Ain Shams Engineering Journal xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Ain Shams Engineering Journal

journal homepage: www.sciencedirect.com

An economic production quantity model for three levels of production with Weibull distribution deterioration and shortage

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ARTICLE INFO

Article history:
Received 26 October 2015
Revised 7 October 2016
Accepted 30 October 2016
Available online xxxx

Keywords:
EPQ
Weibull distribution deterioration
Cycle time
Three levels of production

ABSTRACT

An EPQ model plays an important role in production and manufacturing units. In this paper, we introduce three levels of economic production inventory model for deterioration items in which three different levels of production are considered and the rate of deterioration follows two parameter Weibull distributions. The total cost of production is dependent on production rate, demand rate, and rate of deteriorative items, and it is possible to switch over the production started at one rate to another rate after certain time, such a situation is desirable in the sense that by starting at a low rate of production, a large quantum of stock of manufacturing items at the initial stage is avoided, leading to reduction in the holding cost. The variation in production level provides a way of attaining consumer satisfaction and earning potential profit. The objective of the paper is to find the optimal solution of production time so that total cost of the whole cycle will be minimized. Finally numerical example and sensitivity analysis on parameters are made to validate the results of the proposed inventory system.

1. Introduction

Inventory system is one of the main streams of the operation research which is essential in business enterprises and industries. The Economic Order Quantity model (EOQ) was originally developed by Harris [1] in 1913 and it was extensively applied by R.H. Wilson. The Economic Production Quantity (EPQ) model is a simple mathematical model to deal with inventory management issues in a production inventory system. It is considered to be one of the most popular inventory control model used in the industry [2]. In general, almost all products are found to be deteriorating over time. Sometimes the rate of deterioration is too low, for items such as steel, hardware, glassware and toys, to cause consideration of deterioration in the determination of economic lot sizes. Normally some items have significant rate of deterioration, such as blood, fish, strawberry, alcohol, gasoline, radioactive chemicals, medicine and food grains those deteriorate rapidly over time, which cannot be ignored in the decision making process of production lot size.

The problem of deteriorating inventory has received considerable attention in recent years. Most researchers in deteriorating inventory assumed constant rate of deterioration. In general, the Weibull distribution is used to represent the product in stock which gets deteriorated with time. The deterioration rate increases with age, that is, longer the items remain unused, the higher the rate at which they fail. Deterioration is defined as decay, damage, change or spoilage that prevents items from being used for its original purpose. Some examples of items that deteriorate are fashion goods, foods, mobile phones, chemicals, automobiles, drugs, etc.

Definition. A continuous random variable t (e.g. the time to deterioration of an item) has a Weibull distribution, with parameters α and β if its density function is given by

$$f(t;\alpha,\beta)=\begin{cases}\alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}, t>0,\\ 0,\ elsewhere,\end{cases}\quad \text{where } \alpha>0 \text{ and } \beta>0$$

Wagner and Whitin [3] dealt with inventory model of deteriorating item at the end of shortage period. Thus deterioration functions are of various types, it can be constant type or time dependent function. In our proposed model we have considered the Weibull distribution as the function of deterioration. It is observed by Berrotoni [4] that both the leakage failure for the dry batteries and the life expectancy of ethical drugs could be expressed in term of Weibull distribution. In these cases the rate of deterioration increases with time i.e. longer the items remain unused higher the rate that the item gets deteriorated.

The work of Berrotoni influenced Covert and Philip [5] to develop an inventory model for deteriorating item with variable rate of deterioration. They have used two parameters Weibull

Peer review under responsibility of Ain Shams University.
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http://dx.doi.org/10.1016/j.asej.2016.10.006
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distribution to manifest deterioration as distribution of time. This model was further improved and generalized by Philips by taking three parameter Weibull distributions for deterioration. Misra [6] considered an EPQ model for deteriorating items with both a varying and a constant rate of deterioration. Raafat [7], Goyal and Giri [8] presented a complete survey of the published inventory literature for deteriorating inventory models.

Balki and Benkherouf [9] introduced an inventory model of stock dependent and time varying demand rate for deteriorating items under finite time horizon. Chang [10] modified the model of Balki and Benkherouf by considering the effect of profit in the inventory system. Begam et al. [11] developed an instantaneous replenishment policy for deteriorating items with price-dependent demand. They applied a three parameters Weibull distribution to present time dependency of inventory deterioration rate. Begam et al. [12] carried out another study on the previous model by neglecting shortage and assuming demand as an on linear function of price. Rabbani et al. [13] represented an integrated model for dynamic pricing and inventory control of deteriorating items. Pal et al. [14] considered the effect of inflation when there is no shortage in the stock and the model is under finite time horizon. Sivasankari and Panayappan [15] proposed a production inventory model for deteriorating items in which two different levels of production are considered. Cardenas-Barron et al. [16] proposed an alternative heuristic algorithm for a multi-product Economic Production Quantity (EPQ) vendor-buyer integrated model with just in time philosophy and a budget constraint. Sarkar et al. [17] considered the Economic production quantity model with rework process at a single-stage manufacturing system with planned backorders and three different inventory models are developed for three different distribution density functions such as Uniform, Triangular and Beta. Cardenas-Barron et al. [18] developed a production inventory model for two echelon supply chain consisting of one manufacturing and one retailer. Cardenas-Barron et al. [19] presented a brief introduction to the paper included in the special issue 'celebrating a century of the economic order quantity model in honor of Ford Whitman Harris'. Pasandideh et al. [20] developed an economic production quantity inventory model for a multiproduct single-machine lot-size problem for a multiproduct single-machine lot-size. Cardenas-Barron et al. [21]. Derived the optimal replenishment lot size and shipment policy for an EPQ inventory model with multiple deliveries and rework. Taleizadeh et al. [22] proposed research work deals with the problem of the joint determination of selling price, replenishment lot size and the number of shipment for an economic production quantity model with rework of defective items when multi-shipment policy is used.

The objective is to find the optimal number of boxes of production in different periods to minimize the total inventory cost, (including ordering, holding, and shortage and purchasing cost). Recently, Karthikeyan and Viji [23] developed an economic production quantity inventory model for constant deteriorating items in which three different levels of production are considered. In this paper, we developed an economic production inventory model for three different levels of production with Weibull distribution deterioration and demand, production rates are considered as constants. This paper is organized as follows. Section 2 presents the assumptions and notations. Section 3 is for mathematical modeling. Section 4 gives numerical example and sensitivity analysis. Section 5 concludes the paper.

2. Assumption and notations

2.1. Assumption

- The Production rate of the inventory system is considered to be constant.
- Three rates of productions are considered.
- The demand rate is known, constant and continuous.
- The time horizon of the production system is finite.
- Deterioration varies with time and it is a function of two parameter Weibull distributions of time, i.e., the distributions of time for deterioration of the item is $\theta(t) = \alpha\beta t^{\beta-1}$, where $0 < \alpha \leqslant 1$ is the scale parameter and $\beta > 1$ is the shape parameter. Here we assume $\beta = 2$.
- Shortage is permitted and completely backlogged.
- Production rate is greater than demand rate (D).

2.2. Notation

- I(t): Inventory level at time T.
- P: Production rate in units/unit time.
- D: Demand rate in units/unit time.
- $I_1(t)$: Inventory level for the product during the production and demand i.e. $0 < t < T_1$.
- $I_2(t)$: Inventory level for the product during the production and demand i.e. $T_1 < t < T_2$.
- $I_3(t)$: Inventory level for the product during the production and demand i.e. $T_2 < t < T_3$.
- $I_4(t)$: Inventory level for the product during the period when there is no production only demand with deterioration of the rate Weibull probability distribution. i.e. $T_3 < t < T_4$.
- $I_5(t)$: Inventory level for the production during the period there is no stock in hand and demand is continued and fully backlogged.
- I(t): Inventory level for the production during the period there is production starts and fulfil demand which was backlogged.
- B: Maximum Shortage of the product.
- Q_1: Maximum inventory level at time T_1.
- Q_2: Maximum inventory level at time T_2.
- Q_3: Maximum inventory level at time T_3.
- Q: Optimum production (or optimum quantity).
- θ: Deteriorative items.
- C_p Production cost per unit.
- C_h: Holding cost per unit time.
- C_o Ordering cost per unit time.
- T: Cycle time.
- T_i: Unit time in period (i = 1, 2, 3, ...).
- TC Total cost

3. Inventory models for three levels of production with Weibull distribution of deteriorative items

The description of Fig. 1, on hand inventory of defective items is as follows.

Let us assume that the production starts at time t = 0 and end at time t = T. During the time interval $[0, T_1]$, let the production rate be 'P' and demand rate be 'D' where D is less than P. The stock attains a level Q_1 at time $t = T_1$. During the time intervals $[T_1, T_2]$ and $[T_2, T_3]$. Let the rate of growing to be considered as a(P-D) and b(P-D) where a and b are constants. The inventory level attains levels Q_2 and Q_3 at time T_2 and T_3 respectively. During the decline time T, the product becomes technologically obsolete or customer tastes change. Care should be taken to control the amount of stocks of the product. The inventory level starts to decrease due to demand at a rate of D. Time T needed to consume all units Q at demand rate (for example boxes production).

The governing differential equations of the model are as follows:

$$\frac{dI(t)}{dt} + \theta I(t) = P - D; \;\; 0 \leqslant t \leqslant T_1 \tag{1}$$

Please cite this article in press as: Viji G, Karthikeyan K. An economic production quantity model for three levels of production with Weibull distribution deterioration and shortage. Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.10.006

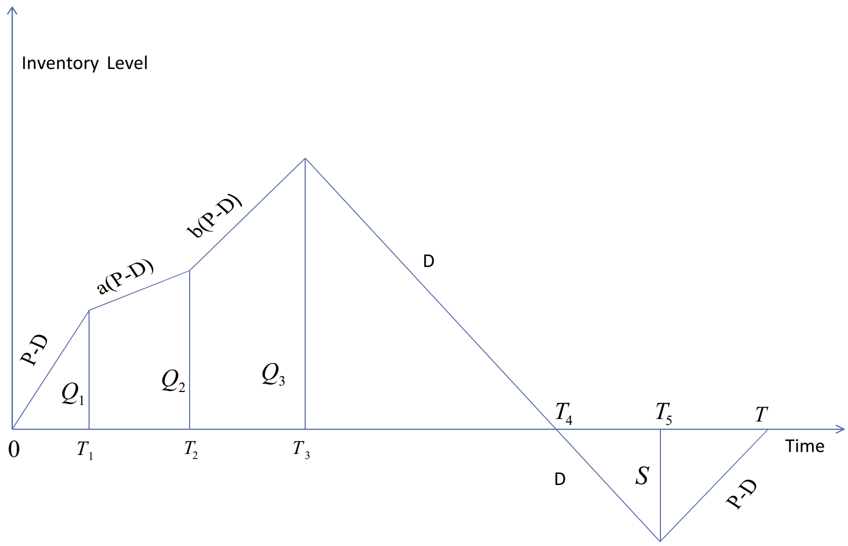

Figure 1. On hand inventory of defective items.

$$\frac{dI(t)}{dt} + \theta I(t) = a(P-D);\ T_1 \leqslant t \leqslant T_2 \tag{2}$$

$$\frac{dI(t)}{dt} + \theta I(t) = b(P-D);\ T_2 \leqslant t \leqslant T_3 \tag{3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D;\ T_3 \leqslant t \leqslant T \tag{4}$$

$$\frac{dI(t)}{dt} = -D;\ T_4 \leqslant t \leqslant T_5 \tag{5}$$

$$\frac{dI(t)}{dt} = P - D,\ T_5 \leqslant t \leqslant T \tag{6}$$

where $\theta(t) = \alpha\beta t^{\beta-1}$ $0 < \alpha \leqslant 1$, $\beta > 1$.

The boundary conditions are

$$I(0) = 0,\ I(T_1) = Q_1,\ I(T_2) = Q_2,\ I(T_3) = Q_3,\ I(T_4) = 0,\ I(T_5) = S,\ I(T) = 0 \tag{7}$$

The solution of Eq. (1) is,

$$I(t) = (P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^{\beta}} \tag{8}$$

The solution of Eq. (2) is,

$$I(t) = a(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^{\beta}} \tag{9}$$

The solution of Eq. (3) is,

$$I(t) = b(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^{\beta}} \tag{10}$$

The solution of Eq. (4) is,

$$I(t) = -D\left\{(t - T_4) + \frac{\alpha}{\beta+1}(t^{\beta+1} - T_4^{\beta+1})\right\}e^{-\alpha t^{\beta}} \tag{11}$$

The solution of Eq. (5) is,

$$I(t) = -D(T_4 - t) \tag{12}$$

The solution of Eq. (6) is,

$$I(t) = (P-D)[T - t] \tag{13}$$

3.1. Maximum inventory: Q_1

During the time T_1 the maximum inventory is computed using Eqs. (7) and (8) we have

$$I(T_1) = Q_1 \Rightarrow I(T_1) = (P-D)\left\{T_1 + \frac{\alpha T_1^{\beta+1}}{\beta+1}\right\}e^{-\alpha T_1^{\beta}}$$

By neglecting second and higher power of α in the expansion of $e^{\alpha T_1^{\beta}}$ for small values of α. We have,

$$Q_1 = (P-D)\left[T_1 - \frac{\alpha\beta}{\beta+1}T_1^{\beta+1}\right] \tag{14}$$

3.2. Maximum inventory: Q_2

During the time T_2 the maximum inventory is computed using Eqs. (7) and (9) we have

$$I(T_2) = Q_2 \Rightarrow I(T_2) = a(P-D)\left\{T_2 + \frac{\alpha T_2^{\beta+1}}{\beta+1}\right\}e^{-\alpha T_2^{\beta}}$$

By neglecting second and higher power of α in the expansion of $e^{\alpha T_2^{\beta}}$ for small values of α. We have,

Therefore,

$$Q_2 = a(P-D)\left[T_2 - \frac{\alpha\beta}{\beta+1}T_2^{\beta+1}\right] \tag{15}$$

3.3. Maximum inventory: Q_3

During the time T_3 the maximum inventory is computed using Eqs. (7) and (10) we have

$$I(T_3) = Q_3 \Rightarrow I(T_3) = b(P-D)\left\{T_3 + \frac{\alpha T_3^{\beta+1}}{\beta+1}\right\}e^{-\alpha T_3^{\beta}}$$

By neglecting second and higher power of α in the expansion of $e^{\alpha T_3^{\beta}}$ for small values of α. We have,

Therefore,

$$Q_3 = b(P-D)\left[T_3 - \frac{\alpha\beta}{\beta+1}T_3^{\beta+1}\right] \tag{16}$$

3.4. Shortage level S

From Eqs. (7), (12), and (13) we have,

$I(T_5) = S \Rightarrow -D(T_4 - T_5) = S$ and

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$I(T_5) = S \Rightarrow (P - D)(T - T_5) = S$

Therefore $(P - D)(T - T_5) = -D(T_4 - T_5)$ that is

$$T_5 = \frac{(P - D)}{P}T + \frac{D}{P}T_4 \tag{17}$$

3.5. Total cost

The total cost comprise of the sum of the production cost, ordering cost, holding cost deteriorating cost and shortage cost. They are grouped together after evaluating the above cost individually,

I. Production cost/unit time $= DC_P$ (18)

II. Ordering cost/unit time $= \frac{C_0}{T}$ (19)

III. Holding cost/unit time $= \frac{C_h}{T}\left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T_2} I(t)dt + \int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt\right]$

$$= \frac{C_h}{T}\left\{\begin{array}{l} \int_0^{T_1}\left[(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^\beta}\right]dt \\ + \int_{T_1}^{T_2}\left[a(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^\beta}\right]dt \\ + \int_{T_2}^{T_3}\left[b(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^\beta}\right]dt \\ + \int_{T_3}^{T_4}\left[-D\left\{(t - T_4) + \frac{\alpha}{\beta+1}(t^{\beta+1} - T_4^{\beta+1}\right\}e^{-\alpha t^\beta}\right]dt \end{array}\right\}$$

Expanding the exponential function and neglecting second and higher power of α for small value of α.

$$= \frac{C_h}{T}\left\{\begin{array}{l} (P-D)\left[\frac{T_1^2}{2} - \frac{\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + a(P-D)\left[\left(\frac{T_2^2}{2} - \frac{T_1^2}{2}\right) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left[T_2^{\beta+2} - T_1^{\beta+2}\right]\right] + \\ b(P-D)\left[\left(\frac{T_3^2}{2} - \frac{T_2^2}{2}\right) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left[T_3^{\beta+2} - T_2^{\beta+2}\right]\right] + \\ D\left[\frac{(T_4-T_3)^2}{2} - \frac{\alpha}{(\beta+1)(\beta+2)}\left[T_4^{\beta+2} - T_3^{\beta+2}\right] - \frac{\alpha}{(\beta+1)}\left[T_4^{\beta+1}T_3 - T_4T_3^{\beta+1}\right] + \frac{\alpha}{(\beta+2)}\left[T_4^{\beta+2} - T_3^{\beta+2}\right]\right] \end{array}\right\} \tag{20}$$

Deteriorating Cost/unit time:

$$\text{Deteriorating cost} = \frac{C_P}{T}\left[\int_0^{T_1}\theta(t)I(t)dt + \int_{T_1}^{T_2}\theta(t)I(t)dt + \int_{T_2}^{T_3}\theta(t)I(t)dt + \int_{T_3}^{T_4}\theta(t)I(t)dt\right]$$

$$= \frac{C_p}{T}\left\{\begin{array}{l} \int_0^{T_1}\left[(\alpha\beta t^\beta)\left[(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^\beta}\right]\right]dt \\ + \int_{T_1}^{T_2}\left[(\alpha\beta t^\beta)\left[a(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^\beta}\right]\right]dt \\ + \int_{T_2}^{T_3}\left[(\alpha\beta t^\beta)\left[b(P-D)\left\{t + \frac{\alpha t^{\beta+1}}{\beta+1}\right\}e^{-\alpha t^\beta}\right]\right]dt \\ + \int_{T_3}^{T_4}\left[(\alpha\beta t^\beta)\left[-D\left\{(t-T_4) + \frac{\alpha}{\beta+1}(t^{\beta+1} - T_4^{\beta+1}\right\}e^{-\alpha t^\beta}\right]\right]dt \end{array}\right\}$$

$$= \frac{C_P}{T}\left[\begin{array}{l} \frac{\alpha\beta}{(\beta+1)}T_1^{\beta+1} + a(P-D)\frac{\alpha\beta}{(\beta+1)}\left(T_2^{\beta+1} - T_1^{\beta+1}\right) + \\ b(P-D)\frac{\alpha\beta}{(\beta+1)}\left(T_3^{\beta+1} - T_2^{\beta+1}\right) + D\left(\frac{\alpha}{(\beta+1)}T_4^{\beta+1} + \frac{\alpha\beta}{(\beta+1)}T_3^{\beta+1} - \alpha T_4 T_3^\beta\right) \end{array}\right] \tag{21}$$

Shortage cost/unit time:

$$\text{Shortage cost} = \frac{C_S}{T}\left[\int_{T_4}^{T_5} I(t)dt + \int_{T_5}^{T} I(t)dt\right]$$
$$= \frac{C_S}{T}\left[\int_{T_4}^{T_5}[-D(T_4 - t)]dt + \int_{T_5}^{T}[(P-D)(T-t)]dt\right]$$

$$\text{Shortage cost} = \frac{C_S}{2TP}\left[D(P-D)(T-T_4)^2\right] \tag{22}$$

Therefore, total cost can be presented as follows

Total Cost = Production Cost + Setup Cost + Holding Cost + Deteriorating Cost + Shortage Cost

$$= \left[\begin{array}{l} DC_P + \frac{C_0}{T} + \frac{C_h}{T}\left[\begin{array}{l}(P-D)\left[\frac{T_1^2}{2} - \frac{\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + a(P-D)\left[\left(\frac{T_2^2}{2}-\frac{T_1^2}{2}\right) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}[T_2^{\beta+2} - T_1^{\beta+2}]\right] + \\ b(P-D)\left[\left(\frac{T_3^2}{2}-\frac{T_2^2}{2}\right) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}[T_3^{\beta+2} - T_2^{\beta+2}]\right] + \\ \left[D\frac{(T_4-T_3)^2}{2} - \frac{\alpha}{(\beta+1)(\beta+2)}\left[T_4^{\beta+2} - T_3^{\beta+2}\right] - \frac{\alpha}{(\beta+1)}[T_4^{\beta+1}T_3 - T_4T_3^{\beta+1}] + \frac{\alpha}{(\beta+2)}\left[T_4^{\beta+2} - T_3^{\beta+2}\right]\right]\end{array}\right] + \\ + \frac{C_P}{T}\left[\begin{array}{l}\frac{\alpha\beta}{(\beta+1)}T_1^{\beta+1} + a(P-D)\frac{\alpha\beta}{(\beta+1)}(T_2^{\beta+1} - T_1^{\beta+1}) \\ + b(P-D)\frac{\alpha\beta}{(\beta+1)}(T_3^{\beta+1} - T_2^{\beta+1}) + D\left(\frac{\alpha}{(\beta+1)}T_4^{\beta+1} + \frac{\alpha\beta}{(\beta+1)}T_3^{\beta+1} - \alpha T_4 T_3^{\beta}\right)\end{array}\right] + \frac{C_S}{2TP}[D(P-D)(T-T_4)^2]\end{array}\right] \quad (23)$$

Let us assume that

$$T_1 = lT_4, T_2 = mT_4, T_3 = nT_4, \beta = 2 \quad (24)$$

$$= \left[\begin{array}{l} DC_P + \frac{C_0}{T} + \frac{C_h}{T}\left[\begin{array}{l}(P-D)\left[\frac{l^2T_4^2}{2} - \frac{\alpha l^4 T_4^4}{6}\right] + a(P-D)\left[(m^2-l^2)\frac{T_4^2}{2} - \frac{\alpha}{6}[m^4 - l^4]T_4^4\right] + \\ b(P-D)[(n^2-m^2)\frac{T_4^2}{2} - \frac{\alpha}{6}(n^4-m^4)T_4^4] + \\ D\left[\frac{(1-n^2)}{2}T_4^2 - \frac{\alpha}{12}(1-n^4)T_4^4 - \frac{\alpha}{3}(n-n^3)T_4^4 + \frac{\alpha}{4}(1-n^4)T_4^4\right]\end{array}\right] + \\ + \frac{C_P}{T}\left[\begin{array}{l}\frac{2\alpha l^3}{3}T_4^3 + \frac{2\alpha}{3}a(P-D)(m^3-l^3)T_4^3 \\ + \frac{2\alpha}{3}b(P-D)(n^3-m^3)T_4^3 + D\left(\frac{\alpha}{3} + \frac{2\alpha n^3}{3} - \alpha n^2\right)T_4^3\end{array}\right] + \frac{C_S}{2TP}[D(P-D)(T-T_4)^2]\end{array}\right] \quad (25)$$

The objective is to determine optimum value of T_4 and T so that TC (T_4, T) is minimum. The value of T_4 and T, for which the total cost TC (T_4, T) is minimum are the solution of equations $\frac{\partial TC(T_4,T)}{\partial T_4} = 0$ and $\frac{\partial TC(T_4,T)}{\partial T} = 0$ satisfy the condition $\left\{\left(\frac{\partial^2 TC(T_4,T)}{\partial T_4^2}\right)\left(\frac{\partial^2 TC(T_4,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(T_4,T)}{\partial T_4 \partial T}\right)^2\right\} > 0$.

The optimum solution of Eq. (22) is obtained by using MATLAB.

4. Numerical example

The above model is illustrated by the following numerical example. Consider the following parameter values. P = 1000, D = 900, C_0 = 100, C_s = 10, C_p = 50, C_h = 6, a = 1, b = 1, α = 0.00, $\beta = 2$, l = 0.4, m = 0.6, n = 0.8. We obtain the optimal value of T_4 = 0.8876, T = 1.028.

4.1. Optimum solution

Production Cost = 45,000; Holding Cost = 114.9184; Setup Cost = 97.2763; Deterioration Cost = 10.9596; Shortage Cost = 8.6289; Total Cost = 45,232.

4.2. Observation

From Table 1, it is observed that when the rate of deteriorative items increases then cycle time (T), ordering quantity (Q) and Holding cost decreases whereas Ordering cost, Deterioration cost and Total cost increases.

4.3. Sensitivity analysis

4.3.1. Observations

The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity, i.e. the effect of making changes in the model parameters over a given optimum solution.

I. As we increase the value of deteriorative items of parameter (α), optimum quantity (Q), production time (T_1, T_2 and T_3), cycle time (T) and Maximum inventory (Q_1, Q_2 and Q_3) are decreased, but shortage cost and total cost are increased.
II. As we increase the value of ordering cost per unit (Co), Optimum quantity (Q), maximum inventory (Q_1, Q_2 and Q_3), Production time (T_1, T_2 and T_3), cycle time (T), shortage cost and total cost are increased.
III. As we increase value of holding cost per unit (Ch), optimum quantity (Q) maximum inventory (Q_1, Q_2 and Q_3), Production time (T_1, T_2 and T_3), and cycle time (T) are decreased, but shortage cost and total cost are increased.

Table 1
Variation of rate of deteriorating items with inventory and total cost.

A	T	Q	Production cost	Ordering cost	Holding cost	Deteriorating cost	Shortage cost	Total cost
0.001	1.0280	925.200	45,000	97.2763	114.9184	10.9596	8.6289	45,232
0.002	1.0150	913.500	45,000	98.5222	111.9148	24.3327	9.2573	45,244
0.003	1.0020	901.800	45,000	99.8004	109.2579	39.8560	9.7443	45,259
0.004	0.9905	891.450	45,000	100.9591	106.6783	57.2965	10.3177	45,275
0.005	0.9796	881.640	45,000	102.0825	104.2984	76.4488	10.8520	45,294
0.007	0.9599	863.910	45,000	104.1775	100.0474	119.1061	11.8517	45,335
0.009	0.9424	848.160	45,000	106.1121	96.0978	167.1929	12.9214	45,382

Table 2
Effect of demand and cost parameters on optimum values.

Parameters		Optimum values										
		T	Q	T_1	T_2	T_3	T_4	Q_1	Q_2	Q_3	S	TC
α	0.001	1.0280	925.200	0.3550	0.5326	0.7101	0.8876	35.5010	53.2459	70.9841	12.6360	45,232
	0.002	1.0150	913.500	0.3482	0.5223	0.6941	0.8705	34.8144	52.2110	69.5950	13.0050	45,244
	0.003	1.0020	901.800	0.3419	0.5128	0.6838	0.8547	34.1800	51.2550	68.3121	13.2570	45,259
	0.004	0.9905	891.450	0.3359	0.5039	0.6718	0.8398	33.5819	50.3539	67.1031	13.5630	45,275
	0.005	0.9796	881.640	0.3304	0.4955	0.6607	0.8259	33.0240	49.5134	65.9759	13.8330	45,294
Co	80	0.9451	850.5900	0.3288	0.4932	0.6576	0.8220	32.8776	49.3120	65.7410	11.0790	45,215
	90	0.9880	889.2000	0.3424	0.5137	0.6849	0.8561	34.2413	51.3570	68.4666	11.8710	45,224
	100	1.0280	925.2000	0.3550	0.5326	0.7101	0.8876	35.5010	53.2459	70.9841	12.6360	45,232
	110	1.0660	959.4000	0.3668	0.5503	0.7337	0.9171	36.6807	55.0419	73.3417	13.4010	45,240
	120	1.101	990.9000	0.3779	0.5668	0.7558	0.9447	37.7844	56.6699	75.5472	14.0670	45,247
C_h	3	1.059	953.1000	0.3733	0.5600	0.7466	0.9333	37.3285	55.9863	74.6363	11.3130	45,171
	6	1.028	925.2000	0.3550	0.5326	0.7101	0.8876	35.5010	53.2459	70.9841	12.6360	45,232
	9	0.9994	899.4600	0.3383	0.5074	0.6766	0.8457	33.8254	50.7333	67.6354	13.8330	45,286
	12	0.9731	875.7900	0.3229	0.4843	0.6458	0.8072	32.2858	48.4244	64.5580	14.9310	45,333
	15	0.9489	854.0100	0.3087	0.4631	0.6174	0.7718	30.8700	46.3014	61.7283	15.9390	45,374
C_s	9	1.0360	932.4000	0.3523	0.5284	0.7046	0.8807	35.2251	52.8322	70.4327	13.9770	45,230
	10	1.0280	925.2000	0.3550	0.5326	0.7101	0.8876	35.5010	53.2459	70.9841	12.6360	45,232
	11	1.0210	918.9000	0.3574	0.5360	0.7147	0.8934	35.7330	53.5937	71.4477	11.4840	45,234
	12	1.0160	914.4000	0.3593	0.5389	0.7186	0.8982	35.9249	53.8816	71.8313	10.6020	45,235
	13	1.0110	909.9000	0.3609	0.5414	0.7218	0.9023	35.0889	54.1274	72.1589	9.7830	45,237
C_p	30	1.0340	930.6000	0.3580	0.5370	0.7160	0.8950	35.7969	53.6897	71.5755	12.5100	27,228
	40	1.0310	927.9000	0.3565	0.5348	0.7130	0.8913	35.6490	53.4678	71.2798	12.5730	36,230
	50	1.0280	925.2000	0.3550	0.5326	0.7101	0.8876	35.5010	53.2459	70.9841	12.6360	45,232
	60	1.0250	922.5000	0.3536	0.5304	0.7072	0.8840	35.3571	53.0301	70.6964	12.6900	54,234
	70	1.0220	919.8000	0.3522	0.5283	0.7044	0.8805	35.2171	52.8202	70.4167	12.7350	63,236
a	1	1.0280	925.2000	0.3550	0.5326	0.7101	0.8876	35.5010	53.2459	70.9841	12.6360	45,232
	2	0.9758	878.2200	0.3320	0.4979	0.6639	0.8299	33.1936	99.5715	66.3725	13.1310	45,255
	4	0.9008	810.7200	0.2981	0.4472	0.5962	0.7453	29.8102	178.8482	59.6099	13.9950	45,293
	5	0.8725	785.2500	0.2851	0.4276	0.5702	0.7127	28.5065	213.7839	57.0036	14.3820	45,309
	7	0.8273	744.5700	0.2639	0.3958	0.5278	0.6597	26.3868	277.0451	52.7662	15.0840	45,338
b	1	1.0280	925.2000	0.3550	0.5326	0.7101	0.8876	35.5010	53.2459	70.9841	12.6360	45,232
	2	0.9573	861.5700	0.3236	0.4855	0.6473	0.8091	32.3617	48.5384	129.4198	13.3380	45,263
	3	0.9057	815.1300	0.3002	0.4504	0.6005	0.7506	30.0222	45.0299	180.1007	13.9590	45,290
	4	0.8659	779.3100	0.2818	0.4227	0.5636	0.7045	28.1785	42.2650	225.3923	14.5260	45,312
	5	0.8338	750.4200	0.2667	0.4001	0.5334	0.6668	26.6707	40.0037	266.6694	15.0300	45,333

IV. As we increase the value of shortage Cost per unit (C_s), Optimum quantity (Q) and cycle time (T) and shortage cost are decreased, but maximum inventory (Q_1, Q_2 and Q_3), Production time (T_1, T_2 and T_3), and total cost are increased.
V. Similarly other parameters 'a' and 'b' and production cost can also be observed from Table 2.

5. Conclusion

In this article we introduced an inventory model for defective items in which three different levels of production are considered. We assumed that the demand rate is constant and the rate of deterioration follows Weibull distribution with two parameters. The proposed model is suitable for newly launched product with constant pattern up to a point in time. Such situation is desirable since by starting at low rate of production, large quantum of stock of manufacturing items at the initial stage to be avoided which will be leading to reduction in the holding cost. Due to this we will get consumer satisfaction and earn potential profit. Here we established mathematical model and a solution for this. To demonstrate the model, numerical example and its sensitivity analysis are given. The proposed inventory model can assist the manufacturer and retailer in determining the optimal order quantity, cycle time and total inventory cost accurately. For further research, this model can be extended in several ways involving different demand rates such as linear, quadratic, cubic, Weibull deterioration with three parameters, time discounting and rework of defective items.

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Please cite this article in press as: Viji G, Karthikeyan K. An economic production quantity model for three levels of production with Weibull distribution deterioration and shortage. Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.10.006