under fuzziness, Omar et al. [73] applied a parametric technique. Based on the concept of extension principle, Liu and Kao [62] discussed a new procedure for solving the FTP by considering the objective value and parameters as fuzzy numbers.

Jershan Chiang [43] discussed the optimal solution of the FTP, in which requirement and product are fuzzy. NagoorGani and Abdul Razak [70] discussed a two-stage cost minimizing FTP in which demands and supplies are trapezoidal fuzzy numbers. Nagoor Gani and Abdul Razak [71] have solved TP with availability and demand as fuzzy values and also with an integration condition imposed on the solution using a procedure. For finding the fuzzy initial basic feasible solution of FTP, Das and Baruah [15] applied a method of VAM. Li et al. [55] discussed a new procedure for solving FTP with fuzzy costs and their idea was based on the concept of goal programming. Chen et al. [14] discussed the methods for solving TP on a fuzzy network. Lin [60] proposed a genetic algorithm for solving TP by considering the coefficients as fuzzy. Dinagar and Palanivel [20] introduced a fuzzy MODI method for finding the optimal solution to the FTP, in which parameters are trapezoidal fuzzy numbers. In [21], they applied trapezoidal membership functions for solving FTP. Pandian and Natarajan [74] introduced a new algorithm namely, fuzzy zero point method for finding the optimal solution to the FTP in which parameters are trapezoidal fuzzy numbers. GuzelNuran [32] investigated the FTP at two stages, in first stage they calculated the level of satisfaction between uncertain supply and demand and in second stage by considering the unit costs of TP from zero to maximum satisfaction level. Kaur and Kumar [45] discussed a heuristic approach for solving FTP by considering that a decision maker is imprecise about the precise values of the unit cost of transportation, availability and requirement of the quantity. For solving FTP, Gani et al. [29] discussed a simplex type procedure. Edward Samuel and Venkatachalapathy [22] discussed VAM for solving FTP. Sobha [83] found the maximum profit cost of some products through a capacity network, when the availability and requirement of nodes and the cost and capacity of nodes are considered as triangular fuzzy numbers. A new dual based procedure for the unbalanced FTP was discussed by Edward Samuel and Venkatachalapathy [23]. Kaur and Kumar [46] proposed a heuristic method for finding the optimal solution to the FTP, in which parameters are generalized trapezoidal fuzzy numbers. In [24], Edward Samuel and Venkatachalapathy investigated a heuristic algorithm for solving generalized trapezoidal FTP. Mohanaselvi and Ganesan [66] presented a new algorithm to find the initial fuzzy feasible solution for the FTP and they applied fuzzy version of MODI method to find the fuzzy optimal solution. Shugani Poonam et al. [82] solved a FTP where the parameters are trapezoidal fuzzy numbers. Robust's ranking function is used to transform the FTP into crisp TP. Fegade et al. [28] solved fuzzy transportation problem using zero suffix and robust ranking method. Manimekalai et al. [65] proposed an algorithm for solving FTP with minimum cost using robust ranking method. Narayanamoorthy et al. [72] proposed fuzzy Russell's method to find the initial basic feasible solution of fuzzy transportation problem. They applied Yager's ranking method to transform fuzzy transportation problem to crisp transportation problem. Edward Samuel and Venkatachalapathy [26] proposed a new method for solving a special type of fuzzy transportation problem on the assumption of the uncertainty of the decision maker about the precise values of transportation cost. They [25] also investigated the improved zero point method for solving fuzzy transportation problems using ranking functions. Solaiappan and Jeyaraman [84] proposed an algorithm for solving a fuzzy transportation problem using zero termination method with trapezoidal fuzzy numbers. Srinivasan and Geetharamani [86] discussed an innovative method for solving fuzzy transportation problem. Removing some variables from equations by the elimination method and applying the Fourier elimination method, Poonam Shugani et al. [77] have found the best compromise solution for the fuzzy transportation problem. The same authors [78] introduced dual simplex method to solve transportation problem with fuzzy objective functions. Edward Samuel and Venkatachalapathy [27] discussed improved zero point method for solving the unbalanced fuzzy transportation problems. Dayi He et al. [18] transformed the fuzzy transportation problem into four types of crisp linear programming problems by the parametric method using possibility theory in fractile and modality approach. Srinivas and Ganeshan [85] used robust ranking indices to convert the fuzzy transportation problem into crisp transportation problem and stepping stone method to find an optimal solution to the fuzzy transportation problem. Thamaraiselvi and Santhi [89] discussed a fuzzy transportation problem in which the values of transportation costs are represented by indiscriminate hexagonal fuzzy numbers. Gaurav Sharma et al.

[31] proposed an algorithm for finding the fuzzy optimal solution for a fuzzy transportation problem in which the parameters are trapezoidal fuzzy numbers. Vimala et al. [93] proposed monalisha's approximation method for solving the fuzzy transportation problem. Ismail Mohideen et al. [40] applied octagon fuzzy numbers with α-cut and ranking technique for solving fuzzy transportation problem .Muruganandam and Srinivasan [67] found the least transportation cost of some commodities through a capacitated network when the origin and destination of nodes and the capacity and cost of edges are represented as fuzzy numbers. Krishna Prabha and Vimala [49] discussed a new algorithm for maximizing the profit of the transportation problem in fuzzy environment. By using the method of magnitude ranking, fuzzy quantities were transformed into crisp quantities. Krishna Prabha and Vimala [48] proposed a new technique for finding the maximum profit cost for fuzzy transportation Problem. The circumcenter of centroids ranking technique was used by them [50] for maximizing the profit for an unbalanced fuzzy transportation problem. Kalpanapriya and Anuradha [44] proposed an algorithm for the two vehicle cost varying balanced transportation problem and unbalanced transportation problem with uncertain data. Darunee Hunwisai and PoomKumam [17] used Robust's ranking technique for transforming uncertain data to precise data. They used allocation table method for finding an initial basic feasible solution to the fuzzy transportation problem.

3.1.Mathematical Statement of SOFTP
Now, the mathematical model of an SOFTP is given as follows

$$\text{Minimize } \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \quad i=1,2,\dots,m \tag{6}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \quad j=1,2,\dots,n$$

$$\tilde{x}_{ij} \geq 0, \quad \text{for all } i \text{ and } j$$

where $\tilde{c}_{ij}$ is the fuzzy cost of transporting one unit from source i to the destination j and $\tilde{x}_{ij}$ is the fuzzy number of units transported from source i to destination j. $\tilde{a}_i$ is the amount of material available at source point i and $\tilde{b}_j$ is the amount of the material required at destination point j.

4.Multi objective fuzzy transportation problem (MOFTP)
In general, real-life transportation problems may be modelled more profitably with the concurrent consideration of multi criteria because a transportation system decision-maker generally pursues multiple goals. For solving LPP involving constraints of equality and objectives conflicting with each other, we use MOTP. Lee and Moore [54] suggested the application of goal programming approach for solving the MOTP. Zimmermann [100] discussed the fuzzy programming technique for solving multi criteria problems. Isermann [38] proposed a new approach for finding all the non-dominated solutions for a linear MOTP. Leberling [52] used hyperbolic membership technique for solving MOLPP and found that the solutions obtained using this method is always efficient. Luhandjula [64] discussed the usage of compensatory operators in fuzzy linear programming with multiple objectives. Zimmermann's [101] developed fuzzy approach to solve TP and MOLPP. Ringuest and Rinks [79] proposed the usage of interactive algorithms to obtain more than k non-dominated and dominated solutions for linear MOTP. Biswal [4] solved multi objective geometric programming problems by using fuzzy programming technique.Bit et al. [7] discussed MOTP by using the fuzzy programming approach with linear membership function, and arrived at efficient solutions as well as an optimal compromise solution for MOTP. In [8], using additive fuzzy programming model, they considered

weights and priorities for all non-equivalent objectives for the TP. Lee and Li [53] investigated the possibility of fuzzy multiple objective programming and compromised programming with paretoo ptimum for solving MOFTP. Bit et al. [9] discussed fuzzy programming technique to chance constrained MOTP. Biswal and Sinha [6] solved MONLPP using fuzzy programming approach. Biswal [5] solved multi-objective FLPP using projective and scaling algorithm. Verma et al. [92] discussed a fuzzy programming technique for solving MOTP with some nonlinear membership functions. Verma et al. [92] tried solving MOTP with some nonlinear membership functions after discussing a fuzzy programming approach. Hussein [37] investigated the complete solutions of MOTP with possibilistic coefficients ofthe objective functions. Das et al. [16]discussed the procedure of deriving solution to the MOTP in which all the parameters have been considered as intervals. Li and Lai [56] obtained a non-dominated compromise solution to the MOTP using a fuzzy compromise programming method. Waiel and Wahed [95] dealt with MOTP under fuzziness. Wahed and Sinna [94] applied a fuzzy technique to find the optimal compromise solution of a MOTP and calculated the closeness degree of the compromise solution to the ideal solution using the concept of distance functions. Gao and Liu [30] used a two-phase fuzzy algorithm for solving MOTP with nonlinear and linear membership functions. Ammar and Youness [1] discussed the efficiency of the solutions and the stability of MOTP in which parameters are fuzzy. Wahed and Lee [2] obtained a compromise solution to the MOTP using an interactive fuzzy goal programming technique. Liang [57] dealt with distribution of planning decisions by applying interactive fuzzy MOTP. Islam and Roy [39] solved the multi objective entropy TP with an additional delivery time constraint where its shipping costs were in the form of generalized trapezoidal fuzzy numbers. Tien-Fu Liang [59] optimized transportation planning decision using an interactive fuzzy MOTP technique. Zangiabadi and Maleki [98] presented a fuzzy goal programming technique with hyperbolic membership function to obtain an optimal compromise solution for the MOTP. Surapati and Roy [88] discussed a priority based fuzzy goal programming technique with membership function for finding a compromise solution of a MOTP with fuzzy coefficients. They converted the membership functions into membership goals, by prioritising the highest degree of a membership function as a level of aspiration and by introducing deviational variables to each of them. For solving TP using fuzzy LPP, Liang [58] proposed an interactive multi objective technique. Lau et al. [51] solved the MOTP using fuzzy logic guided non-dominated sorting genetic algorithm. Deshabrata Roy Mahapatra et al. [19] apprehensive about the usage of fuzzy programming approach to the objective function, the stochastic technique was used for the randomness of supply and demand parameters in inequality type of constraints of multi objective stochastic unbalanced TP. Lohgaonkar and Bajaj [63] applied fuzzy programming approach with membership function(linear, hyperbolic and exponential) to obtain the optimal compromise solution of a multi objective capacitated transportation problem. Hale GonceKocken and Mehmet Ahlatcioglu [33] investigated fuzziness in the objective functions. They presented a compensatory technique to solve the MOLTP with cost coefficients as fuzzy. Their method generated both compensary and Pareto optimal compromise solutions. Venkatasubbaiah et al. [91] used fuzzy goal deviation function to identify a compromise solution for the MOTP and introduced a fuzzy max-min operator, an auxiliary variable, the equivalent fuzzy interactive goal programming technique was formulated to maximize λ. Peidro and Vasant [76] used modified S-curve non-linear membership function to determine a compromise solution for the MOTP. Zangiabadi and Maleki [99] proposed a fuzzy goal programming technique with special type of nonlinear membership function to find an optimal compromise solution for the linear MOTP. Thorani and Ravi Shankar [90] discussed an algorithm for analyzing a FMOTP by applying a linear programming model based on a heuristic approach for ranking generalized LR fuzzy numbers. Subhran and Goswami et al. [87] proposed two-vehicle cost varying MOTP. They transformed the cost varying MOTP to MOTP by NWCR method and then used MODI method for finding the optimal solution. Khan and Das [47] proposed a review of the connection between modern era approaches and fuzzy multi objective optimization to deal with its shortcoming and fuzzy multi objective optimization used in TP. Hale GonceKocken [34] proposed a compensatory fuzzy technique to the MOLTP in which unit cost supply and demand quantities are

triangular fuzzy numbers. This technique had three stages. In the first stage, by using Zimmermann's "min" operator, the uncertain availability and requirement amount was removed, that is, the precise availability and requirement amount were obtained from uncertain amount to satisfy the balance condition. In the second stage, breaking points and cost-satisfaction interval sets were fixed for each objective. In the third stage, considering cost-satisfaction interval sets of all objectives, an overall cost-satisfaction interval set was identified. Muruganandam and Srinivasan [68] presented a heuristic approach for finding the optimal solution to the two stage cost minimizing FTP with multi objective constraints. For MOTP, SaruKumari and Priyamvada Singh [80] applied fuzzy efficient interactive goal programming technique.

4.1.Mathematical Statement of MOFTP
Now, the mathematical model of an MOFTP is given as follows

$$\text{Minimize } \tilde{z}_r(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{r}\tilde{x}_{ij}, \quad r = 1,2,...,K$$

subject to

$$\sum_{j=1}^{n}\tilde{x}_{ij} = \tilde{a}_i \ , \ i=1,2,...,m \tag{7}$$

$$\sum_{i=1}^{m}\tilde{x}_{ij} = \tilde{b}_j \ , \ j=1,2,...,n$$

$$\tilde{x}_{ij} \geq 0, \textit{ for all i and j}$$

where $\tilde{a}_i$ is the amount of the material available at the ith source and $\tilde{b}_j$ is the amount of the material required at jth destination. r is the number of the objective function of MOFTP. $\tilde{c}_{ij}^{r}$ is the uncertain unit transportation cost from source i to destination j for the objective r and $\tilde{x}_{ij}$ is the uncertain number of units shipped from source i to destination j.

4.2.Fuzzy Goal Programming Approach with Linear and Non-linear Membership functions
Using the linear membership function as defined in (3) than an equivalent linear model for the model (7) can be formulated as:

$$\min : \Phi$$

subject to

$$\frac{z_m - L_m}{U_m - L_m} + d_m^- + d_m^+ = 1,$$

$$\Phi \geq d_m^-, \ m=1,2,...,k,$$

$$d_m^+ d_m^- = 0,$$

$$\sum_{j=1}^{n}x_{ij} = a_i, \ i=1,2,...,m; \ \sum_{i=1}^{m}x_{ij} = b_j, \ j=1,2,...,n,$$

$$d_m^+ d_m^- \geq 0,$$

$$\Phi \leq 1, \Phi \geq 0,$$

$$x_{ij} \geq 0, \text{ for all i,j,}$$

where the equlibrium condition

$$\sum_{i=1}^{m}a_i = \sum_{j=1}^{n}b_j$$

Using the exponential membership function as stated in (4) then an equivalent linear model for the model (7) can be formulated as:

$$\min : \Phi$$

subject to

$$\frac{e^{-k\psi_m(y)} - e^{-k}}{1 - e^{-k}} + d_m^- + d_m^+ = 1,$$

$$\Phi \geq d_m^-, \ m=1,2,...,M,$$

$$d_m^+ d_m^- = 0,$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \ i=1,2,...,m; \ \sum_{i=1}^{m} x_{ij} = b_j, \ j=1,2,...,n,$$

$$d_m^+ d_m^- \geq 0,$$

$$\Phi \leq 1, \Phi \geq 0,$$

$$x_{ij} \geq 0, \ \text{for all i,j,}$$

where the equilibrium condition

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Using the Hyperbolic membership function as stated in (5) than an equivalent linear model for the model (7) can be formulated as:

$$\min : \Phi$$

subject to

$$\frac{1}{2} + \frac{1}{2} \frac{e^{\left\{\frac{U_m + L_m}{2} - z_m\right\}\alpha_m} - e^{-\left\{\frac{U_m + L_m}{2} - z_m\right\}\alpha_m}}{e^{\left\{\frac{U_m + L_m}{2} - z_m\right\}\alpha_m} + e^{-\left\{\frac{U_m + L_m}{2} - z_r\right\}\alpha_m}} + d_m^- + d_m^+ = 1,$$

$$\Phi \geq d_m^-, \ m=1,2,...,M,$$

$$d_m^+ d_m^- = 0,$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \ i=1,2,...,m; \ \sum_{i=1}^{m} x_{ij} = b_j, \ j=1,2,...,n,$$

$$d_m^+ d_m^- \geq 0,$$

$$\Phi \leq 1, \Phi \geq 0,$$

$$x_{ij} \geq 0, \ \text{for all i,j,}$$

where the equlibrium condition

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

5. Conclusion
This paper provides the survey on single objective FTP and multi-objective FTP with its mathematical models. The survey also studies the approaches used to solve such problems.

References

[1]   Ammar E E and Youness E A 2005 *Appl. Math. Comput.* **166** 241–253
[2]   Abd W F and El-Wahed Lee S M 2006 *Omega* **34** 158-166
[3]   Bellman R E and Zadeh L 1970 *J. Manag. Sci.* **17** 141-164
[4]   Biswal M P 1992 *Fuzzy Set Syst.* **51** 67-71
[5]   Biswal M P 1997 *J. Fuzzy Math.* **5** 439-448
[6]   Biswal M P and Sinha S B 1996 *J. Fuzzy Math.* **4** 315-321
[7]   Bit A K, Biswal M P and Alam S S 1992 *Fuzzy Set Syst.* **50** 135-141
[8]   Bit A K, Biswal M P and Alam S S 1993 *Fuzzy Set Syst.* **57** 313-319
[9]   Bit A K, Biswal M P and Alam S S 1994 *J. Fuzzy Math.* **2** 117-130
[10] Chanas S, Delgado M, Verdegay J and Vila M 1993 *Transport. Plan.* Techn. **17** 203–218
[11] Chanas S, KolodziejczykW and Machaj A 1984 *Fuzzy Set Syst.* **13** 211–221
[12] Chanas S and Kuchta D 1996 *Fuzzy Set Syst.* **82** 299-305
[13] Chanas S and Kuchta D 1998 *Fuzzy Set Syst.* **98** 291-298
[14] Chen M, Ishii H and Wu C 2008 *Int. J. Innov. Comput. Inf. Control* **4** 1105-1109
[15] Das M K and Baruah H K 2007 *Jl. Fuzzy Math* **15** 79–95
[16] Das S K, Goswami A and Alam S S 1999 *Eur. J. Oper. Res.* **117** 100-112
[17] DaruneeHunwisai and PoomKumam 2017 *CogentMathematics* 1-12
[18] Dayi He, Ran Li, Huang Qi and Ping Lei 2014 *PLOS* **9** 1-12
[19] Deshabrata Roy Mahapatra, Sankar Kumar Roy and Mahendra Prasad Biswal 2010 *AMO* **12** 205-223
[20] Dinagar D S and Palanivel K 2009 *Inter. J.Algor. Comput. Math.* **2** 65-71
[21] Dinagar D and Stephen Palanival K 2009 *Inter. J. Comput. Phy. Sci.* **1** 1-12
[22]   Edward Samuel A and Venkatachalapathy M 2011 *App. Math. Sci.* **5** 1367-1372
[23] Edward Samuel A and Venkatachalapathy M A 2012 *App. Math. Sci.* **6** 4443-4455
[24] Edward Samuel A and Venkatachalapathy M 2012 *Adv.Fuzzy Set Syst.* Accepted
[25] Edward Samuel A and Venkatachalapathy M 2013 *Far East J. Math. Sci.* **75** 85-100
[26] Edward Samuel A and Venkatachalapathy M 2013 *Inter. J. Pure App. Math.* **83** 91-100
[27] Edward Samuel A and Venkatachalapathy M 2014 *Inter. J. Pure App. Math.* **94** 419-424
[28] Fegade M R, Jadhav V A and Muley A A 2012 *IOSR J. Eng.* **2** 36-39.
[29] Gani A N, Samuel A E and Anuradha D 2011 *Tamsui Oxford Jl. Math. Sci* **27** 89–98
[30] Gao S P and Liu S A 2004 *J. Fuzzy Math.* **12** 147–155
[31] Gaurav Sharma , Abbas S H and Vijay Kumar Gupta 2015 *Scitech Res. Organ.* **4** 386 – 392
[32] GuzelNuran 2010 *World Appl. Sci.* **8** 543-549
[33] Hale Gonce Kocken and Mehmet Ahlatcioglu 2011 Math. *Probl. Eng.* **2011** 1-19
[34] Hale GonceKocken, BeyzaAhlatciogluOzkok and Tiryaki F 2014 *Eur. J. Pure Appl. Math.* **7** 369-386
[35] Heigeartaigh O H 1982 *Fuzzy Set Syst.* **8** 235-243
[36] Hitchcock F L 1941 *J.Math.Phys* **20** 224-230
[37] Hussien M L 1998 *Fuzzy Set Syst.* **93** 293-299
[38] Isermann H 1979 *Nav. Res. Log.* **26** 123-139
[39] Islam S and Roy T K 2006 *Eur. J. Oper. Res.* **173** 387–404
[40] Ismail Mohideen S, Prasannadevi K and Devi Durga M 2016 *J. Computer.* **01** 60-67
[41] Jain R 1976 Trans *Syst Man Cybern Syst.* **6** 698-703
[42] Jain R 1977 *Int. J. Syst. Sci.* **8** 1-7
[43] Jershan Chiang 2005 *J. Inf. Sci. Eng.* **21** 439-451
[44] Kalpanapriya D and Anuradha D 2016 *Int. J. Pharm. Technol.* **8** 16254-16260
[45] Kaur A and Kumar A 2011 *App. Math. Modeling* **35** 5652-5661
[46] Kaur A and Kumar A 2012 *App. soft comput.* **12** 1201-1213
[47] Khan A J and Das D K 2014 *Recent Res. Sci. Tech.* **6** 274-282
[48] Krishna Prabha S and Vimala S 2016 *Inter. J. Pure Appl. Math.* **106** 45-52
[49] Krishna Prabha S and Vimala S 2016 *ISROJ publication* **01** 1-6

[50]   Krishna Prabha S and Vimala S 2016 *Inter. J. Appl. Eng. Res.* **11** 282-287
[51]   Lau H C W, Chan T M, Tsui W T , Chan F T S , Ho G T S and Choy K L 2009 *Expert Syst. Appl.* **36** 8255-8268
[52]   Leberling H 1981 *Fuzzy Set Syst.* **6** 105-118
[53]   Lee E S and Li R J 1993 *Fuzzy Set Syst.* **53** 275-288
[54]   Lee S M and Moore L J 1973 *AIEE transactions* **5** 333-338
[55]   Li L, Huang Z, Da Q and Hu J 2008 *J. Hu, Inter. Sympos. Inf. Pro.* 3–8
[56]   Li L and Lai K 2000 *Comput.Oper.Res* **27** 43-57
[57]   Liang T 2006 *Fuzzy Set Syst.* *157* 1303-1316
[58]   Liang T 2008 *Asia-Pacific J. Oper. Res.* **25** 11-31
[59]   Liang T 2006 *J. Inf. Optim. Sci.* 27 107-126
[60]   Lin F T 2009 *Inter. Conf. Fuzzy Syst.* 1468-1473
[61]   Liou T S and Wang M J 1992 *Fuzzy Set Syst.* **50** 247-255
[62]   Liu S T and Kao C 2004 *Eur. J. Oper. Res.* **153** 661-674
[63]   Lohgaonkar M H and Bajaj V H 2010 *Inter. J. Bioinfor. Res.* **2** 10-14
[64]   Luhandjula M K 1982 *Fuzzy Set Syst.* **8** 245-252
[65]   Manimekalai S, Revathy M and Gokilamani S 2013 *IJSR* **4** 2319-7064
[66]   Mohanaselvi S and Ganesan K 2012 *Int. J. computer sci. eng.* **4** 367−375
[67]   Muruganandam S and Srinivasan R 2016 *IJRTER* **2** 428-437
[68]   Muruganandam S and Srinivasan R 2016 *Asian J. Res. Social Sci. and Humanities* **6** 744-752
[69]   NagoorGani A and Abdul Razak K 2004 *Bull. Pure Appl. Sci.* **23** 281-289
[70]   Nagoor GaniA and Abdul Razak K 2006 *J. Phy. Sci.* **10** 63 – 69
[71]   NagoorGani A and AbdulRazak K 2007 *J. Pure Appl. Math. Sci.* **LXVI** 27-35
[72]   Narayanamoorthy S, Saranya S and Maheswari S 2013 *IJISA* **02** 71-75
[73]   Omar Saad M and Samir Abbas A 2003 *J. Fuzzy Math.* **11** 115-124
[74]   Pandian P and Natrajan G 2010 *Appl. Math. Sci* **4** 79–90
[75]   Parra M A, Terol A B and Uria M V R 1999 *Eur. J. Oper. Res.* **117** 175–182
[76]   Peidro D and Vasant P 2011 *Appl. Soft Comput.* **11** 2656-2663
[77]   Poonam Shugani , Abbas S H and Vijay Gupta 2014 *IJRSSET* **1** 1-5
[78]   PoonamShugani Abbas S H and Vijay Gupta 2014 *IJSRP* **4** 1-3
[79]   Ringuest J L and Rinks D B 1987 *Eur. J. Oper. Res.* **32** 96-106
[80]   SaruKumari and Priyamvada Singh 2016 *J. Appl. Comput. Math.* 1-21
[81]   Sengupta A and Pal T K 2003 *VU J. Phy. Sci.* **9** 71 – 81
[82]   ShuganiPoonam Abbas S H and Gupta V K 2012 *Inter. J. Fuzzy Math. Syst.* **2** 263−267
[83]   Sobha K R 2012 *IJSR* **3** 2300-2302
[84]   Solaiappan S and Jeyaraman K 2014 *Inter. J. Adv. Res.* **2** 933−942
[85]   Srinivas B and Ganeshan G 2015 *Inter. J. IT Eng.* **03** 185-198
[86]   Srinivasan A and Geetharamani G 2014 *Indian J. Appl. Res.* **4** 377-384
[87]   Subhrananda Goswami, Arpita Panda Chandan and Bikash Das 2014 *Ann. Pure Appl.Math.* **7** 47-52
[88]   Surapati P and Roy T K 2008 *J. Trans. Syst. Eng. and Inf. Tech.* **08** 40-48
[89]   Thamaraiselvi A and Santhi R 2015 *IOSR J.Math.* **11** 08-13
[90]   Thorani Y L P and Ravi Shankar N 2014 *Appl.Math. Sci.* **8** 6849 – 6879
[91]   Venkatasubbaiah K, Acharyulu S G and Chandra Mouli K V V 2011 *GJRE* **11** 5-10
[92]   Verma R, Biswal M P and Verma A B 1997 *Fuzzy Set Syst.* **91** 37–43
[93]   Vimala S and Krishna Prabha S 2016 *British J. Math. computer sci.* **17** 1-11
[94]   Wahed W F and Sinna M A 2001 *Fuzzy Set Syst.* **119** 71-85
[95]   Waiel and Wahed 2001 *Fuzzy Set Syst.* **117** 27–31
[96]   Yager R R 1981 *J.Inf. Sci.* **24** 143–161
[97]   ZadehL A 1978 *Fuzzy Set Syst.* **1** 3-28
[98]   Zangiabadi M and Maleki H R 2007 *J. Appl. Math. and Computing* **24** 449-460

[99]   Zangiabadi M and Maleki H R 2013 *Iranian J. Fuzzy Syst.***10** 61-74
[100] Zimmermann H J 1978 *Fuzzy Set Syst.* **1** 45-55
[101] Zimmermann H J 1983 *Eur. J. Oper. Res.* **13** 201-216
[102] Zimmermann H J 1985 J.Inf. Sci **36** 29-58