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Domination and Total domination in Wrapped Butterfly networks

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Abstract

A set S of vertices in a graph G is a dominating set if every vertex of G is either in S or in adjacent to some vertex of S. If S is independent, then S is called an independent dominating set. The domination problem is to determine a dominating set of minimum cardinality. Independent domination problem is defined similarly. A Wrapped butterfly network $WBF(n)$, $n \geq 3$, is obtained by merging the first and last levels of a butterfly network $BF(n)$, $n \geq 3$. In this paper we determine upper bounds for the domination and total domination numbers of $WBF(n)$.

Keywords: Dominating set; Domination number; Total domination; Butterfly graph

1. Introduction

Domination in graphs is currently a very important research branch of graph theory. Historically, the domination type problems mainly arise from chess game to obtain minimum number of queens needed to attack or dominate every square on the chessboard. Domination problems used to find the sets of representatives, in monitoring communication or electrical networks, and in land surveying where it is necessary to minimize the number of places a surveyor must stand in order to take height measurements for an entire region. It also plays a vital role in parallel processing and supercomputing, which continues to exert great influence in the development of modern science and engineering. The network of processors and interconnections play a vital role in facilitating the communication between processors in parallel computers. Some of the popular interconnection networks are rings, toroids, hypercube, Butterfly Graphs and wrapped Butterfly networks. The domination problem has been proved to be NP-complete [4]. In this paper we consider the domination and independent domination problems for the wrapped butterfly network $WBF(n), n \geq 3$.

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2. Basic Concepts

Definition 2.1[1]: A dominating set S of a graph G is a subset of vertices of G with the condition that every vertex in $V \backslash S$ is adjacent to some vertex in S. Such a set with the minimum cardinality yields the domination number denoted by $\gamma(G)$.

Definition 2.2[7]: If every vertex $v \in V$ is adjacent to some vertex is a set S of vertices in G, then S is said to be a total dominating set. The total domination number $\gamma_t(G)$ is the minimum cardinality of a total dominating set.

Definition 2.3[7]: If no two vertices in a dominating set are adjacent, then S is called an independent dominating set. Such a set with minimum cardinality yields the independent domination number denoted by $\gamma_i(G)$.

Following results on domination number and total domination number for connected graphs exist in the literature already.

Theorem 2.1[2]**:** For any graph G of order p and maximum degree Δ, we have $\gamma(G) \geq p/(\Delta + 1)$.

Theorem 2.2[3]**:** If G is a connected graph with $p \geq 3$ vertices, then we have $\gamma_t(G) \leq 2p/3$.

Theorem 2.3[6]**:** If G is a 4-regular graph of order n, then $\gamma(G) \leq \frac{4}{11} n$.

Theorem 2.4[5]**:** If G has p vertices and no isolates, then we have $\gamma_t(G) \leq p - \Delta(G) + 1$.

Theorem 2.5[5]**:** If G is connected and $\Delta(G) < p - 1$, then $\gamma_t(G) \leq p - \Delta(G)$.

We also observe the following result:

Theorem 2.6: Let G be an r-regular graph of order n. Then $\gamma_t(G) \geq 2 \left\lceil \frac{n}{2r} \right\rceil$.

Proof: An optimal total dominating set is obtained when the induced subgraph of the total dominating set is a matching. Since the end vertices of an edge, together can dominate at most $2r$ vertices $\gamma_t(G) \geq 2 \left\lceil \frac{n}{2r} \right\rceil$.

3. Main Results

In this section we determine an upper bound for the domination number of wrapped butterfly networks which improves the bound given by Theorem 2.3.

Definition 3.1[8]: The n-dimensional butterfly network $BF(n)$ has vertex set $V = \{(x; i) \,/\, x = (x_1, x_2, \dots \dots x_n)$, $x_i = 0 \; or \; 1,\; 1 \leq i \leq n\}$. Two vertices $(x;i)$ and $(y;j)$ are linked by an edge in $BF(n)$ if and only if $j = i+1$ and either (i) $x = y$, or (ii) x differs from y in precisely the j^{th} bit.

Wrapped butterfly, denoted by $WBF(n)$ is an n- level graph with $n.2^n$ vertices and each vertex of degree 4.

Theorem 3.1: Let G connected undirected graph $WBF(n),\ n \geq 3$. Then we have $\gamma(G) \leq n.2^{n-2}$.

Proof: $WBF(n)$ has n rows, each containing 2^n vertices representing the columns. We divide the columns into two halves H_1 and H_2 as the columns represented by the first 2^{n-1} vertices and the columns represented by the next 2^{n-1} vertices respectively. We select vertices from H_1 in a set D as follows:

1. Divide the first row of H_1 into 4 sets of 2^{n-3} vertices from the 4 quarters of the consecutive columns in H_1.
   Select the first quarter and the third quarter vertices in D.
2. Divide the i^{th} row of H_1 into 2^{i+1} sets, $S_1,\ S_2,\ S_3, \dots\dots, S_{2^{i+1}}$, each consisting of 2^{n-i-2} vertices from the 2^{i+1} sets of consecutive columns of size 2^{n-i-2} in Row $i,\ 2 \leq i \leq n-3$. Select 2^{i-1} sets from among $S_1,\ S_2, S_3, \dots\dots, S_{2^{i+1}}$, in D such that the vertices in the selected set S_j are not dominated by end vertices in Level $(i-1)$, of straight edges and oblique edges incident at them, $2 \leq i \leq n-3$. The number of selected vertices H_1 in Level i is $2^{i-1} \times 2^{n-i-2} = 2^{n-3}$.
3. Select 2^{n-3} number of vertices at Level $n-2$ such that they are consecutive pairs satisfying the condition that they are not dominated by end vertices of straight edges and oblique edges in Level $n-3$.
4. Include in D, the mirror images in H_2 of the already selected vertices in H_1.
   We claim that D is a dominating set of G. It is clear from the choice of vertices in D, that none of the 4 neighbouring vertices of any vertex in D belongs to D. Hence every vertex in D belonging to Level i dominates 2 vertices in Level $(i-1)$ and 2 vertices in Level $i,\ 2 \leq i \leq n-2$. This implies that all vertices in Level i which are not in D are dominated by the vertices in D belonging to Level $(i+1)$, $2 \leq i \leq n-2$. Vertices in D from Level 1 dominate vertices in Level n. Thus D is a dominating set of $WBF(n)$, $n \geq 3$. The cardinality of D is $2(2^{n-1} + (n-4)2^{n-3}) = n.2^{n-2}$. Hence $\gamma(WBF(n)) \leq n.2^{n-2}$ ■

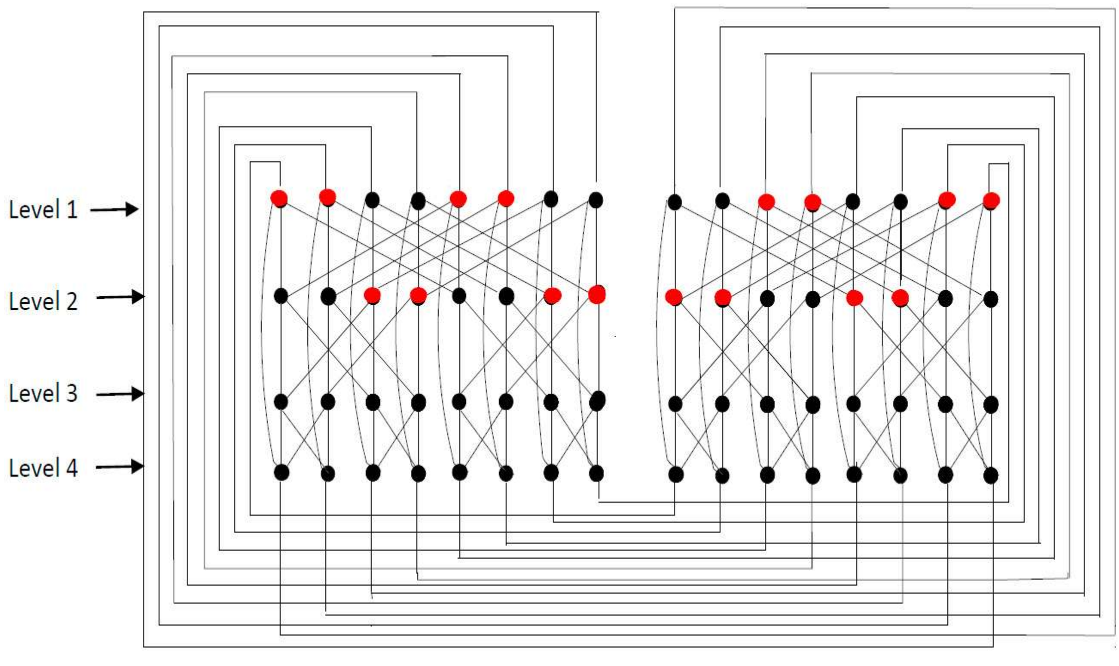

Fig.1 $WBF(4)$ with dominating set marked in red

Remark 1: Let G represent $WBF(n)$, $n \geq 3$. By Theorem 2.3, $\gamma(G) \leq \frac{4}{11}\, n.2^n$. In other words, $\gamma(G) \leq \frac{16}{11}\, n.2^{n-2}$. By Theorem 3.1 we have $\gamma(G) < n.2^{n-2}$. Hence the bound obtained in Theorem 3.1 is approximately $\frac{7}{10}^{th}$ the bound obtained using Theorem 2.3.

Remark 2: Let G represent $WBF(n)$, $n \geq 3$. By Theorem 2.1, $\gamma(G) \geq \left\lfloor \frac{n.2^n}{5} \right\rfloor$. Thus we have the following result:
Theorem 3.1: Let G connected undirected graph $WBF(n)$, $n \geq 3$. Then $\left\lfloor \frac{n.2^n}{5} \right\rfloor \leq \gamma(G) \leq n.2^{n-2}$.

Remark 3: The approximation ratio of an algorithm is the ratio between the result obtained by the algorithm and optimal value. This implies that the approximation ratio for the algorithm given in Theorem 3.1 is $n.2^{n-2} / \left\lceil \frac{n.2^n}{5} \right\rceil$ which is approximately 1.25.

The choice of vertices in the dominating set constructed in Theorem 3.1 happens to be an independent set. This yields the following result:

Theorem 3.2: Let G represent the Wrapped Butterfly network $WBF(n)$, $n \geq 3$. Then the independent domination number $\gamma_i(G)$ satisfies the relation $\gamma_i(G) \leq n.2^{n-2}$.

A natural extension of the dominating set to a total domination is to obtain a matching of edges with one end at the already chosen vertices of the dominating set. This observation leads to the following result:

Theorem 3.3: Let G represent the Wrapped Butterfly network $WBF(n)$, $n \geq 4$. Then $\gamma_t(G) \leq 3n.2^{n-3}$.

Remark 4: Combining Theorem 3.3 and Theorem 2.6, we arrive at the following result:
Let G represent the Wrapped Butterfly network $WBF(n)$, $n \geq 3$. Then $2\left\lceil \frac{n.2^n}{8} \right\rceil \leq \gamma_t(G) \leq 3n.2^{n-3}$.

Conclusion

In this paper we give the domination number, total domination number and independence domination number for $WBF(n)$, where $n \geq 3$.

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