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Analysis of multi-pulse position modulation free space optical communication system employing wavelength and time diversity over Malaga turbulence channel

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ABSTRACT

Free space optics (FSO) communication has proved to be a useful technology both in the scientific and commercial domains in response to exponential growth in demand for ultrahigh bandwidth for applications such as urban broadband services, cellular backhaul and earth to satellite links among others. However, atmospheric turbulence, prevailing adverse weather conditions and pointing errors between the transmitter and receiver affect significantly this technology. To mitigate these challenges, different approaches have been considered in literature such as error control coding, spatial diversity, the use of relays and so on alongside different modulation techniques. Therefore in this paper, we have considered the use of multipulse position modulation (MPPM) modulation technique first with wavelength diversity and then with time diversity techniques over the Malaga (M) distribution as a way of mitigating against turbulence fading and pointing error effects. Closed form expressions for the average bit error rate (BER) and outage probability have been derived and later on used to analyze the system performance. The effect of diversity order, normalized jitter and beam width has been scrutinized, all as functions of the transmitted power. Beside the fact that the increase in diversity order improves both the BER and outage probability, it has been noted that the system performance is highly degraded when the normalized jitter is increased beyond 4 for any given diversity order. The BER decreases while the receiver beam width, W_z increases for all diversity orders, reaching to an optimum value of 0.5 m and 1.2 m for normalized jitter of 2 and 5 respectively. Ultimately, the conclusion drawn from the analytical results of this paper support the application of wavelength as well as time diversity as means of enhancing the FSO communication system performance

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Introduction

Over the years, it has been recognized and accepted that free space optical (FSO) communication is of increasing importance both in the research as well as commercial domains. This has been due to the fact that FSO combines a variety of advantages such as the availability of huge unlicensed bandwidth, considerably improved security, at the same time with much reduced installation costs among others [1]. Hence many applications of FSO technology has immerged from intra and inter chip connection [2] to the deep space applications [3,4], including terrestrial applications. The basic idea in free space optical communication systems is to transmit a data laden laser beam through the atmosphere and collecting the same using a photodetector at the receiver. Therefore, the data modulated laser beam propagating in free space suffers adverse effects which are functions of the channel state between the FSO communication transceivers such as absorption, scattering and displacement. The refractive index of the atmosphere is not constant, but rather changes constantly resulting from the constant atmospheric temperature and pressure fluctuations. This introduces a signal loss referred to in the literature as scintillation [5]. The mobility and sway of the high-rise structures on which FSO transceivers are installed results in misalignment between the transmitter and receiver, ultimately leading to pointing errors [6].

Different approaches and techniques have been applied in the literature in order to combat the above-mentioned deterioration effects on FSO communication mainly atmospheric turbulence and pointing errors. Some of the successful approaches include the use of adaptive optics at the receiver when OOK modulation is used, increasing the receive aperture diameter known as aperture averaging and the use of diversity techniques. [7,8]. Diversity methods, popular in wireless radio communication refers the scenario in which the receiver is provided with multiple copies of the same signal. It mainly depends on the fact that the signals propagating in different channel path are affected differently, hence the depths and duration of the fades experienced by the receiver can be reduced considerably. Diversity can be applied in space, polarization, angle, time or wavelength among others [9]. For the case of spatial diversity, multiple transmitter-receiver pairs are used for transmitting the same information data with the aim of reducing the probability of bit error rate. Using a single transmitter and a single receiver, the same data is retransmitted in different time slots in order to implement time diversity. Finally, in wavelength diversity, using the same time slot a composite transmitter transmits the same information using different wavelengths, which will be received by wavelength specific receivers [10].

Different digital modulation schemes have been applied in FSO studies, but pulse position modulation (PPM) has gained popularity due to lower duty cycle consequently leading to high peak to average power ratio compared to other Pulse modulation signaling techniques. The mapping in PPM is such that a block of L bits is placed into one of the $M = 2^L$ time slots. Because of the orthogonality of symbols, the PPM has a remarkable improvement in BER. In [10], it has been concluded that high energy efficient PPM schemes require a correspondingly huge bandwidth. This has necessitated the introduction of multi-pulse position (MPPM) in [11]. In this arrangement, more pulses are placed within a symbol of M slots, consequently increasing the data carrying capacity without sacrifice on the performance [12]. In FSO communication, MPPM has been analyzed in [13] over the gamma-gamma channel. However, in a more recent study, the MPPM has been investigated over the generalized channel including pointing errors using the BER, outage probability and ergodic channel capacity [14].

In [10], the use of wavelength diversity to mitigate fog effects on the received FSO signal was carried out. The source information was encoded onto three different wavelength carriers of 0.85 μm, 1.55 μm and, 10 μm and then propagated through fog conditions. When compared to a single carrier, such a system exhibited a considerable improvement in the average received power.

A free space optical communication employing the on off keying influenced by the gamma-gamma fading channel model was enhanced through the use of both wavelength and time diversity [11]. After deriving the closed form expressions for the BER, the results clearly demonstrated that the use of these two diversity techniques significantly improves the BER. However, the major disadvantage of such a system was increased complexity. Using OOK signaling over Malaga channel, the probability of fade has been analyzed and improved by using time diversity technique [12]. In another study [15], polarization shift keying signaling has been used with wavelength diversity over the gamma-gamma channel by deriving closed form expressions for the BER and outage probability. Analytical results proved that wavelength and time diversity are options for dealing with scintillation effects. The performance of an OOK FSO links using wavelength diversity over the exponentiated Weibull channel has been carried out [16]. Likewise, the research in [17], extended the previous analysis by considering the K channel, using three carrier wavelengths of 1.55 μm 1.31 μm and 0.85 μm.

More recently, Balaji et al. [18] analyzed an FSO system using polarization shift keying (PolSK) modulation techniques influenced by Malaga (M) atmospheric turbulence model. The results demonstrated that wavelength as well as time diversity schemes are candidates for inclusion in an FSO communication system to mitigate turbulence induced fading as well as weather induced signal losses. In [19], the outage probability and BER of OOK signaling over the gamma-gamma channel, FSO channel has been analyzed considering wavelength diversity with order 1, 2 and 3.

Motivated by the above analysis, in this article, we carry out an analysis of the MPPM FSO system influenced by Malaga atmospheric turbulence conditions including pointing errors using wavelength and time diversity schemes to deal with atmospheric turbulence signal deterioration. It is clear from the literature, that although the wavelength and time diversity has been analyzed using different modulation schemes and channel models, such studies have not been done with MPPM signaling over the generalized Malaga or simply M FSO turbulence channel. From the present study, the analysis can be reduced to any desired earlier proposed channel model such as K, gamma-gamma and so on.

The remainder of this manuscript is organized as follows: in Sect.2, the system model is considered. In Sect.3 the FSO system that incorporate wavelength and then time diversity has been discussed. In Sect. 4, the average BER analysis without diversity, with wavelength diversity and time diversity has been carried out. In Sect.5, the outage probability has been analyzed likewise. The results have been presented and discussed in Sect.6. Finally, the paper is concluded in Sect.7.

FSO channel model

We consider that the free space optical channel model h_m is a product of h_a, h_p and h_l expressed as [20]:

$$h_m = h_l h_a h_p \tag{1}$$

Where h_l represents the geometric atmospheric loss, h_a the atmospheric turbulence loss and h_p represents the misalignment or pointing error effects.

Atmospheric loss

The atmospheric loss represents the exponentiation extinction of the laser beam, which can be expressed in terms of Beer-Lambert%s law as [15]:

$$h_l = e^{-\sigma L} \tag{2}$$

Here, σ is the signal attenuation coefficient, whereas the FSO link is denoted by L. However, without loss of generality, we have considered the value of h_l to be equal to 1.

FSO channel model

We consider that the atmospheric turbulence scintillation is modeled using the generalized Malaga (M) distribution FSO model. The advantage of this distribution is that it is valid for almost all channel conditions from weak to strong turbulence and further takes care of the misalignment or pointing errors.

In this model, the optical beam is considered to consist of three components. The first is the line of sight component, having power expressed as Ω, and the second is the scattered component which is coupled to the line of sight with power $2\rho bo$. The third component is the scattered component which is independent of the previous two components with power $2(1-\rho)bo$. Therefore, from this, it is apparent that the two scattered components carry a total power of $2bo$. In this representation, the degree of coupling between the scattered and the line of sight components is represented by the parameter ρ. The PDF of the Malaga distribution free space turbulence model is expressed as [21]:

$$f_{h_n}(h_n) = A\sum_{k=1}^{\beta_n} a_k h_n^{\frac{\alpha_n+k}{2}-1} K_{\alpha-k}\left(2\sqrt{\frac{\alpha_n\beta_n h_n}{\gamma\beta_n+\Omega'}}\right) \tag{3}$$

where

$$A = \frac{2\alpha_n^{\alpha_n/2}}{\gamma^{1+\frac{\alpha_n}{2}}\,\Gamma(\alpha_n)}\left(\frac{\gamma\beta_n}{\gamma\beta_n+\Omega'}\right)^{\beta_n+\frac{\alpha_n}{2}} \tag{4}$$

$$a_k = \binom{\beta_n-1}{k-1}\frac{\left(\gamma\beta_n+\Omega'\right)^{1-\frac{k}{2}}}{(k-1)!}\left(\frac{\Omega'}{\Upsilon}\right)^{k-1}\left(\frac{\alpha_n}{\beta_n}\right)^{\frac{k}{2}} \tag{5}$$

In Eq. (3), $\Gamma(\cdot)$ is the gamma function, $K_\upsilon(\cdot)$ is the modified Bessel function of second kind and order ν. Also α_m represents the effective number of large-scale scattering processes. Hence it is a positive number. β_n is the amount of fading parameter, which in this case is a natural number. It should be noted that the case for β_n being a real number can be derived and has also been provided in literature such as in [21], however, the model has been proposed with a high degree of freedom therefore the case for the natural number is chosen. Moreover, to simplify the representation, we have considered $\gamma = 2(1-\rho)b_o$. Further, $\Omega' = \Omega + 2\rho b_o + 2\sqrt{2b_o\Omega\rho}\cos(\phi_A - \phi_B)$ is the average power of the coherent distribution. ϕ_A and ϕ_B are the deterministic phases of the LOS and coupled to LOS components respectively.

Pointing errors

The performance of any given FSO system depends also on the degree of alignment between the transceivers

Existence of dynamic wind loads, thermal expansions and building or structural sways on which FSO transceivers are installed leads to pointing errors denoted by h_p, and its PDF is expressed as [22]:

$$f_{h_p} = \frac{g^2}{A_o^{g^2}}(h_p)^{g^2-1},\ 0 \le h_p \le A_o \tag{6}$$

In Eq. (6), $A_o = [\mathrm{erf}(\nu)]^2$ is the fraction of the collected optical power. $\nu = \sqrt{\frac{\pi}{2}} \cdot \frac{a}{\omega_z}$ in which a denotes the receiver telescope radius, and ω_z represents the width of the data carrying laser beam, measured at a free space distance of L. Furthermore, ω_{zeq} denotes the equivalent beam width which can be further expressed as $\omega_{zeq} = [\frac{\sqrt{\pi}\mathrm{erf}(\nu).\omega_z^2}{2\nu e^{-\nu^2}}]^{\frac{1}{2}}$. Finally, $g = \frac{\omega_{zeq}}{2\sigma_s}$, where σ_s denotes the jitter standard deviation.

Combined channel model

A complete statistical channel model is obtained by taking into account the turbulence induced fading, pointing errors and the unconditional channel PDF and is obtained as:

$$f_{h_n}(h_n) = \int f_{h_n|h_a}(h_n|h_a).f_{h_a}(h_a)dh_a \tag{7}$$

where $f_{h_n|h_a}(h_n h_a)$ is the conditional probability for a given turbulence state h_a. By substituting Eq. (3) and (6) in Eq. (7), the combined channel fading model can be expressed as:

$$f_{h_n}(h_n) = \frac{g^2 A}{2h_n}\sum_{k=1}^{\beta_n}\left(a_k\left[\frac{1}{B_n}\right]^{\frac{\alpha_n+k}{2}} G_{1,3}^{3,0}\left[\frac{h_n}{BA_o h_l}\middle| \begin{matrix} 1+g^2 \\ g^2,\ \alpha_n,\ k \end{matrix}\right]\right) \tag{8}$$

In Eq. (8), $B_m = (\frac{\Omega'+\gamma_n}{\alpha_n \beta_n})$ and $G_{p,q}^{m,n}[\cdot]$ is Meijer%s G function. The CDF of the M channel model can be expressed as [20]

$$F_{h_n}(h_n) = \frac{g^2 A}{2}\sum_{k=1}^{\beta_n}\left(a_k\left[\frac{1}{B_n}\right]^{\frac{\alpha+k}{2}} G_{2,4}^{3,1}\left(\frac{h_n}{BA_o I_l}\middle| \begin{matrix} 1,\ 1+g^2 \\ g^2,\ \alpha_n,\ k,\ 0 \end{matrix}\right)\right) \tag{9}$$

FSO system with MPPM and wavelength diversity

We consider an intensity modulation with direct detection (IM/DD) FSO communication system based on MPPM signaling.

At the transmitter side, the incoming serial data bits are first mapped to parallel bit stream using a serial to parallel converter. This process is followed by converting the parallel bits to MPPM symbols which also involves placing a symbol in any of the available M-time slots. It is now this MPPM symbol which is finally used to modulate the intensity of the laser beam. The beam is subsequently transmitted into the FSO turbulent channel by means of a transmitting telescope. The transmitting telescope also helps in steering the beam width and direction for correct transmission towards the receiver. At the receiver, a receiving telescope captures the optical signal and pass it to the photodetector which do the optical to electrical conversion. Finally, a low pass filter conditions the electrical signal by removing unwanted noise

The low pass filter output signal can be expressed as:

$$y(t) = RP_R x(t) + n(t) = R\frac{M}{k}P_R\sum_{k=0}^{M-1} C_k\, rect\left(t - \frac{kT}{M}\right) + n(t) \tag{10}$$

where

$$rect(t) = \begin{cases} 1, & if\ 0 \le t \le \frac{T}{M} \\ 0, & otherise \end{cases}$$

In Eq. (10), P_R is the average received optical power, x is the transmitted data R is the photodetector responsivity and $n(t)$ is the additive white Gaussian noise with variance σ_n^2. Furthermore, $C_k = 1$ for signal time slot and zero for non signal time slot. Using the link equation, the received optical signal can be expressed as [23]:

$$P_R(h_n) = P_T\eta_T\eta_R G_T G_R\left(\frac{\lambda_n}{4\pi L}\right)^2 h_n \tag{11}$$

In Eq. (11), the transmitter optical power is denoted by P_T, the efficiency of the transmitter is denoted by η_T, whilst the efficiency of the receiver optics is denoted by η_R. Where, P_T is the transmitted optical power, η_T and η_R are the efficiencies of the transmitter and receiver optics respectively. G_T is the gain of the transmitter and G_R is the gain of the receiver telescope. λ_n is the operating wavelength, L is the transmission length and h_n is the channel state due to atmospheric turbulence. If we assume that the transmitter and receiver telescope gains are equal, then;

$$G_T = G_R = \left(\frac{\pi D}{\lambda_n}\right)^2 \tag{12}$$

D is the diameter of the receiving telescope. Next, we also make the assumption that $\eta_T = \eta_R = \eta$ and then substitute Eq. (12) into (11), we get:

$$P_R(h_n) = P_T\left(\frac{\eta A_r}{nL}\right)^2 h_n \tag{13}$$

For the case of multiple receivers, the received power can be expressed as $P_R(h_n) = P_T(\frac{\eta A_r}{\lambda_n L})^2 \sum_{n=1}^{N} h_n$

Eq (13) gives the received power at the photodiode. Moreover, for MPPM signaling under consideration, the receiver signal to noise ratio (SNR) is given as:

$$SNR(h_n) = R^2 {P_T}^2 \left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\, Log_2 M}{2N\, \sigma_n^2} h_n \tag{14}$$

In Eq. (14), σ_n is the variance of the channel noise, M is the MPPM modulation order, N is the number of transceivers and $A_r = \frac{\pi D^2}{4}$, where D is the diameter of the receiving aperture.

Average BER

This section is devoted to the derivation of the closed form expressions for the considered performance metrics, i.e., average bit error rate and the outage probability for the MPPM FSO communication system over Malaga atmospheric turbulence channel without diversity, with wavelength diversity and with time diversity, with pointing errors taken into consideration.

Without diversity

When no diversity is considered, the FSO system uses one transmitter and one receiver. For the multi-pulse position signaling FSO, the conditional probability of error is expressed as [24]:

$$BER(h_n) = \frac{M}{4}\, erfc\left[R P_R(h_n) \frac{\sqrt{M\, Log_2 M}}{2\sigma_n} \right] \tag{15}$$

In order to derive the closed form expression for the average BER for the MPPM FSO system, the following relation is used [25]:

$$Pe = \int_o^\infty BER(h_n) f_{h_n}(h_n) dh_n \tag{16}$$

By substituting Eq. (15) and Eq. (8) in Eq. (16), we get:

$$\begin{aligned} Pe = &\int_0^\infty \frac{M}{4}\, erfc\left[P_T \left(\frac{\eta A_r}{\lambda_n L}\right)^2 \frac{\sqrt{M\, Log_2 M}}{2N\sigma_n} h_n \right] \\ &\times \frac{g^2 A}{2h_n} \sum_{k=1}^{\beta_n} \left(a_k \left[\frac{1}{B_n}\right]^{\frac{\alpha_n + k}{2}} G_{1,3}^{3,0}\left[\frac{h_n}{BA_o h_l} \middle| \begin{matrix} 1+g^2 \\ g^2,\ \alpha_n,\ k \end{matrix} \right] \right) dh_n \end{aligned} \tag{17}$$

The complimentary error function $erfc(\cdot)$ can be expressed in terms of the Meijer G functions as [26]:

$$erfc(z) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left[z^2 \middle| \begin{matrix} 1 \\ 0,\ 1/2 \end{matrix} \right] \tag{18}$$

Therefore Eq. (17) can be expressed as:

$$\begin{aligned} Pe = &\frac{Mg^2 A}{8\sqrt{\pi}} \sum_{k=1}^{\beta} a_k \left[\frac{1}{B}\right]^{\frac{\alpha + k}{2}} \int_0^\infty {h_n}^{-1}\, G_{1,2}^{2,0}\left[\frac{R^2 P_T^2\, M\, Log_2 M}{4(\lambda L/\eta A_r)^4 \sigma_n^2} {h_n}^2 \middle| \begin{matrix} 1 \\ 0,\ 1/2 \end{matrix} \right] \\ &\times G_{1,3}^{3,0}\left[\frac{h_n}{BA_o h_l} \middle| \begin{matrix} 1+g^2 \\ g^2,\ \alpha_n,\ k \end{matrix} \right] \end{aligned} \tag{19}$$

Using Eq (21) in [26], the final expression for the BER can be expressed as:

$$\begin{aligned} Pe = &\frac{2^{\alpha-3} M g^2 A}{8\sqrt{\pi^3}} \sum_{k=1}^{\beta} 2^k a_k \left[\frac{1}{B}\right]^{\frac{\alpha + k}{2}} \\ &G_{7,4}^{2,6}\left[\frac{4R^2 P_T^2\, M\, Log_2 M (BA_o I_l)^2}{(\lambda L/\eta A)^4 \sigma_n^2} \middle| \begin{matrix} \frac{1-g^2}{2},\ \frac{2-g^2}{2},\ \frac{1-\alpha}{2},\ \frac{2-\alpha}{2},\ \frac{1-k}{2},\ \frac{2-k}{2},\ 1 \\ 0,\ \frac{1}{2},\ -\frac{g^2}{2},\ \frac{1-g^2}{2} \end{matrix} \right] \end{aligned} \tag{20}$$

Wavelength diversity

In order to apply wavelength diversity, we assume independent and identically distributed data transmission. Therefore, the conditional probability for the multi-channel, $(n = 1, 2, \cdots M)$ FSO system is expressed as [27];

$$P_{ec}(h_n) = Q\left(\sqrt{SNR}\right) \tag{21}$$

As explained under Eq (14) and using Eq (21), the conditional BER for MPPM FSO communication using multiple wavelength with optimal combining can be expressed as;

$$P_{ec}(h_n) = Q\left(RP_T\left(\frac{\eta A_r}{L\lambda_n}\right)^2\sqrt{\frac{M\ Log_2 M}{2\sigma_n^2}\sum_{n=1}^{N} h_n}\right) \tag{22}$$

In Eq (22), $Q(\cdot)$ is the Gaussian Q function and σ_n^2 is the channel noise variance. Then the average BER for the MPPM signaling with wavelength diversity FSO can be expressed as:

$$Pe = \int_o^\infty P_{ec}(h_n) f_{h_n}(h_n) dh_n \tag{23}$$

Upon substituting Eq (8) and (22) into Eq. (23), we get;

$$\begin{aligned} Pe = \frac{g_n^2 A_n}{2}\int_o^\infty \sum_{k=1}^{\beta_n} h_n^{-1}\left(a_{k_n}\left[\frac{1}{B_n}\right]^{\frac{\alpha_n+k}{2}} G_{1,3}^{3,0}\left[\frac{h_n}{B_n A_o h_l}\middle| \begin{matrix} 1+g_n^2 \\ g_n^2,\ \alpha_n,\ k_n \end{matrix}\right]\right) \\ \times Q\left(RP_T\left(\frac{\eta A_r}{L\lambda_n}\right)^2\sqrt{\frac{M\ Log_2 M}{2\sigma_n^2}\sum_{n=1}^{N} h_n}\right) dh_n \end{aligned} \tag{24}$$

In order to simplify Eq. (24), the following approximation for the Q function is used: $Q(x) = \frac{1}{12}e^{-\frac{x^2}{2}} + \frac{1}{4}e^{-\frac{2x^2}{3}}$ [28] to obtain the average BER as:

$$\begin{aligned} Pe = \frac{g_n^2 A_n}{2} \times \frac{1}{12}\int_o^\infty \sum_{k=1}^{\beta_n}\prod_{n=1}^{N} h_n^{-1}\left(a_{k_n}\left[\frac{1}{B_n}\right]^{\frac{\alpha_n+k}{2}} G_{1,3}^{3,0}\left[\frac{h_n}{B_n A_o h_l}\middle| \begin{matrix} 1+g_n^2 \\ g_n^2,\ \alpha_n,\ k_n \end{matrix}\right]\right) \\ \times e^{-\left[R^2 {P_T}^2\left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{4N\ \sigma_n^2} h_n\right]} dh_n + \frac{g_n^2 A_n}{2} \\ \times \frac{1}{4}\int_o^\infty \sum_{k=1}^{\beta_n}\prod_{n=1}^{N} h_n^{-1}\left(a_{k_n}\left[\frac{1}{B_n}\right]^{\frac{\alpha_n+k}{2}} G_{1,3}^{3,0}\left[\frac{h_n}{B_n A_o h_l}\middle| \begin{matrix} 1+g_n^2 \\ g_n^2,\ \alpha_n,\ k_n \end{matrix}\right]\right) \\ \times e^{-\left[R^2 {P_T}^2\left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{3N\ \sigma_n^2} h_n\right]} dh_n \end{aligned} \tag{25}$$

Using the characteristics of Meijer G function $e^{-x} = G_{0,1}^{1,0}[x|_0^{\cdot}]$ [26], hence Eq. (25) can be further expressed as:

$$\begin{aligned} Pe = \frac{g_n^2 A_n}{2} \times \frac{1}{12}\int_o^\infty \sum_{k=1}^{\beta_n}\prod_{n=1}^{N} h_n^{-1}\left(a_{k_n}\left[\frac{1}{B_n}\right]^{\frac{\alpha_n+k}{2}} G_{1,3}^{3,0}\left[\frac{h_n}{B_n A_o h_l}\middle| \begin{matrix} 1+g_n^2 \\ g_n^2,\ \alpha_n,\ k_n \end{matrix}\right]\right) \\ \times G_{0,1}^{1,0}\left[R^2 {P_T}^2\left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{4N\ \sigma_n^2} h_m \middle| \begin{matrix} \cdot \\ 0 \end{matrix}\right] dh_n + \frac{g_n^2 A_n}{2} \\ \times \frac{1}{4}\int_o^\infty \sum_{k=1}^{\beta_n}\prod_{n=1}^{N} h_n^{-1}\left(a_{k_n}\left[\frac{1}{B_n}\right]^{\frac{\alpha_n+k}{2}} G_{1,3}^{3,0}\left[\frac{h_n}{B_n A_o h_l}\middle| \begin{matrix} 1+g_n^2 \\ g_n^2,\ \alpha_n,\ k_n \end{matrix}\right]\right) \\ \times G_{0,1}^{1,0}\left[R^2 {P_T}^2\left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{3N\ \sigma_n^2} h_n \middle| \begin{matrix} \cdot \\ 0 \end{matrix}\right] dh_n \end{aligned} \tag{26}$$

Using Eq (21) in Ref [26], the above integral can be estimated so that the final average BER for MPPM signaling with wavelength diversity can be expressed as:

$$P_e = \frac{g_n^2 A_n M}{24} \sum_{k_n=1}^{\beta_m} \prod_{n=1}^{N} a_{k_n} \left[\frac{1}{B_n}\right]^{\frac{\alpha_n + k}{2}} G_{3,2}^{1,3}\left[R^2 {P_T}^2 \left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{4N\ \sigma_n^2} \middle| \begin{matrix} 1-g_n^2,\ 1-\alpha_n,\ 1-k_n \\ 0,\ -g_n^2 \end{matrix} \right] + \frac{g_n^2 A_n M}{8} \sum_{k_n=1}^{\beta_m} \prod_{n=1}^{N} a_{k_n} \left[\frac{1}{B_n}\right]^{\frac{\alpha_n + k}{2}} G_{3,2}^{1,3}\left[R^2 {P_T}^2 \left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{3N\ \sigma_n^2} \middle| \begin{matrix} 1-g_n^2,\ 1-\alpha_n,\ 1-k_n \\ 0,\ -g_n^2 \end{matrix} \right] \tag{27}$$

Time diversity

When a time diversity scheme is used, a single wavelength is used. Therefore, $\lambda_1 = \lambda_2 = \cdots \lambda_n$, $\alpha_1 = \alpha_2 = \cdots \alpha_n$ and $\beta_1 = \beta_2 = \cdots \beta_n$. The average BER with time diversity is obtained by incorporating this assumption in Eq. (19) as:

$$P_e = \frac{g_n^2 A_n M}{24} \sum_{k_n=1}^{\beta_n} \left[a_{k_n} \left[\frac{1}{B_n}\right]^{\frac{\alpha_n + k}{2}} \right]^N \times \left\{ \left[G_{3,2}^{1,3}\left[R^2 {P_T}^2 \left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{4N\ \sigma_n^2} \middle| \begin{matrix} 1-g_n^2,\ 1-\alpha_n,\ 1-k_n \\ 0,\ -g_n^2 \end{matrix} \right] \right]^N + 3\left[G_{3,2}^{1,3}\left[R^2 {P_T}^2 \left(\frac{\eta A_r}{\lambda_n L}\right)^4 \frac{M\ Log_2 M}{3N\ \sigma_n^2} \middle| \begin{matrix} 1-g_n^2,\ 1-\alpha_n,\ 1-k_n \\ 0,\ -g_n^2 \end{matrix} \right] \right]^N \right\} \tag{28}$$

Therefore, Eq. (28) is a closed form expression for the average BER for MPPM FSO system over the M turbulence channel.

Outage probability

The outage probability refers to the probability that the endwise SNR is less than the critical threshold value. It one of the important performance metrics for a given FSO communication system. In order to obtain the outage probability, the cumulative distribution (CDF) is used.

Without diversity

When considering a fading channel, the outage probability is given as:

$$P_{out} = P(SNR(h) \le SNR_{th}) \tag{29}$$

For MPPM,

$$P_{out} = P\left(RP_T \left(\frac{\eta A}{L\lambda}\right)^2 \frac{\sqrt{M\ Log_2 M}}{2\sigma_n} h\ \le SNR_{th} \right) = P\left(h \le 2\sigma_n^2 \left(\frac{\lambda L}{\eta A}\right)^4 \frac{SNR_{th}}{R^2 P_T^2\ MLog_2 M} \right) \tag{30}$$

$$P_{out} = F_h\left(2\sigma_n^2 \left(\frac{\lambda L}{\eta A}\right)^4 \frac{SNR_{th}}{R^2 P_T^2\ MLog_2 M} \right) \tag{31}$$

In Eq (31), $F_h(\cdot)$ denotes the concerned channel model CDF, which for the case of the channel model under consideration, is expressed as given in Eq. (10). By combining Eq. (10) and (31), the outage probability of the MPPM FSO system over the generalized atmospheric turbulence channel can be expressed as:

$$P_{out} = \frac{Ag^2}{2} \sum_{k=1}^{\beta} a_k \left[\frac{1}{B}\right]^{\frac{\alpha + k}{2}} G_{2,4}^{3,1}\left[\frac{2\sigma_n^2}{BA_o h_l} \left(\frac{\lambda L}{\eta A}\right)^4 \frac{SNR_{th}}{R^2 P_T^2\ MLog_2 M} \middle| \begin{matrix} 1, 1+g^2 \\ g^2,\ \alpha,\ k, 0 \end{matrix} \right] \tag{32}$$

Wavelength diversity

In order to derive the outage probability for the case of wavelength diversity, we first consider the outage probability for each of the N wavelengths, given as:

$$P_{out,n} = \frac{g_n^2 A_n}{2} \sum_{k=1}^{\beta_n} a_k \left[\frac{1}{B_n}\right]^{\frac{\alpha_n + k}{2}} G_{2,4}^{3,1}\left[\frac{2\sigma_n^2}{B_n A_o h_l} \left(\frac{\lambda_n L}{\eta A_r}\right)^4 \frac{SNR_{th}}{R^2 P_T^2\ MLog_2 M} \middle| \begin{matrix} 1,\ 1+g_n^2 \\ g_n^2,\ \alpha_n,\ k_n, 0 \end{matrix} \right] \tag{33}$$

Table 1
Comparison of the wavelength and time diversity in strong atmospheric turbulence.

Transmitted power (dBm)	BER			
	Wavelength diversity		Time diversity	
	$R = 2$	$R = 3$	$R = 2$	$R = 3$
−10	1.831×10^{-5}	2.304×10^{-7}	2.772×10^{-6}	1.219×10^{-9}
−5	2.871×10^{-7}	4.662×10^{-10}	4.741×10^{-8}	2.851×10^{-12}
0	4.129×10^{-9}	8.192×10^{-13}	7.145×10^{-10}	5.641×10^{-15}
5	5.615×10^{-11}	1.316×10^{-15}	1.022×10^{-11}	9.89×10^{-18}
10	7.328×10^{-13}	1.981×10^{-18}	1.393×10^{-13}	1.593×10^{-20}

In considering the FSO system that uses N wavelengths, we assume independent outage probability for each channel. Therefore, for the MPPM FSO system under consideration, the total outage probability can be expressed as [16]:

$$P_{out,\,N} = \prod_{n=1}^{N} P_{out,n} \tag{34}$$

Substituting Eq. (33) into Eq. (34) we get:

$$P_{out,\,N} = \prod_{n=1}^{N}\left(\frac{g_n^2 A_n}{2}\sum_{k=1}^{\beta_n} a_k \left[\frac{1}{B_n}\right]^{\frac{\alpha_n+k}{2}} G_{2,4}^{3,1}\left[\frac{2\sigma_n^2}{B_n A_o h_l}\left(\frac{\lambda_n L}{\eta A_r}\right)^4 \frac{SNR_{th}}{R^2 P_T^2\ MLog_2 M}\middle|\begin{matrix}1,\ 1+g_n^2\\ g_n^2,\ \alpha_n,\ k_n, 0\end{matrix}\right]\right) \tag{35}$$

Time diversity

As mentioned before, for the case of time diversity, a single wavelength is used, however, the transmitter retransmit the same information signal in different time slots. Therefore, $\lambda_1 = \lambda_2 = \cdots \lambda_n$, $\alpha_1 = \alpha_2 = \cdots \alpha_n$ and $\beta_1 = \beta_2 = \cdots \beta_n$, so the total outage probability is obtained as:

$$P_{out,\,N} = \left\{\frac{Ag^2}{2}\sum_{k=1}^{\beta} a_k \left[\frac{1}{B}\right]^{\frac{\alpha+k}{2}} G_{2,4}^{3,1}\left[\frac{2\sigma_n^2}{BA_o h_l}\left(\frac{\lambda L}{\eta A}\right)^4 \frac{SNR_{th}}{R^2 P_T^2\ MLog_2 M}\middle|\begin{matrix}1, 1+g^2\\ g^2,\ \alpha,\ k, 0\end{matrix}\right]\right\}^N \tag{36}$$

Results and discussion

Following the derivations performed in the previous section, Eq. (20), (27) and Eq. (28), can be used directly to evaluate the average BER while Eq. (32), (35) and Eq. (36) can be used to evaluate the outage probability for the system under consideration without diversity, with wavelength diversity and then with time diversity respectively. Using available literature, the MPPM FSO system is assumed to have noise standard deviation $\sigma = 5 \times 10^{-7}\ A/Hz$, photodetector responsivity of $0.5\ A/W$. Furthermore, we also consider the average beam radius at a distance L to be $\omega_L = 2.5\ m$ at a distance of $1\ km$ and the jitter standard deviation is assumed to be $\sigma_s = 30\ cm$ [22]. The BER and outage probability performance have been plotted and analyzed for strong atmospheric turbulence conditions ($\alpha = 1,\ \beta = 2,\ C_n^2 = 2 \times 10^{-13}$), moderate atmospheric turbulence, ($\alpha = 4,\ \beta = 5,\ C_n^2 = 6 \times 10^{-14}$) and weak turbulence ($\alpha = 4,\ \beta = 9,\ C_n^2 = 2 \times 10^{-15}$) [11,16]. Moreover, for the M channel, the optical power has been normalized as $\Omega' = \Omega + 2b0 = 1$. The aperture diameter has been considered as 0.01 m and assumed that for the wavelength diversity case, the commonly used wavelengths in FSO are $\lambda_1 = 1.55\ \mu m,\ \lambda_2 = 0.85\ \mu m$ and $\lambda_2 = 1.31\ \mu m$ [29]. The numerical analysis results achieved for an FSO system derived above are considered by taking into account the various factors, which are known to affect the FSO communication systems.ca

First, we estimate the average BER versus the transmitted power in dBm, for $N = 1,\ 2, 3$ for wavelength diversity. The results are shown in Fig. 1, for a link length of 5 km. From Fig. 1, it is clear that the average BER of the MPPM FSO system is improving as the number N of different wavelength channels is increasing. For the case of $N = 1$, i.e., diversity order 1, a single wavelength of $\lambda_1 = 1.55\ \mu m$ has been used, for the case of a diversity order of 2, two wavelengths $\lambda_1 = 1.55\ \mu m,\ \lambda_2 = 0.85\ \mu m$ has been used whilst for the case of diversity order of 3, all the three wavelengths were used. At a transmitted power of 0 dBm, for the case of $N = 1$, the BER is 3.37×10^{-5} in strong turbulence and 7.55×10^{-7} in moderate turbulence. For the case of $N = 2$, the BER is 7.16×10^{-10} in strong turbulence and 1.15×10^{-13} in moderate turbulence. For the case of $N = 3$, the BER is 2.83×10^{-14} in strong turbulence and 5.13×10^{-21} in moderate turbulence. In all these cases, it can be observed that for any given diversity order, the difference in the BER performance of the MPPM FSO communication system in moderate and weak atmospheric turbulence is not very much significant.

In Fig. 2, the average BER has been plotted versus transmitted power for the case of time diversity in moderate atmospheric turbulence. It is clear that there is a significant improvement in the BER for a given transmitted power when the

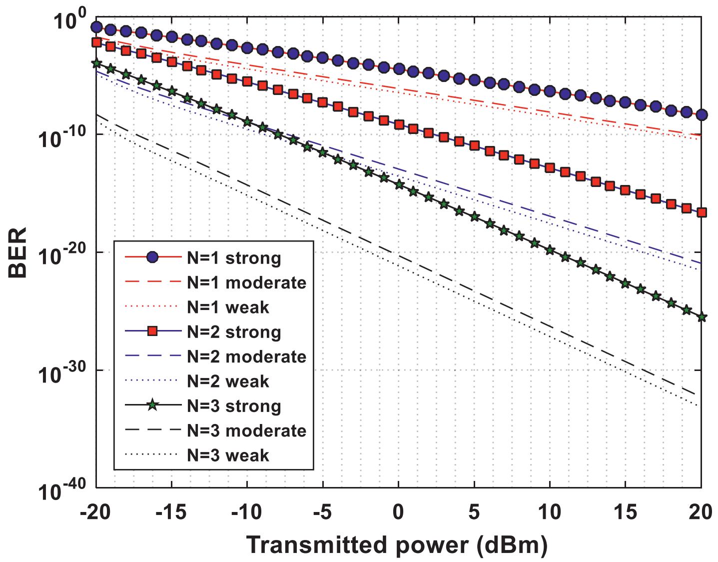

Fig. 1. BER versus transmitted power in dBm for different with wavelength diversity for no diversity ($N = 1$) and diversity order = 2and 3 in different turbulence strength.

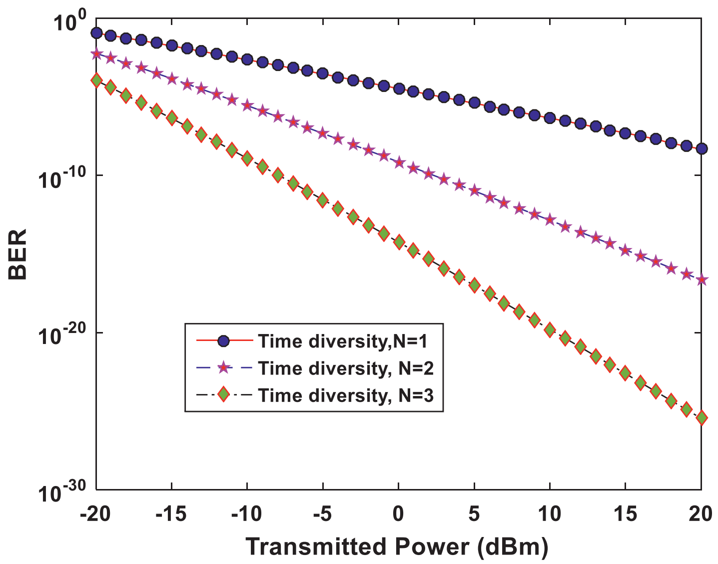

Fig. 2. BER versus transmitted power in dBm with Time diversity.

time diversity order is increased. When the transmitted power is 0 dBm, the BER achieved for a diversity order of 1 is 3.38×10^{-5}, when the diversity order increases to 2, the BER reduces to 7.17×10^{-10}. For a diversity order of 3, the BER is still reduced to 5.64×10^{-15}. These results encourage the use of time diversity in FSO communication systems.

In Fig. 3, the dependence of the BER on the normalized jitter σ_s/a with different wavelength diversity at a link length of 1 km has been illustrated. Again, it can be noted that for any given normalized jitter, the BER is improves as the diversity order increases. The system performance for all diversity order is highly degraded when the normalized jitter is increased beyond 4. When normalized jitter of 1,2 and 3 are considered, the average BER remains constant at a value of 6.51×10^{-5}, 1.02×10^{-9} and 1.51×10^{-14} for diversity orders 1, 2 and 3 respectively.

The BER versus the laser beam width at the receiver W_z considering different wavelength diversity orders of 1,2 and 3, as well as normalized jitter $\frac{\sigma_s}{a} = 2$ (for small pointing errors) and $\frac{\sigma_s}{a} = 5$ (for increased pointing errors) has been illustrated in Fig. 4. the transmitter power has been considered as 10 dBm. As can be seen from the figure, the BER decreases while W_z increases for all diversity orders, reaching to an optimum value of 0.5 m and 1.2 m for $\frac{\sigma_s}{a} = 2$ and $\frac{\sigma_s}{a} = 5$ respectively. Again, increasing the diversity order improves the BER. Therefore, it is possible to adjust the beam divergence at the transmitter

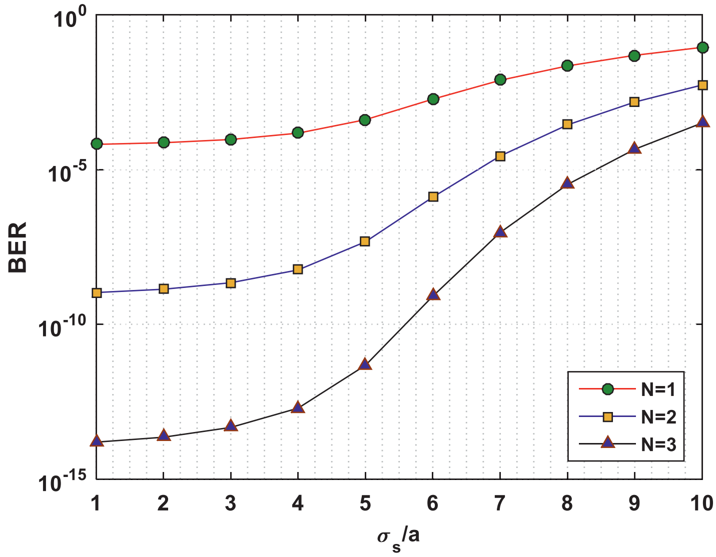

Fig. 3. Average BER versus normalized jitters, σ_s/a for FSO with receiver wavelength diversity order 1,2 and 3.

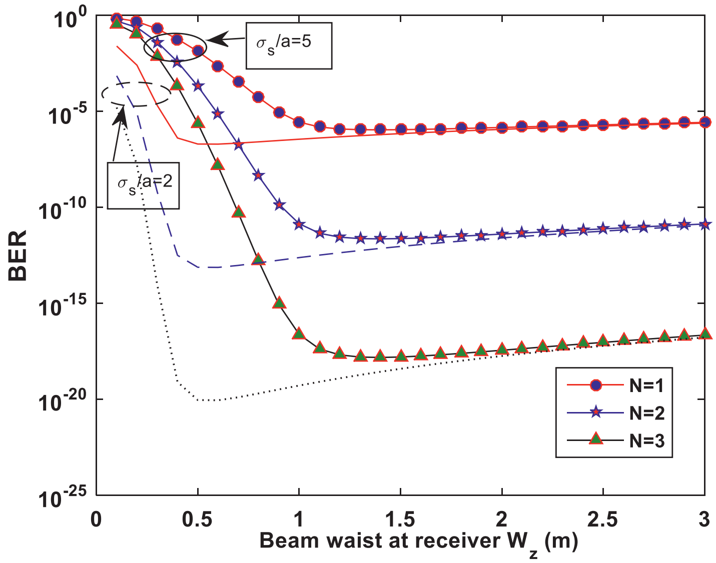

Fig. 4. BER versus receiver beam waist radius W_m for the FSO with wavelength diversity order 1,2 and 3.

based on the maximal radial displacement to come up with a desired receiver beam radius and consequently improves the FSO system BER. In Table 1, the BER as a function of transmitted power using wavelength and time diversity has been compared. It can be observed that at all considered diversity order, time diversity performs better than wavelength diversity.

In Fig. 5, the outage probability versus the threshold SNR with wavelength diversity of order 1,2 and 3 has been illustrated in moderate turbulence. Different normalized jitter standard deviation σ_s/a of 5 and 2 have also been considered. Generally, as the threshold SNR, SNR_{th} increases, the outage probability P_{out} also increases. Whist the $SNR_{th} = 10\ dBm$, $P_{out} = 9.18 \times 10^{-4}$, 4.53×10^{-7} and 4.61×10^{-11} for $N = 1$,2 and 3 respectively, whilst $\frac{\sigma_s}{a} = 2$. However, when $\frac{\sigma_s}{a} = 5$, P_{out} increases to 4.11×10^{-3}, 9.71×10^{-5} and 5.35×10^{-9} for $N = 1$,2 and 3 respectively.

In Fig. 6, the outage performance of wavelength and time diversity are compared in moderate turbulence conditions. It can be observed that for diversity order of 2 and 3, the Time diversity performs better than wavelength diversity. However, the difference is very small for $N = 2$ and increases as N increases to 3.

The outage probability as a function of the transmitted power with wavelength diversity considering different FSO link distances and diversity order of 1 and 3 has been illustrated in Fig. 7. It can be observed that the outage probability de-

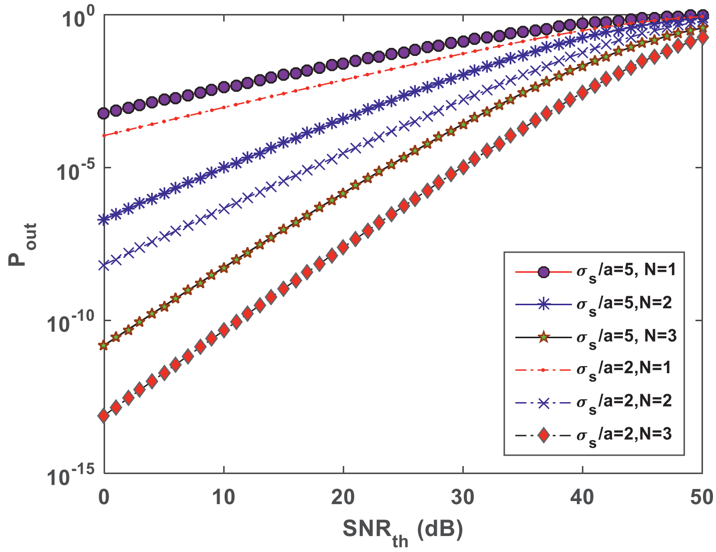

Fig. 5. Outage probability versus the threshold SNR with wavelength diversity.

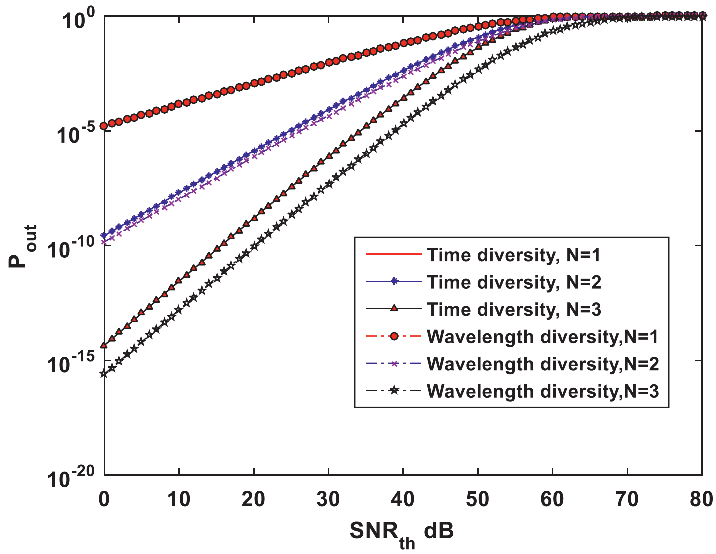

Fig. 6. Outage probability versus threshold SNR for wavelength and time diversity.

creases while the transmitter power increases. For any given wavelength diversity order, as the FSO link distance increases, the outage probability also increases.

In Fig. 8, the outage probability has been examined as a function of the transmitted power for different wavelength diversity cases, also considering different MPPM modulation order of $M = 2$ and 4. The outage probability decreases with increasing power for any given modulation and diversity order. However, for a given diversity order, 2PPM performs better than 4PPM and the difference in outage probability performance increases as the diversity order increases.

Conclusion

In this work, the performance of a free space optical communication system using MPPM signaling influenced by combined effects of the Malaga atmospheric turbulence and misalignment has been analyzed in terms of the average BER and outage probability when the transmitter power is varied. To improve the performance of the FSO link, diversity has been applied first in the wavelength and then in time using the optimal combining method at the receiver. The closed form expressions for the average BER and outage probability have been derived and then later on used for the performance analysis of such a system. It has been observed that for any given diversity order, there is a maximum value for the normalized jitter standard deviation, beyond which the system performance is highly degraded. The results obtained reveal that the use of wavelength or time diversity in FSO communication systems is a clear option toward atmospheric turbulence mitigation.

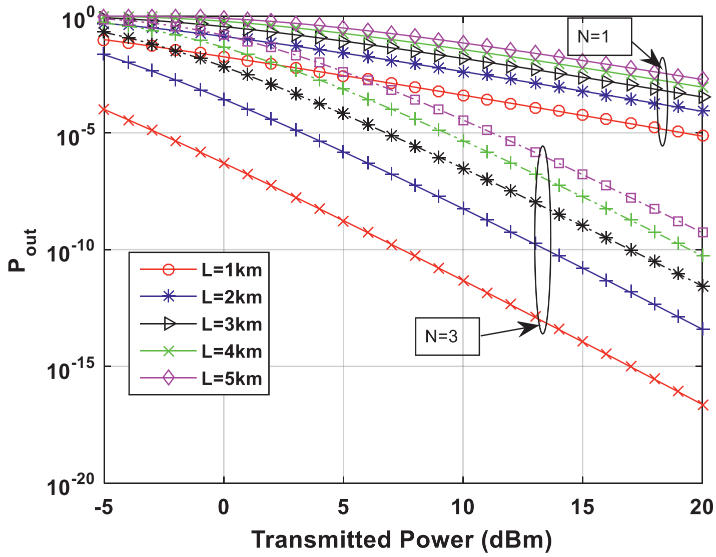

Fig. 7. Outage probability versus transmitted power for different FSO link distances and diversity orders.

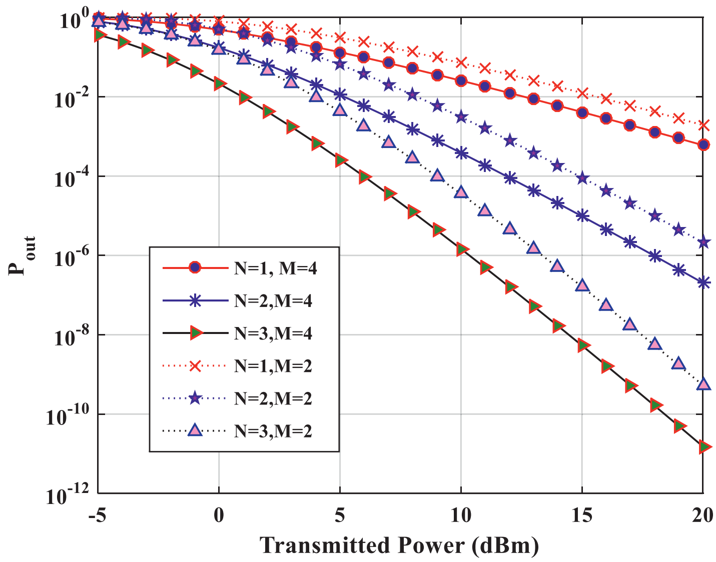

Fig. 8. Outage probability versus transmitter power considering different Modulation order M and time diversity order N.

Declaration of competing interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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References

[1] Debbie Kedar, Shlomi Arnon, Urban optical wireless communication networks: the main challenges and possible solutions, IEEE Commun Mag 42 (5) (2004) S2–S7.
[2] Jing Xue, et al., An intra-chip free-space optical interconnect, ACM SIGARCH Computer Architecture News 38 (3) (2010) 94–105.
[3] Keith Wilson, Michael Enoch, Optical communications for deep space missions, IEEE Commun Mag 38 (8) (2000) 134–139.

[4] B.L. Edwards, NASA%s current activities in free space optical communications, International Conference on Space Optics—ICSO 2014, 10563, International Society for Optics and Photonics, 2017.
[5] W.O. Popoola, Zabih Ghassemlooy, BPSK subcarrier intensity modulated free-space optical communications in atmospheric turbulence, J Lightwave Technol 27 (8) (2009) 967–973.
[6] K. Prabu, Sumanta Bose, D.S. Kumar, BPSK based subcarrier intensity modulated free space optical system in combined strong atmospheric turbulence, Opt Commun 305 (2013) 185–189.
[7] Liang Yang, Xiqi Gao, Mohamed-Slim Alouini, Performance analysis of free-space optical communication systems with multiuser diversity over atmospheric turbulence channels, IEEE Photonics J 6 (2) (2014) 1–17.
[8] Junyi Jiang, et al., Aperture selection for ACO-OFDM in free-space optical turbulence channel, IEEE Trans Veh Technol 65 (8) (2015) 6089–6100.
[9] Ronen Rachmani, Shlomi Arnon, Wavelength diversity in turbulence channels for sensor networks, 2010 IEEE 26-th Convention of Electrical and Electronics Engineers in Israel, IEEE, 2010.
[10] Eric Wainright, H.H. Refai, J.J.Sluss Jr, Wavelength diversity in free-space optics to alleviate fog effects, Free-Space Laser Communication Technologies XVII, International Society for Optics and Photonics, 2005 Vol. 5712.
[11] H.E. Nistazakis, G.S. Tombras, On the use of wavelength and time diversity in optical wireless communication systems over gamma–gamma turbulence channels, Opt Laser Technol 44 (7) (2012) 2088–2094.
[12] K. Prabu, Sanchal Thakkar, Analysis of FSO link with time diversity over M-distribution channel model with pointing errors and GVD effects, Opt Commun 421 (2018) 115–124.
[13] A.E. Morra, et al., Performance analysis of both shot-and thermal-noise limited multipulse PPM receivers in gamma–gamma atmospheric channels, J Lightwave Technol 31 (19) (2013) 3142–3150.
[14] Simbarashe Magidi, Jabeena Afthab, Performance analysis of multipulse position modulation free space optical communication system under generalized atmospheric turbulence conditions including pointing errors, Opt Eng 59 (1) (2020) 016107.
[15] K. Prabu, Shashidhar Cheepalli, D. Sriram Kumar, Analysis of PolSK based FSO system using wavelength and time diversity over strong atmospheric turbulence with pointing errors, Opt Commun 324 (2014) 318–323.
[16] Dhaval Shah, Dilipkumar Kothari, Anjan Ghosh, Performance of free-space optical link with wavelength diversity over exponentiated Weibull channel, Opt Eng 55 (11) (2016) 116112.
[17] Dhaval Shah, Dilip Kumar Kothari, A.K. Ghosh, Bit error rate analysis of the K channel using wavelength diversity, Opt Eng 56 (5) (2017) 056106.
[18] K.A. Balaji, K. Prabu, Performance evaluation of FSO system using wavelength and time diversity over malaga turbulence channel with pointing errors, Opt Commun 410 (2018) 643–651.
[19] Varun Srivastava, Abhilash Mandloi, G.G. Soni, Outage probability and average BER estimation of FSO system employing wavelength diversity, Opt Quantum Electr 51 (7) (2019) 229.
[20] Ping Wang, et al., On the performances of relay-aided FSO system over M distribution with pointing errors in presence of various weather conditions, Opt Commun 367 (2016) 59–67.
[21] Antonio Jurado-Navas, et al., A unifying statistical model for atmospheric optical scintillation, Numerical simulations of physical and engineering processes 181 (2011).
[22] A.A. Farid, Steve Hranilovic, Outage capacity optimization for free-space optical links with pointing errors, J Lightwave Technol 25 (7) (2007) 1702–1710.
[23] Anna Polishuk, Shlomi Arnon, Optimization of a laser satellite communication system with an optical preamplifier, JOSA A 21 (7) (2004) 1307–1315.
[24] Xiang Yi, et al., BER performance analysis for M-ary PPM over gamma-gamma atmospheric turbulence channels, 2010 6th International Conference on Wireless Communications Networking and Mobile Computing (WiCOM), IEEE, 2010.
[25] Dhaval Shah, Dilipkumar Kothari, Performance Analysis of Free Space Optical Link With Wavelength Diversity Under Weak and Moderate Turbulence Conditions, Sens Lett 17 (2) (2019) 137–143.
[26] V.S. Adamchik, O.I. Marichev, The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system, in: Proceedings of the international symposium on Symbolic and algebraic computation, ACM, 1990.
[27] Xuan Tang, et al., Coherent heterodyne multilevel polarization shift keying with spatial diversity in a free-space optical turbulence channel, J Lightwave Technol 30 (16) (2012) 2689–2695.
[28] M. Chiani, D. Dardari, M.K. Simon, New exponential bounds and approximations for the computation of error probability in fading channels, IEEE Trans Wireless Commun 2 (4) (2003) 840–845.
[29] A.K. Majumdar, Free-space laser communication performance in the atmospheric channel, Journal of Optical and Fiber Communications Reports 2 (4) (2005) 345–396.