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Comparative study of digital holography reconstruction methods

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Abstract

Digital holograms which are recorded in the CCD sensors is subjected to numerical reconstruction for calculation of amplitude and phase using numerical reconstruction algorithms. This reconstruction process offers high flexibility and provides new possibilities of exploitation in domains like pattern recognition, 3D microscopy, encryption etc. In this paper,a comparative analysis of different holography reconstruction methods is presented .

Keywords: Holography, interferometry, digital holography, CCD recording, numerical reconstruction methods, Fresnel transform, convolution.

1. Introduction

HOLOGRAPHY is a two step process of recording the optical wave fields on a photographic plate followed by its reconstruction. Conventional holography is time consuming and requires wet chemical processing. In interferometric analysis a comparison is usually performed between the wave fields before and after a variation has occurred and this result is applied for various analysis like surface contour measurement, stress calculation and refractive index determination .The recording of interference pattern on a CCD array and its subsequent storing is analogous to optical imaging system requires knowledge of Fourier optics. This paper mainly emphasizes on the different methods employed for numerical evaluation of digitally recorded holograms. Various numerical computation algorithms and their typical characteristics are analyzed in a comparative manner.

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A. Holography Principle and Methods

Holography is a technique which allows three dimensional objects to be reconstructed. It uses a Laser and involves physical phenomenon Interference, Diffraction, light intensity recording and suitable illumination of recording. Depending on the experimental set ups there are different types of holography. The inline holography in which the reference beam is made collinear to the object beam. Digital Phase shifting holography is an example of inline geometry where multiple interferograms are recorded each depicting the phase relationship between the reflected object wave and collinear reference wave. In off axis holography a small angle is included between the object wave and the reference wave. In this case from a single recorded hologram reconstruction of the original object is possible. The concept of using CCD cameras as recording medium is exploited in Fresnel holography and Fraunhofer holography and shown in Fig 1. and Fig 2. respectively.

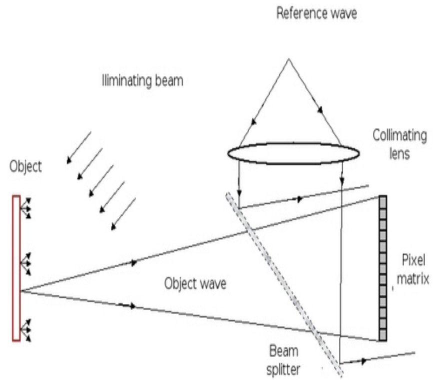

Fig 1: Fresnel Holography

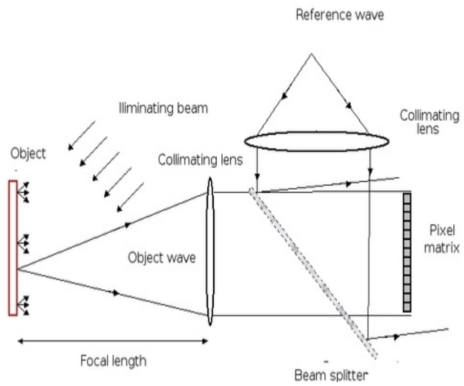

Fig 2: Fraunhofer Holography

In Fresnel holography the near field regime is utilized, whereas in Fraunhofer holography the far field regime is considered. Thus Fraunhofer holography is a special case of Fresnel holography at infinity distance. The diagrammatic representation of geometry of Fresnel holography is shown below. It has three planes mainly the object plane, the hologram plane and the image plane. A plane rough surface(x,y) is chosen that reflects the wave field b(x,y). The hologram plane which corresponds to a CCD target is in the (Ɛ,η) plane at a distance d from the object surface. The image plane (x',y') is located at a distance d' from the hologram plane. The microinterference at each point in the hologram can be determined by an angle θ between the reference and the object wave[1]. This can be expressed as

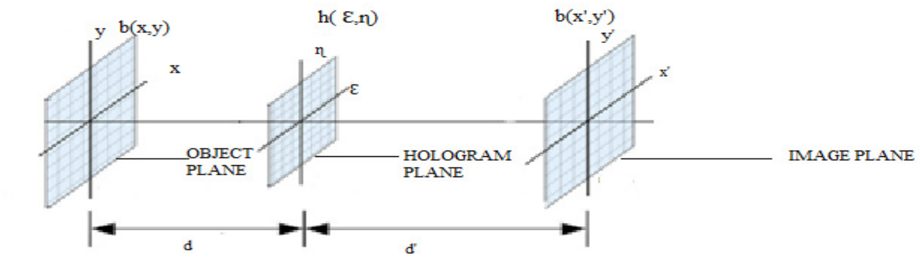

Fig. 3 Diagrammatic representation of different planes

$$\delta = \frac{\lambda}{2\sin\frac{\theta}{2}}$$

Where δ is the fringe spacing and λ is the wavelength used. For proper reconstruction sampling theorem should be obeyed which states that for a CCD array having pixel spacing $\Delta\mathcal{E}$, at least two pixel should sample the fringe period.

$$\theta < \frac{\lambda}{2\Delta\mathcal{E}}$$

Here θ needs to be very small. This is achieved by using a small object, by keeping the object at a large distance from the CCD or by optical demagnification using negative lens. In order to calculate the diffracted field by Rayleigh-Sommerfeld diffraction formula is employed, the mathematical expression of which is

$$b'(x',y') = \frac{1}{i\lambda}\iint h(\mathcal{E},\eta)\, r(\mathcal{E},\eta)\, \exp\frac{\{ik\rho\}}{\rho}\cos\theta d\mathcal{E}\, d\eta \quad (1)$$

With the following assumption

$$\rho = \sqrt{d'^2 + (\mathcal{E} - x')^2 + (\eta - y')^2} \quad (2)$$

Here h(Ɛ,η) denotes the recorded hologram, r(Ɛ,η) is the reference wave and k is the wave number. The obliquity factor cosθ is set to 1 because of small angles between hologram normal and rays diffracted from hologram to the image points.

1.1. Fresnel-Approximation Approach

In Fresnel-approximation ρ in the denominator of Rayleigh-Sommerfeld diffraction formula is replaced by d' provided d' is very large compared to (Ɛ-x') and (η-y'). But the rapidly varying phase induced by ρ in the numerator leads to errors. So expanding (2) binomially and retaining only the first two terms ρ is approximated as [1]

$$\rho \approx d' \left[1 + \frac{1}{2}\left(\frac{\mathcal{E}-x'}{d'}\right)^2 + \frac{1}{2}\left(\frac{\eta-y'}{d'}\right)^2\right]$$

So putting this value in (1) we get the diffraction integral as

$$b'(x',y') = \frac{1}{i\lambda d'}\iint h(\mathcal{E},\eta)\, r(\mathcal{E},\eta) \exp\{ikd'[1 + \frac{1}{2}(\frac{\mathcal{E}-x'}{d'})^2 + \frac{1}{2}(\frac{\eta-y'}{d'})^2]\}\, d\mathcal{E}\, d\eta \quad (3)$$

$$= \frac{\exp\{ikd'\}}{i\lambda d'}\iint h(\mathcal{E},\eta)\, r(\mathcal{E},\eta) \exp\{\frac{ik}{2d'}[(\mathcal{E}-x')^2 + (\eta-y')^2]\}\, d\mathcal{E}\, d\eta$$

$$= \frac{\exp\{ikd'\}\exp\{i\Pi d'\lambda(\upsilon^2+\mu^2)\}}{i\lambda d'} \iint h(\mathcal{E},\eta)\, r(\mathcal{E},\eta) \exp\left\{\frac{i\Pi}{d'\lambda}\left(\mathcal{E}^2+\eta^2\right)\right\} \exp\{-2i\Pi(\mathcal{E}\upsilon+\eta\mu)\}\, d\mathcal{E}\, d\eta$$

Here υ and μ can be represented by

$$\upsilon = \frac{x'}{d\lambda}, \quad \mu = \frac{y'}{d\lambda}$$

The discrete form of (3) for numerical analysis is shown below where the spatially constant factors are omitted.

$$b'(n, m) = \exp\left\{-\frac{i\Pi d'\lambda}{N^2}\left(\frac{n^2}{\Delta\mathcal{E}^2}+\frac{m^2}{\Delta\eta^2}\right)\right\} \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} h(k\Delta\mathcal{E}, l\Delta\eta)\, r(k\Delta\mathcal{E}, l\Delta\eta) \exp\left\{\frac{i\Pi}{d'\lambda}\left(k^2\Delta\mathcal{E}^2+l^2\Delta\eta^2\right)\right\} \times \exp\left\{2i\Pi\left(\frac{kn}{N}+\frac{lm}{N}\right)\right\}$$

In abbreviated form we can write the above equation as

$$b' = z \cdot \mathcal{F}^{-1}\{h \cdot r \cdot w\}$$

where $\mathcal{F}^{-1}$ is the inverse Fourier transform ,h(k$\Delta\mathcal{E}$, lΔn) is the digitized hologram, r(k$\Delta\mathcal{E}$, l$\Delta\eta$) denotes the reference wave, w(k$\Delta\mathcal{E}$, l$\Delta\eta$) is the two-dimensional chirp function which is expressed as

$$w(k\Delta\mathcal{E}, l\Delta\eta) = \exp\left\{\frac{i\Pi}{d'\lambda}\left(k^2\Delta\mathcal{E}^2+l^2\Delta\eta^2\right)\right\}$$

And the phase factor z (nΔx', mΔy') can be written as

$$z(n\Delta x', m\Delta y') = \exp\left\{-\frac{i\Pi d'\lambda}{N^2}\left(\frac{n^2}{\Delta\mathcal{E}^2}+\frac{m^2}{\Delta\eta^2}\right)\right\} = \exp\left\{-\frac{i\Pi}{d\lambda'}\left(n^2\Delta x'^2+m^2\Delta y'^2\right)\right\}$$

The pixel size in the reconstructed image is given by

$$\Delta x' = \frac{d'\lambda}{N\Delta\mathcal{E}}, \quad \Delta y' = \frac{d'\lambda}{N\Delta\eta}$$

A typical example of numerical reconstruction using Fresnel-approximation is illustrated below. The hologram used has a dimension of N $\times$ N = 576 $\times$ 768 having pixel size $\Delta\mathcal{E} = \Delta\eta = 6.7\mu m$. A plane reference wave is used in this case. The digital hologram is post processed to avoid DC terms by mean value subtraction.

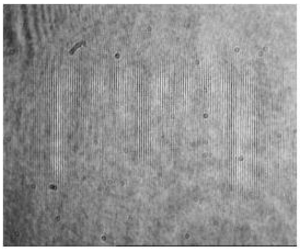
Fig 4a. Original Hologram


Fig 4b. Reconstructed hologram

1.2. Diffraction Integral as Convolution

Rewriting the Rayleigh-Sommerfeld which is a superposition integral by including an impulse response g(x',y',$\mathcal{E}$,η) equation (1) is expressed as [1]

$$b'(x', y') = \iint h(\mathcal{E},\eta)\, r(\mathcal{E},\eta)\, g(x', y', \mathcal{E}, \eta)\, d\mathcal{E}\, d\eta$$

Where

$$g(x', y', \mathcal{E}, \eta) = \frac{1}{i\lambda}\frac{\exp\{ik\rho\}}{\rho}\cos\theta$$

$$= \frac{d'}{i\lambda}\frac{\exp\left\{ik\sqrt{d'^2+(\mathcal{E}-x')^2+(\eta-y')^2}\right\}}{d'^2+(\mathcal{E}-x')^2+(\eta-y')^2}$$

Here, $\cos\theta = \frac{d'}{\rho}$

The convolution of h .r with g can be expressed as the product of the individual Fourier transforms of Ƒ{h.r} and Ƒ{g}.So b'(x',y') can be obtained by taking the inverse Fourier transform of this product. Thus numerical reconstruction by implementing efficient FFT algorithm is possible. The function g(x',y',Ɛ,ɳ) is the impulse response of free space propagation, the numerical realization of which is shown mathematically as

$$\mathrm{g(k,l)} = \frac{1}{i\lambda}\frac{\exp\{\frac{2i\Pi}{\lambda}\sqrt{d'^2 + (k-\frac{N}{2})^2 \Delta \mathcal{E}^2 + \left(l-\frac{N}{2}\right)^2 \Delta n^2\}}}{\sqrt{d'^2 + (k-\frac{N}{2})^2 \Delta \mathcal{E}^2 + \left(l-\frac{N}{2}\right)^2 \Delta n^2\}}}$$

The Fourier transform of the function g(k,l) is denoted by G(n,m) which may be defined as

$$\mathrm{G(n,m)} = \exp\{\frac{2\Pi i d'}{\lambda}\sqrt{1 - \lambda^2 \frac{\left(n + \frac{N^2 \Delta \varepsilon^2}{2d'\lambda}\right)^2}{N^2 \Delta \varepsilon^2} - \frac{\lambda^2 \,(m + \frac{N^2 \Delta \eta^2}{2d'\lambda})^2}{N^2 \Delta \eta^2}}$$

This gives numerical freedom to save one Fourier transform. The reconstructed image pixel size is given by $\Delta x' = \Delta \varepsilon$ and $\Delta y' = \Delta \eta$. The resulting image therefore has an area of $N \Delta \varepsilon \, x \, N \Delta \eta$ .The larger image size implies that the reconstructed hologram contains more information by using a single convolution integral. An example illustrating numerical reconstruction of a hologram using the convolution approach is shown below. The original hologram used has a dimension of N x N = 576 x 768 which has been zero padded to 1024 x 1024.This is shown in the figure below

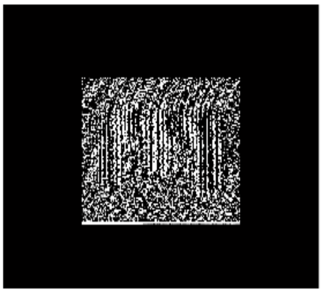

Fig 5a Zero padded input image

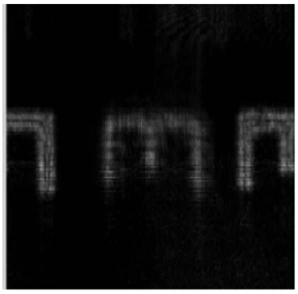

Fig 5b. Reconstructed image

1.3. In-Line Holograms Reconstruction using Two Intensity Measurements

In in-line holography the set up configuration is simple. Here the object, the recording surface and a light source is kept in a collinear fashion. A coherent light source like He-Ne Laser is made incident on a diverging lens. The resultant parallel beam of light illuminates an object $\boldsymbol{a(x,y)}$ and the hologram is recorded on the CCD surface $\boldsymbol{A(x,y)}$ located at a distance $\boldsymbol{d}$ from the object. The CCD is connected to a computer where numerical analysis and reconstruction is performed. A typical configuration is shown below [3]

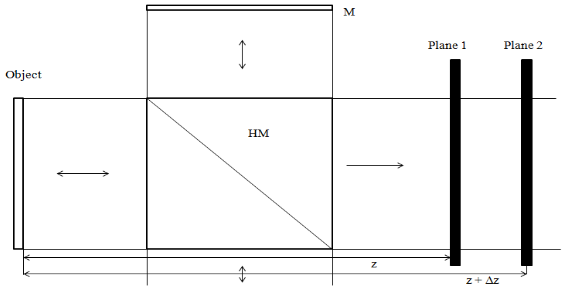

Fig 6. Two intensity measurement experimental setup

A half mirror divides the laser beam into two parts while one illuminates the object; the other is used as a reference beam. Mirror (M) reflects the reference beam and the half mirror (HM) combines the object beam and the reference beam. The interference pattern is recorded by CCD at plane 1 which is located at a distance z from the object. The mathematical semantics [3] are expressed as

$$A(x,y,z) = A_0 \exp\left(-i\frac{2\Pi d}{\lambda}\right)[1 + u(x,y,z)] \quad (4)$$

Here A_0 is the amplitude of the reference wave and $u(x,y,z)$ denotes the object wave which can be written as

$$u(x,y,z) = \iint U(f_x, f_y, 0)\, H(f_x, f_y, z) \text{ x } \exp[i2\Pi(f_x x + f_y y)]df_x\, d f_y$$

where,

$$U(f_x, f_y, 0) = \iint u(x,y,0) \exp[-i2\Pi(f_x x + f_y y)\text{dxdy}$$

When the object wave is weak compared to the reference wave (4) can be reduced as

$$A(x,y,z) = A_0 \exp\left(-i\frac{2\Pi d}{\lambda}\right)\exp[u(x,y,z)]$$

The intensity on plane 1 can be represented by the following equation

$$\text{I(x,y,z)} = |\,(A(x,y,z)|^2$$

$$= A_0 \exp[u(x,y,z) + u^*(x,y,z)]$$

Taking logarithm of the above equation

$$\text{l(x,y,z)} = \log[(\text{I(x,y,z)}/A_0^2] = \text{u(x,y,z)} + \text{u*(x,y,z)}$$

The Fourier transform of the above equation yields

$$L(x,y,z) = \iint l(x,y,z) \exp[-i2\Pi(f_x x + f_y y)\text{dxdy}$$

$$= U(f_x, f_y, 0) H(f_x, f_y, z) + U(-f_x, -f_y, 0)\, H * (f_x, f_y, z)$$

$L(x,y,z)$ contains both the object function along with its conjugate. For reconstruction of the object function and to eliminate the conjugate images another hologram is recorded at a distance $z + \Delta z$ from the object which leads to the following function

$$\Delta L(x,y,z) = L(x,y,z) - L(x,y,z+\Delta z)\, H(f_x,f_y,z)$$

$$= U(f_x,f_y,0)H(f_x,f_y,z) \text{ x } [1- H(f_x,f_y,2\Delta z)$$

Thus this distribution contains the Fourier transform of the object function $U((f_x,f_y,0))$ along with transfer functions $H(f_x,f_y,z)$ and $H(f_x,f_y,2\Delta z)$. The object wave is reconstructed by taking an inverse Fourier transform .This method requires two interferograms to be recorded at distances z and $z+\Delta z$ simultaneously. A computer simulation is done using the above method with a standard test image (Lena image) having size of 512 x 512. It is zero padded to a dimension of 1024 x 1024 with the image located in the middle and the background having intensity 0. The pixel size is 7.6mm x 7.6mm. Two holograms are recorded at distances $z = 80mm$ and $z+\Delta z = 80 + 0.1\ mm$. The free space propagation transfer function is denoted as

$$H(f_x,f_y,z) = \exp[\ -i\frac{2\Pi z}{\lambda}(1-\lambda^2 f_x^2 - \lambda^2 f_y^2)^{1/2}$$

The following are the standard images

Fig 7a. Original input image

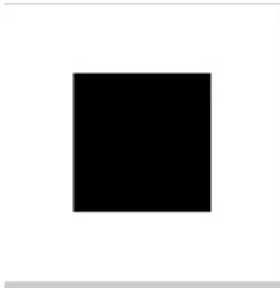

Fig 7b. Intensity and phase of image

Fig 7c. Reconstructed intensity

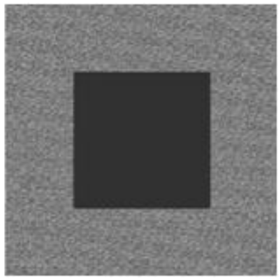

Fig 7d. Reconstructed phase

1.4. Phase Shifting Holography

In phase shifting holography multiple interferograms are captured each of which indicates phase relationship between light reflected from an object surface and a reflected wave which is collinear to the object wave. From this various recorded interferograms, the hologram of the object can be obtained by numerical computation. The numerical calculation is done by using Fresnel Kirchoff Integral [2]. The wave from the object and the reference wave interfere at the surface of a CCD. The refererence wave can be guided with the help of a mirror mounted on a piezoelectric transducer (PZT). Change in phase of the reference wave can be induced by the PZT. Thus multiple interferograms having mutual phase shifts are recorded. The object phase ϕ can be then calculated from these phase shifted interferograms. In the recording plane (x_0,y_0,z_0) the complex amplitude of the object can be written as [4]

$$U(x, y) = Aexp\ (i\phi)$$

$$= \frac{A_0}{,z_0} \exp\ (i\phi_0 + ikz_0 + ik\frac{(x-x_0)^2+(y-y_0)^2}{2z}$$

Where A_0 exp($i\phi_0$) is the complex amplitude of the object and k is the wave number. The reference wave can be represented by $U_R\phi_R = A_R$ exp ($i\phi_R$). These two waves superimpose and the resultant intensity which is to be recorded is given by

$$I(x, y, \phi_R) = |\ (U_R\ \phi_R + U(x, y)|^2$$

$$= A_R^2 + A^2 + 2A_RA\ cos(\phi_R - \phi)$$

The object phase can be calculated by the following equation

$$\phi(x\ ,\ y) = \tan^{-1}\frac{I\left(x,y;\frac{3\Pi}{2}\right) - I(x,y;\frac{\Pi}{2})}{I(x,y;0) - I(x,y,\Pi)}$$

In the illustration below, we show position phase shifting digital holography and its results (Figures 8a-8h). A standard test image (Lena image) of size 512 x 512 is simulated with pixel size of 7.6mm x 7.6mm . The image is zero padded to a size of 1024 x 1024. Four interferograms are recorded. The third interferogram is deducted from the first interferogram and similarly the fourth interferogram is deducted from the second interferogram. The recording distance is $z = 140mm$ and $z + \Delta\ z = 140 + 0.5mm$ with wavelength of light at 632.8µm.

Fig 8a. Input Image

Fig 8b. Intensity and phase of image

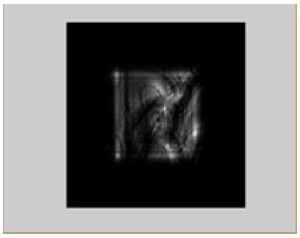

Fig 8c. First Interferogram

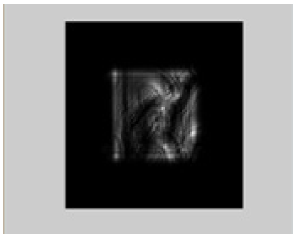

Fig 8d. Second Interferogram

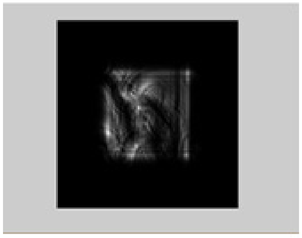

Fig 8e. Third Interferogram

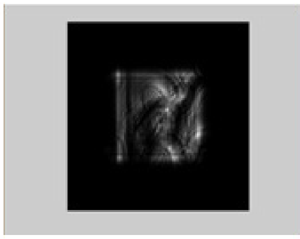

Fig 8f. Fourth Interferogram

Fig 8g. Reconstructed Intensity

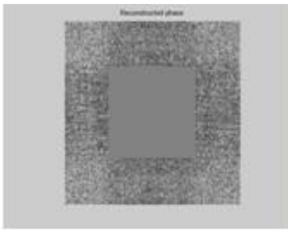

Fig 8h. Reconstructed Phase

2. Comparison of different holography reconstruction methods

The Fresnel reconstruction involves the use of a single Fourier Transform with either a chirp function or Impulse response. The reconstructed image size depends on *d'* and λ. The convolution approach on the other hand involves the use of two or three FFT with transfer function and impulse response. The multiplication of the hologram and the reference wave followed by its multiplication with a transfer function in spatial frequency domain and later transformed back in spatial domain again results in the reconstructed image size to be independent of the reconstruction depth *d'* and λ. The in-line holography has a simple set up and offers greater flexibility but suffers from presence of DC terms and twin image problem. The off axis holography eliminates the twin image and DC problem but at the cost of inefficient use of CCD pixels. The Phase-Shifting Holography also solves the twin image and DC problem by taking more than two interferogram but at the cost of system complexity. Moreover recording of different interferograms consumes a considerable amount of time. The two intensity inline holography has a fast computation time and Fast Fourier algorithms can be effectively employed. Thus it serves to be useful for real time imaging.

3. Conclusion

Digital holography has become an indispensible tool for application in 3D image processing, surface contour measurements, microscopy and label free monitoring of Biological cells. In this paper an attempt is made to compare various reconstruction algorithms used for digital holograms and the results are presented

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