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Commutativity of nonassociative rings with identities in the center

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Abstract. Let R be a nonassociative ring with center U. In this paper, it is shown that nonassociative ring R of char. ≠ 2 with unity is commutative if it satisfies any one of the following identities:
(i) $(xy)x + x(xy) + y \in U$, (ii) $(xy)^2 - x^2 y - xy^2 - xy \in U$, (iii) $(xy)^2 - x^2 y - xy^2 - yx \in U$
(iv) $(xy)^2 - xy^2 \in U$, (v) $(xy)^2 - y^2 x \in U$, (vi) $(x^2y^2)z^2 - (xy)z \in U$,
(vii) $(x^2y^2)z^2-(xy)z \in U$ for all x, y, and for fixed z in R.

1. Introduction

Giriet.al. [2], have proved if R is a nonassociative ring with char. ≠ 2. with unity satisfying the condition $(xy)^2 - xy \in U$ for all x, y in R. This paper contains the generalization of nonassociative ring R with char. ≠ 2 with unity satisfying $(xy)x + x(xy) + y \in U$, $(xy)^2 - y^2 x \in U$ and $(xy)^2 - xy^2 \in U$ then R is commutative and also, we also proved the commutativity of nonassociative ring R with char. ≠ 2 with unity satisfying $(x^2y^2)z^2 - (xy)z \in U$, $(x^2y^2) z^2 - (xy)z \in U$ for all x, y, and for fixed z in R. Giri [3 and 4] proved if R is a 2-torsion free nonassociative semi-simple ring with unity satisfying $(xy)^2 - x^2y - xy^2 - xy$ in center for all x, y in R, then R is commutative. Suvarna [7] also proved the commutativity of nonassociative ring R with char. ≠ 2 with unity satisfying $(xy)^2 - x^2 y - xy^2 - y^2 x^2$ for all x, y in R. This paper includes the commutative of nonassociative ring R with char. ≠ 2 with unity satisfying $(xy)^2 - x^2 y - xy^2 - xy \in U$ and $(xy)^2 - x^2 y - xy^2 - yx \in U$.

Throughout this paper, R represents nonassociative ring with char. ≠ 2. The center of R is defined as $U = \{u \in R \,/\, [u, R] = 0\}$. It is also called as a commutative center. A ring R is of characteristic ≠ n if $nx = 0$ implies $x = 0$ for all x in R and n a natural number.

2. Main results

2.1 Theorem1:

Let R be a nonassociative ring of char. ≠ 2 with unity satisfying $(xy)x + x(xy) + y \in U$ for all x, y in R. Then R is commutative.

Proof:

By hypothesis $(xy)x + x(xy) + y \in U$ (1)

Now by replacing $y = y + 1$ in (1), we get $2x^2 + 1 \in U$

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Since R is of char. $\neq 2$, we have $x^2 \in U$ (2)
By taking $x = x + 1$ in (2) and using (2), we have $2x \in U$
Since R is of char. $\neq 2$, we have $x \in U$
Therefore $xy = yx$ for all x in R.
Hence R is commutative.

2.2 Theorem 2:
Let R be a nonassociative ring of char. $\neq 2$ with unity satisfying $(xy)^2 - x^2 y - xy^2 - xy \in U$ for all x, y in R. Then R is commutative.
Proof:
By hypothesis $(xy)^2 - x^2 y - xy^2 - xy \in U$ (3)
Now by replacing $x = x + 1$ in (3) and using (3), we get
$(xy)y + y(xy) - 2xy - 2y \in U$ (4)
Put $x = x + 1$ in (4) and using (4), we get
$2y^2 - 2y \in U$
Since R is of char. $\neq 2$, $2y^2 - 2y \in U$ (5)
Now by replacing $y = y + 1$ in (5) and using R is of char. $\neq 2$, we get
$y \in U$
Therefore $xy = yx$ for all x in R.
Hence R is commutative.

2.3 *Theorem3*:
Let R be a nonassociative ring of char. $\neq 2$ with unity satisfying $(xy)^2 - x^2 y - xy^2 - yx \in U$ for all x, y in R. Then R is commutative.
Proof:
By hypothesis $(xy)^2 - x^2 y - xy^2 - yx \in U$ (6)
Now by replacing $x = x + 1$ in (6) and using (6), we get
$(xy)y + y(xy) - 2xy - 2y \in U$ (7)
Put $x = x + 1$ in (7) and using (7), we get
$2y^2 - 2y \in U$
Since R is of char. $\neq 2$, $2y^2 - 2y \in U$ (8)
Now by replacing $y = y + 1$ in (5) and using R is of char. $\neq 2$, we get
$y \in U$
Therefore $xy = yx$ for all x in R.
Hence R is commutative.

2.4 *Theorem4*:
Let R be a nonassociative ring of char. $\neq 2$ with unity satisfying $(xy)^2 - xy^2 \in U$ for all x, y in R. Then R is commutative.
Proof:
By hypothesis $(xy)^2 - xy^2 \in U$ (9)
Now by replacing $x = x + 1$ in (9) and using (9), we get
$(xy)y + y(xy) \in U$ (10)
Put $x = x + 1$ in (10) and using (10), we get
$2y^2 \in U$
Since R is of char. $\neq 2$, we get $y^2 \in U$ (11)
$y = y + 1$in (11) and using R is of char. $\neq 2$, we get
Now by replacing $y = y + 1$ in (11) and using R is of char. $\neq 2$, we get
$y \in U$

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Therefore $xy = yx$ for all x in R.
Hence R is commutative.

2.5 *Theorem5*:

Let R be a nonassociative ring of char. $\neq 2$ with unity satisfying $(xy)^2 - y^2 x \in U$ for all x, y in R. Then R is commutative.
Proof:
By hypothesis $(xy)^2 - y^2 x \in U$ (12)
Now by replacing $x = x + 1$ in (12) and using the Theorem (4)
Therefore $xy = yx$ for all x in R.
Hence R is commutative.

2.6 *Theorem 6*:

Let R be a nonassociative ring of char. $\neq 2$ with unity satisfying $(xy)^2 z^2 - (xy)z \in U$ for all x, y, z in R. Then R is commutative.
Proof:
By hypothesis $(xy)^2 z^2 - (xy)z \in U$ (13)
Now by replacing $z = z + 1$ in (13), we get
$(xy)^2 z^2 + 2(xy)^2 z + (xy)^2 - (xy)z - xy \in U.$ (14)
Using (13) in (14), we have
$2(xy)^2 z + (xy)^2 - xy \in U.$ (15)
Again, by replacing $z = z + 1$ in (15) and using (15), we obtain
$2(xy)^2 \in U.$ (16)
Since R is of char. $\neq 2$, we have
$(xy)^2 \in U.$ (17)
Now by replacing $x = x + 1$ in (17), we have
$(xy+y)^2 \in U$
or $(xy)^2 + (xy)y + y(xy) + y^2 \in U.$ (18)
Using (17) in (18), we obtain
$(xy)y + y(xy) + y^2 \in U.$ (19)
Again, by replacing $x = x + 1$in (19) and using (19), we get $2y^2 \in U$.
Since R is of char. $\neq 2$, we get
$y^2 \in U$ (20)
Now by taking $y = y + 1$ in (20) and using (20), we get $2y \in U$.
Since R is of char. $\neq 2$, we have,$y \in U$.
Therefore $xy = yx$ for all x in R.
Hence R is commutative.

2.7 *Theorem 7*:

Let R be a nonassociative ring of char. $\neq 2$ with unity satisfying $(x^2 y^2)\, z^2 - (xy)z \in U$ for all x, y, z in R. Then R is commutative.
Proof:
By hypothesis $(x^2 y^2)\, z^2 - (xy)z \in U$. (21)
Now by replacing z with $z + 1$ in (21) and using (21), we obtain
$2(x^2 y^2)z + x^2 y^2 - xy \in U.$ (22)
Again, replacing $z = z + 1$ in (22) and using (22), we get $2(x^2 y^2) \in U$.
Since R is of char. $\neq 2$, we obtain
$x^2 y^2 \in U.$ (23)

By taking $x = x + 1$ in (23) and using (23), we have
$2xy^2 + y^2 \in U$. (24)
Now again by replacing x with $x + 1$ in (24) and using (24), we get
$2y^2 \in U$. Since R is of char. $\neq 2$, we obtain
$y^2 \in U$. (25)
By replacing $y = y + 1$ in (46) and using (46), we get $2y \in U$.
Since R is of char. $\neq 2$, we have $y \in U$ or $xy = yx$ for all x in R.
Hence R is commutative.

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