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An optimal EOQ inventory model for non-instantaneous deteriorating items with various time dependent demand rates and time dependent holding cost

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Abstract. An economic order quantity inventory model for non-instantaneous deteriorating items has been developed with cubic demand rate, cubic deterioration rate and time dependent holding cost. Shortages in inventory are allowed in this model. In shortage period, partial and complete backlogging cases are considered. The main objective of this model is to develop an optimal policy that minimizes the total average inventory cost and optimal order quantity. Numerical examples are used to illustrate the developed model

1. Introduction

Products which losses its utility or its marginal rate that decreases the usefulness from the original one. This phenomenon is termed as deterioration. Also it may be defined as damage, decay, dryness, spoilage and vaporization. Examples: fashion goods, foods, electronic items, chemicals, etc.
In real life situation, most of the products have a period of maintaining quality or the original condition. During that time lag, there was no deterioration takes place. (E.g. vegetables, fruits, meat, fish and so on). This phenomenon is defined as "non-instantaneous deterioration".

It has been observed that only few researchers considered inventory models for non-instantaneous deteriorating items with time dependent demand rate and time dependent deterioration rate. Optimal policies for inventory models [10] [2] are developed for non- instantaneous deteriorating items with permissible delay in payments. Optimal replenishment policies [1] for non-instantaneous deteriorating items are considered with stock dependent demand. An inventory model for non-instantaneous deteriorating items [9] is developed with with stock dependent demand and partial backlogging. A partial backlogging inventory model [4] for non-instantaneous deteriorating items are developed with stock dependent consumption rate under inflation. [3] is the extension work of [1] model with delay in payment permissible. [11] is the extension work of [3] model by considering with salvages and all possible replenishment cycle, which may be shorter than the period of non-deterioration. Inventory model for non-instantaneous deteriorating items is developed [5] with partial backlogging, permissible delay in payments, inflation. A partial backlogging inventory model deteriorating items is considered [8] with ramp type demand. Inventory model for non-instantaneous deteriorating items [6] with stock dependent demand, partial backlogging and inflation over a finite time horizon. EOQ model for non-instantaneous deteriorating items [7] is developed with cubic demand rate, time dependent holding cost and permissible delay in payments.

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In the present model, an optimal EOQ inventory model for non-instantaneous deteriorating items is developed with the following considerations:

1. Both demand and deterioration rates are considered as piecewise cubic function of time
2. Shortages are allowed. The demand rate is considered as cubic function of time in without shortage period while the demand rate is considered as constant rate in shortage period.
3. Replenishment rate is instantaneous and infinite .
4. The time gap between placing an order and its actual arrival is zero.
5. A finite planning horizon is assumed.

Further, the present model is extended with various time dependent demand rates such as quadratic, linear and constant demands.

1.1 Assumptions and notations

1. Demand of each product is assumed as cubic function of time.
   (i.e) $D(t) = a + bt + ct^2 + dt^3$ at any time $t \geq 0$, $a > 0, b > 0, c > 0$ and $d > 0$.
   where a = initial rate of demand, b = initial rate of change of demand, c = acceleration of demand and d = rate of change of acceleration of demand.
2. Shortages are allowed. During the shortage period, the backlogging rate is dependent on the customers waiting time for the next replenishment.
   i.e., for the negative inventory the backlogging rate is defined by
   $\mathrm{B}(t) = \frac{1}{1+\delta(T-t)}$; $\delta > 0$ denotes the backlogging parameter and $t_2 \leq t \leq T$.
3. Deterioration of each product is also assumed as cubic function of time.
   (i.e) $\theta(t) = \theta_1 + \theta_2 t + \theta_3 t^2 + \theta_4 t^3$, where $\theta_1, \theta_2, \theta_3$ and θ_4 are real numbers, $\theta_4 \neq 0$ and $\theta_1 > 0, \theta_2 > 0, \theta_3 > 0, \theta_4 > 0$ so that $\theta'(t) = \theta_2 + 2\theta_3 t + 3\theta_4 t^2$, $\theta''(t) = 2\theta_3 + 6\theta_4 t$ and $\theta'''(t) = 6\theta_4$,
   where θ_1 = initial deterioration,
   θ_2 = initial rate of change of deterioration,
   θ_3 = acceleration of deterioration and
   θ_4 = rate of change of acceleration of deterioration.
4. $I(t)$: Stock level at any time t, $0 \leq t \leq T$
5. Q_1 : Stock level at time $t = 0$.
6. Q_2 : Stock level at time $t = T$.
7. c_1 : Unit purchasing cost of an item
8. c_2 : Unit shortage cost of an item
9. c_3 : Unit lost sale cost of an item
10. T : Time interval between two successive orders.
11. HC : Holding Cost is assumed as a linear function of time.
12. TC : Total average inventory cost per unit cycle.
13. A : Ordering cost per unit order is known and constant.

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2. Mathematical formulation and solution of the model

During the cycle time $(0, t_1)$, the stock level $I_1(t)$ is defined by the following differential equation

$$\frac{dI_1(t)}{dt} = -(a + bt + ct^2 + dt^3), 0 \le t \le t_1 \tag{1}$$

Solution of equation (1) with $I_1(0) = Q_1$ is

$$I_1(t) = Q_1 - \left(at + \frac{bt^2}{2} + \frac{ct^3}{3} + \frac{dt^4}{4}\right), 0 \le t \le t_1 \tag{2}$$

During the cycle time (t_1, t_2), the stock level $I_2(t)$ is defined by the following differential equation

$$\frac{dI_2(t)}{dt} + (\theta_1 + \theta_2 t + \theta_3 t^2 + \theta_4 t^3) I_2(t) = -(a + bt + ct^2 + dt^3),\ t_1 \le t \le t_2 \tag{3}$$

Solution of equation (3) with $I_2(t_2) = 0$ is

$$I_2(t) = \left[\begin{array}{c} a(t_2 - t) + \frac{(b + a\theta_1)(t_2^2 - t^2)}{2} \\ + \frac{(2c + 2b\theta_1 + a\theta_2)(t_2^3 - t^3)}{6} + \frac{(6d + 6c\theta_1 + 3b\theta_2 + 2a\theta_3)(t_2^4 - t^4)}{24} \\ + \frac{(12d\theta_1 + 6c\theta_2 + 4b\theta_3 + 3a\theta_4)(t_2^5 - t^5)}{60} + \frac{(6d\theta_2 + 4c\theta_3 + 3b\theta_4)(t_2^6 - t^6)}{72} \\ + \frac{(4d\theta_3 + 3c\theta_4)(t_2^7 - t^7)}{84} + \frac{d\theta_4(t_2^8 - t^8)}{32} \end{array}\right] \tag{4}$$

Due to continuity of stock level at $t = t_1$, it follows from equation (2) and (4) that $I_1(t_1) = I_2(t_1)$

$$Q_1 = \left[\begin{array}{c} at_2 + \frac{bt_2^2}{2} + \frac{ct_2^3}{3} + \frac{dt_2^4}{4} + \frac{(a\theta_1)(t_2^2 - t_1^2)}{2} \\ + \frac{(2b\theta_1 + a\theta_2)(t_2^3 - t_1^3)}{6} + \frac{(6c\theta_1 + 3b\theta_2 + 2a\theta_3)(t_2^4 - t_1^4)}{24} \\ + \frac{(12d\theta_1 + 6c\theta_2 + 4b\theta_3 + 3a\theta_4)(t_2^5 - t_1^5)}{60} \\ + \frac{(6d\theta_2 + 4c\theta_3 + 3b\theta_4)(t_2^6 - t_1^6)}{72} \\ + \frac{(4d\theta_3 + 3c\theta_4)(t_2^7 - t_1^7)}{84} + \frac{d\theta_4(t_2^8 - t_1^8)}{32} \end{array}\right] \tag{5}$$

$$\therefore I_1(t) = \left[\begin{array}{c} a(t_2 - t) + \frac{b(t_2^2 - t^2)}{2} + \frac{c(t_2^3 - t^3)}{3} + \frac{d(t_2^4 - t^4)}{4} + \frac{(a\theta_1)(t_2^2 - t_1^2)}{2} \\ + \frac{(2b\theta_1 + a\theta_2)(t_2^3 - t_1^3)}{6} + \frac{(6c\theta_1 + 3b\theta_2 + 2a\theta_3)(t_2^4 - t_1^4)}{24} \\ + \frac{(12d\theta_1 + 6c\theta_2 + 4b\theta_3 + 3a\theta_4)(t_2^5 - t_1^5)}{60} \\ + \frac{(6d\theta_2 + 4c\theta_3 + 3b\theta_4)(t_2^6 - t_1^6)}{72} \\ + \frac{(4d\theta_3 + 3c\theta_4)(t_2^7 - t_1^7)}{84} + \frac{d\theta_4(t_2^8 - t_1^8)}{32} \end{array}\right], 0 \le t \le t_1 \tag{6}$$

During the cycle time (t_2, T), the stock level $I_3(t)$ is defined by the following differential equation

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$$\frac{dI_3(t)}{dt} = -\frac{\omega}{1+\delta(T-t)}, \; t_2 \leq t \leq T \tag{7}$$

Solution of equation (7) with boundary condition $I_3(t_2) = 0$ is

$$I_3(t) = \frac{\omega}{\delta} log\left[\frac{1+\delta(T-t)}{1+\delta(T-t_2)}\right], t_2 \leq t \leq T \tag{8}$$

With boundary conditions $I_3(T) = Q_2$, we get the negative inventory is

$$I_3(T) = -Q_2 = -\frac{\omega}{\delta} log[1+\delta(T-t_2)] \tag{9}$$

$$\text{Total Inventory}, Q = Q_1 + Q_2 \tag{10}$$

(i) Ordering Cost : $OC = \frac{A}{T}$

(ii) Holding Cost :

$$HC = \frac{1}{T}\int_0^T (\alpha + \beta t) I(t) dt$$

$$HC = \frac{\alpha}{T}\int_0^{t_1} I_1(t)dt + \frac{\alpha}{T}\int_0^{t_1} t\, I_1(t)dt + \frac{\beta}{T}\int_{t_1}^{T} I_2(t)dt + \frac{\beta}{T}\int_{t_1}^{T} t\, I_2(t)dt$$

$$HC = HC_1 + HC_2 + HC_3 + HC_4$$

where

$$HC_1 = \frac{\alpha}{T}\left[\begin{array}{c} \frac{a(2t_2t_1-t_1^2)}{2} + \frac{b(3t_2^2t_1-t_1^3)}{6} + \frac{c(4t_2^3t_1-t_1^4)}{12} + \frac{d(5t_2^4t_1-t_1^5)}{20} \\ + \frac{a\theta_1(t_2^2t_1-t_1^3)}{2} + \frac{(2b\theta_1+a\theta_2)(t_2^3t_1-t_1^4)}{6} \\ + \frac{(6c\theta_1+3b\theta_2+2a\theta_3)(t_2^4t_1-t_1^5)}{24} + \frac{(12d\theta_1+6c\theta_2+4b\theta_3+3a\theta_4)(t_2^5t_1-t_1^6)}{60} \\ + \frac{(6d\theta_2+4c\theta_3+3b\theta_4)(t_2^6t_1-t_1^7)}{72} + \frac{(4d\theta_3+3c\theta_4)(t_2^7t_1-t_1^8)}{84} + \frac{d\theta_4(t_2^8t_1-t_1^9)}{32} \end{array}\right]$$

$$HC_2 = \frac{\beta}{T}\left[\begin{array}{c} \frac{a(3t_2t_1^2-2t_1^3)}{2} + \frac{b(2t_2^2t_1^2-t_1^4)}{8} + \frac{c(5t_2^3t_1^2-2t_1^5)}{30} + \frac{d(3t_2^4t_1^2-t_1^6)}{24} \\ + \frac{a\theta_1(t_2^2t_1^2-t_1^4)}{4} + \frac{(2b\theta_1+a\theta_2)(t_2^3t_1^2-t_1^5)}{12} \\ + \frac{(6c\theta_1+3b\theta_2+2a\theta_3)(t_2^4t_1^2-t_1^6)}{48} + \frac{(12d\theta_1+6c\theta_2+4b\theta_3+3a\theta_4)(t_2^5t_1^2-t_1^7)}{120} \\ + \frac{(6d\theta_2+4c\theta_3+3b\theta_4)(t_2^6t_1^2-t_1^8)}{144} + \frac{(4d\theta_3+3c\theta_4)(t_2^7t_1^2-t_1^9)}{168} + \frac{d\theta_4(t_2^8t_1^2-t_1^{10})}{64} \end{array}\right]$$

$$HC_3 = \frac{\alpha}{T}\left[\begin{array}{c} \frac{a(t_2^2-2t_2t_1+t_1^2)}{2} + \frac{(b+a\theta_1)(2t_2^3-3t_2^2t_1+t_1^3)}{6} \\ + \frac{(2c+2b\theta_1+a\theta_2)(3t_2^4-4t_2^3t_1+t_1^4)}{24} + \frac{(6d+6c\theta_1+3b\theta_2+2a\theta_3)(4t_2^5-5t_2^4t_1+t_1^5)}{120} \\ + \frac{(12d\theta_1+6c\theta_2+4b\theta_3+3a\theta_4)(5t_2^6-6t_2^5t_1+t_1^6)}{360} + \frac{(6d\theta_2+4c\theta_3+3b\theta_4)(6t_2^7-7t_2^6t_1+t_1^7)}{504} \\ + \frac{(4d\theta_3+3c\theta_4)(7t_2^8-8t_2^7t_1+t_1^8)}{672} + \frac{d\theta_4(8t_2^9-9t_2^8t_1+t_1^9)}{288} \end{array}\right]$$

$$HC_4 = \frac{\beta}{T}\left[\begin{array}{c} \frac{a(t_2^3-3t_2t_1^2+2t_1^3)}{6} + \frac{(b+a\theta_1)(t_2^4-2t_2^2t_1^2+t_1^4)}{8} \\ + \frac{(2c+2b\theta_1+a\theta_2)(3t_2^5-5t_2^3t_1^2+2t_1^5)}{60} + \frac{(6d+6c\theta_1+3b\theta_2+2a\theta_3)(2t_2^6-3t_2^4t_1^2+t_1^6)}{144} \\ + \frac{(12d\theta_1+6c\theta_2+4b\theta_3+3a\theta_4)(5t_2^7-7t_2^5t_1^2+2t_1^7)}{840} + \frac{(6d\theta_2+4c\theta_3+3b\theta_4)(3t_2^8-4t_2^6t_1^2+t_1^8)}{576} \\ + \frac{(4d\theta_3+3c\theta_4)(7t_2^9-9t_2^7t_1^2+2t_1^9)}{1512} + \frac{d\theta_4(4t_2^{10}-5t_2^8t_1^2+t_1^{10})}{320} \end{array}\right]$$

(iii) Deterioration Cost :

$$DC = \frac{C_1}{T}\left(I_2(t_1) - \int_{t_1}^{t_2} D(t)dt\right)$$

$$DC = \frac{C_1}{T}\left[\begin{array}{c} at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} \\ + \frac{a\theta_1(t_2^2-t_1^2)}{2} + \frac{(2b\theta_1+a\theta_2)(t_2^3-t_1^3)}{6} \\ + \frac{(6c\theta_1+3b\theta_2+2a\theta_3)(t_2^4-t_1^4)}{24} + \frac{(12d\theta_1+6c\theta_2+4b\theta_3+3a\theta_4)(t_2^5-t_1^5)}{60} \\ + \frac{(6d\theta_2+4c\theta_3+3b\theta_4)(t_2^6-t_1^6)}{72} + \frac{(4d\theta_3+3c\theta_4)(t_2^7-t_1^7)}{84} + \frac{d\theta_4(t_2^8-t_1^8)}{32} \end{array}\right]$$

(iv) Shortage Cost :

$$SC = \frac{C_2}{T}\int_{t_2}^{T}[I_3(t)]\, dt$$

$$SC = \frac{C_2\omega}{T\delta}[log\{1+\delta(T-t_2)\}\{(T-t_2)(1-\delta)-1\}+\delta(T-t_2)]$$

(v) Cost due to lost sales :

$$CLS = \frac{C_3}{T}\int_{t_2}^{T}\left[\omega - \frac{\omega}{1+\delta(T-t)}\right]dt$$

$$CLS = \frac{C_3}{T}\left\{T - t_2 - \frac{\omega}{\delta}log[1+\delta(T-t_2)]\right\}$$

Total average inventory cost per unit time, $TC = OC + HC + DC + SC + CLS$ (11)

The main objective of this model is to find the minimum total average inventory cost.

Necessary conditions for minimum total average inventory cost are

$(i)\ \frac{d(TC)}{dt_2} = 0$ and $(ii)\ \frac{d^2(TC)}{dt_2^2} > 0$

3. Numerical analysis

3.1 Partial backlogging models

Model 1. EOQ model with cubic deterioration and cubic demand

Let $A = 1000, a = 0.2, b = 0.1, c = 0.05, d = 0.025, \theta_1 = 0.4, \theta_2 = 0.2, \theta_3 = 0.1, \theta_4 = 0.05,$ $\omega = 1, \alpha = 1,\ \beta = 0.5,\ \delta = 0.125,\ C_1 = 0.39,\ C_2 = 0.6,\ C_3 = 0.8,\ t_1 = 0.2$ and $T = 2$ in appropriated units. Using MATLAB software, we obtain the following optimum solutions:
$t_2{}^* = 1.2606,\ Q^* = 1.2188$ and $TC^* = 498.2534$.

Model 2. EOQ model with cubic deterioration and quadratic demand

Let $A = 1000,\ a = 0.2,\ b = 0.1,\ c = 0.05,\ \theta_1 = 0.4,\ \theta_2 = 0.2,\ \theta_3 = 0.1,\ \theta_4 = 0.05,\ \omega = 1,\ \alpha = 1,\ \beta = 0.5,\ \delta = 0.125,\ C_1 = 0.4,\ C_2 = 0.6,\ C_3 = 0.8,\ t_1 = 0.2$ and $T = 2$
in appropriated units. Using MATLAB software, we obtain the following optimum solutions:
$t_2{}^* = 1.2825, Q^* = 1.2022$ and $TC^* = 498.2440$.

Model 3. EOQ model with cubic deterioration and linear demand

Let $A = 1000, a = 0.2, b = 0.1, \theta_1 = 0.4, \theta_2 = 0.2, \theta_3 = 0.1, \theta_4 = 0.05, \omega = 1, \alpha = 1, \beta = 0.5,$ $\delta = 0.125,\ C_1 = 0.4,\ C_2 = 0.6, C_3 = 0.8, t_1 = 0.2$ and $T = 2$
in appropriated units. Using MATLAB software, we obtain the following optimum solutions:
$t_2{}^* = 1.3718,\ Q^* = 1.1202$ and $TC^* = 498.2003$.

Model 4. EOQ model with cubic deterioration and constant demand

Let $A = 1000,\ a = 0.5,\ \theta_1 = 0.4,\ \theta_2 = 0.2,\ \theta_3 = 0.1,\ \theta_4 = 0.05,\ \omega = 1, \alpha = 1,\ \beta = 0.5,$ $\delta = 0.125, C_1 = 0.4,\ C_2 = 0.6, C_3 = 0.8, t_1 = 0.2$ and $T = 2$
in appropriated units. Using MATLAB software, we obtain the following optimum solutions:
$t_2{}^* = 1.6008,\ Q^* = 0.8541$ and $TC^* = 498.0734$.

3.2 Complete backlogging models

Model 5. EOQ model with cubic deterioration and cubic demand

Let $A = 1000, a = 0.2, b = 0.1, c = 0.05, d = 0.025, \theta_1 = 0.4, \theta_2 = 0.2, \theta_3 = 0.1, \theta_4 = 0.05,$ $\omega = 1, \alpha = 1,\ \beta = 0.5, \delta = 1, C_1 = 0.4, C_2 = 0.6, C_3 = 0.8, t_1 = 0.2$ and $T = 2$
in appropriated units. Using MATLAB software, we obtain the following optimum solutions:
$t_2{}^* = 1.0983. Q^* = 1.0489$ and $TC^* = 500.2642$.

Model 6. EOQ model with cubic deterioration and quadratic demand

Let $A = 1000,\ a = 0.2,\ b = 0.1,\ c = 0.05,\ \theta_1 = 0.4,\ \theta_2 = 0.2,\ \theta_3 = 0.1,\ \theta_4 = 0.05,\ \omega = 1,\ \alpha = 1,\ \beta = 0.5,\ \delta = 1,\ C_1 = 0.4,\ C_2 = 0.6,\ C_3 = 0.8,\ t_1 = 0.2$ and $T = 2$
in appropriated units. Using MATLAB software, we obtain the following optimum solutions:
$t_2{}^* = 1.1169,\ Q^* = 1.0371$ and $TC^* = 500.2558$.

Model 7. EOQ model with cubic deterioration and linear demand

Let $A = 1000, a = 0.2, b = 0.1, \theta_1 = 0.4, \theta_2 = 0.2, \theta_3 = 0.1, \theta_4 = 0.05, \omega = 1, \alpha = 1, \beta = 0.5,$ $\delta = 1,\ C_1 = 0.4,\ C_2 = 0.6, C_3 = 0.8\,, t_1 = 0.2$ and $T = 2$
in appropriated units. Using MATLAB software, we obtain the following optimum solutions:
${t_2}^* = 1.204, Q^* = 1.0023$ and $TC^* = 500.2325$.

Model 8. EOQ model with cubic deterioration and constant demand

Let $A = 1000,\ a = 0.5,\ \theta_1 = 0.4,\ \theta_2 = 0.2,\ \theta_3 = 0.1,\ \theta_4 = 0.05,\ \omega = 1, \alpha = 1,\ \beta = 0.5,$ $\delta = 1, C_1 = 0.4,\ C_2 = 0.6, C_3 = 0.8\,, t_1 = 0.2$ and $T = 2$
in appropriated units. Using MATLAB software, we obtain the following optimum solutions: ${t_2}^* = 1.4725,\ Q^* = 0.8358$ and $TC^* = 500.1473$.

3.2 Observations:

The following results are observed from the numerical examples

1. The optimum total average cost in Model 1 is 0.401% less than that of Model 5 and optimum order quantity in Model 1 is 13.9% more than that of Model 5.

2. The optimum total average cost in Model 2 is 0.402% less than that of Model 6 and optimum order quantity in Model 2 is 13.7% more than that of Model 6.

3. The optimum total average cost in Model 3 is 0.406% less than that of Model 7 and optimum order quantity in Model 3 is 10.5% more than that of Model 7.

4. The optimum total average cost in Model 4 is 0.416% less than that of Model 8 and optimum order quantity in Model 4 is 2.5% more than that of Model 8.

4. Conclusion

In this model, both demand rate and deterioration rate are considered as a cubic function of time. Because, when deterioration and demand for a product starts then both deterioration and demand are accelerated with time. This situation happens in the case of non-instantaneous deteriorating items such as meat, fresh vegetables, fresh fruits, dairy and allied products (milk, curd cheese, etc.), bread, sweets, food, and high nitrogen containing chips, pharmaceuticals, blood and new electronic items. In shortage period, partial and complete backlogging cases are considered and compared with each other. Numerical examples are also provided to illustrate the proposed model.

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