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Analysis on preemptive priority retrial queue with two types of customers, balking, optional re-service, single vacation and service interruption

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Abstract. This paper concerned with performance analysis of single server preemptive priority retrial queue with a single vacation where two types of customers are considered and they are called priority customers and ordinary customers**.** The ordinary customers arrive in batch into the system. The priority customers do not form any queue. After the completion of regular service, the customers may demand re-service for the previous service without joining the orbit or may leave the system. As soon as the system is empty, the server goes for vacation and the regular busy server can be subjected to breakdown. By using the supplementary variable technique, we obtain the steady- state probability generating functions for the system/orbit size. Some important system performance measures and the stochastic decomposition are discussed. Finally, numerical examples are presented to visualize the effect of parameters on system performance measures.

1. Introduction

In recent times retrial queues in queueing theory are recognized as an important research area due to many applications in several areas. Artalejo and Gomez-Corral [1], Artalejo [2] and Gomez-Corral [11], are given the general models of queueing in various aspects. Retrial queueing systems are consuming queues with repeated trials which are characterized by the fact that a new customer who finds the server busy is requested to leave the service area and join a trial group, called orbit. After some delay of time, the customer in the orbit can repeat their request for service according to FCFS. Any customer in the orbit who repeats the request for service is independent of the rest of the customers in the orbit. Upon the arrival of a customer, if the server is busy or under repair or on vacation, the customer will join the waiting space. This kind of (retrial) queue plays a superior role in computer networks, telecommunication, telephone systems communication protocols, retail shopping queues, etc.

In the earlier years, two varieties of customers have been widely studied by many of the researchers like Artalejo et al [3], Liu et al [18], and Wang [22]. The high priority customers are formed in the queue or not queue and served according to the discipline of preemptive or non-preemptive. If the blocked pool of customers, low priority customers (called as ordinary customers) leave the system and join the retrial group to retry its service after some time when the server is free. Moreover, in some of the systems, an arriving higher priority customer may push out the lower priority customers whose service is continuing to the queue or the orbit. In the above features, Choi and Chang [5] first studied a non- preemptive priority retrial queue, in which priority customers have non- preemptive priority over ordinary customers and are queued in FCFS discipline. Krishna Kumar et al. [13] considered a single server retrial queueing system with two-phase service and preemptive resume. Liu and Wu [17] considered a Markovian arrival process of queues with negative customers, and multiple vacations which present the significance of a preemptive queue in real-world situations. . Stochastic decomposition has been widely studied by Fuhrman and Cooper [8]. In latest times, Wu and Lian [23] considered an unreliable retrial queue with priority and unreliable server, negative customer under Bernoulli vacation schedule. For a comprehensive analysis of priority queueing models, the reader may refer Liu and Gao[18], Senthilkumar et al. [20] and Gao [10].

A vacation queueing model is considered as an extension of the classical queueing system in which server may not be available for a period of time due to many reasons, like being checked for maintenance, any damage occurs to the server or simply it is taking a break. The period of time taken to come out of the server absence is considered as a server vacation. The queue with vacation models is essential for the queueing structure because the server can use the idle period for many purposes. Various authors have analyzed queueing models of server vacations with many combinations. A literature survey on queueing systems with server vacations is done by Doshi [7]. Most vacation models deal with the exhaustive policies (Doshi et al 1986) that are the system must be empty when the server starts a vacation. Arivudainambi et al. [4], Krishnakumar et al. [12] and Yuvarani et al [24] have studied a single server retrial queue with general retrial times, single vacations. Recently Geo and Wang [9], Rajadurai et al. [16] have analyzed about the M/G/1 retrial queue with vacations, balking and server breakdown

In most of the literature survey related to queueing theory, it is assumed that the server is available always in the system. Most of the cases, the server is assumed to be liable and always available for the customers to be served. Sometimes we come across the cases where the server may breakdown and resume its service after repair. In an instance, in manufacturing systems, the machine may breakdown due to Mechanical or job-related problems. The computer systems may breakdown due to software related problems, like a virus. The repairs and server failures in retrial queues are analyzed by Kulkarni and choi [14]. Choudhury and Deka [6] and Rajadurai et al. [15] are discussed about the single server queue with two phases of service and the server is focus to the service interruption while providing the service to the customers. Thangaraj et al. [21] studied an M/G/1 queue with two stage heterogeneous service, compulsory vacations, and random breakdowns. The application of server vacation model can be found in manufacturing systems, designing of local area networks, data communication systems. Breakdown in queues are very common in Manufacturing systems and computer Priority networks.

The remaining part of this paper is organised in following manner. In section 2, the model under consideration is explained in detail. The condition for the model to be stable is discussed in this section 3. In section 4, we obtained all the steady state solutions. In section 5, some important and interesting performance measures of the system are discussed. Special cases in our results are derived in section6. In 7^{th} section, the obtained results on the system performance are numerically justified. Conclusion of the paper is discussed in section 8.

2. Description of the model

In this section, we consider a preemptive priority retrial queue with two types of customers with the batch arrival, Re-service (optional), single vacation, and breakdown. The detailed explanation of the model is given as follows:

2.1. The arrival process:

In this model, priority and ordinary customers are arriving into the system. Priority customers have more priorities than ordinary customers in service time of the busy server. The ordinary customer

arrives in batches according to a compound Poisson process with rate λ. Let X_k denote the number of customers belonging to the kth arrival batch with a common distribution, Pr $[X_k=n]=\chi_n$, n=1, 2, 3…
$X(z)$ represents PGF of X and $X_{[1]}$, $X_{[2]}$ are the first and second factorial moments of X. We assume that the priority customers arrive at rate δ according to Poisson process.

2.2. *The retrial process:*
If the server is free, an arriving priority (or ordinary) customer begins its service instantly. Otherwise in the arrival time of a priority customer, if any priority customer is in service then the newly arriving priority customer will leave the system. If the server is busy on ordinary customer then the arriving priority customer will stop the service and instant priority customers will start the service. Also we assume that the interrupted customer who was just being served before waits in the service area complete his remaining service.Inter-retrial times have an arbitrary distribution $R(t)$ with corresponding Laplace Stieltjes- Transform (LST) $R^*(\vartheta)$. The retrial ordinary customer is required to give up the attempt for service if an external priority customer or ordinary customer arrives first. In that case, the retrial ordinary customer goes back to its position in the retrial queue.

2.3. The vacation process: The server begins vacation each time when the orbit becomes empty. In vacation period, the service time follows a general random variable V with distribution function $V(x)$. and LST $V^*(\vartheta)$ and finite k^{th} moment "$v^{(k)}$ (k = 1,2)".

2.4. The breakdown process:
Server may breakdown at any epoch while server is on any kind of service. The service will be stopped for a short period. Let $\propto$ be the breakdown rate.

2.5. Repair process:
When server is on breakdown, it will be sent for repair. During the repair time server stops the service to the customers till system got repaired. The customer who is served before breakdown will wait on the server till he completes the remaining service. The repair time (denoted by G) of the server is generally distributed with d.f, LST "$G^*(\vartheta)$," and finite k^{th} moment "$g^{(k)}$ (k = 1,2)"

2.6. The regular service process:
The Service time of Priority customers follows a general distribution with distribution function $S_p(t)$, having Laplace Stielgies Transform $S_p^*(\vartheta)$. The 1st and 2nd moment are $\beta_p^{(1)}$ and $\beta_p^{(2)}$. The service time of ordinary customers follows a general distribution with distribution function $S_b(t)$, having Laplace Stielgies Transform $S_b^*(\vartheta)$. The 1st and 2nd moments are $\beta_b^{(1)}$ and $\beta_b^{(2)}$. As soon as positive customer completes his service, he may repeat the same service with probability r or may leave the system with probability (1-r).

3. Stability condition

The state of the queueing system can be described by the bivariate Markov process $\{C(t), N(t);\ t \geq 0\}$, where $C(t)$ denotes the server state (0,1,2,3,4,5,6,7,8) depending on the server is free, busy on priority customers, busy on preemptive priority customers, busy on ordinary customers, on re-service, on vacation, repair. $N(t)$ denotes the number of ordinary customers in the orbit. In addition, let $R^0(t)$, $S_p^0(t)$, $S_b^0(t)$, $V^0(t)$ and $G^0(t)$ be the elapsed time of the retrial, of the priority customer, of the ordinary customer, vacation time of any customer and repair time of any customer at an arbitrary time t. Now we introduce the random variable,

$$C(t)=\begin{cases}0, & \text{if the server is idle at time t}\\ 1, & \text{if the server is busy with a priority customer without preempting}\\ & \text{an ordinary customer and in regular service period at time } t,\\ 2, & \text{if the server is busy with a priority customer with preempting}\\ & \text{an ordinary customer and in regular service period at time } t,\\ 3, & \text{if the server is busy with an ordinary customer}\\ & \text{and in regular service period at time } t,\\ 4, & \text{if the server is busy with priority customer in}\\ & \text{optional re-service period at time } t,\\ 5, & \text{if the server is busy with preemptive priority customer in}\\ & \text{optional re-service period at time } t,\\ 6, & \text{if the server is busy with an ordinary customer in}\\ & \text{optional re-service period at time } t,\\ 7, & \text{if the server is on vacation at time } t,\\ 8, & \text{if the server is on repair at time } t,\end{cases}$$

Then the system of random vectors $Z_n=\{C(t_n+),\, N(t_n+)\}$ forms a Markovian chain in the system of retrial.

*Theorem 3.1***:**
"The embedded Markov chain $\{Z_n;\, n\in N\}$ *is ergodic if and only if* $\rho<R^*(\lambda+\delta)$

$$where\ \rho=\left\{\begin{array}{l}\delta\bar{R}^*(\lambda+\delta)\Big[[(1-r)+rS_p^*(\alpha)]\Big]S_p^*(\alpha)+(1+rS_p^*(\alpha))A_b(1)[1-S_p^*(\alpha)]\\ +\Big(R^*(\lambda+\delta)+\lambda R^*(\lambda+\delta)\Big)\left\{\begin{array}{l}[(1-r)+rS_b^*(\tau)]S_b^*(\tau)\\ +(1+rS_b^*(\tau))[1-S_p^*(\alpha)]\Big[(1+rS_p^*(\tau))A_b(1)+[1-S_b^*(\tau)]\Big]\end{array}\right\}\end{array}\right\}$$

Proof We observe that $\{Z_n;\, n\in N\}$ satisfies the following fundamental equation

$Z_{n+1}=Z_n - B_{n+1}+ V_{n+1,}$ where V_{n+1} is the number of the jobs arriving during the (n+1)th service time and

$$B_{n+1}=\begin{cases}1, & \text{if the n}^{th}\text{ job in service proceeds from orbit}\\ 0, & \text{otherwise}\end{cases}$$

We also know that $\{Z_n;\, n\in N\}$ is an irreducible and aperiodic Markov chain. To prove the sufficient condition of ergodicity, it is very convenient to use Foster's criterion, which states that the chain $\{Z_n;\, n\in N\}$ is an irreducible and aperiodic Markov chain is ergodic if there exists a non-negative function $f(j)$, $j\in N$ and $\varepsilon>0$, such that mean drift $\psi_j=E\big[f(z_{n+1})-f(z_n)/z_n=j\big]$ is finite for all $j\in N$ and $\psi_j\le-\varepsilon$ for all $j\in N$, except perhaps for a finite number j's. In our case, we consider the function $f(j)=j$. then we have

$$\psi_j=\begin{cases}\rho-R^*(\lambda+\delta), & \text{if } j=0,\\ \rho<1, & \text{if } j=1,2...\end{cases}$$

Clearly, the inequality $\rho < R^*(\lambda+\delta)$, is sufficient condition for Ergodicity and this inequality is also necessary for ergodicity. To prove the necessary condition, As noted in Sennott et al. [21], if the Markov chain $\{Z_n;\ n\geq 1\}$ satisfies Kaplan's condition, namely, $\psi_j < \infty$ for all $j \geq 0$ and there exists $j_0 \in$ N such that $\psi_j \geq 0$ for $j \geq j_0$. Notice that, in our case, Kaplan's condition is satisfied because there is a k such that $m_{ij} = 0$ for $j < i - k$ and $i > 0$, where $M = (m_{ij})$ is the one step transition matrix of $\{Z_n;\ n\in N\}$. Then, the inequality $\rho \geq R^*(\lambda+\delta)$, implies the non-Ergodicity of the Markov chain.

Since the arrival follows a Poisson process, from the Burke's theorem it is clear that the steady state probabilities of $\{(C(t), X(t)), t>0\}$ exist and are positive if and only if $\rho < R^*(\lambda+\delta)$. From the mean drift $\psi_j = \rho - R^*(\lambda+\delta)$ for j ≥ 1, we arrive at the decision that the mean number of ordinary customers who arrive per service equals ρ and the expected number of customers enters service from orbit at an epoch in which a service starts given that the previous service time left j customers in orbit, equals. For the stability condition we must have $\rho < R^*(\lambda+\delta)$, and so the above Theorem 3.1, indicates that the arrival of ordinary customers per service interval is less than the customers from orbit enters service at the epoch at which service starts".

4. Steady state analysis of the system

In steady state, we assume that " $R(0)=0$, $R(\infty)=1$, $S_p(0) = 0$, $S_p(\infty) = 1$, $S_b(0) = 0$, $S_b(\infty) = 1$, $V(0)=0$, $V(\infty)=1$, $G(0)=0$, $G(\infty)=1$"are continuous at $x=0$. So that the functions "$a(x)$, $\mu_p(x)$, $\mu_b(x)$, $\gamma(x)$ and $\xi(x)$" are the hazard rates for retrial, service of a priority customer and ordinary customer, vacation completion rate, repair completion rate of a customer respectively.

$$i.e.,\ a(x)dx = \frac{dR(x)}{1-R(x)};\ \mu_p(x)dx = \frac{dS_p(x)}{1-S_p(x)};\ \mu_b(x)dx = \frac{dS_b(x)}{1-S_b(x)};\ \gamma(x)dx = \frac{dV(x)}{1-V(x)};\xi(x)dx = \frac{dG(x)}{1-G(x)}.$$

For the process $\{N(t),\ t\geq 0\}$, the limiting probabilities are defined as

$$P_0(t) = P\{X(t)=0,\ N(t)=0\}$$
$$P_n(x,t)dx = P\left\{C(t)=0, N(t)=n,\ x<R^0(t)\leq x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0 \text{ and } n\geq 1.$$
$$\Pi_{1,n}(x,t)dx = P\left\{C(t)=1, N(t)=n,\ x<S_p^0(t)\leq x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0,\ n\geq 0.$$
$$\Pi_{2,n}(x,y,t)dx = P\left\{C(t)=2, N(t)=n,\ x<S_p^0(t)\leq x+dx,\ y<S_b^0(t)\leq y+dy\right\},\ \text{for } t\geq 0,\ x\geq 0,\ y\geq 0,\ n\geq 0.$$
$$\Pi_{3,n}(x,t)dx = P\left\{C(t)=3, N(t)=n,\ x<S_b^0(t)\leq x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0,\ n\geq 0.$$
$$Q_{1,n}(x,t)dx = P\left\{C(t)=4, N(t)=n,\ x<S_p^0(t)\leq x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0,\ n\geq 0.$$
$$Q_{2,n}(x,y,t)dx = P\left\{C(t)=5, N(t)=n,\ x<S_p^0(t)\leq x+dx,\ y<S_b^0(t)\leq y+dy\right\},\ \text{for } t\geq 0,\ x\geq 0,\ y\geq 0,\ n\geq 0.$$
$$Q_{3,n}(x,t)dx = P\left\{C(t)=6, N(t)=n,\ x<S_b^0(t)\leq x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0,\ n\geq 0.$$
$$\Omega_n(x,t)dx = P\left\{C(t)=7,\ N(t)=n,\ x<V^0(t)\leq x+dx\right\}, \text{for } t\geq 0,\ x\geq 0 \text{ and } n\geq 0.$$
$$R_n(x,t)dx = P\left\{C(t)=8, N(t)=n,\ x<G^0(t)\leq x+dx\right\},\ \text{for } t\geq 0,\ x\geq 0 \text{ and } n\geq 0.$$

We assume that the stability condition $\rho < R^*(\lambda+\delta)$ is fulfilled and so that we can set $P_0 = \lim_{t\to\infty} P_0(t)$; and densities at limiting case, for $t \geq 0, x \geq 0$ and $n \geq 1$ are,

$$P_n(x)=\lim_{t\to\infty}P_n(x,t); \quad \Pi_{1,n}(x)=\lim_{t\to\infty}\Pi_{1,n}(x,t); \quad \Pi_{2,n}(x,y)=\lim_{t\to\infty}\Pi_{2,n}(x,y,t);$$
$$\Pi_{3,n}(x)=\lim_{t\to\infty}\Pi_{3,n}(x,t)\ ; \quad Q_{1,n}(x)=\lim_{t\to\infty}Q_{1,n}(x,t); \quad Q_{2,n}(x)=\lim_{t\to\infty}Q_{2,n}(x,t);$$
$$Q_{3,n}(x)=\lim_{t\to\infty}Q_{3,n}(x,t); \quad \Omega_n(x)=\lim_{t\to\infty}\Omega_n(x,t); \quad R_n(x)=\lim_{t\to\infty}R_n(x,t).$$

4.1 The steady state equations:

The governing equations of the proposed model are derived by using "supplementary variable technique" as given below:

$$(\lambda+\delta)P_0=\int_0^\infty \Omega_0(x)\gamma(x)dx+[1-r]\left\{\int_0^\infty \Pi_{3,0}(x)\mu_b(x)dx+\int_0^\infty \Pi_{1,0}(x)\mu_p(x)dx\right\}$$
$$+\int_0^\infty Q_{3,0}(x)\mu_b(x)dx+\int_0^\infty Q_{1,0}(x)\mu_p(x)dx+\int_0^\infty R_0(x)\xi(x)dx \tag{4.1}$$

$$\frac{dP_n(x)}{dx}+\left(\lambda+\delta+\alpha+a(x)\right)P_n(x)=0,\ n\ge 1 \tag{4.2}$$

$$\frac{d\Pi_{1,0}(x)}{dx}+\left(\lambda+\alpha+\mu_p(x)\right)\Pi_{1,0}(x)=\lambda(1-b)\Pi_{1,0}(x,y),\ n=0, \tag{4.3}$$

$$\frac{d\Pi_{1,n}(x)}{dx}+\left(\lambda+\alpha+\mu_p(x)\right)\Pi_{1,n}(x)=\lambda(1-b)\Pi_{1,n}(x)+\lambda b\sum_{k=1}^{n}\chi_k\Pi_{1,n-k}(x),\ n\ge 1, \tag{4.4}$$

$$\frac{\partial\Pi_{2,0}(x,y)}{\partial x}+\left(\lambda+\alpha+\mu_p(x)\right)\Pi_{2,0}(x,y)=\lambda(1-b)\Pi_{2,0}(x,y),\ n=0, \tag{4.5}$$

$$\frac{\partial\Pi_{2,n}(x,y)}{\partial x}+\left(\lambda+\alpha+\mu_p(x)\right)\Pi_{2,n}(x,y)=\lambda(1-b)\Pi_{2,n}(x,y)+\lambda b\sum_{k=1}^{n}\chi_k\Pi_{2,n-k}(x),\ n\ge 1, \tag{4.6}$$

$$\frac{d\Pi_{3,0}(x)}{dx}+\left(\lambda+\delta+\mu_b(x)\right)\Pi_{3,0}(x)=\int_0^\infty \Pi_{2,0}(y,x)\mu_p(y)dy+\lambda(1-b)\Pi_{3,0}(x),\ n=0 \tag{4.7}$$

$$\frac{d\Pi_{3,n}(x)}{dx}+\left(\lambda+\delta+\mu_b(x)\right)\Pi_{3,n}(x)=\lambda(1-b)\Pi_{3,n}(x)+\int_0^\infty \Pi_{2,n}(y,x)\mu_p(y)dy$$
$$+\lambda b\sum_{k=1}^{n}\chi_k\Pi_{3,n-k}(x),\ n\ge 1 \tag{4.8}$$

$$\frac{dQ_{1,0}(x)}{dx}+\left(\lambda+\alpha+\mu_p(x)\right)Q_{1,0}(x)=\lambda(1-b)Q_{1,0}(x),\ n=0 \tag{4.9}$$

$$\frac{dQ_{1,n}(x)}{dx}+\left(\lambda+\alpha+\mu_p(x)\right)Q_{1,n}(x)=\lambda(1-b)Q_{1,n}(x)+\lambda b\sum_{k=1}^{n}\chi_kQ_{1,n-k}(x),\ n\ge 1 \tag{4.10}$$

$$\frac{dQ_{2,0}(x,y)}{dx}+\left(\lambda+\alpha+\mu_p(x)\right)Q_{2,0}(x,y)=\lambda(1-b)Q_{2,0}(x,y),\ n=0 \tag{4.11}$$

$$\frac{\partial Q_{2,n}(x,y)}{\partial x}+\left(\lambda+\alpha+\mu_p(x)\right)Q_{2,n}(x,y)=\lambda(1-b)Q_{2,n}(x,y)+\lambda b\sum_{k=1}^{n}\chi_k Q_{2,n-k}(x),\ n\geq 1, \tag{4.12}$$

$$\frac{dQ_{3,0}(x)}{dx}+\left(\lambda+\alpha+\mu_p(x)\right)Q_{3,0}(x)=\lambda(1-b)Q_{3,0}(x),\ n=0 \tag{4.13}$$

$$\frac{dQ_{3,n}(x)}{dx}+\left(\lambda+\delta+\mu_b(x)\right)Q_{3,n}(x)=\lambda(1-b)Q_{3,n}(x)+\lambda b\sum_{k=1}^{n}\chi_k Q_{3,n-k}(x),\quad n\geq 1 \tag{4.14}$$

$$\frac{d\Omega_n(x)}{dx}+\left(\lambda+\gamma(x)\right)\Omega_{j,n}(x)=\lambda(1-b)\Omega_n(x)+\lambda b\sum_{k=1}^{n}\chi_k\Omega_{n-k}(x),\ n\geq 1 \tag{4.15}$$

$$\frac{dR_n(x)}{dx}+\left(\lambda+\xi(x)\right)R_n(x)=\lambda(1-b)R_n(x)+\lambda b\sum_{k=1}^{n}\chi_k R_{n-k}(x),\ n\geq 1 \tag{4.16}$$

The steady state boundary conditions at $x = 0$ and $y = 0$ are ,

$$\begin{aligned}P_n(0)&=\int_0^\infty\Omega_n(x)\gamma(x)dx+[1-r]\left\{\int_0^\infty\Pi_{3,n}(x)\mu_b(x)dx+\int_0^\infty\Pi_{1,n}(x)\mu_p(x)dx\right\}\\&\quad+\int_0^\infty Q_{3,n}(x)\mu_b(x)dx+\int_0^\infty Q_{1,n}(x)\mu_p(x)dx+\int_0^\infty R_n(x)\xi(x)dx\qquad ,n\geq 1\end{aligned} \tag{4.17}$$

$$\Pi_{1,n}(0)=\delta\int_0^\infty P_n(x)dx+\delta P_0,\ n\geq 0 \tag{4.18}$$

$$\Pi_{2,n}(0,x)=\delta\left[\Pi_{3,n}(x)+Q_{3,n}(x)\right],\qquad n\geq 0 \tag{4.19}$$

$$\Pi_{3,\mathrm{n}}(0)=\int_0^\infty P_{n+1}(x)a(x)dx+\lambda b\sum_{k=1}^{n}\chi_k\int_0^\infty P_{n+1-k}(x)dx+\lambda b\chi_{n+1}P_0,\ n\geq 0 \tag{4.20}$$

$$Q_{1,n}(0)=r\int_0^\infty\Pi_{1,n}(x)\mu_p(x)dx,\ n\geq 0 \tag{4.21}$$

$$Q_{2,n}(0,x)=r\int_0^\infty Q_{2,n}(0,x)\mu_p(y)dy,\ n\geq 0 \tag{4.22}$$

$$Q_{3,n}(0)=r\int_0^\infty\Pi_{3,n}(x)\mu_b(x)dx,\ n\geq 0 \tag{4.23}$$

$$\Omega_n(0)=\begin{cases}[1-r]\left\{\int_0^\infty \Pi_{3,0}(x)\mu_b(x)dx+\int_0^\infty \Pi_{1,0}(x)\mu_p(x)dx\right\}+\int_0^\infty Q_{3,0}(x)\mu_b(x)dx+\int_0^\infty Q_{1,0}(x)\mu_p(x)dx, & n=0,\\ 0, & n\ge 1\end{cases} \tag{4.24}$$

$$R_n(0)=\alpha\left(\begin{array}{l}\int_0^\infty \Pi_{1,n}(x)dx+\theta\left\{\int_0^\infty \Pi_{3,n-1}(x)dx+\int_0^\infty Q_{3,n-1}(x)dx\right\}+\int_0^\infty \Pi_{2,n}(x,y)dx\\ +\int_0^\infty Q_{2,n}(x,y)dx+\int_0^\infty Q_{1,n}(x)dx+(1-\theta)\left\{\int_0^\infty \Pi_{3,n}(x)dx+\int_0^\infty Q_{3,n}(x)dx\right\}\end{array}\right),\quad n\ge 1 \tag{4.25}$$

The normalizing condition is

$$P_0+\sum_{n=1}^{\infty}\int_0^\infty P_n(x)dx+\sum_{n=0}^{\infty}\left(\begin{array}{l}\int_0^\infty \Pi_{1,n}(x)dx+\int_0^\infty\int_0^\infty \Pi_{2,n}(x,y)dxdy+\int_0^\infty \Pi_{3,n}(x)dx\\ \int_0^\infty Q_{1,n}(x)dx+\int_0^\infty\int_0^\infty Q_{2,n}(x,y)dxdy+\int_0^\infty Q_{3,n}(x)dx\\ +\int_0^\infty \Omega_n(x)dx+\int_0^\infty R_n(x)dx\end{array}\right)=1 \tag{4.26}$$

4.2 The steady state solution:

In order to solve the above equations, the PGFs are defined for $|z|\le 1$ as given below:

$$P(x,z)=\sum_{n=1}^{\infty}P_n(x)z^n;P(0,z)=\sum_{n=1}^{\infty}P_n(0)z^n;\Pi_1(x,z)=\sum_{n=0}^{\infty}\Pi_{1,n}(x)z^n;\Pi_1(0,z)=\sum_{n=0}^{\infty}\Pi_{1,n}(0)z^n;\Pi_2(x,y,z)=\sum_{n=0}^{\infty}\Pi_{2,n}(x,y)z^n;$$

$$\Pi_2(x,0,z)=\sum_{n=0}^{\infty}\Pi_{2,n}(x,0)z^n;\ \Pi_3(x,z)=\sum_{n=0}^{\infty}\Pi_{3,n}(x)z^n;\Pi_3(0,z)=\sum_{n=0}^{\infty}\Pi_{3,n}(0)z^n;Q_1(x,z)=\sum_{n=0}^{\infty}Q_{1,n}(x)z^n;\ X(z)=\sum_{n=0}^{\infty}\chi_n z^n$$

$$Q_1(0,z)=\sum_{n=0}^{\infty}Q_{1,n}(0)z^n;Q_2(x,y,z)=\sum_{n=0}^{\infty}Q_{2,n}(x,y)z^n;\ Q_2(x,0,z)=\sum_{n=0}^{\infty}Q_{2,n}(x,0)z^n;Q_3(x,z)=\sum_{n=0}^{\infty}Q_{3,n}(x)z^n;$$

$$Q_3(0,z)=\sum_{n=0}^{\infty}Q_{3,n}(0)z^n;\Omega(x,z)=\sum_{n=0}^{\infty}\Omega_n(x)z^n;\Omega(0,z)=\sum_{n=0}^{\infty}\Omega_n(0)z^n;R(x,z)=\sum_{n=0}^{\infty}R_n(x)z^n;\ R(0,z)=\sum_{n=0}^{\infty}R_n(0)z^n;$$

Assume that,

$$A_p(z)=\lambda b(1-X(z))+\alpha,\quad A_b(z)=\left(A_p(z)+\delta\left(1-S_p^*\left(A_p(z)\right)\right)\right)\quad\text{and}\quad \mathrm{b}(z)=\lambda b(1-X(z))$$

From the Eqns. (4.17) - (4.25), we can obtain

$$P(0,z)=\left\{\begin{array}{l}(1-r)\left[\int_0^\infty \Pi_1(x,z)\mu_p(x)dx+\int_0^\infty \Pi_3(x,z)\mu_b(x)dx\right]+\int_0^\infty Q_1(x,z)\mu_p(x)dx\\ +\int_0^\infty Q_3(x,z)\mu_b(x)dx+\int_0^\infty \Omega(x,z)\gamma(x)dx+\int_0^\infty R(x,z)\xi(x)dx-(\lambda+\delta)P_0\end{array}\right\} \tag{4.27}$$

$$\Pi_1(0,z)=\delta\int_0^\infty P(x,z)dx+\delta P_0, \tag{4.28}$$

$$\Pi_2(0,x,z)=\delta\Pi_3(x,z)+\delta Q_3(x,z) \tag{4.29}$$

$$\Pi_3(0,z)=\left(\frac{1}{z}\int_0^\infty P(x,z)a(x)dx+\frac{\lambda bX(z)}{z}\int_0^\infty P(x,z)dx+\frac{\lambda bX(z)}{z}P_0\right), \tag{4.30}$$

$$Q_1(0,z)=rS_p^*(A_p(z))\Pi_1(0,z) \tag{4.31}$$

$$Q_2(0,x,z)=rS_p^*(A_p(z))\left[\Pi_3(x,z)+Q_3(x,z)\right] \tag{4.32}$$

$$Q_3(0,z)=rS_b^*(A_b(z))\Pi_3(0,z) \tag{4.33}$$

$$\Omega(0,z)=\left[(1-r)+rS_p^*(A_p(z))\right]\int_0^\infty \Pi_1(x,z)\mu_p(x)dx+\left[(1-r)+rS_b^*(A_b(z))\right]\int_0^\infty \Pi_3(x,z)\mu_b(x)dx \tag{4.34}$$

$$R(0,z)=\alpha\left\{\begin{array}{l}\dfrac{\left[1-S_p^*(A_p(z))\right]}{A_p(z)}\left[\Pi_1(0,z)+\Pi_2(0,y,z)+Q_1(0,z)+Q_2(0,y,z)\right]\\ +(1-\theta+\theta z)\dfrac{\left[1-S_b^*(A_b(z))\right]}{A_b(z)}\left[\Pi_3(0,z)+Q_3(0,z)\right]\end{array}\right\} \tag{4.35}$$

Inserting the Eqn. (4.27) in (4.37), we get

$$\Pi_1(0,z)=\delta P(0,z)\bar{R}^*(\lambda+\delta)+\delta P_0, \tag{4.36}$$

where $\bar{R}^*(\lambda+\delta)=\left(\dfrac{1-R^*(\lambda+\delta)}{\lambda+\delta}\right)$

Inserting equation (4.27) in (4.30) and make some manipulation, finally we

get, $\Pi_3(0,z)=\dfrac{P(0,z)}{z}\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)+\lambda P_0$ (4 .37)

Using (4.28)-(4.35) in (4.27) and make some manipulation, we get

$$\begin{aligned}P(0,z)= {} & S_p^*\left(A_p(z)\right)\left[\Pi_1(0,z)+Q_1(0,z)\right]+S_b^*\left(A_b(z)\right)\left[\Pi_3(0,z)+Q_3(0,z)\right]\\ & +\Omega(0,z)V^*\left(\mathrm{b}(z)\right)+R(0,z)G^*\left(\mathrm{b}(z)\right)-(\lambda+\delta)bP_0\end{aligned} \tag{4.38}$$

Using the Eqns. (4.28)-(4.35) in (4.38), we get

$$P(0,z)=\frac{Nr(z)}{Dr(z)} \tag{4.39}$$

$$Nr(z) = P_0 \left\{ \begin{array}{l} \delta z A_b(z) \left[\begin{array}{l} \left[(1-r) + rS_p^*(A_p(z))\right] S_p^*(A_p(z)(1+V^*(b(z)))A_p(z) \\ +G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z))) \end{array} \right] \\ +\lambda b X(z) \left[\begin{array}{l} \left[(1-r) + rS_b^*(A_b(z))\right] S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z) \\ +G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z))) \left(\begin{array}{l} (1+rS_p^*(A_p(z))A_b(z) \\ +(1-\theta+\theta z) \end{array} \right) (1-S_b^*(A_b(z)) \end{array} \right] \end{array} \right\}$$

$$Dr(z) = zA_b(z)A_p(z) - \left\{ \begin{array}{l} \delta z A_b(z)\bar{R}^*(\lambda+\delta) \left[\begin{array}{l} \left[(1-r) + rS_p^*(A_p(z))\right] S_p^*(A_p(z)(1+V^*(b(z)))A_p(z) \\ +G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z))) \end{array} \right] \\ +\left(R^*(\lambda+\delta) + \lambda X(z)\bar{R}^*(\lambda+\delta)\right) \left[\begin{array}{l} \left[(1-r) + rS_b^*(A_b(z))\right] S_b^*(A_b(z) \\ (1+V^*(b(z)))zA_b(z)A_p(z) \\ +G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z))) \\ \left(\begin{array}{l} (1+rS_p^*(A_p(z))A_b(z) \\ +(1-\theta+\theta z)(1-S_b^*(A_b(z)) \end{array} \right) \end{array} \right] \end{array} \right\}$$

Using the equation (4.39) in (4.28), we get

$$\Pi_1(0,z) = \delta p_0 \left\{ z[1-(\lambda+\delta)b](A_b(z)A_p(z) - \bar{R}^*(\lambda+\delta) \left\{ \begin{array}{l} \left[(1-r)+rS_b^*(A_b(z))\right] S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z) \\ +G^*(b(z))(1+rS_b^*(A_b(z))(1-S_p^*(A_p(z))) \\ \left(\begin{array}{l} (1+rS_p^*(A_p(z))A_b(z) \\ +(1-\theta+\theta z)(1-S_b^*(A_b(z)) \end{array} \right) \end{array} \right\} \right\} \Big/ Dr(z) \quad (4.40)$$

Using the equation (4.39) in (4.30), we get

$$\Pi_3(0,z) = P_0 \left\{ \begin{array}{l} [\lambda b X(z) - (\lambda+\delta)bz]A_p(z) \\ -R^*(\lambda+\delta) \left\{ \begin{array}{l} \left\{ \delta\left[(1-r)+rS_p^*(A_p(z))\right] S_p^*(A_p(z))(1+V^*(b(z)))zA_p(z) \right\} \\ +\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z))) \end{array} \right\} \end{array} \right\} \Big/ Dr(z) \quad (4.41)$$

Using the equation (4.39) in (4.32), we get

$$Q_3(0,z) = rS_b^*(A_b(z))P_0 \left\{ [\lambda b X(z) - (\lambda+\delta)bz]A_p(z) - R^*(\lambda+\delta) \left\{ \begin{array}{l} \left\{ \begin{array}{l} \delta\left[(1-r)+rS_p^*(A_p(z))\right] S_p^*(A_p(z)) \\ (1+V^*(b(z)))zA_p(z) \end{array} \right\} \\ +\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z))) \end{array} \right\} \right\} \Big/ Dr(z) \quad (4.42)$$

Using the equation (4.39) in (4.29), we get

$$\Pi_2(0,x,z) = \delta e^{-A_b(z)x}[1-S_b(x)][1+rS_b^*(A_b(z))]P_0 \left\{ \lambda b X(z)A_b(z)A_p(z) - R^*(\lambda+\delta) \left\{ \begin{array}{l} \left\{ \begin{array}{l} \delta\left[(1-r)+rS_p^*(A_p(z))\right] S_p^*(A_p(z)) \\ (1+V^*(b(z)))zA_b(z)A_p(z) \end{array} \right\} \\ +\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))A_b(z) \end{array} \right\} \right\} \Big/ Dr(z) \quad (4.43)$$

Using the equation (4.39) in (4.31), we get

$$Q_2(0,x,z)=\delta r S_p^*(A_p(z))e^{-A_b(z)x}[1-S_b(x)][1+rS_b^*(A_b(z))]P_0\left\{\begin{array}{l}\lambda bX(z)A_b(z)A_p(z)\\ -R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\begin{array}{l}\delta\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z))\\(1+V^*(b(z)))zA_b(z)A_p(z)\end{array}\right\}\\+\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))A_b(z)\end{array}\right\}\end{array}\right\}\Bigg/ Dr(z) \tag{4.44}$$

Using the equation (4.39) in (4.32) we get

$$Q_1(0,z)=\delta r S_p^*(A_p(z))p_0\left\{z[1-(\lambda+\delta)b]A_b(z)A_p(z)-\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\+G^*(b(z))(1+rS_b^*(A_b(z))(1-S_p^*(A_p(z)))\\\left((1+rS_p^*(A_p(z)))A_b(z)+(1-\theta+\theta z)(1-S_b^*(A_b(z)))\right)\end{array}\right\}\right\}\Bigg/ Dr(z) \tag{4.45}$$

Using the equation (4.39) in (4.33) we get

$$\Omega(0,z)=P_0\left[(1-r)+rS_p^*(A_p(z))\right]\left\{\begin{array}{l}[\lambda bX(z)+\delta z]A_b(z)A_p(z)-\delta R^*(\lambda+\delta)G^*(b(z))\left\{\begin{array}{l}(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\\(1-\theta+\theta z)(1-S_b^*(A_b(z))\end{array}\right\}\\+\lambda bX(z)\left[(1-r)+rS_b^*(A_b(z))\right]A_p(z)A_b(z)\end{array}\right\}\Bigg/ D_r(z) \tag{4.46}$$

Using the equation (4.39) in (4.34) we get

$$R(0,z)=\frac{\alpha\delta P_0\left[(1+rS_p^*(A_p(z))\right]\left[(1-S_p^*(A_p(z))\right]}{b(z)}\left\{\left\{\begin{array}{l}(z-\lambda bX(z))A_b(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}A_b(z)[(1-r)+rS_b^*(A_b(z))]\\(1+V^*(b(z))S_b^*(A_b(z))\end{array}\right\}\\\lambda bX(z)(1-\theta+\theta z)\left[1-S_b^*(A_b(z))\right](1+rS_b^*(A_b(z))\end{array}\right\}\right\}\Bigg/ Dr(z) \tag{4.47}$$

Using the equations (4.39)-(4.47) in (4.1)-(4.17), then we get the results for the following PGFs $P(x,z)$, $\Pi_1(x,z)$, $\Pi_2(x,y,z)$, $\Pi_b(x,z)$, $\Omega(x,z)$ and $Q_v(x,z)$. Next we are interested in investigating the marginal orbit size distributions due to system state of the server.

Theorem 4.1. "*The marginal probability distributions of the number of customers in the orbit when server being idle, busy serving priority customers with preempting an ordinary customer, busy serving priority customers without preempting an ordinary customer, busy serving ordinary customers, on vacation and repair is given by*"

$$P(z)=\frac{Nr(z)}{Dr(z)} \tag{4.48}$$

$$N(z)=P_0\,\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\delta zA_b(z)\left[\begin{array}{l}\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z)(1+V^*(b(z)))A_p(z)\\+G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))\end{array}\right]\\+\lambda bX(z)\left[\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\+G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\left(\begin{array}{l}(1+rS_p^*(A_p(z))A_b(z)\\+(1-\theta+\theta z)\end{array}\right)(1-S_b^*(A_b(z))\end{array}\right]\end{array}\right\}$$

$$D_r(z)=zA_b(z)A_p(z)-\left\{\begin{array}{l}\delta zA_b(z)\bar{R}^*(\lambda+\delta)\left[\begin{array}{l}\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z)(1+V^*(b(z)))A_p(z)\\+G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))\end{array}\right]\\+\left(R^*(\lambda+\delta)+\lambda X(z)\bar{R}^*(\lambda+\delta)\right)\left[\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\+G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\left(\begin{array}{l}(1+rS_p^*(A_p(z))A_b(z)\\+(1-\theta+\theta z)(1-S_b^*(A_b(z))\end{array}\right)\end{array}\right]\end{array}\right\}$$

$$\Pi_1(z)=\frac{\delta p_0[1-S_p^*(A_p(z))]}{A_p(z)}\left\{zA_b(z)A_p(z)-\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\left[\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\right.\\+G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\\\left.\left((1+rS_p^*(A_p(z))A_b(z)+(1-\theta+\theta z)(1-S_b^*(A_b(z))\right)\right]\end{array}\right\}\right\}\Big/Dr(z)\qquad(4.49)$$

$$\Pi_2(z)=\frac{\delta P_0[1-S_b^*(A_b(z))][1-S_p^*(A_p(z))}{A_b(z)A_p(z)}\left\{\begin{array}{l}\lambda bX(z)A_b(z)A_p(z)\\-R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\delta\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z))(1+V^*(b(z)))zA_b(z)A_p(z)\right\}\\+\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))A_b(z)\end{array}\right\}\end{array}\right\}\Big/Dr(z)\qquad(4.50)$$

$$\Pi_3(z)=P_0[1-S_b^*(A_b(z))]\left\{\lambda bX(z)A_p(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\begin{array}{l}\delta\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z))\\(1+V^*(b(z)))zA_p(z)\end{array}\right\}\\+\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))\end{array}\right\}\right\}\Big/Dr(z)\qquad(4.51)$$

$$Q_1(z)=\delta rp_0[1-S_p^*(A_p(z))]S_p^*(A_p(z))\left\{zA_b(z)A_p(z)-\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\left[\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\right.\\+G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\\\left.\left((1+rS_p^*(A_p(z))A_b(z)+(1-\theta+\theta z)(1-S_b^*(A_b(z))\right)\right]\end{array}\right\}\right\}\Big/A_p(z)Dr(z)\qquad(4.52)$$

$$Q_2(z)=\frac{\delta rP_0[1-S_b^*(A_b(z))][1-S_p^*(A_p(z))]S_p^*(A_p(z))}{A_b(z)A_p(z)}\left\{\lambda bX(z)A_b(z)A_p(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\begin{array}{l}\delta\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z))\\(1+V^*(b(z)))zA_b(z)A_p(z)\end{array}\right\}\\+\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))A_b(z)\end{array}\right\}\right\}\Big/Dr(z)\qquad(4.53)$$

$$Q_3(z)=rP_0S_b^*(A_b(z))[1-S_b^*(A_b(z))]\left\{\lambda bX(z)A_p(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\begin{array}{l}\delta\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z))\\(1+V^*(b(z)))zA_p(z)\end{array}\right\}\\+\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))\end{array}\right\}\right\}\Big/Dr(z)\qquad(4.54)$$

$$\Omega(z)=P_0[1-V^*(b(z))]\left[(1-r)+rS_p^*(A_p(z))\right]\left\{\begin{array}{l}zA_b(z)A_p(z)-\delta R^*(\lambda+\delta)G^*(b(z))\left\{\begin{array}{l}(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\\(1-\theta+\theta z)(1-S_b^*(A_b(z))\end{array}\right\}\\+\lambda bX(z)\left[(1-r)+rS_b^*(A_b(z))\right]A_p(z)A_b(z)\end{array}\right\}\Big/D_r(z)b(z)\qquad(4.55)$$

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$$R(z)=\frac{\alpha\delta P_0\left[1-G^{*}(b(z))\right]\left[(1+rS_p^{*}(A_p(z))\right]\left[(1-S_p^{*}(A_p(z))\right]}{b(z)}\left\{\begin{Bmatrix}(z-\lambda bX(z))A_b(z)\\ -R^{*}(\lambda+\delta)\begin{Bmatrix}A_b(z)[(1-r)+rS_b^{*}(A_b(z))]\\(1+V^{*}(b(z))S_b^{*}(A_b(z))\end{Bmatrix}\\ \lambda bX(z)(1-\theta+\theta z)\left[1-S_b^{*}(A_b(z))\right](1+rS_b^{*}(A_b(z))\end{Bmatrix}\right\}\Bigg/Dr(z) \tag{4.56}$$

Where,

$$P_0=\frac{1-\rho}{\left\{\begin{array}{l}\alpha\lambda^2\delta\beta_p^{(1)}\left\{\tau(1-\bar{R}^{*}(\lambda+\delta))+\bar{\alpha}\tau\left(R^{*}(\lambda+\delta)+\delta\bar{R}^{*}(\lambda+\delta)\right)-\alpha\bar{R}^{*}(\lambda+\delta)[\tau S_p^{*}(\beta)-\beta[1-S_p^{*}(\beta)]\right\}\\ +\alpha\lambda[1-S_p^{*}(\beta)]\left\{R^{*}(\lambda+\delta)[N'(1)-1+A_b'(1)]+\bar{\alpha}\left[\bar{\alpha}\lambda\theta\tau+\left(R^{*}(\lambda+\delta)+\lambda\bar{R}^{*}(\lambda+\delta)\right)[A_b'(1)-\lambda\tau h^{(1)}\right]\right\}\\ +\alpha\lambda\left\{\beta_b^{(1)}A_b'(1)\right\}\left[\left(R^{*}(\lambda+\delta)+\lambda\bar{R}^{*}(\lambda+\delta)\right)((\lambda+\delta)+\bar{\alpha}\delta)+\lambda+\alpha\delta R^{*}(\lambda+\delta)S_b^{*}(\tau)\right]\\ +\alpha\lambda[1-S_b^{*}(\tau)]\left\{\lambda\bar{R}^{*}(\lambda+\delta)\{(-(\lambda+\delta)+\bar{\alpha}\delta)\}+\left(R^{*}(\lambda+\delta)+\lambda\bar{R}^{*}(\lambda+\delta)\right)\left[(\lambda+\delta)N'(1)-\lambda\alpha\delta\bar{R}^{*}(\lambda+\delta)\beta_p^{(1)}\right]\right\}\end{array}\right\}} \tag{4.57}$$

$$\rho=\left\{\begin{array}{l}\delta\bar{R}^{*}(\lambda+\delta)\left[[(1-r)+rS_p^{*}(\alpha)]\right]S_p^{*}(\alpha)+(1+rS_p^{*}(\alpha))A_b(1)[1-S_p^{*}(\alpha)]\\ +\left(R^{*}(\lambda+\delta)+\lambda R^{*}(\lambda+\delta)\right)\left\{[(1-r)+rS_b^{*}(\tau)]S_b^{*}(\tau)+(1+rS_b^{*}(\tau))[1-S_p^{*}(\alpha)]\left[(1+rS_p^{*}(\tau))A_b(1)+[1-S_b^{*}(\tau)]\right]\right\}\end{array}\right\};$$

$$b(z)=\lambda b(1-X(z));\ A_p(z)=\left(\lambda b(1-X(z))+\alpha(1-G^{*}(b(z))\right);\ A_b(z)=\left(\lambda b(1-X(z))+\alpha(1-G^{*}(b(z))+\delta\left(1-S_p^{*}\left(A_p(z)\right)\right)\ \right)$$

Proof. Integrating the Eqns. (4.44)-(4.49) with respect to x, we define the PGFs as,

$$P(z)=\int_0^\infty P(x,z)dx,\ \Pi_1(z)=\int_0^\infty \Pi_1(x,z)dx,\ \Pi_2(z)=\int_0^\infty \Pi_2(x,z)dx,\ \Pi_3(z)=\int_0^\infty \Pi_3(x,z)dx,$$

$$\Omega(z)=\int_0^\infty \Omega(x,z)dx, Q_1(z)=\int_0^\infty Q_1(x,z)dx,\ Q_2(z)=\int_0^\infty Q_2(x,z)dx,\ Q_3(z)=\int_0^\infty Q_3(x,z)dx,\ R(z)=\int_0^\infty R(x,z)dx,$$

By using the normalized condition, we can be determined the probability that the server is idle (P_0). Thus, by setting z = 1 in (4.54)-(4.63) and applying L-Hospital's rule whenever necessary and we get $P_0+P(z)+\Pi_1(z)+\Pi_2(z)+\Pi_3(z)+Q_1(z)+Q_2(z)+Q_3(z)+R(z)+\Omega(z)=1.$

Corollary 4.1.: "*The probability generating function of number of customers in the system and orbit size distribution at stationary point of time is"*

$$K_s(z)=\frac{Nr_s(z)}{Dr_s(z)}=P_0+P(z)+z\left(\Pi_1(z)+\Pi_2(z)+\Pi_3(z)+Q_1(z)+Q_2(z)+Q_3(z)+R(z)\right)+\Omega(z) \tag{4.58}$$

$$
Nr_s(z)=P_0\left\{\begin{array}{l}
A_b(z)A_p(z)b(z)D_r(z)+\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}
\delta zA_b(z)\left[\begin{array}{l}\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z)(1+V^*(b(z)))A_p(z)\\ +G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))\end{array}\right]\\
+\lambda bX(z)\left[\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\ +G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\left(\begin{array}{l}(1+rS_p^*(A_p(z))A_b(z)\\ +(1-\theta+\theta z)\end{array}\right)(1-S_b^*(A_b(z))\end{array}\right]
\end{array}\right\}\\
+z\delta A_b(z)b(z)[1-S_p^*(A_p(z))]\left\{\begin{array}{l}
zA_b(z)A_p(z)\\
-\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\ +G^*(b(z))(1+rS_b^*(A_b(z))(1-S_p^*(A_p(z)))\\ \left((1+rS_p^*(A_p(z))A_b(z)+(1-\theta+\theta z)(1-S_b^*(A_b(z))\right)\end{array}\right\}
\end{array}\right\}\\
+z\left(\begin{array}{l}A_b(z)b(z)[1-S_b^*(A_b(z))][1+rS_b^*(A_b(z))]\\ \left\{A_p(z)+\delta[1-S_p^*(A_p(z))][1+rS_p^*(A_p(z))]\right\}\end{array}\right)\left\{\lambda bX(z)A_b(z)A_p(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\begin{array}{l}\delta\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z))\\ (1+V^*(b(z)))zA_b(z)A_p(z)\end{array}\right\}\\ +\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))A_b(z)\end{array}\right\}\right\}\\
+[1-V^*(b(z))]\left[(1-r)+rS_p^*(A_p(z))\right]A_b(z)A_p(z)\left\{\begin{array}{l}zA_b(z)A_p(z)-\delta R^*(\lambda+\delta)G^*(b(z))\left\{\begin{array}{l}(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\\ (1-\theta+\theta z)(1-S_b^*(A_b(z))\end{array}\right\}\\ +\lambda bX(z)\left[(1-r)+rS_b^*(A_b(z))\right]A_p(z)A_b(z)\end{array}\right\}\\
+z\left[1-G^*(b(z))\right]\delta\left[(1+rS_p^*(A_p(z))\right]\left[(1-S_p^*(A_p(z))\right]\left\{\left\{\begin{array}{l}(z-\lambda bX(z))A_b(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}A_b(z)[(1-r)+rS_b^*(A_b(z))]\\ (1+V^*(b(z))S_b^*(A_b(z))\end{array}\right\}\\ \lambda bX(z)(1-\theta+\theta z)\left[1-S_b^*(A_b(z))\right](1+rS_b^*(A_b(z))\end{array}\right\}\right\}
\end{array}\right\}
$$

$$
D_r(z)=zA_b(z)A_p(z)-\left\{\begin{array}{l}
\delta zA_b(z)\bar{R}^*(\lambda+\delta)\left[\begin{array}{l}\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z)(1+V^*(b(z)))A_p(z)\\ +G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))\end{array}\right]\\
+\left(R^*(\lambda+\delta)+\lambda X(z)\bar{R}^*(\lambda+\delta)\right)\left[\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\ +G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\left(\begin{array}{l}(1+rS_p^*(A_p(z))A_b(z)\\ +(1-\theta+\theta z)(1-S_b^*(A_b(z))\end{array}\right)\end{array}\right]
\end{array}\right\}
$$

$$
K_o(z)=\frac{Nr_o(z)}{Dr_s(z)}=P_0+P(z)+\Pi_1(z)+\Pi_2(z)+\Pi_3(z)+\Omega(z)+Q_1(z)+Q_2(z)+Q_3(z)+R(z) \qquad (4.59)
$$

$$Nr_o(z)=\frac{P_0}{A_b(z)A_p(z)b(z)D_r(z)}\left\{\begin{array}{l}
A_b(z)A_p(z)b(z)D_r(z)+\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\delta zA_b(z)\left[\begin{array}{l}\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z)(1+V^*(b(z)))A_p(z)\\+G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))\end{array}\right]\\+\lambda bX(z)\left[\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\+G^*(b(z))(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\left(\begin{array}{l}(1+rS_p^*(A_p(z))A_b(z)\\+(1-\theta+\theta z)\end{array}\right)(1-S_b^*(A_b(z))\end{array}\right]\end{array}\right\}\\
\delta A_b(z)b(z)[1-S_p^*(A_p(z))]\left\{\begin{array}{l}zA_b(z)A_p(z)\\-\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\\+G^*(b(z))(1+rS_b^*(A_b(z))(1-S_p^*(A_p(z)))\\\left((1+rS_p^*(A_p(z))A_b(z)+(1-\theta+\theta z)(1-S_b^*(A_b(z))\right)\end{array}\right\}\end{array}\right\}\\
+\left(\begin{array}{l}A_b(z)b(z)[1-S_b^*(A_b(z))][1+rS_b^*(A_b(z))]\\\left\{A_p(z)+\delta[1-S_p^*(A_p(z))][1+rS_p^*(A_p(z))]\right\}\end{array}\right)\left\{\lambda bX(z)A_b(z)A_p(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\begin{array}{l}\delta\left[(1-r)+rS_p^*(A_p(z))\right]S_p^*(A_p(z))\\(1+V^*(b(z)))zA_b(z)A_p(z)\end{array}\right\}\\+\delta G^*(b(z))(1+rS_p^*(A_p(z))(1-S_p^*(A_p(z)))A_b(z)\end{array}\right\}\right\}\\
[1-V^*(b(z))]\left[(1-r)+rS_p^*(A_p(z))\right]A_b(z)A_p(z)\left\{\begin{array}{l}zA_b(z)A_p(z)-\delta R^*(\lambda+\delta)G^*(b(z))\left\{\begin{array}{l}(1+rS_b^*(A_p(z))(1-S_p^*(A_p(z)))\\(1-\theta+\theta z)(1-S_b^*(A_b(z))\end{array}\right\}\\+\lambda bX(z)\left[(1-r)+rS_b^*(A_b(z))\right]A_p(z)A_b(z)\end{array}\right\}\\
\left[1-G^*(b(z))\right]\delta\left[(1+rS_p^*(A_p(z))\right]\left[(1-S_p^*(A_p(z))\right]\left\{\begin{array}{l}(z-\lambda bX(z))A_b(z)-R^*(\lambda+\delta)\left\{\begin{array}{l}A_b(z)[(1-r)+rS_b^*(A_b(z))]\\(1+V^*(b(z))S_b^*(A_b(z))\end{array}\right\}\\\lambda bX(z)(1-\theta+\theta z)\left[1-S_b^*(A_b(z))\right](1+rS_b^*(A_b(z))\end{array}\right\}
\end{array}\right\}$$

where P_0 is given in Eq. (4.57).

5. System performance measures

In this section, we derive some system probabilities, a mean number of customers in the orbit/system, mean busy period and the busy cycle of the model.

5.1. System state probabilities

From Eqns. (4.48) – (4.56), by setting $z\to 1$ and applying L-Hospital's rule whenever necessary, then we get the following results,

(i) "The probability that the server is idle during the retrial, is given by",

$$P=P(1)=\frac{P_0\bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\delta A_b(1)\left[\left[(1-r)+rS_p^*(\alpha))\right]\alpha S_p^*(\alpha)+(1+rS_p^*(\alpha)(1-S_p^*(\alpha))\right]\\+\lambda b\left[\left[(1-r)+rS_b^*(A_b(1))\right]\alpha S_b^*(A_b(1))A_b(1)+(1+rS_b^*(\alpha))(1-S_p^*(\alpha))\left((1+rS_p^*(\alpha))A_b(1)+(1-S_b^*(A_b(1))\right)\right]\end{array}\right\}}{[1-\rho]}$$

(ii) "The probability that the server is busy serving priority customers without pre-empting an ordinary customer, is given by",

$$\Pi_1 = \Pi_1(1) = \frac{\delta P_0[1-S_p^*(\alpha)]\left\{\alpha A_b(1) - \bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\left[(1-r)+rS_b^*(A_b(1))\right]S_b^*(A_b(1)\alpha A_b(1) + (1+rS_b^*(A_b(1))(1-S_p^*(\alpha)) \\ \left((1+rS_p^*(\alpha))A_b(1) + (1-S_b^*(A_b(1))\right)\end{array}\right\}\right\}}{[1-\rho]}$$

(iii) "The probability that the server is busy serving priority customers with pre-empting an ordinary customer", is given by,

$$\Pi_2 = \Pi_2(1) = \frac{\delta P_0[1-S_b^*(A_b(1))][1-S_p^*(\alpha)]\left\{\lambda b\alpha A_b(1) - R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\delta\left[(1-r)+rS_p^*(\alpha)\right]S_p^*(\alpha)\alpha A_b(1)\right\} \\ +\delta(1+rS_p^*(\alpha)(1-S_p^*(\alpha))A_b(1)\end{array}\right\}\right\}}{[1-\rho]}$$

(iv) "The probability that the server is busy serving ordinary customers", is given by,

$$\Pi_3 = \Pi_3(1) = \frac{P_0[1-S_b^*(A_b(1))]\left\{\lambda b\alpha - R^*(\lambda+\delta)\left\{\left\{\delta\left[(1-r)+rS_p^*(\alpha)\right]\alpha S_p^*(\alpha)\right\} + \delta(1+rS_p^*(\alpha))(1-S_p^*(\alpha))\right\}\right\}}{[1-\rho]}$$

(v) "The probability that the server is busy serving re-service for priority customers without pre-empting an ordinary customer", is given by,

$$Q_1 = Q_1(1) = \frac{\delta r p_0[1-S_p^*(\alpha)]S_p^*(\alpha)\left\{z\alpha A_b(1) - \bar{R}^*(\lambda+\delta)\left\{\begin{array}{l}\left[(1-r)+rS_b^*(A_b(1))\right]S_b^*(A_b(1)\alpha A_b(1) + (1+rS_b^*(\alpha))(1-S_p^*(\alpha)) \\ \left((1+rS_p^*(\alpha)A_b(1) + (1-S_b^*(A_b(1))\right)\end{array}\right\}\right\}}{(1-\rho)}$$

(vi) "The probability that the server is busy serving re-service for priority customer", is given by,

$$Q_2 = Q_2(1) = \frac{\delta r p_0[1-S_b^*(A_b(1))][1-S_p^*(\alpha)]S_p^*(\alpha)\left\{\lambda b\alpha A_b(1) - R^*(\lambda+\delta)\left\{\begin{array}{l}\left\{\delta\left[(1-r)+rS_p^*(\alpha)\right]S_p^*(\alpha)\alpha A_b(1)\right\} \\ +\delta(1+rS_p^*(\alpha)(1-S_p^*(\alpha))A_b(1)\end{array}\right\}\right\}}{(1-\rho)}$$

(vii) "The probability that the server is busy serving re-service for ordinary customer", is given by,

$$Q_3 = Q_3(1) = \frac{rp_0[1-S_b^*(A_b(1))]S_b^*(A_b(1))\left\{\begin{array}{l}\lambda b\alpha - R^*(\lambda+\delta) \\ \left\{\left\{\delta\left[(1-r)+rS_p^*(\alpha)\right]S_p^*(\alpha)\alpha A_b(1)\right\} + \delta(1+rS_p^*(\alpha)(1-S_p^*(\alpha))A_b(1)\right\}\end{array}\right\}}{(1-\rho)}$$

(viii) "The probability that the server is on vacation", is given by

$$\Omega = \Omega(1) = \frac{v^{(1)} p_0 S_p^*(\alpha)\left[(1-r)+rS_p^*(\alpha)\right]\left\{\begin{array}{l}\alpha A_b(1)\left[1+\lambda b S_b^*(A_b(1))\left[(1-r)+rS_b^*(A_b(1))\right]\right] \\ -\delta\bar{R}^*(\lambda+\delta)\left\{\left[1+rS_b^*(A_b(1))\right](1-S_p^*(\alpha))(1-S_b^*(A_b(1)))\right\}\end{array}\right\}}{\lambda b X_{[1]}(1-\rho)}$$

(ix) "The probability that the server is on repair", is given by,

$$R = R(1) = \frac{\alpha\delta P_0 g^{(1)}\left[1-S_p^*(\alpha)\right]\left[1+rS_p^*(\alpha)\right]\left\{\left\{\begin{array}{l}(1-\lambda b)A_b(1) - R^*(\lambda+\delta)\left\{\alpha[(1-r)+rS_b^*(A_b(1))]S_b^*(A_b(1))\right\} \\ \lambda b\left[1-S_b^*(A_b(1))\right](1+rS_b^*(A_b(1))\end{array}\right\}\right\}}{\lambda b X_{[1]}[1-\rho]}$$

Where

$$A_b^{'}(1)=-\lambda bX_{[1]}[1+\delta\beta_p^{(1)}];\ A_b^{''}(1)=-\lambda bX_{[2]}[1+\delta\beta_p^{(1)}]-(\lambda bX_{[1]})^2\delta\beta_p^{(2)};A_p^{'}=-\lambda bX_{[1]};A_p^{''}=-\lambda bX_{[2]}$$

5.2. Mean system size and orbit size

In steady state, we have

(i) The expected quantity of customers in the waiting space (L_q) is calculated as,

$$L_q=K_o'(1)=\lim_{z\to 1}\frac{d}{dz}K_o(z)=P_0\left[\frac{Nr_q'''(1)Dr_q''(1)-Dr_q'''(1)Nr_q''(1)}{3\left(Dr_q''(1)\right)^2}\right]$$

The expected quantity of customers in the system (L_s) is derived using the formula as given below

$$L_s\ =\ K_s'(1)\ =\ \lim_{z\to 1}\frac{d}{dz}K_s(z)\ =\ P_0\left[\frac{Nr_s'''(1)Dr_q''(1)-Dr_q'''(1)Nr_q''(1)}{3\left(Dr_q''(1)\right)^2}\right]$$

6. Special cases

In this section, we analyze briefly some special cases of our model, which are consistent with the existing literature

***Case* (i):** *"No vacation and No breakdowns"*

In this case, we put "*Pr* [*V* = 0]=1; *α* =r= 0", our model will be modified to a single server retrial queueing system and $K_s(z)$ can be obtained as follows,

$$K_s(z)=\frac{P_0R^*(\lambda+\delta)S_b^*\left(A_b(z)\right)(z-1)\left[\left\{\delta zA_bS_p^*(A_p(z))+\lambda\left[S_b^*(A_b(z)A_b(z)\right]\right\}\right]\left(1+\delta S_p^*\left(A_p(z)\right)\right)}{\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)S_b^*\left(A_b(z)\right)-z\delta\bar{R}^*(\lambda+\delta)\left(S_p^*\left(A_p(z)\right)\right)\right\}}$$

This is same as the result of Gao [11].

***Case* (ii):** *"No priority arrival and No breakdowns"*

In this case, we consider $\delta=\alpha=0$, our model can be reduced to a single server retrial queueing system with balking, optional re-service, single vacation and $K_s(z)$ can be obtained as

$$K_s(z)=\frac{\left\{\begin{array}{l}\bar{R}^*(\lambda+\delta)\left\{\lambda bX(z)\left[\left[(1-r)+rS_b^*(A_b(z))\right]S_b^*(A_b(z)(1+V^*(b(z)))zA_b(z)A_p(z)\right]\right\}\\+z\left(A_b(z)b(z)[1-S_b^*(A_b(z))][1+rS_b^*(A_b(z))]\right)\left\{\lambda bX(z)A_b(z)A_p(z)\right\}\\+[1-V^*(b(z))]\left[(1-r)+rS_p^*(A_p(z))\right]A_b(z)A_p(z)\left\{zA_b(z)A_p(z)\right\}\end{array}\right\}}{\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)S_b^*\left(A_b(z)\right)-z\delta\bar{R}^*(\lambda+\delta)\left(S_p^*\left(A_p(z)\right)\right)\right\}}$$

follows,

***Case* (iii):** *"No priority arrival, No vacation, and No optional re-service"*

In this case, we put *Pr* [*V* = 0] =1; $r=\delta=0$, our model can be reduced to a single server retrial queueing system with breakdowns and repairs and $K_s(z)$ can be obtained as follows,

$$K_s(z)=\frac{\bar{R}^*(\lambda+\delta)\left\{\left[S_b^*(A_b(z)+G^*(b(z))(1-S_b^*(A_b(z))\right]\right\}+\left(A_b(z)[1-S_b^*(A_b(z))]\right)\left\{\lambda bX(z)-1\right\}}{\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)S_b^*\left(A_b(z)\right)-z\delta\bar{R}^*(\lambda+\delta)\left(S_p^*\left(A_p(z)\right)\right)\right\}}$$

7. Numerical examples

We present some numerical examples to see the influence of different parameters of the system where all are exponentially distributed. We assume that the value of the parameters satisfies the condition for steady state. We have used "MATLAB Software" to illustrate the results numerically, where the "exponential distribution is $f(x) = \upsilon e^{-\upsilon x}, x > 0$".

To check the effect of the parameters λ, δ, ξ and γ on the system performance measures, three dimensional graphs are illustrated in Figure 1 and Figure 2. In Figure 1, the surface shows an upward trend as expected for increasing the value of arrival rate (λ) and priority arrival rate (δ) against the mean orbit size (L_q). Figure 2 shows that the probability that server is idle (Lq) increases for increasing the value of repair rate (ξ) and vacation rate (γ).

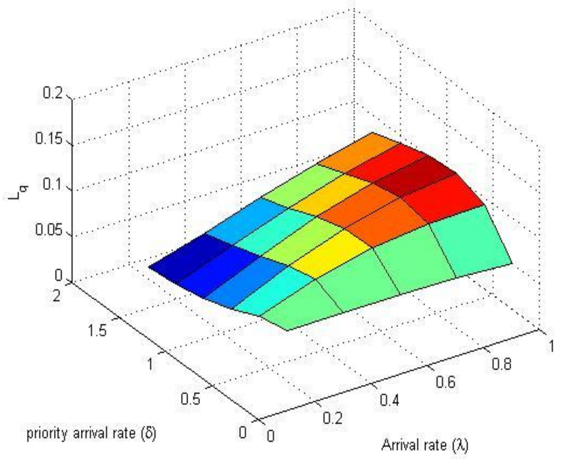

Figure 1. L_q versus λ and δ

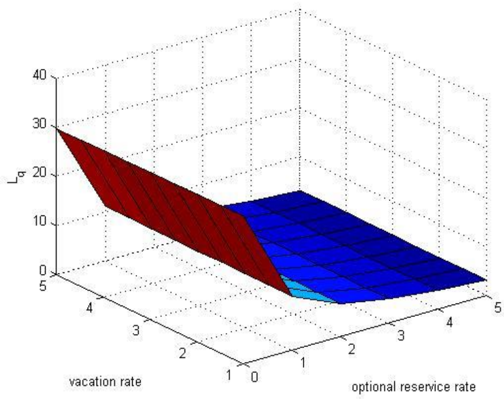

Figure 2. L_q versus ξ and γ

From the above numerical examples, we can find the influence of parameters on the performance measures in the system and know that the results are coincident with the practical situations.

8. Conclusion

In this paper, we have analyzed a single server retrial queueing system with balking, optional re-service, single vacation and service interruption, where the server is subject to breakdown and repair. Using the method of supplementary variable technique, the probability generating functions for the numbers of customers in the system when it is free, busy, on optional re-service, on vacations, and under repairs is found. Some important system performance measures and stochastic decomposition law are discussed. The explicit expressions for the average queue length of orbit and system have been obtained. Finally, the analytical results are validated with the help of numerical illustrations.

The present investigation includes features simultaneously such as,

- Preemptive priority retrial queue
- Balking
- Batch arrival
- Optional re-service

Our suggested model and its results have a potential application in the field of Telephone service, computer processing system and Manufacturing systems. This work can be further extended in many directions by developing the concepts of

- Working vacation

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- Orbital search
- At most J vacations
- Starting Failure

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