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# Application of frequency response curvature method for damage detection in beam and plate like structures

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**Abstract:** There has been a lot of research in recent years to find effective methods for damage detection in structures, usually in following steps: First, localization of damage and second, quantification of the severity of the damage. Present days, there are already a several methods to better understand this domain, some very simple, others quite sophisticated. In this paper, a method is proposed to locate damage based on the estimation of curvatures of the frequency response functions (FRF's). Similarly procedures have been developed in the past, using curvature method of mode shapes. However, the application of the curvatures of the FRF's shows better in damage detection process. Some numerical models like simple beam and plate are presented to estimate the efficiency of the proposed method.

## 1. Introduction

Major work has been completed in the area of detecting damage in structures with the help of modifying the dynamic response of the structure. The mass and stiffness distribution has influence on natural frequencies and mode shapes of a structure, if any consequent changes in mass and stiffness should, theoretically, dominated in the changes in natural frequencies and mode shapes and their sensitivities to damage level. Thus, structural safety and functionally will be significantly improved and a condition-based maintenance procedure can be developed.

In recent times resonant methods based on modal data have been used to identify that damage exists, and its location. Few techniques, such as those treat frame work as discrete systems, and evaluate the modal behavior of the damaged and undamaged structure [1] Experimental modal analysis preferred in [5] referred to here as modal testing. This explains not only detecting but also locating a structural fault [7] and also offered a technique based on the decreases in modal strain energy between two structural degrees of freedom as distinct by the curvature of the measured mode shapes. This technique has been successfully applied to data from a damaged bridge [12]. The absolute changes in mode shape curvature is a reasonable indicator of defect for the FEM beam structure they taken into account [9]. The displacement functions transformed into curvature functions which are more processed to yield a damage index.

A wide-ranging literature evaluation of damage detection methods using vibration signals for structural and mechanical system is explained [4]. Afterward in place of using the displacement mode shapes. Strain or curvature shapes (surface strain in a beam is proportional to curvature) are most effective at identifying the location of damage [8]. A method which requires that the mode shapes before and after damage be known, but the modes do not need to be mass normalized and only a limited to structures that are



characterized by one dimensional curvature [3]. A method for locating structural damage using experimental vibration data measured frequency response functions to obtain displacement as function of frequency [10]. Experimental aspects of dynamic response-based damage detection technique on carbon/epoxy composites are addressed. Smart piezoelectric materials are used as sensors (or) actuators too obtain the curvature mode shapes of the structures. These materials are surface bonded to the beams. An impulse hammer is used as an actuating source as well as four types of damage algorithms are evaluated for several possible damage configurations with two different detection algorithms are explained [6]. A neural network approach that utilizes the vibration and thermal signatures to conclude the condition of a composite sandwich structure is presented. This method can work jointly to balance each other in detecting the state of sandwich composite structure [2]. Structural damage identification methods based on changes in the dynamic characteristics of the structure are evaluated and novel methodology is also developed [11]. Acceleration responses energy based damage detection strategy; numerical analysis on long-span cable stayed bridge is performed by using the proposed method and the traditional mode shape curvature strategy, and at the same time damage quantification analysis and robustness analysis for noise pollution are carried out [14]. An overview of some of the methods of health monitoring for damage detection, applications of different techniques to bridges and critical issues for further research and development [13].

The effect of accelerometer's mass on tool point FRFs and stability diagrams is demonstrated for several cases with different tool-to-accelerometer mass ratios by using laser velocity sensor measurements. The structural modification method can be used to correct the FRFs measured with accelerometers, and thus the resulting stability diagrams [15]. Damage identification method for beam-like structures with a real fatigue crack is subjected with varying excitation forces with the help of frequency response function that derivates defined damage index [16]. 3D point-tracking and image-based frequency response functions (FRFs) are compared to those obtained by collocated accelerometers.<sup>[17]</sup> Steel beam was modeled in ANSYS and harmonic analysis was used to obtain FRFs at different locations of the beam. The results were checked for different slot depths by adding 5–10% noise in the simulated results. [18].

The non-linear harmonic components and the emerging anti-resonances in Higher-Order Frequency Response Functions can provide useful information on about on-line crack monitoring system for small levels of damage. Even the numerical examples for a pipeline beam including breathing crack [19]. The dynamic properties of the joint are investigated using both methods through a finite element (FE) simulation and experimental tests [20]. Modal Flexibility Method and Modal Curvature Method are used to detect crack damage in finite element models of reinforced concrete beams. The crack damage is simulated using discontinuing element model method [21]. FRF based damage detection technique is employed subsequently along with PSO. It is observed that the use of FRF as response of damaged structure has led to better accuracy, since it contains data related to mode shape in addition to natural frequencies [22]. The Interpolation Damage Detection Method is used to give an interpretation of frequency response function (FRF) measurements performed on a reinforced concrete single span bridge subject to increasing levels of concentrated damage. A comparison with the results obtained by the Modal Curvature Method is also presented [23]. A noise-robust damage identification method is presented for localization of structural damage in presence of heavy noise influences. Damage detection experiments of a fixed–fixed steel beam are presented to illustrate the feasibility and effectiveness of the proposed method. The method can overcome the problems of output measurement noise and deliver encouraging results on damage localization [24]. FRF-based statistical method in combination with the non-destructive hammer test measurements has the potential to be employed to identify the characteristic frequencies of damaged conditions in railway tracks in the frequency range of 300–3000Hz [25].

Effective studies in vibration analysis on damaged structures explore the Frequency response function and its curvature behavior it shows even it has ability to identify small damage. This paper focuses on the

study of the FRF curvature energy damage index method for damage identification in beam and plate like structural models. The results explores the proposed method is quite sensitive in defect identification and locations.

## 2. Formulation

### 2.1 Frequency response function (FRF)

The general mathematical representation of a single degree of freedom (SDOF) system is expressed by

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \tag{1}$$

Assuming that the forcing is harmonic of the form  $F(t) = F_0 e^{i\Omega t}$

In more general case the Receptance matrix for MDOF systems with viscous damping, can be expressed as

$$[H(\Omega)] = [[K] - \Omega^2[M] + i\Omega[c]]^{-1} \tag{2}$$

Similarly without viscous damping the above equation can written as

$$[H(\Omega)] = [[K] - \Omega^2[M]]^{-1} \tag{3}$$

The Receptance matrix is systematic for linear systems and therefore

$$H_{rz}(\Omega) = \frac{\vec{X}_r}{F_z} = H_{zr} = \frac{\vec{X}_s}{F_z} \tag{4}$$

Where  $\vec{X}_k$  and  $F_k$ , are respectively the Fourier transform of the displacement and applied force time histories at  $k_{th}$  degree of freedom. The functions  $H_{rz}(\Omega)$  can be arranged in matrix form. This leads to a Receptance matrix defined as

$$[H(\Omega)] = \begin{bmatrix} H_{11} & H_{12} & \dots & H_{1n} \\ H_{21} & H_{22} & \dots & H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1} & H_{n2} & \dots & H_{nn} \end{bmatrix} \tag{5}$$

### 2.2 The frequency response function (FRF) curvature method

This method presented by [9] have found that in place of using a displacement mode shape, strain or curvature shapes (surface strain in a beam is to proportional to curvature) are more effective identifying the location of damage.[8] showed in place of using displacement mode shapes. Strain or curvature shapes (surface strain in a beam is proportional to curvature) are more effective at identifying the location of damage. This paper is extended to plate like structures by using FRF curvature data rather than mode shape data. The FRF curvature for any frequency is defined by

$$H''_{ij}(\Omega) = \frac{-H_{i-2,j}(\Omega) + 16H_{i-1,j}(\Omega) - 30H_{i,j}(\Omega) + 16H_{i+1,j}(\Omega) - H_{i+2,j}(\Omega)}{2h^2}, \tag{6}$$

### 2.3 The FRF curvature energy damage index

In this section a new damage index based on the concept of FRF curvature energy is proposed. The damage indices are based on the variation of the FRF curvature energy at the element of the structure for a element of the structure for a given excitation frequency. In the plate structure the FRF curvature energy can be defined as

$$\eta(\Omega) = \int_0^L [H''(x; \Omega)]^2 dx, \quad (7)$$

Where L is Span of the plate and  $H''(\Omega)$  is FRF curvature for a frequency  $\Omega$ , x and y the horizontal and vertical directions of the plate. In above equation showed only x direction. The structure system is assumed to be divided into n elements ( $j=1, 2, 3, \dots, n$ ). For  $j^{\text{th}}$  element the FRF curvature energy can be written as

$$\eta(\Omega) = \int_{x_k}^{x_{k+1}} [H''(x; \Omega)]^2 dx, \quad (8)$$

Where  $x_k$  and  $x_{k+1}$ : Coordinates of the nodes of the elements j.

For applied force at point j, the absolute difference between the FRF curvature energy of damaged structure at a location i, in a predetermined frequency range is defined as

$$\Delta\eta = \sum_{\Omega} |\eta^*(\Omega) - \eta(\Omega)|, \quad (9)$$

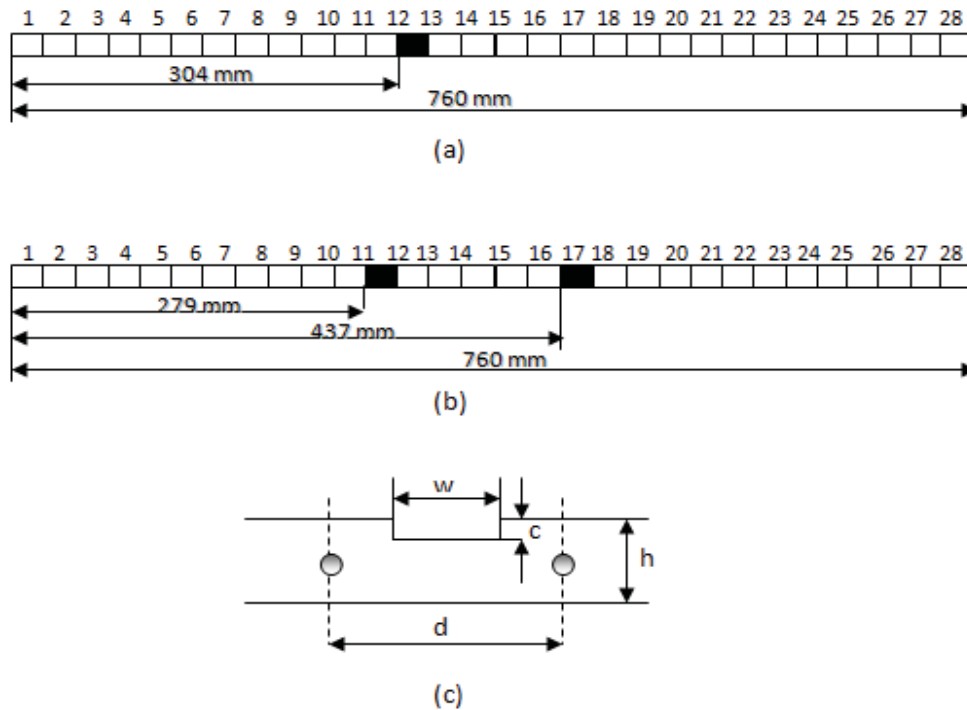
$\eta^*(\Omega)$ : FRF curvature energy of damaged plate.

$\eta(\Omega)$ : FRF curvature energy of undamaged plate.

## 3. Simulation results and discussions

### 3.1. Beam model and damage scenarios

A beam of rectangular cross section of dimension 760 x 30 x 20 mm with Young's Modulus of 200 GPa and density of 7860 Kg/m<sup>3</sup> is used for numerical simulation to evaluate methods based on changes in the modal properties of the structure, modal strain energy and frequency response function. For finite element purpose, the beam is divided into 30 two node one-dimensional elements as shown in (Figure 1a), Damage is defined as c/h ratio, where c gives the depth of damage and h is the height of beam as shown in (Figure 1b).



**Figure 1.** Damage simulation (a) damage at Single location (b) damage at two locations (c) Damage geometry

The parameter  $d$  represents the spatial sampling distance which is the distance between successive measurement to obtain mode shape and  $w$  is the width of cut which is equal to the width of an element. Five simulated damage scenarios are given in (Table 1) where first three cases corresponds to damage at single location and last two corresponds to damage at two location. In the present study damage is simulated by reducing the area moment of inertia of a desired element. Reduction in stiffness is calculated as the ratio of difference between the area moment of inertia of undamaged and damaged element to undamaged area moment of inertia.

**Table 1.** Simulated damage scenarios and reduction in stiffness

Damage scenario	Damaged element Number	$c/h$	Reduction in stiffness (Damage severity)
FD1	13	0.2	0.48
FD2	13	0.15	0.38
FD3	13	0.1	0.27
FD4	12 and 18	0.15	0.38
FD5	12 and 18	0.1	0.27

**3.2. Effect of damage on modal parameters**

For each damage scenario, the dynamic characteristics (frequencies and mode shapes) before and after the damage were numerically evaluated, with programs coded in MATLAB 12. The first five frequencies are listed in (Table 2). Only bending (flexural) modes are considered for analysis.

**Table 2.** First five Natural frequencies for all damage scenarios

Damage Scenario	Natural Frequency (Hz)				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode5
Undamaged	180.88	498.61	977.49	1615.9	2413.9
FD1	174.76	492.82	967.09	1578.5	2410.8
FD2	176.78	494.69	970.44	1590.1	2411.8
FD3	178.43	496.24	973.23	1600	2412.7
FD4	173.38	487.68	969.07	1559.5	2402.6
FD5	176.33	492.08	972.34	1581	2407

According to the results in the table the simulated damage scenarios cause the first modal frequency to shift from 1.3% to 4.1 % of the undamaged frequency. Also, it can be noted that the change in the frequencies for the cases FD2 (single damage) and FD5 (double damage) is practically the same. The decrease in the natural frequencies of the higher modes range from 0.47 to 2.19 % for the second mode, 0.43 to 1.06 % for mode 3, 0.98 to 3.49 % for mode 4, and 0.04 to 0.46 % for the last mode considered. Thus changes in natural frequencies cannot provide information about the location of structural damage. This conclusion is in agreement with the observations in the studies carried out in [9].

In this section a new damage index based on the concept of Receptance energy, is proposed which is similar to modal strain energy. The index was conceived to predict the damage location and to estimate the severity of the damage in a structure directly from the measured FRF (Receptance or Accelerance). In the formulation presented here the Receptance function will be used.

The damage indices are based on the variation of the Receptance energy at the elements of the structure for a given excitation frequency. In a one-span beam, the Receptance energy can be defined as:

$$\zeta_{(\Omega)} = \int_0^L [H''(x; \Omega)]^2 dx \tag{10}$$

Where, L: span of the beam.

$H''(x; \Omega)$  Receptance-curvature for a frequency  $\Omega$ .

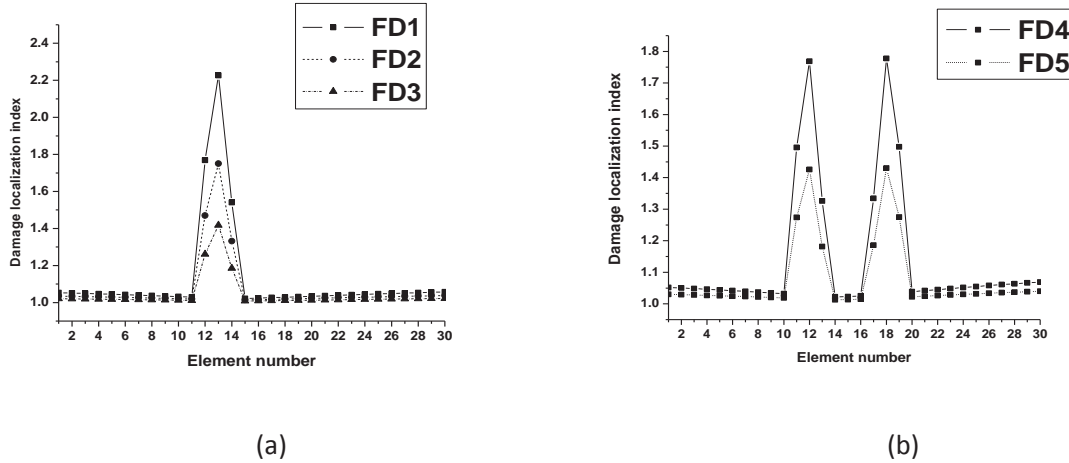
The structural system is assumed to be divided into  $n_e$  elements ( $j=1,2,\dots,n_e$ ). For the  $j^{\text{th}}$  element the Receptance energy can be written as

$$\zeta_{(\Omega)} = \int_{x_k}^{x_{k+1}} [H''(x; \Omega)]^2 dx \tag{11}$$

$x_k$  and  $x_{k+1}$ : coordinates of the nodes of element  $j$ .

For a given frequency range, the damage localization index for the  $j^{\text{th}}$  location and for an external force applied at point  $p$  is defined as follows

$$\Psi_{j,p} = \sum_{\Omega} |\tilde{\zeta}_j - \zeta_j| \tag{12}$$

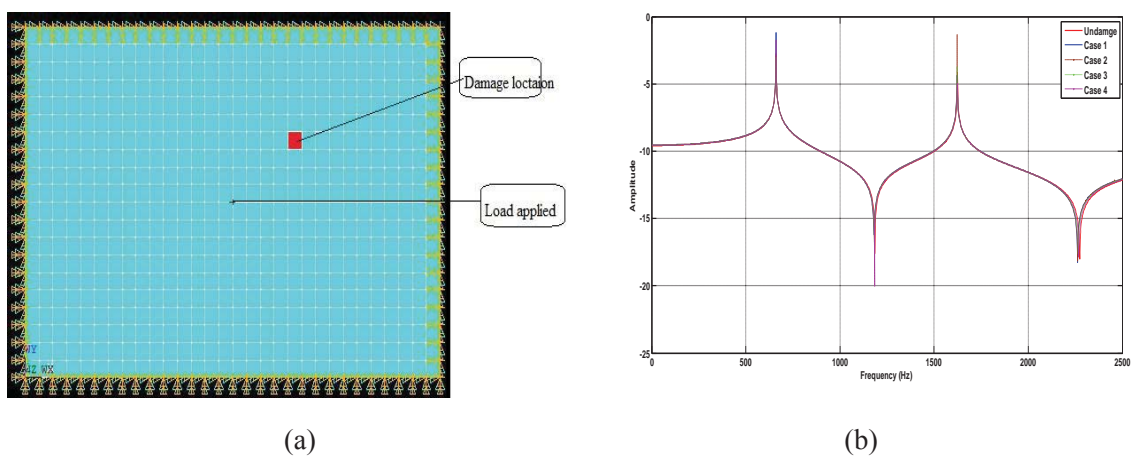


**Figure 2.** Variation of Damage localization index Vs element number for (a) single and (b) double location damage case

(Figure 2), is the plot showing variation of damage localization index with element number and the damage is indicated by sudden peak at the damaged element. In contrast to method based on natural frequency and mode shape this method locates and quantifies the damages correctly in case of both single and two damages as seen in (Figures 2a and 2b).

**3.3 Plate model and damage scenarios**

The dimension of plate under consideration is 300 x 200 x 4mm with fixed-fixed boundary conditions shown in figure 1. The material properties used in modeling the plate are  $E = 68.9 \text{ Gpa}$ ,  $\mu = 0.33$  and  $\rho = 2710 \text{ kg/m}^3$  which refer to Young Modulus, Poisson ratio and Mass Density respectively. The length and width of plate is equally divided into 15 elements so the simulated plate model has total 225 elements as shown in (Figure 3a). In the present study damage is simulated by reducing the thickness of one element. The different damage cases with percentage of thickness reduction are highlighted in (Table 3).



**Figure 3.** (a) Finite element model of a fixed-fixed plate with damage location. (b). Frequency Response function data for undamaged and damaged cases



The frequency response function data of the plate with damage of different cases are obtained by using MATLAB 12. The initial five frequencies resulting from free vibration analysis for cases are tabulated in Table 4. Simultaneously harmonic excitation analysis is carried out with frequency range 0-2500 Hz with applied load on structure 150N and the Frequency Response Function (FRF's) data is collected from same point, where load applied is shown in (Figure 3b) for undamaged and all damaged cases.

**Table 3.** Represents different damage cases with element number

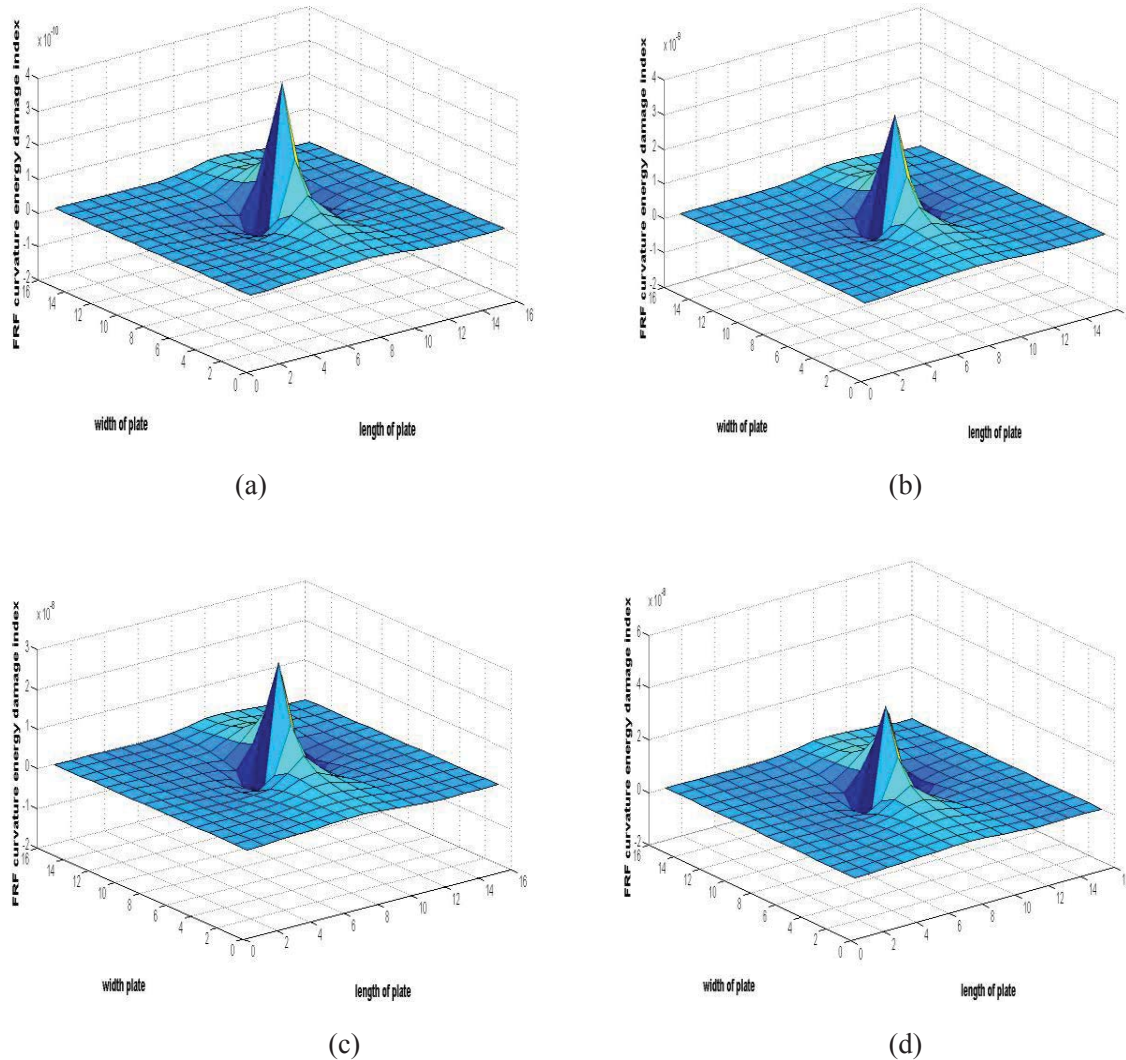
Damage cases	% of thickness reduction	Damaged element number
1	80	129
2	60	129
3	40	129
4	20	129
5	10	129
6	5	129
7	1	129

**Table 4.** Comparison of first five natural frequencies (Hz) for undamaged and damaged cases

Damage cases	Frequencies (Hz) for the mode number				
	1	2	3	4	5
Undamaged	660.27	1018.8	1615.9	1624.4	1947.2
Case 1	660.06	1018.6	1615.7	1622.5	1945.0
Case 2	659.67	1018.6	1615.7	1622.0	1945.1
Case 3	659.63	1018.6	1615.7	1622.2	1945.5
Case 4	659.90	1018.7	1615.8	1623.2	1946.3
Case 5	660.09	1018.7	1615.8	1623.8	1946.7
Case 6	660.18	1018.8	1615.9	1624.1	1947.0
Case 7	660.25	1018.8	1615.9	1624.3	1947.2

From the (Table 4), It is clearly shows that there is no considerable shift in natural frequencies due to damage and thus damage localization is very difficult. Also observed that third and fourth modes are almost same frequencies because the structure is symmetric and it shows the coupling modes.

From (Figure 3) shows the FRF curvature energy damage index values for different damage cases in the first fundamental frequency range of 0 to 700 Hz. (Figure 4a) highlights the peak with bigger amplitude at damage location for Case 1 where the more severity of thickness reduction as 80% of one element.



**Figure 4.** FRF curvature energy damage index for a frequency range of 0-700 Hz (a) Case 1 (b) Case 2  
(c) Case 5 (d) Case 7

Similarly (Figures 4b, 4c and 4d) represents the damage identification with decrease in peak amplitude for damage cases 2, 5 and 7. From this analysis observed that even smallest damage severity as one percent reduction in thickness of an element also detected by this method. From this proved that FRF curvature energy method having well reliable in detecting and estimating the damage severity.

**5. Conclusions**

The methods based on FRF curvature energy damage index method were able to identify damage correctly using the difference data between the undamaged and damaged beam and plate models. This method requires numerical differentiation and integration of the input spatial data. The damage index is a function of the frequency range and variation of damage index values versus band width seems to provide further

information in choice of optimum frequency range response analysis. The results proved that this method is successful in damage identification with quantification. It can be concluded that the method is promising technique and further experimentally studies are to be justified in order to better assess the possible method applications.

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