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An exact solution on unsteady MHD free convection chemically reacting silver nanofluid flow past an exponentially accelerated vertical plate through porous medium

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Abstract. This article studies, an exact solution of unsteady MHD free convection boundary-layer flow of a silver nanofluid past an exponentially accelerated moving vertical plate through aporous medium in the presence of thermal radiation, transverse applied amagnetic field, radiation absorption and Heat generation or absorption with chemical reaction are investigated theoretically. We consider nanofluids contain spherical shaped nanoparticle of silverwith a nanoparticle volume concentration range smaller than or equal to 0.04. This phenomenon is modeled in the form of partial differential equations with initial boundary conditions. Some suitable dimensional variables are introduced. The corresponding dimensionless equations with boundary conditions are solved by using Laplace transform technique. The exact solutions for velocity, energy, and species are obtained, also the corresponding numerical values of nanofluid velocity, temperature and concentration profiles are represented graphically. The expressions for skin friction coefficient, the rate of heat transfer and mass transfer are derived. The present study finds applications involving heat transfer, enhancement of thermal conductivity and other applications like transportation, industrial cooling applications, heating buildings and reducing pollution, energy applications and solar absorption. The effect of heat transfer is found to be more pronounced in a silver–water nanofluid than in the other nanofluids.

Nomenclature.

ρ_f Density of the base nanofluid	T' Temperature of the fluid near the plate
ρ_s Density of the solid base nanofluid	T'_∞ Temperature of the fluid far away from the plate
μ_{nf} Dynamic viscosity of the nanofluid	T'_w Temperature of the plate
μ_f Dynamic viscosity of the base fluid	θ Dimensionless temperature
v_f Kinematic viscosity of the base fluid	C'_w Concentration near the plate
β_{nf} Coefficient of thermal expansion of the nanofluid	

g Acceleration due to gravity	C Dimensionless species concentration
$(C_P)_{nf}$ Specific heat capacity of the nanofluid at constant pressure	K' Dimensional permeability parameter
φ Solid volume fraction of nanoparticles	K Dimensionless permeability parameter
N Thermal radiation parameter	M Magnetic field parameter
κ_{nf} is the thermal conductivity of the nanofluid	K_r Dimensional chemically reaction parameter
y' axis is taken normal to the plate	k Dimensionless chemical reaction parameter
y Dimensionless coordinate axis normal to the plate	D Chemical molecular diffusivity
t' Dimensional Time	G_r Thermal Grashof number
$(C_P)_{nf}$ Specific heat capacity of the nanofluid at constant pressure	B_0 External magnetic field
φ Solid volume fraction of nanoparticles	u_0 Velocity of the plate
N Thermal radiation parameter	u Dimensionless velocity
κ_{nf} is the thermal conductivity of the nanofluid	$\Pr$ Prandtl number
y' axis is taken normal to the plate	Sc Schmidt number
t' Dimensional Time	∞ Free stream conditions
	Q_1' Coefficient of proportionality for
	Q_1 Radiation absorption parameter
	Q_0 Dimensional heat absorption
	Q Heat absorption
	$erfc$ Complementary error function
	t Non-dimensional time

1. Introduction

Many industrial processes involve the transfer of heat by means of a flowing fluid either the laminar or turbulent regime as well as flowing or stagnant boiling fluids. The processes cover alarge range of temperatures and pressures. Many of these applications would benefit from a decrease in a thermal resistance of the heat transfer fluid. This situation would lead to smaller heat transfer systems with lower capital costs and improved energy efficiencies. Nanofluids have the potential to reduce such thermal resistances, and the industrial groups that would benefit such improved heat transfer fluids are quite varied. They include transportation, electronics, medical, food, manufacturing of many types. In addition to the numerous experimental investigations into nanofluid thermal properties and heat transfer, various investigators have proposed physical mechanisms and mathematical models to describe and predict the phenomena.

Recent improvements in nanotechnology made it possible to produce solid particles with diameters smaller than 100 nm. As a result, an innovative idea of preparing liquid suspensions by dispersing these nanoparticles instead of amillimeteror micrometer-sized particles in a base fluid and utilizing them for heat transfer enhancement was proposed (Masuda et al [1] and Choi [2]). These liquid suspensions are called nanofluids. An important feature of nanofluids is that since nanoparticles are very small, they behave like fluid molecules and this solves the problem of clogging of small passages in case of the usage of larger particles. Many researchers proposed theoretical models to explain and predict those anomalous thermal conductivity ratios, defined as theeffective thermal conductivity of the nanofluid (keff) divided by the thermal conductivity of the base fluid (kf).

One of the major parameters in heat transfer is the thermal conductivity of the working fluid. Commonly used fluids in heat transfer applications; such as water, ethylene glycol, and engine oil have low thermal conductivities when compared to thermal conductivities of solids, especially metals. As a consequence, researchers have tried to find a way of improving thethermal conductivity of these commonly used fluids. A nanofluid is a term first introduced by Choi and refers to a base liquid with suspended solid nanoparticles. Several others studies Chamkha and Aly [3] studied MHD free convection flow of a nanofluid past a vertical plate in the presence of heat generation or absorption effects. Chamkha et al [4] investigated the Radiation effects on mixed convection over a wedge embedded in a porous medium filled with a nanofluid by the keller-box method he discussed the Nusselt number and Sherwood number. Akilu and Narahari [5-6] studied free convection flow of a nanofluid past an isothermal inclined plate by Keller-box method and also he discussed the velocity and temperature. He extended the same work on the Effects of heat generation or absorption on free convection flow of a nanofluid past an isothermal inclined plate that at the rate of heat transfer decreases with increasing heat generation. Satya Narayana et al. [7-8] studied Thermal radiation and heat source effects on an MHDnanofluid past a vertical plate in a rotating system with a porous medium. He investigated the effect of various important parameters entering into the problem on velocity and temperature fields within the boundary layer are discussed for three different water-based nanofluids such as Cu, Al2O3, and TiO2. The present work shows the need for immediate attention in next-generation solar film collectors, heat-exchanger technology, material processing exploiting vertical surface, geothermal energy storage, and all those processes which are greatly exaggerated by heat enhancement concepts. Turkyilmazoglu [9-11] investigated unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer he found the Closed-form analytic solutions for the flow and heat transfer parameters are obtained also he extended the work with radiation effect. Makinde et al. [12] investigated the Buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet and he analysis the combined effects of buoyancy force, convective heating, Brownian motion, thermophores is and magnetic field on stagnation point flow and heat transfer due to nanofluid flow towards a stretching sheet. Rajesh et al. [13-14] studied the Transient MHD free convection flow and heat transfer of nanofluid past an impulsively started vertical porous plate in the presence of viscous dissipation also he studied Unsteady MHD free convection flow of nanofluid past an accelerated vertical plate with variable temperature and thermal radiation by using Laplace transform technique. Sarit Kumar Das et al. [15] gave a review of heat transfer in nanofluids.Ahmad and Pop [16] studied the steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids using different types of nanoparticles as Cu (copper), Al2O3 (alumina) and TiO2 (titania).Hamad and Pop [17] studied Unsteady MHD free convection flow past a vertical permeable flat plate in a rotating frame of reference with a constant heat source in a nanofluid. Loganathan et al. [18] studied the transient natural convective flow of a nanofluid past a vertical plate in the presence of heat generation by Laplace transform technique also he studied skin friction and Nusselt number. Srinivas et al. [19] investigated the MHD flow of a nanofluid in an expanding or contracting porous pipe with chemical reaction and heat source/sink by using HAM (Homotopy Analysis Method). Ramana Reddy et al. [20] studied Influence of chemical reaction, radiation, and rotation on MHD nanofluid flow past a permeable flat plate in a porous medium. Aly and Ebaid [21] studied Exact Analytical Solution for the Peristaltic Flow of Nanofluids in an Asymmetric Channel with Slip Effect of the Velocity, Temperature, and Concentration. Bourantas et al. [22] studied Heat transfer and natural convection of nanofluids in porous media. Sandeep et al. [23] studied Unsteady MHD radiative flow and heat transfer of a dusty nanofluid over an exponentially stretching surface he investigated two types of nanofluids namely Cu-water and CuO-water embedded with conducting dust particles. Mahdi et al. [24] give the practical heat transfer process applications of porous media with nanofluid. Das and Jana [25] studied Natural convective magneto-nanofluid flow and radiative heat transfer past a moving vertical plate by using Rosseland approximation applies to optically thick media and gives the net radiation heat flux. Venkateswarlu and Satya Narayana [26] investigated Chemical reaction and radiation absorption effects on the flow and heat transfer of a nanofluid in a rotating system. He concludes that the copper oxide nanofluid velocity and temperature profiles increase with increasing values of heat absorption. Haile and Shankar [27] investigated a steady MHD boundary-layer flow of water-based nanofluids over a moving permeable flat plate solved numerically by shooting method with Runge-Kutta integration scheme. Ganga et al. [28] studied MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. Khalid et al. [29] studied exact solutions for free convection flow of nanofluids with ramped wall temperature by using Laplace transform technique. Li [30] studied exact solutions for free convective heat transfer in nanofluids in a vertical channel. Hamilton and Crosser [31] studied thermal conductivity of heterogeneous two-component systems.

By taking into an account of all the above-cited articles, we studied an exact solution of unsteady MHD free convection boundary-layer flow of a silver nanofluid past an exponentially accelerated moving vertical plate through porous medium in the presence of thermal radiation, transverse applied magnetic field, radiation absorption and Heat generation or absorption with chemical reaction is investigated theoretically. The linear partial differential equations governing the flow of heat and mass transfer problem have been solved analytically by using Laplace transform technique with suitable boundary conditions. An analytic solution is presented and discussed the influence along with the friction factor, local Nusselt and Sherwood numbers.

2. Mathematical analysis

We consider the unsteady laminar two-dimensional boundary layer flow of a viscous incompressible electrically conducting nanofluid past an exponentially accelerated vertical permeable plate in the presence of an applied magnetic field, chemical reaction, thermal radiation, and radiation absorption are considered. The x-axis is taken along the plate in the vertically upward direction, and y-axis is taken normal to the plate. Initially the nanofluid and the plate are having the same temperature T_∞' and the concentration C_∞' at all the points in a stationary state for a time $t' \le 0$. At the time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in the vertical direction versus the gravitational field. At the same time, plate temperature and concentration are raised linearly with time t. A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis. It is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state. It is assumed that there is no applied voltage which implies the absence of an electric field. Also, it is assumed that the induced magnetic field is small compared to the external magnetic field. This is valid when the magnetic Reynolds number is small. The plate coincides with the plane $y = 0$ and the flow being confined to $y > 0$. It is assumed that the pressure gradient is neglected in this problem. It is also assumed that a radiative heat flux q_r is applied in the normal direction to the plate. Furthermore, the flow variables are functions of y and t only. The coordinate system and the flow configuration are shown in Fig. 1. The basic unsteady momentum, thermal energy and species equations based on the model for nanofluids [32] satisfying the Boussinesq's approximation [33] in the presence of magnetic field, thermal radiation, heat generation and radiation absorption are as follows:

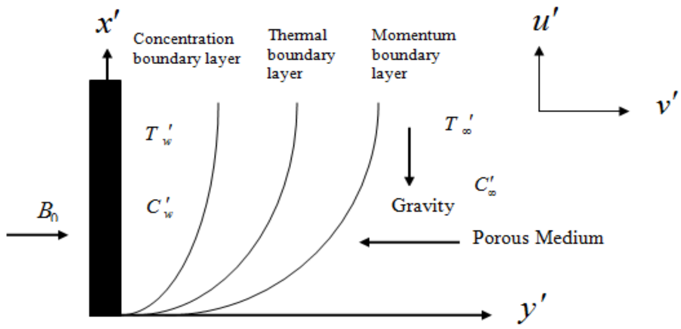

Figure 1. Physical coordinate system

The governing equations of this investigation are given by:

$$\rho_{nf}\frac{\partial u}{\partial t'}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+(\rho\beta)_{nf}\,g\left(T'-T_\infty'\right)-\sigma B_0^2 u-\frac{v_f u}{k} \tag{1}$$

$$\frac{\partial T'}{\partial t'}=\frac{1}{(\rho C_P)_{nf}}\left\{k_{nf}\frac{\partial^2 T'}{\partial y^2}-\frac{\partial q_r}{\partial y}-Q_0\left(T'-T_\infty'\right)-\right\}+Q_1'\left(C'-C_\infty'\right) \tag{2}$$

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y^2}-k_r\left(C'-C_\infty'\right) \tag{3}$$

The initial boundary conditions are given by

$$t'\le 0: u=0, T'=T_\infty', C'=C_\infty' \text{ for all } y$$

$$t'>0: u=u_0\exp(a't'), T'=T_\infty'+\left(T_w'-T_\infty'\right)At', C'=C_\infty'+\left(C_w'-C_\infty'\right)At' \text{ at } y=0$$

$$u\to 0, T'\to T_\infty', C'\to C_\infty' \text{ as } y\to\infty \tag{4}$$

Where $A=\frac{u_o^2}{v_f}$, For an optically thick fluid, we can adopt Rosseland approximation for radiative flux q_r by the expression $q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y}$ (5)

The flow is caused by the stretching of the sheet which moves in its own plane with the surface velocity $U_w(x)=ax$, where a (stretching rate) is positive constant. The dynamic viscosity μ_{nf}, density ρ_{nf}, heat capacitance, thermal conductivity k_{nf} of the nanofluid and kinematic viscosity v_f of the base fluid are defined as follows:where σ^* is the Stefan-Boltzmann constant and k^* the Rosseland mean absorption coefficient. We assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about a free stream temperature T_∞' as follows:

$$T'^4=T_\infty'^4+4T_\infty'^3(T'-T_\infty')+6T_\infty'^2(T'-T_\infty')^2+.......................... \tag{6}$$

Neglecting higher-order terms in Eq. (6) beyond the first order in $(T'-T_\infty')$, we get

$$T'^4\cong 4T_\infty'^3T'-3T_\infty'^4 \tag{7}$$

In view Eqs. (6) and (7) reduces to

$$\frac{\partial T'}{\partial t'}=\frac{1}{(\rho C_P)_{nf}}\left[k_{nf}+\frac{16\sigma^*T_\infty'^3}{3k^*}\right]\frac{\partial^2 T'}{\partial y^2}-\frac{Q_0\left(T'-T_\infty'\right)}{(\rho C_P)_{nf}}+Q_1'\left(C'-C_\infty'\right) \tag{8}$$

Introducing non-dimensional variables

$$U=\frac{u}{u_0}, Y=\frac{yu_0}{v_f}, t=\frac{t'u_0^2}{v_f}, M=\frac{\sigma B_0^2 v_f}{\rho_f u_0^2}, Gr=\frac{g\beta_f v_f\left(T_w'-T_\infty'\right)}{u_0^3}, \theta=\frac{T'-T_\infty'}{T_w'-T_\infty'}, C=\frac{C'-C_\infty'}{C_w'-C_\infty'}, N=\frac{16\sigma^*T_\infty'^3}{3k^*k_f},$$

$$\Pr=\frac{v_f(\rho C_P)_f}{\kappa_f}, k=\frac{v_f K_r}{u_o^2}, Sc=\frac{v_f}{D}, Q=\frac{v_f Q_0}{u_o^2(\rho C_p)_f}, Q_1=\frac{v_f Q_1'(C_w'-C_\infty')}{u_o^2(T_w'-T_\infty')} \tag{9}$$

The properties of nanofluids are given as follows

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},\ \rho_{nf}=(1-\varphi)_{\rho f}+\varphi\rho_s,\ (\rho C_P)_{nf}=(1-\varphi)(\rho C_P)_f+\varphi(\rho C_P)_s,\ (\rho\beta)_{nf}=(1-\varphi)(\rho\beta)_f+\varphi(\rho\beta)_s$$

$$k_{nf}=k_f\left[\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}\right] \tag{10}$$

The Equations (1)-(3) are reduced to the following forms by using equations (8) and (9) as follows

$$\frac{\partial U}{\partial t}=\alpha_1\frac{\partial^2 U}{\partial Y^2}+\alpha_2 Gr\theta-\alpha_3\left(M+\frac{1}{K}\right) \tag{11}$$

$$\frac{\partial\theta}{\partial t}=\alpha_4\frac{\partial^2\theta}{\partial Y^2}-\alpha_5 Q\theta+Q_1 C \tag{12}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2}-kC \tag{13}$$

where

$$x_1=(1-\varphi)+\varphi\left(\frac{\rho_s}{\rho_f}\right),\ x_2=(1-\varphi)+\varphi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right),\ x_3=(1-\varphi)+\varphi\left(\frac{(\rho C_P)_s}{(\rho C_P)_f}\right),\ x_4=\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}$$

$$\alpha_1=\frac{1}{(1-\varphi)^{2.5}x_1},\ \alpha_2=\frac{x_2}{x_1},\ \alpha_3=\frac{1}{x_1}\ \alpha_4=\frac{x_4+N}{x_3\Pr},\ \alpha_5=\frac{1}{x_3}$$

In non-dimensional form, the conditions are reduced to as follows

$$\begin{aligned}&t\le 0: U=0,\quad \theta=0,\ C=0\ \text{ for all } Y\\ &t>0: U=\exp(at),\theta=t,\ \ C=t \text{ at } Y=0\\ &U\to 0,\ \ \theta\to 0,\ \ C\to 0 \text{ as } Y\to\infty\end{aligned} \tag{14}$$

Solution of the problem

The non-dimensional equations (11)-(13) associated with the boundary conditions (14) are solved by the method of Laplace transforms technique and the solutions of velocity, temperature and concentration are given below

$$U=\frac{\exp(at)}{2}\left[\exp\left(Y\sqrt{\alpha_3BP+aB}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}+\sqrt{(\alpha_3P+a)t}\right)+\exp\left(-Y\sqrt{\alpha_3BP+aB}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}-\sqrt{(\alpha_3P+a)t}\right)\right]$$

$$-\frac{A_1}{2}\left[\exp\left(Y\sqrt{\alpha_3BP}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}+\sqrt{\alpha_3Pt}\right)+\exp\left(-Y\sqrt{\alpha_3BP}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}-\sqrt{\alpha_3Pt}\right)\right]$$

$$+A_2\left[\left(\frac{t}{2}+\frac{YB}{4\sqrt{\alpha_3BP}}\right)\exp\left(Y\sqrt{\alpha_3BP}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}+\sqrt{\alpha_3Pt}\right)+\left(\frac{t}{2}-\frac{YB}{4\sqrt{\alpha_3BP}}\right)\exp\left(-Y\sqrt{\alpha_3BP}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}-\sqrt{\alpha_3Pt}\right)\right]$$

$$+\frac{A_3\exp(-a_2t)}{2}\left[\exp\left(Y\sqrt{\alpha_3BP-a_2B}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}+\sqrt{(\alpha_3P-a_2)t}\right)+\exp\left(-Y\sqrt{\alpha_3BP-a_2B}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}-\sqrt{(\alpha_3P-a_2)t}\right)\right]$$

$$+\frac{A_4\exp(-a_4t)}{2}\left[\exp\left(Y\sqrt{\alpha_3BP-a_4B}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}+\sqrt{(\alpha_3P-a_4)t}\right)+\exp\left(-Y\sqrt{\alpha_3BP-a_4B}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}-\sqrt{(\alpha_3P-a_4)t}\right)\right]$$

$$-\frac{A_5\exp(-a_8t)}{2}\left[\exp\left(Y\sqrt{\alpha_3BP-a_8B}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}+\sqrt{(\alpha_3P-a_8)t}\right)+\exp\left(-Y\sqrt{\alpha_3BP-a_8B}\right)erfc\left(\frac{Y\sqrt{B}}{2\sqrt{t}}-\sqrt{(\alpha_3P-a_8)t}\right)\right]$$

$$+\frac{A_6}{2}\left[\exp\left(Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}+\sqrt{\alpha_5Qt}\right)+\exp\left(-Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}-\sqrt{\alpha_5Qt}\right)\right]$$

$$-A_7\left[\left(\frac{t}{2}+\frac{YA}{4\sqrt{\alpha_5AQ}}\right)\exp\left(Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}+\sqrt{\alpha_5Qt}\right)+\left(\frac{t}{2}-\frac{YA}{4\sqrt{\alpha_5AQ}}\right)\exp\left(-Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}-\sqrt{\alpha_5Qt}\right)\right]$$

$$+\frac{A_8\exp(-a_2t)}{2}\left[\exp\left(Y\sqrt{\alpha_5AQ-a_2A}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}+\sqrt{(\alpha_5Q-a_2)t}\right)+\exp\left(-Y\sqrt{\alpha_5AQ-a_2A}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}-\sqrt{(\alpha_5Q-a_2)t}\right)\right]$$

$$-\frac{A_4\exp(-a_4t)}{2}\left[\exp\left(Y\sqrt{\alpha_5AQ-a_4A}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}+\sqrt{(\alpha_5Q-a_4)t}\right)+\exp\left(-Y\sqrt{\alpha_5AQ-a_4A}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}-\sqrt{(\alpha_5Q-a_4)t}\right)\right]$$

$$-\frac{A_9}{2}\left[\exp\left(Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\exp\left(-Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\right]$$

$$+A_{10}\left[\left(\frac{t}{2}+\frac{Y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\left(\frac{t}{2}-\frac{Y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\right]$$

$$-\frac{A_{11}\exp(-a_2t)}{2}\left[\exp\left(Y\sqrt{kSc-a_2Sc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(k-a_2)t}\right)+\exp\left(-Y\sqrt{kSc-a_2Sc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(k-a_2)t}\right)\right]$$

$$+\frac{A_5\exp(-a_8t)}{2}\left[\exp\left(Y\sqrt{kSc-a_8Sc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(k-a_8)t}\right)+\exp\left(-Y\sqrt{kSc-a_8Sc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(k-a_8)t}\right)\right]$$

$$\theta=(1+A_{12})\left[\left(\frac{t}{2}+\frac{YA}{4\sqrt{\alpha_5AQ}}\right)\exp\left(Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}+\sqrt{\alpha_5Qt}\right)+\left(\frac{t}{2}-\frac{YA}{4\sqrt{\alpha_5AQ}}\right)\exp\left(-Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}-\sqrt{\alpha_5Qt}\right)\right]$$

$$-\frac{A_{13}}{2}\left[\exp\left(Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}+\sqrt{\alpha_5Qt}\right)+\exp\left(-Y\sqrt{\alpha_5AQ}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}-\sqrt{\alpha_5Qt}\right)\right]$$

$$+\frac{A_{13}\exp(-a_2t)}{2}\left[\exp\left(Y\sqrt{\alpha_5AQ-a_2A}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}+\sqrt{(\alpha_5Q-a_2)t}\right)+\exp\left(-Y\sqrt{\alpha_5AQ-a_2A}\right)erfc\left(\frac{Y\sqrt{A}}{2\sqrt{t}}-\sqrt{(\alpha_5Q-a_2)t}\right)\right]$$

$$+\frac{A_{13}}{2}\left[\exp\left(Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\exp\left(-Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\right]$$

$$-A_{12}\left[\left(\frac{t}{2}+\frac{Y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\left(\frac{t}{2}-\frac{Y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\right]$$

$$-\frac{A_{13}\exp(-a_2t)}{2}\left[\exp\left(Y\sqrt{kSc-a_2Sc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(k-a_2)t}\right)+\exp\left(-Y\sqrt{kSc-a_2Sc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(k-a_2)t}\right)\right]$$

$$C=\left(\frac{t}{2}+\frac{Y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\left(\frac{t}{2}-\frac{Y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-Y\sqrt{kSc}\right)erfc\left(\frac{Y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)$$

Where

$$P=M+\frac{1}{K},\ A=\frac{1}{\alpha_4},B=\frac{1}{\alpha_1},\ a_1=\frac{AQ_1}{(Sc-A)},a_2=\frac{kSc-A\alpha_5Q}{(Sc-A)},a_3=\frac{\alpha_2BGr}{(A-B)},$$

$$a_4=\frac{A\alpha_5Q-\alpha_3BP}{(A-B)},a_5=\frac{a_1\alpha_2BGr}{a_2^2(A-B)},a_6=\frac{a_1\alpha_2BGr}{a_2(A-B)},a_7=\frac{a_1\alpha_2BGr}{a_2^2(Sc-B)},a_8=\frac{kSc-\alpha_3BP}{(Sc-B)},$$

$$a_9=\frac{a_1\alpha_2BGr}{a_2(Sc-B)}\ A_1=\frac{a_8^2(a_3+a_4a_5+a_6)-a_4^2(a_7a_8+a_9)}{a_4^2a_8^2},A_2=\frac{a_8(a_3+a_6)-a_4a_9}{a_4a_8},$$

$$A_3=\frac{a_7(a_2-a_4)-a_5(a_2-a_8)}{(a_2-a_4)(a_2-a_8)},A_4=\frac{a_2(a_3+a_4a_5+a_6)-a_4(a_3+a_6)}{a_4^2(a_2-a_4)},$$

$$A_5=\frac{a_2a_7a_8+a_9(a_2-a_8)}{a_8^2(a_2-a_8)},A_6=\frac{a_3+a_4a_5+a_6}{a_4^2},A_7=\frac{a_3+a_4}{a_4},A_8=\frac{a_5}{a_2-a_4},A_9=\frac{a_7a_8+a_9}{a_8^2},$$

$$A_{10}=\frac{a_9}{a_8},A_{11}=\frac{a_7}{a_2-a_8},A_{12}=\frac{a_1}{a_2},A_{13}=\frac{a_1}{a_2^2}$$

Effects of parameters on the shear stress at the plate

For the sake of engineering purpose, one is usually interested to evaluate the shear stress (or skin friction). The increased shear stress is generally a disadvantage in the technical applications. The non-dimensional shear stress at the plate $y=0$ due to the flow is given by $C_f = \frac{\tau_w}{\rho_f {u_0}^2}$, Where τ_w skin-friction or shear stress is given by $\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}$

Using non-dimensional variables (9), we get the skin-friction

$$C_f = \frac{1}{(1-\varphi)^{2.5}}\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$$

$$C_f = \frac{1}{(1-\varphi)^{2.5}}\Bigg\{\sqrt{\frac{B}{\pi t}}\exp(-\alpha_3 Pt) + \exp(at)\sqrt{(\alpha_3 P + a)B}\, erf\left(\sqrt{(\alpha_3 P + a)t}\right)$$

$$-A_1\left[\sqrt{\frac{B}{\pi t}}\exp(-\alpha_3 Pt) + \sqrt{\alpha_3 BP}\, erf\left(\sqrt{\alpha_3 Pt}\right)\right]$$

$$+A_2\left[\sqrt{\frac{Bt}{\pi}}\exp(-\alpha_3 Pt) + t\sqrt{\alpha_3 BP}\, erf\left(\sqrt{\alpha_3 Pt}\right) + \frac{\sqrt{B}}{2\sqrt{\alpha_3 P}}\, erf\left(\sqrt{\alpha_3 Pt}\right)\right]$$

$$+A_3\left[\sqrt{\frac{B}{\pi t}}\exp(-\alpha_3 Pt) + \exp(-a_2 t)\sqrt{(\alpha_3 P - a_2)B}\, erf\left(\sqrt{(\alpha_3 P - a_2)t}\right)\right]$$

$$+A_4\left[\sqrt{\frac{B}{\pi t}}\exp(-\alpha_3 Pt) + \exp(-a_4 t)\sqrt{(\alpha_3 P - a_4)B}\, erf\left(\sqrt{(\alpha_3 P - a_4)t}\right)\right]$$

$$-A_5\left[\sqrt{\frac{B}{\pi t}}\exp(-\alpha_3 Pt) + \exp(-a_8 t)\sqrt{(\alpha_3 P - a_8)B}\, erf\left(\sqrt{(\alpha_3 P - a_8)t}\right)\right]$$

$$-A_6\left[\sqrt{\frac{A}{\pi t}}\exp(-\alpha_5 Qt) + \sqrt{\alpha_5 AQ}\, erf\left(\sqrt{\alpha_5 Qt}\right)\right]$$

$$+A_7\left[\sqrt{\frac{At}{\pi}}\exp(-\alpha_5 Qt) + t\sqrt{\alpha_5 AQ}\, erf\left(\sqrt{\alpha_5 Qt}\right) + \frac{\sqrt{A}}{2\sqrt{\alpha_5 Q}}\, erf\left(\sqrt{\alpha_5 Qt}\right)\right]$$

$$-A_8\left[\sqrt{\frac{A}{\pi t}}\exp(-\alpha_5 Qt) + \exp(-a_2 t)\sqrt{(\alpha_5 Q - a_2)A}\, erf\left(\sqrt{(\alpha_5 Q - a_2)t}\right)\right]$$

$$+A_4\left[\sqrt{\frac{A}{\pi t}}\exp(-\alpha_5 Qt) + \exp(-a_4 t)\sqrt{(\alpha_5 Q - a_4)A}\, erf\left(\sqrt{(\alpha_5 Q - a_4)t}\right)\right]$$

$$+A_9\left[\sqrt{\frac{Sc}{\pi t}}\exp(-kt) + \sqrt{kSc}\, erf\left(\sqrt{kt}\right)\right]$$

$$-A_{10}\left[\sqrt{\frac{tSc}{\pi}}\exp(-kt) + t\sqrt{kSc}\, erf\left(\sqrt{kt}\right) + \frac{Sc}{2\sqrt{kSc}}\, erf\left(\sqrt{kt}\right)\right]$$

$$+A_{11}\left[\sqrt{\frac{Sc}{\pi t}}\exp(-kt) + \exp(-a_2 t)\sqrt{(k - a_2)Sc}\, erf\left(\sqrt{(k - a_2)t}\right)\right]$$

$$-A_5\left[\sqrt{\frac{Sc}{\pi t}}\exp(-kt) + \exp(-a_8 t)\sqrt{(k - a_8)Sc}\, erf\left(\sqrt{(k - a_8)t}\right)\right]$$

Effects of parameters on rate of heat transfer to the plate

The local Nusselt number Nu is defined by $Nu = \frac{xq_w}{k_f(T'_w - T'_\infty)}$ where q_w is the wall heat flux is given by $q_w = -k_{nf}\left(\frac{\partial T'}{\partial y}\right)_{y=0}$, using non-dimensional variables (9), we get the Nusselt number

$$Nu = -\frac{k_{nf}}{k_f}\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0}$$

$$\begin{aligned}Nu = x_4 \Bigg\{ &(1+A_{12})\left[\sqrt{\frac{At}{\pi}}\exp(-\alpha_5 Qt) + t\sqrt{\alpha_5 AQ}\, erf\left(\sqrt{\alpha_5 Qt}\right) + \frac{\sqrt{A}}{2\sqrt{\alpha_5 Q}}\, erf\left(\sqrt{\alpha_5 Qt}\right)\right] \\ &-A_{13}\left[\sqrt{\frac{A}{\pi t}}\exp(-\alpha_5 Qt) + \sqrt{\alpha_5 AQ}\, erf\left(\sqrt{\alpha_5 Qt}\right)\right] \\ &+A_{13}\left[\sqrt{\frac{A}{\pi t}}\exp(-\alpha_5 Qt) + \exp(-a_2 t)\sqrt{(\alpha_5 Q - a_2)A}\, erf\left(\sqrt{(\alpha_5 Q - a_2)t}\right)\right] \\ &+A_{13}\left[\sqrt{\frac{Sc}{\pi t}}\exp(-kt) + \sqrt{kSc}\, erf\left(\sqrt{kt}\right)\right] - A_{12}\left[\sqrt{\frac{tSc}{\pi}}\exp(-kt) + t\sqrt{kSc}\, erf\left(\sqrt{kt}\right) + \frac{Sc}{2\sqrt{kSc}}\, erf\left(\sqrt{kt}\right)\right] \\ &-A_{13}\left[\sqrt{\frac{Sc}{\pi t}}\exp(-kt) + \exp(-a_2 t)\sqrt{(k-a_2)Sc}\, erf\left(\sqrt{(k-a_2)t}\right)\right]\Bigg\}\end{aligned}$$

Effects of parameters on rate of mass transfer at the plate

The local Sherwood number Sh is defined by $Sh = \frac{xq_m}{D(C'_w - C'_\infty)}$ where q_m is the wall mass flux is given by $q_m = -\mu_{nf}\left(\frac{\partial C'}{\partial y}\right)_{y=0}$, using non-dimensional variables (9), we get the Sherwood number

$$Sh = -\mu_{nf}\left(\frac{\partial C}{\partial Y}\right)_{Y=0}$$

$$Sh = \frac{1}{(1-\varphi)^{2.5}}\left\{\sqrt{\frac{tSc}{\pi}}\exp(-kt) + t\sqrt{kSc}\, erf\left(\sqrt{kt}\right) + \frac{Sc}{2\sqrt{kSc}}\, erf\left(\sqrt{kt}\right)\right\}$$

Thermo physical properties of water based nanoparticles

Thermo physical properties	Base fluid(water)	Cu (Copper)	Ag (Silver)	TiO_2 (Titanium dioxide)
$\rho(kg/m^3)$	997.1	8933	10500	4250
$C_P(J/kg\,K)$	4179	385	235	686.2
$k(W/m\,K)$	0.613	401	429	8.9538
$\beta\times 10^5(K^{-1})$	21	1.67	1.89	0.90
φ	0.0	0.05	0.04	0.2

3. Results and Discussion

A theoretical study on the effect of the silver nanoparticle on unsteady MHD free convective fluid flow through porous medium past an infinite vertical porous plate in the presence of transverse applied amagneticfield, thermal radiation and first order chemical reactionhave been performed in this paper. The effects of nanoparticles on the velocity, temperature and concentration profiles as well as on the skin friction coefficient, the local Nusselt number, and Sherwood number are discussed numerically. We have chosen here Gr = 10, M=3, N=4, Pr= 6.2, Q=5, Q1=0.5, K=0.5, k= 0.5, Sc==0.78 and a = 0.5 at time t=1are varied over a range, which are listed in the figures legends. In this study, the nanoparticle volume fraction is considered in the range of 0< φ<0.04, as sedimentation takes place when the nanoparticle volume fraction exceeds8%. Also, we consider spherical nanoparticles with thermal conductivity and dynamic viscosity shown in Table 1. The Prandtl number*Pr*of the base fluid is kept constant at 6.2. When $\varphi = 0$ the model contracts to the governing equations for a regular viscous fluid i.e. nanoscale characteristics are eliminated.The velocity profile for different values of magnetic field parameter M is shown in Figure 2. From this figure,it is found that the velocity of Ag-water nanofluid decreases with the increase in M. The reason behind this phenomenon is that application of magnetic field to an electrically conducting nanofluid gives rise to a resistive type force called the Lorentz force. This force has the tendency to slow down the motion of the nanofluid in the boundary layer. Figure.3 represents the influence of radiation parameter N on the nanofluid velocity profile. It is observed that the nanofluid velocity distributions enhance with the increase in radiation parameter (N). This is because the thermal boundary layer thickness increases with an increase in the thermal radiation. Thus, it is pointed out that, the radiation should be minimized to have the cooling process at a faster rate.

The effect of the radiation absorption parameter has shown in Figure 4. It is shownthe velocity increases with increases of radiation absorption parameter. Figure 5has been plotted to find the variation of ananofluid velocity profile for different values of heat generation parameter Q. It is clear that there is a decrease in thevelocity with anincrease of Q. This implies that heat absorption tends to retard fluid velocity throughout the boundary layer region. This may be attributed to the fact that the tendency of heat absorption (thermal sink) is to reduce the fluid temperature which causes the strength of thermal buoyancy force to decrease resulting in a net reduction in the fluid velocity.It is clear from Figure 6 that an increase in porosity parameter leads to enhance the velocity profile because it reduces the drag force.Figure 6 shows the effects of thermal Grashof number on the velocity profile. From this figure, it is found that the parameter Gr signifies the relative influence of thermal buoyancy force and viscous force in the boundary layer regime. It is also seen that the velocity of Ag -water nanofluid increases with the increase in Gr. This means that the buoyancy force accelerates velocity field. This gives rise to an increase in the induced flow transport. It is observed in figure 8 that the velocity increases with increasing exponential accelerated parameter also when we take exponential accelerating parameter equal to zero the whole problem converted into nanofluid flow over impulsively started plate with variable temperature and variable concentration in the presence of radiation absorption and heat generation. It is found in figure 9, that the dimensionless velocity increases with increasing time.Figures 10-13 describes the effects of N, Q1, Q and t with temperature. It is evident from the figures that a temperature of the fluid increases with increasing values of N (Radiation parameter), Q1 (Radiation absorption) and t (time), but the reverse effect will happen for Q (Heat generation).

The concentration profiles are presented for different values Sc k and $t$are depicted in Figures 14, 15 and 16. It is noticed that increase of Schmidt number and chemical reaction parameter leads to the decrease in concentration of the species. And, it is also observed that the concentration will be zero as it moves far away from the plate. Since the concentration is considered as time dependent, therefore this figure clearly reflects thesituation that the concentration increases with theincrease of time$t$.

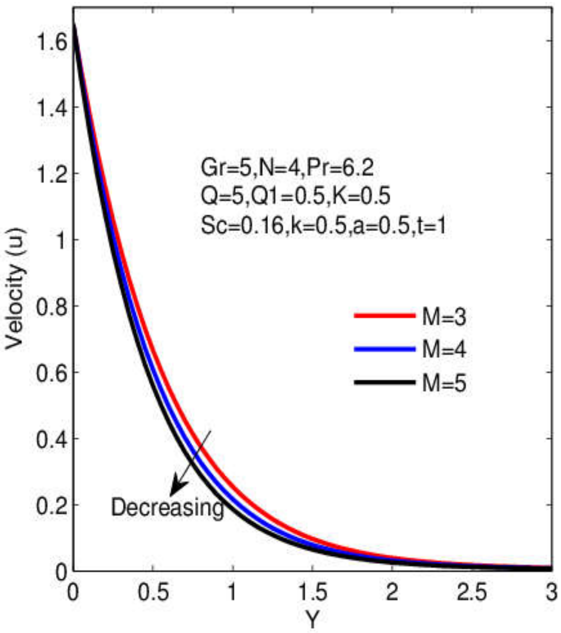

Figure2. Velocity profiles for different values of M

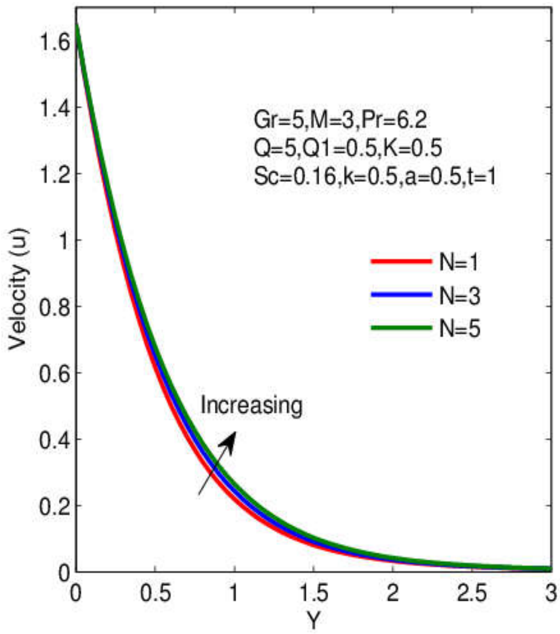

Figure3. Velocity profiles for different values of N

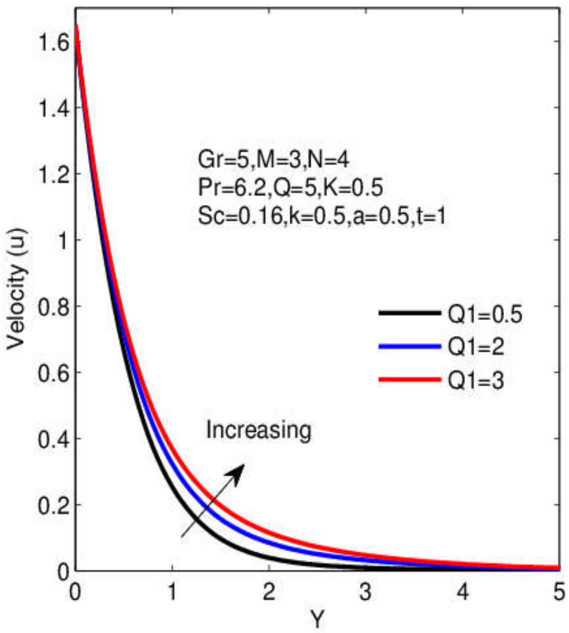

Figure 4. Velocity profiles for different values of Q1

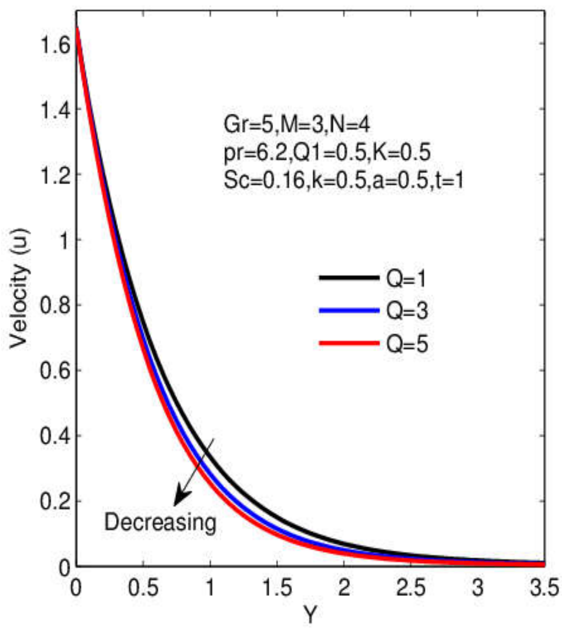

Figure 5. Velocity profiles for different values of Q

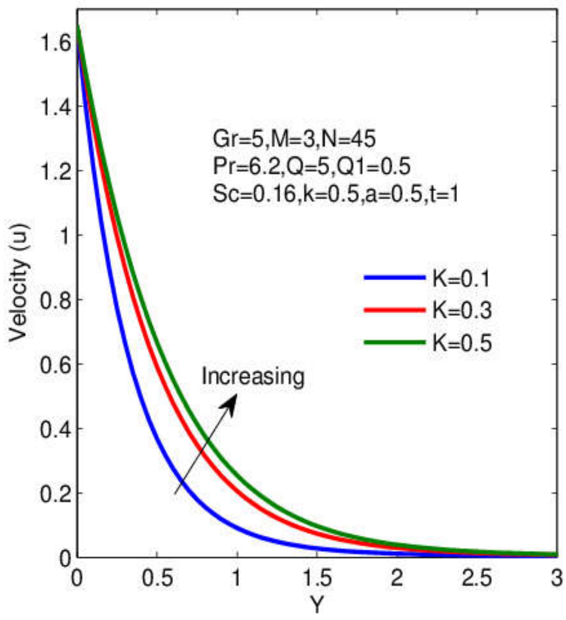

Figure 6. Velocity profiles for different values of K

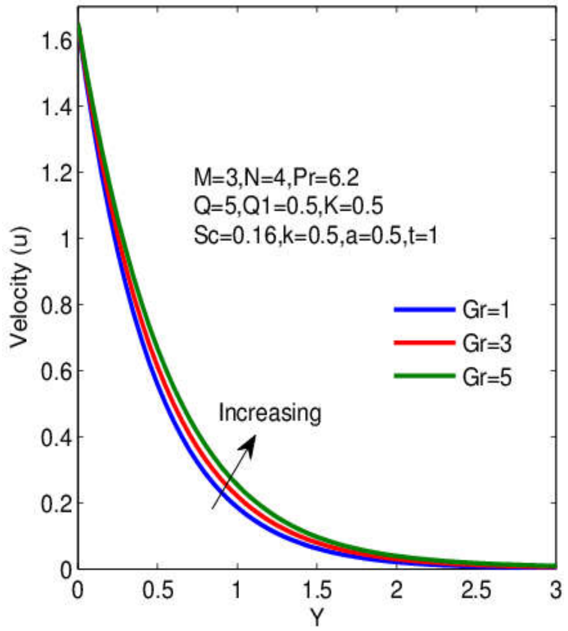

Figure7. Velocity profiles for different values of Gr

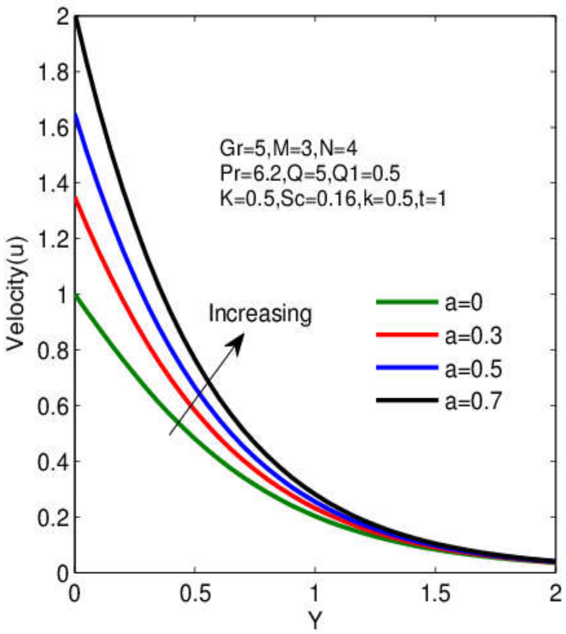

Figure 8. Velocity profiles for different values of a

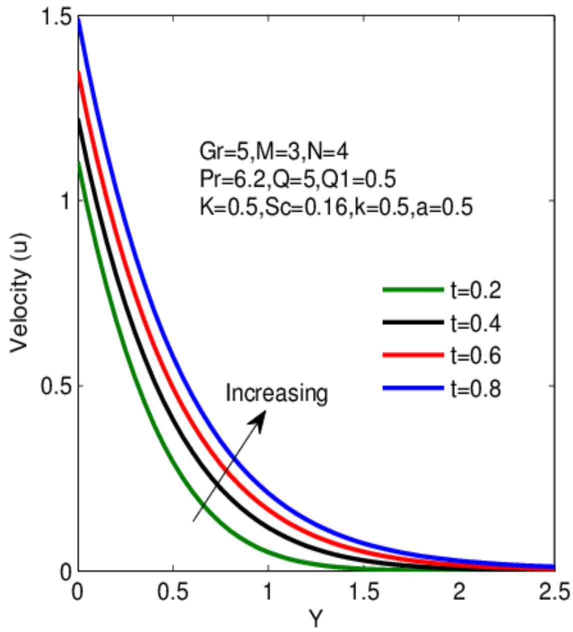

Figure9. Velocity profiles for different values of t

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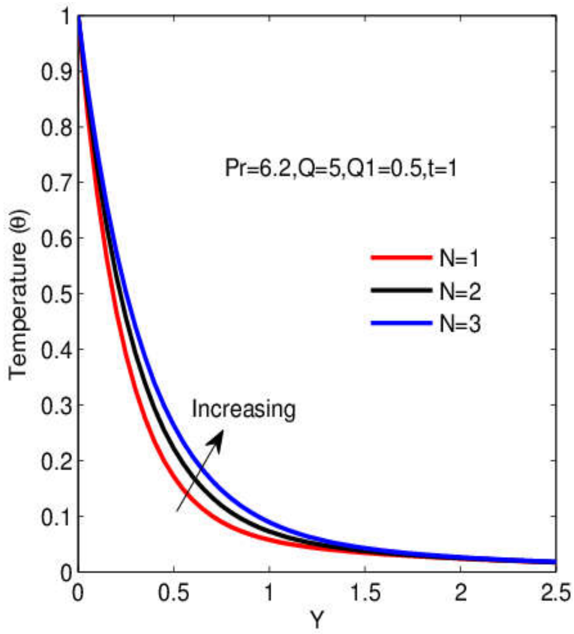

Figure 10. Temperatureprofiles for different values of N

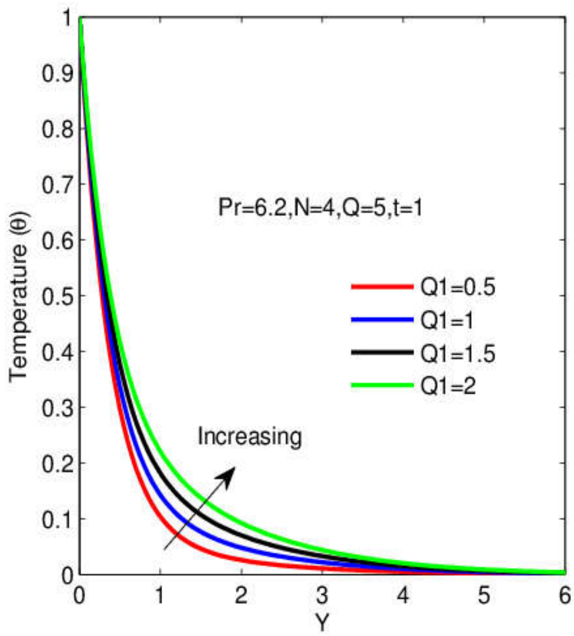

Figure11. Temperature profiles for different values of Q1

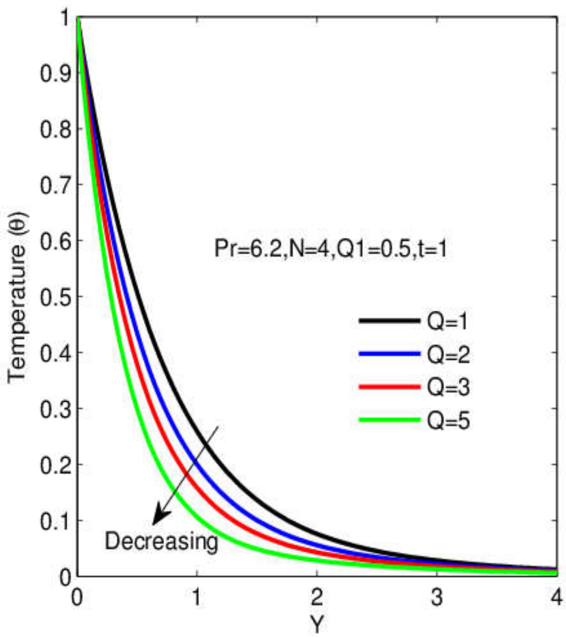

Figure12. Temperature profiles for different values of Q

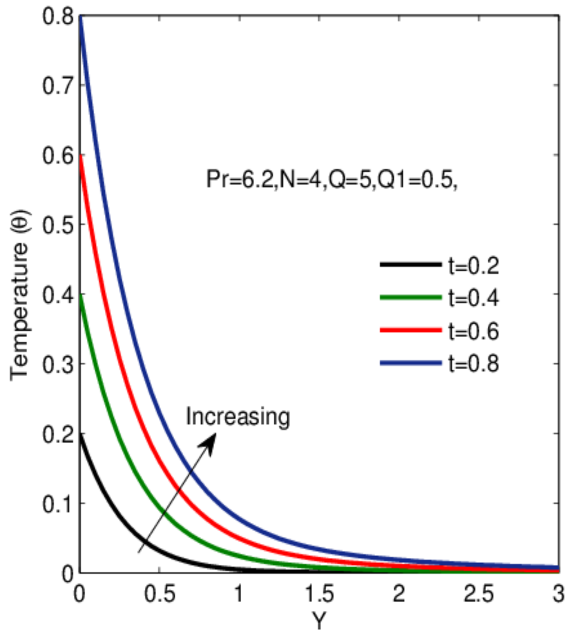

Figure13. Temperature profiles for different values of t

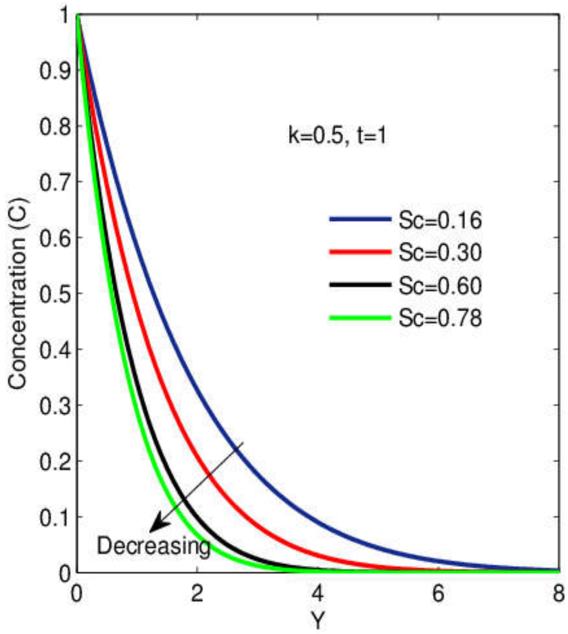

Figure14. Concentration profiles for different values of Sc

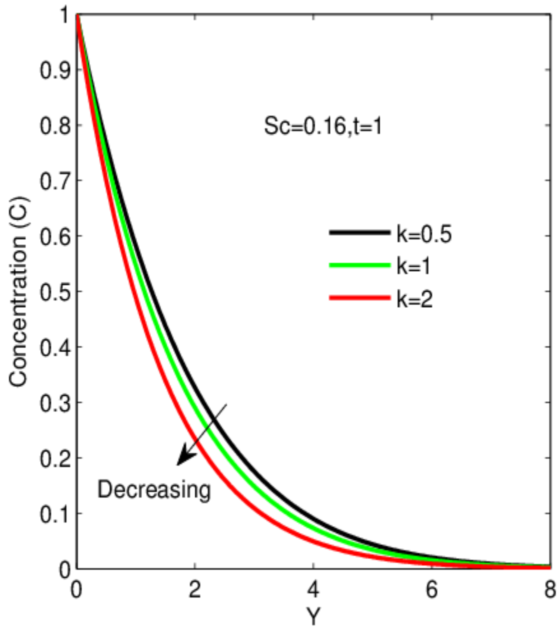

Figure15. Concentration profiles for different values of k

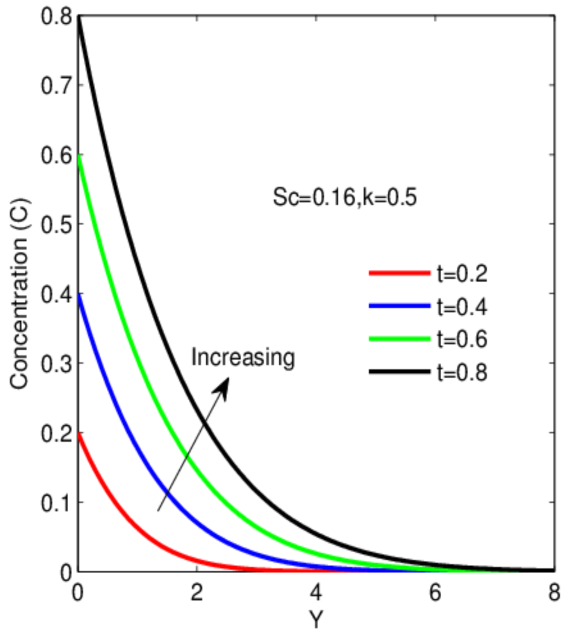

Figure16. Concentration profiles for different values of t

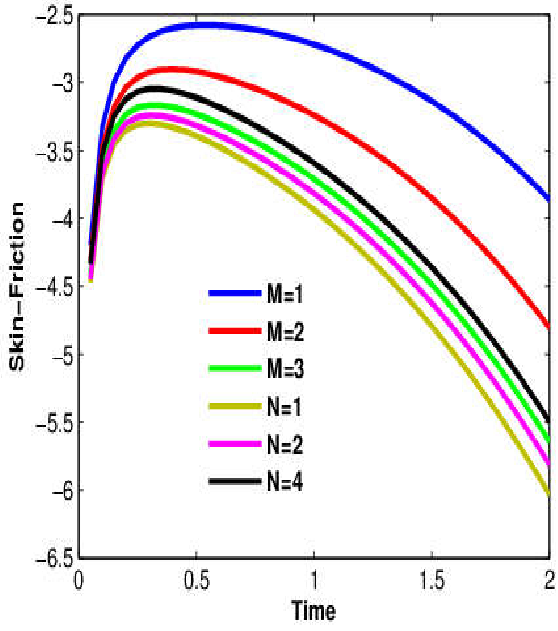

Figure17. Skin-friction coefficient for various values of M and N

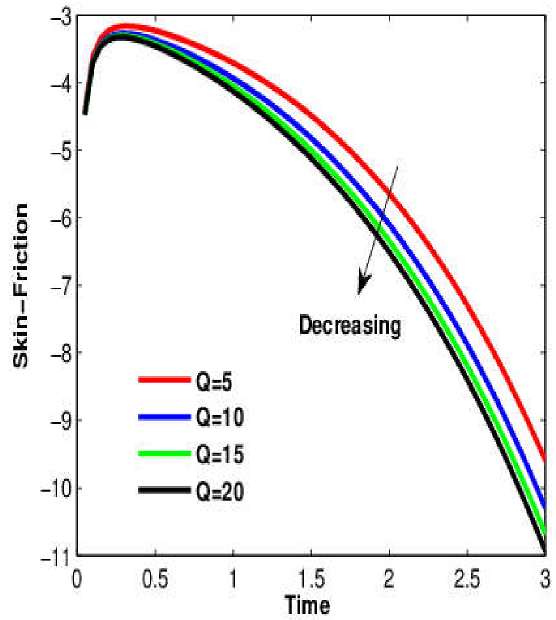

Figure18. Skin-friction coefficient for various values of Q

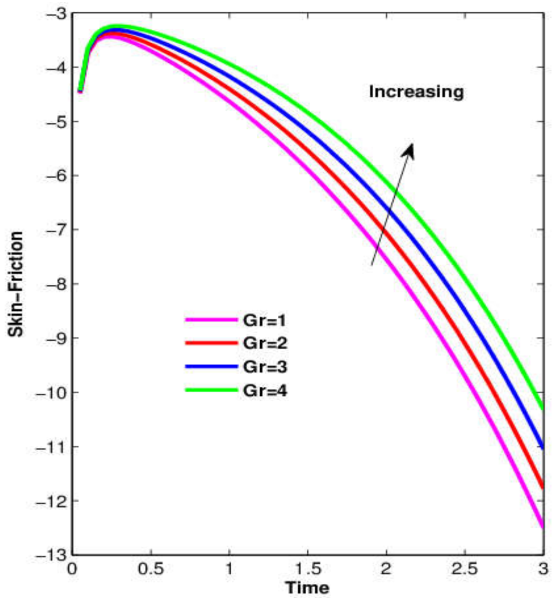

Figure19. Skin-friction coefficient for various values of Gr

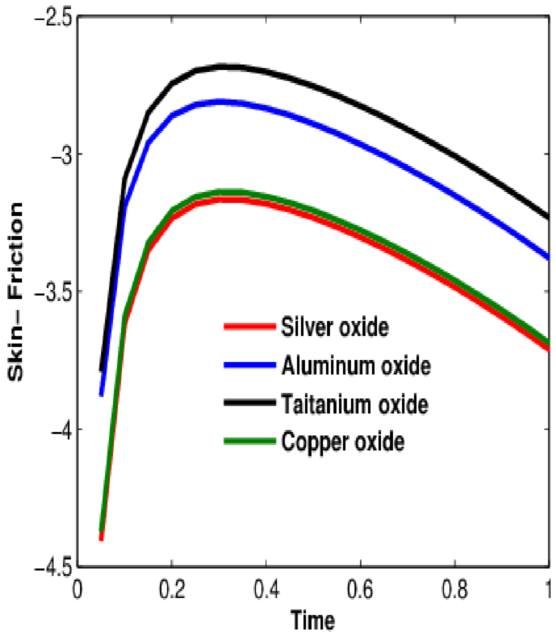

Figure20. Skin-friction coefficient for various Nanoparticles

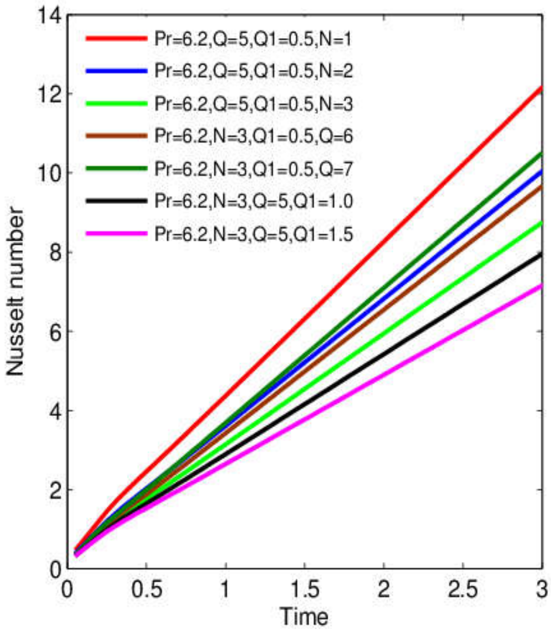

Figure21. Nusselt number

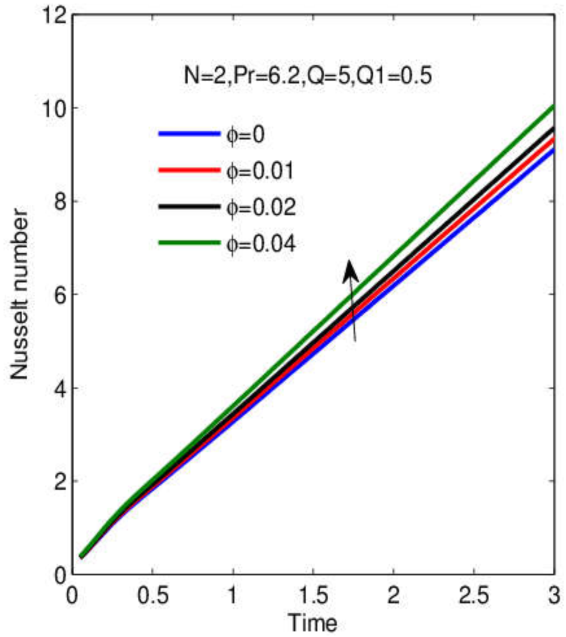

Figure 22. Nusselt number

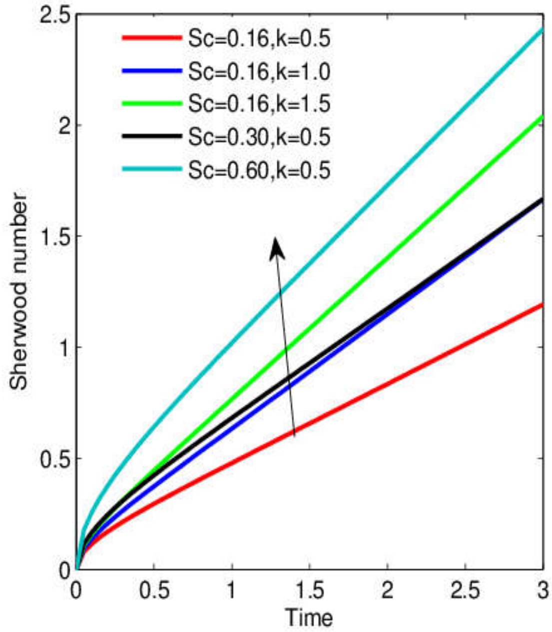

Figure23. Sherwood number

Figure 17illustrates skin friction coefficients for different values of M and N. It is observed that friction coefficient is decreased for increasing magnetic field parameter, but the reverse effect for increasing radiation parameter. From Figure 18 it is observed that the rate of velocity of the silver nanofluid decreases with the increase of heat source parameter. Figure 19 shows the effect of thermal Grashof number. It is found that the skin friction coefficient increases with theincrease of Gr. The skin friction coefficient for different nanoparticles is studied from Figure 20. It is observed that the friction coefficient increases for different nanofluids namely silver oxide, copper oxide, Aluminium oxide and titanium oxide. Therefore the friction coefficient is more for titanium oxide comparatively the other nanofluids.On the other hand variation of Nusselt number and Sherwood number versus time is plotted in Figures 21, 22 and 23 for various parameters of interest. It is found from Figure 21 that Nusselt number increases with increasing value of heat generation parameter, whereas it decreases with increasing values of radiation parameter and radiation absorption parameter when Pr= 6.2. From figure 22 it is observed that the nanoparticle volume fraction parameter is increased, the Nusselt number increased for silver nanofluid. From Figure 23 it is observed that Sherwood number increases as increasing Schmidt parameter and Chemical reaction parameter increases.

4. Conclusions

The main objective of the present work is to study the unsteady MHD free convection chemically reacting Silver nanofluid flow past an exponentially accelerated vertical plate through aporous medium in the presence of heat absorption and radiation absorption has been studied. The dimensionless governing partial differential equations are solved by the usual Laplace transform technique. The effect of different parameters such as Magnetic field parameter, thermal radiation parameter, Prandtl number, Schmidt number, Porosity parameter, heat generation parameter, radiation absorption parameter, thermal Grashof number, nanoparticle volume fraction parameter and Chemical reaction parameters are studied.

Conclusions of the study are as follows:

- The Thermal Grashofnumber (Gr) increases the velocity and Skin-friction coefficient increases for Silvernanofluid.
- When the Magnetic field parameter (M) increases the both velocity and Skin-friction isdecreases.
- The Thermal radiation parameter (N) increases the velocity, temperature and Skin friction coefficient are increases but the Nusselt number is decreasing.
- The dimensionless velocity, temperature, and concentration of silver nanofluid increased with the increase in dimensionless time (t).
- The exponentially accelerated parameter (a) increased, the velocity of silver water nanofluid increased.
- The heat absorption parameter (Q) increases the velocity, temperature and Skin friction parameter is decreases, but the Nusselt number isincreasing.
- The radiation absorption parameter (Q1) increases the velocity, the temperature is increasing but the Nusselt number is decreasing.
- The dimensionless velocity is increased for increasing porosity parameter (K).
- The concentration isdecreased for increasing both the parameters Schmidt number and chemical reaction parameter (k), but the reverse effect will be found in the rate of mass transfer.
- The friction coefficient values of titaniumoxidena nofluid are maximum than other nanofluids like silver oxide nanofluid, copperoxide nanofluid, aluminum oxide nanofluid.
- As the nanoparticle volume fraction parameter is increased, the Nusselt number increased for silver nanofluid.

These results might find wide applications in engineering, such as transportation, industrial cooling applications, heating buildings and reducing pollution, energy applications and solar absorption.

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