Journal of Applied Fluid Mechanics, Vol. 9, No. 2, pp. 555-563, 2016.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.68.225.24784

Dual Solutions of MHD Boundary Layer Flow past an Exponentially Stretching Sheet with Non-Uniform Heat Source/Sink

C. S. K. Raju[1], N. Sandeep[2], C. Sulochana[2†] and M. Jayachandra Babu[1]

[1]*Fluid Dynamics Division, VIT University, Vellore-632014, Tamil Nadu, India.*
[2]*Department of Mathematics, Gulbarga University, Gulbarga-585106, Karnataka, India.*

†*Corresponding Author Email:math.sulochana@gmail.com*

(Received March 10, 2015; accepted May 27, 2015)

ABSTRACT

In this study we analyzed the momentum and heat transfer characteristics of MHD boundary layer flow over an exponentially stretching surface in porous medium in the presence of radiation, non-uniform heat source/sink, external pressure and suction/injection. Dual solutions are presented for both suction and injection cases. The heat transfer analysis is carried out for both prescribed surface temperature (PST) and prescribed heat flux (PHF) cases. The governing equations of the flow are transformed into system of nonlinear ordinary differential equations by using similarity transformation and solved numerically using bvp4c Matlab package. The impact of various non-dimensional governing parameters on velocity, temperature profiles for both PST and PHF cases, friction factor and rate of heat transfer is discussed and presented with the help of graphs and tables. Results indicate that dual solutions exist only for certain range of suction or injection parameters. It is also observed that the exponential parameter have tendency to increase the heat transfer rate for both PST and PHF cases.

Keywords: Exponential stretching sheet; MHD;Non-uniform heat source/sink; Thermal Radiation; Suction/injection.

1. INTRODUCTION

The convective flow over stretching surfaces immersed in porous media has paramount importance because of its potential applications in industrial purposes like soil physics, filtration of solids from liquids, chemical engineering and biological systems. In addition with the recent improvements in modern technology many researchers are concentrating on the study of heat and mass transfer in fluid flows due to its broad applications in geothermal engineering as well as other geophysical and astrophysical studies. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, etc.

Ajay Kumar (2008) investigated the two-dimensional MHD flow of viscoelastic fluid through a stretching surface in the presence of heat source and radiation effects by using Kummer's functions. Subhas Abel *et al*. (2009) analyzed the MHD heat transfer analysis of a power law fluid through a vertical stretching surface with non-uniform heat source and thermal buoyancy effects in the presence of variable thermal conductivity. Mahantesh *et al*. (2010) analyzed the heat transfer analysis of Walter's B liquid fluid flow past an impermeable stretching surface in the presence of viscoelastic deformation and non-uniform heat source/sink and concluded that the viscoelastic parameter increases the thermal boundary layer in both PHF and PST cases. The steady two-dimensional stagnation point flow of a nanofluid past a stretching/shrinking surface in its own plane was investigated by Bachok *et al.* (2011). The boundary layer flow and heat transfer characteristics of nonlinearly stretching surface in the presence of variable wall temperature and non-uniform heat source was discussed numerically by Mahentesh *et al*. (2011).

Sandeep *et al*. (2012) discussed the radiation effect on MHD flow over a vertical plate in presence of heat source. Hajipour and Dehkordi (2012) are discussed the heat transfer analysis of a nanofluid over parallel plate and vertical channels by considering partially filled porous medium. They concluded that by mixing of nanofluid with base fluid enhances the heat transfer rate significantly. Boundary layer flow and heat transfer characteristics of a Maxwell fluid past a continuous moving exponential surface was discussed by Abbas *et al*. (2014). Ahmad *et al*. (2014) analyzed the radiation and magnetic effect on the boundary layer viscous flow past an exponential stretching sheet in porous medium. The three-dimensional flow of a waterbased nanofluid past an exponential stretching surface was studied by Nadeem *et al*. (2014). Hussian *et al*. (2014) illustrated the viscous dissipation and Brownian motion effects of two dimensional boundary layer flow of Jeffry nanofluid past an exponentially stretching sheet. A numerical Analysis on unsteady MHD mixed convective boundary layer flow of a nanofluid past an exponential stretching sheet through porous medium was presented by Anwar *et al*. (2014). Steady laminar MHD boundary layer flow of a viscoelastic fluid past a permeable stretching surface in presence of heat absorption/generation was examined by Cortell (2014). The effects of thermal radiation and wall mass transfer of MHD flow through a porous stretching sheet with non-uniform heat source/sink was analytically studied by Abdul Hakeem *et al*. (2014). A homotopy analysis of convective steady two-dimensional magnetic fluid flow through a permeable vertical stretching surface with existing of radiation and buoyancy effects was presented by Rashidi *et al* (2014). Singh and Agarwal (2014) discussed the variable thermal conductivity and non-uniform heat source/sink effects on the MHD flow of Maxwell fluid through porous medium. MHD boundary layer flow and heat transfer investigation of a second grade nanofluid past permeable stretching surface with heat generation in the presence of partial slip conditions was theoretically studied by Bhargava and Goyal (2014).

Pal and Mandal (2014) analyzed the mixed convective heat and mass transfer analysis of stagnation-point flow of a nanofluid around a stretching surface in a porous medium with thermal radiation and chemical reaction effects. They highlighted that the copper-water nanofluid has higher mass transfer rate compared with the other combination of fluids. Chandramandal and Mukhopadhyay (2013) discussed heat transfer analysis for fluid flow over an exponentially porous stretching sheet. Sandeep and Sugunamma (2014) discussed the effects of radiation and inclined magnetic field on unsteady MHD convective flow past an impulsively moving vertical plate in a porous medium. Mohan Krishna *et al*. (2014) studied the radiation and magnetic field effects on the flow past a vertical plate. Mustafa *et al*. (2015) discussed the comparative analysis of a laminar axisymmetric flow of a nanofluid past a nonlinearly stretching surface. In this study they concluded that the heat transfer rate at the wall is extraneous to the flow. Sandeep and Sulochana (2015) presented dual solutions for radiative MHD nanofluid flow over an exponentially stretching sheet. Murshed *et al*. (2015) discussed the radiation effect on boundary layer flow of a nanofluid over a nonlinear stretching sheet. Choudhury and Kumar Das (2014) studied MHD free convective flow through porous media in presence of radiation and chemical reaction effects.

Present study is the extension work of Ahmad *et al*. (2014). In this study we analyzed the flow and heat transfer characteristics of MHD boundary layer flow over an exponentially stretching surface in porous medium in the presence of radiation, non-uniform heat source/sink, external pressure, suction/injection. The heat transfer analysis is carried out for both prescribed surface temperature (PST) and prescribed heat flux (PHF) cases. The governing equations of the flow are transformed in to nonlinear ordinary differential equations by using similarity transformation and then solved numerically. The influence of various non-dimensional governing parameters on velocity, temperature profiles for PST and PHF cases, skin friction coefficients, rate of heat transfer are discussed and presented through graphs and tables.

2. MATHEMATICAL FORMULATION

Consider a steady two-dimensional MHD boundary layer flow of an electrically conducting viscous fluid towards an exponentially stretching sheet in a porous medium in the presence of non-uniform heat generation/absorption, radiation and suction/injection. The fluid occupies the space $y > 0$ and is flowing in the x direction and $y - axis$ is normal to the flow. External pressure p is applied along $x\, and\, y$ directions. A constant magnetic field of strength B_0 is applied in the normal to the flow and the induced magnetic field is neglected which is valid assumption when the magnetic Reynolds number is small. As per the assumptions made above, the boundary-layer equations that govern the present flow are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k'}u - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$\frac{\partial p}{\partial y} + g\beta(T - T_\infty) = 0, \tag{3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) - \frac{\partial q_r}{\partial y} + q''', \tag{4}$$

Where u and v are the components of velocity in x and y directions respectively, ρ is density, p is the pressure, ν represents kinematic viscosity, k' is the permeability of porous medium, σ is the electrical conductivity, g is the acceleration due to gravity , β is the thermal expansion coefficient, T is the fluid temperature, c_p is the specific heat, q_r is the radiative heat flux, q''' is the non-uniform heat generation/absorption and k is the thermal conductivity defined in the following way:

$$k = \begin{cases} k_\infty\left[1 + \delta\theta(\eta)\right] in \quad PST\ case \\ k_\infty\left[1 + \delta\phi(\eta)\right] in \quad PHF\ case \end{cases} \tag{5}$$

in which δ is the small parameter ($\delta > 0$ for gases and $\delta < 0$ for solids and liquids), $\theta(\eta)$ and $\phi(\eta)$ are dimensionless temperature distributions for PST and PHF cases respectively. The relevant boundary conditions for the present problem are:

$$\left.\begin{aligned} &u = U_0 e^{Nx/l},\ v = v_w, T = T_\infty + Ae^{Nax/2l}(PST),\\ &-k\frac{\partial T}{\partial y} = Be^{(b+l)Nx/2l}(PHF), \qquad at\ \ y = 0,\\ &u \to 0,\ T \to T_\infty, p \to p_\infty, \qquad as\ \ y \to \infty, \end{aligned}\right\} \tag{6}$$

where U_0 is a characteristic velocity, l is the characteristic length scale, and $A, a, B\ and\ b$ are the parameters of temperature profiles depending on the properties of the liquid, v_w is the suction/injection parameter, $v_w < 0$ for suction and $v_w > 0$ for injection. The radiative heat flux given by Brewster (1972)

$$q_r = -\frac{4\sigma_1}{3m}\frac{\partial T^4}{\partial y}, \tag{7}$$

in the above equation σ_1 is the Stefan-Boltzmann constant and m is the coefficient of mean absorption. Assuming T^4 as a linear combination of temperature, then

$$T^4 \cong -3T_\infty{}^4 + 4T_\infty{}^3 T \tag{8}$$

The time dependent non-uniform heat source/sink q''' is defined as (Abdul Hakeem (2014))

$$q''' = \frac{k_f U_w(x)}{l\nu}\left(A^*(T_w - T_\infty)f' + B^*(T - T_\infty)\right) \tag{9}$$

In the above equation $A^*, B^* > 0$ corresponds to heat generation and $A^*, B^* < 0$ corresponds to heat absorption.

Substituting (7) and (8) in (4) we get

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{16T_\infty{}^3\sigma_1}{3m}\frac{\partial^2 T}{\partial y^2} + q''', \tag{10}$$

For similarity solution of equations (1)-(4), subject to the boundary conditions (6), we introduce the similarity variables as

$$\left.\begin{aligned} &\eta = \left(\frac{U_0}{2\nu l}\right)^{1/2} e^{Nx/2l} y, \psi = \left(2\nu l U_0\right)^{1/2} e^{Nx/2l} f(\eta),\\ &p - p_\infty = \rho U_0{}^2 e^{Nx/2l} h(\eta),\\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}(PST), \phi(\eta) = \frac{T - T_\infty}{T_w - T_\infty}(PHF), \end{aligned}\right\} \tag{11}$$

where η is the similarity variable ψ is the stream function defined as $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$, which is identically satisfies continuity equation (1).

By defining η in this form, the boundary conditions at $y = 0$ reduce to the boundary conditions at $\eta = 0$ which is more convenient for numerical computations. Substituting (11) into equations (2), (3) and (4) we get the following nonlinear ordinary differential equations:

$$f''' - 2(K + M)f' + Nff'' - 2Nf'^2 - (h'N\eta + 4hN) = 0, \tag{12}$$

$$h' = Gr_x\theta, \tag{13}$$

$$(1 + R_a + \delta\theta)\theta'' + N\Pr(f\theta' - af'\theta) + \delta\theta'^2 - (A^* f' + B^*\theta) = 0 \quad (PST), \tag{14}$$

$$(1 + R_a + \delta\phi)\phi'' + N\Pr(f\phi' - bf'\phi) + \delta\phi'^2 - (f'A^* + B^*\phi) = 0 \quad (PHF), \tag{15}$$

subject to the boundary conditions

$$f(0) = S, f'(0) = 1, \theta(0) = 1, \phi'(0) = -\frac{1}{1+\delta}, \text{ at } \eta = 0$$
$$f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0, h(\infty) \to 0 \quad \text{as } \eta \to \infty \tag{16}$$

where dimensionless parameters are given as K is the porosity parameter, $\Pr$ is the Prandtl number parameter, M is the magnetic field parameter, Gr_x is the Grashof number, A^* and B^* are non-uniform internal heat generation/absorption coefficients , N is the exponential parameter (Chandramandal and Mukhopadhyay , 2013), S is the suction/injection parameter, $S > 0$ for suction and $S < 0$ for injection.

$$\left.\begin{aligned} &K = \frac{\nu l}{k'U_0}e^{-Nx/l}, M = \frac{l\sigma B_0{}^2}{\rho U_0}e^{-Nx/l}\\ &,\Pr = \frac{\mu c_p}{k_\infty}, R_a = \frac{16\sigma_1 T_\infty{}^3}{3k_\infty m},\\ &Gr_x = \frac{g\beta(T_w - T_\infty)\sqrt{2\nu l}}{U_0{}^{5/2}e^{5Nx/2l}}, v_w = -S\left(\frac{\nu U_0}{2l}\right)^{1/2}, \end{aligned}\right\} \tag{17}$$

The main physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x are given by

$$\begin{aligned} &C_f \operatorname{Re}_x^{1/2} = f''(0), Nu_x / \operatorname{Re}_x^{1/2} = -\theta'(0)\ (PST),\\ &Nu_x / \operatorname{Re}_x^{1/2} = 1/\phi(0)\ (PHF), \end{aligned} \tag{18}$$

3. RESULTS AND DISCUSSION

The system of nonlinear ordinary differential equations (12) to (15) with the boundary conditions (16) are solved numerically. The results obtained shows the influences of the non dimensional governing parameters namely thermal radiation parameter R_a , exponential parameter N , magnetic field parameter M , non-uniform heat source/sink parameter A^* ,thermal conductivity parameter δ and temperature distribution parameters a and b on velocity, temperature, skin friction coefficient and local Nusselt numbers are thoroughly investigated for suction and injection cases and presented through graphs and tables. In this study for numerical results we considered $N = M = \eta = \Pr = 1,\ Gr_x = 5,$

$A^* = B^* = \delta = a = b = 0.1,\ K = 0.2, \eta = 1,$

$and\ R_a = 0.5$. These values are kept as common in entire study except the varied vales as displayed in respective figures and tables. In this study blue color profiles indicates suction case and red color corresponds to injection case.

PHF cases are shown in Figs. 1-3. It is evident from the figures that increasing in the magneticfield parameter depreciates the velocity profiles and enhances the temperature profiles for both PST and PHF cases. It is also observed that the velocity and thermal boundary layers enhances in injection case compared with suction case. The Lorentz force on the flow is reason for decrease in the velocity boundary layer and increases the thermal boundary layer for both PST and PHF cases. Figs. 4 -6 illustrate the influence of the non-uniform heat source/sink parameter on the velocity and temperature profiles for both PST and PHF cases. It is clear from the figures that a raise in the value of A^* increase the velocity and temperature profiles for PST and PHF cases.

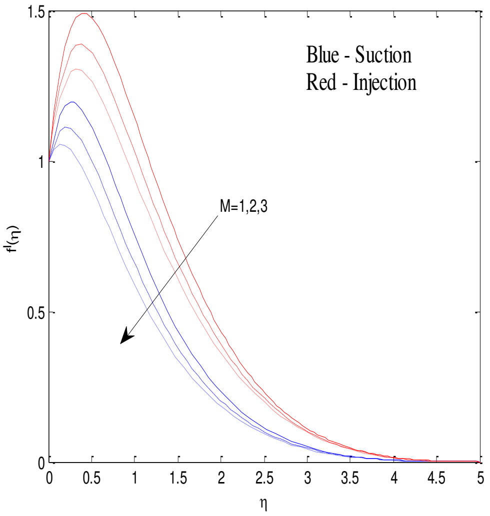

Fig. 1. Velocity profiles for different values of Magnetic field parameter M .

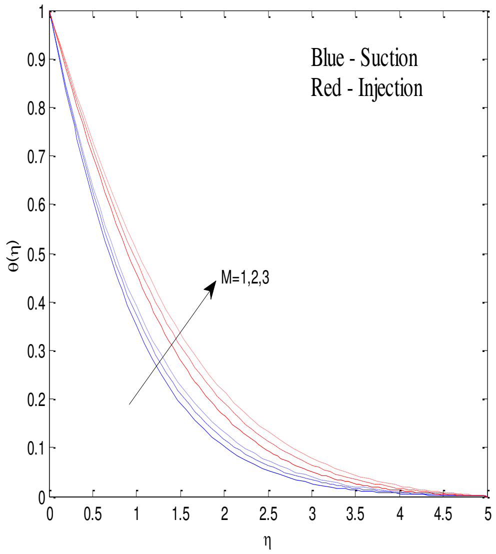

Fig. 2. Temperature profiles (PST) for different values of Magnetic field parameter M .

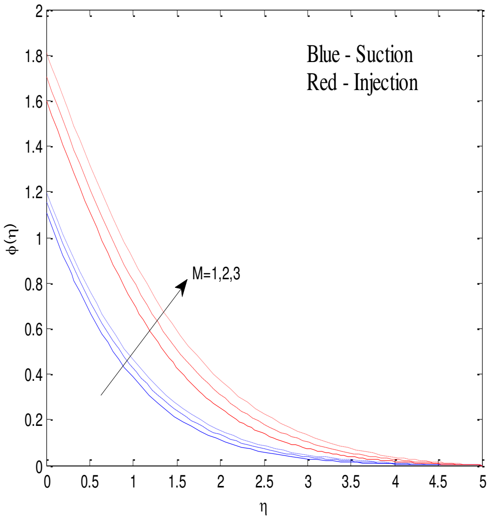

Fig. 3. Temperature profiles (PHF) for different values of Magnetic field parameter M

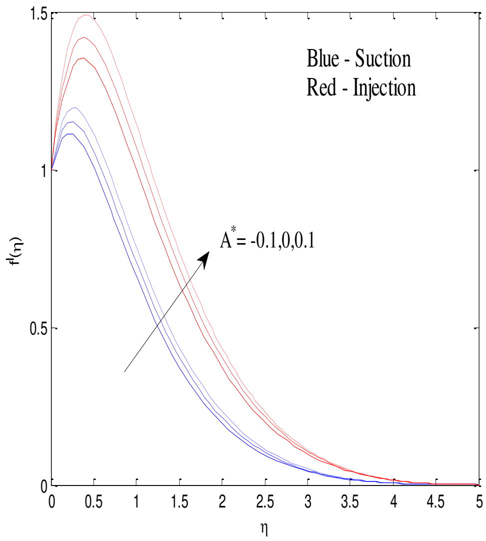

Fig. 4. Velocity profiles for different values of non-uniform heat source/sink parameter A^* .

The influence of the magnetic field parameter on velocity, temperature profiles for both PST and

The effect of Grashof number on the velocity and temperature profiles of both PST and PHF cases are displayed in Figs. 7-9. It is evident from these plots that the thermal boundary becomes thin for higher values of Gr_x and momentum boundary layer thickness increases with the increase in Gr_x. Figs.10 and 11depicts the impact of variable thermal conductivity on the temperature (both for PST and PHF) profiles for suction and injection cases. It is clear from the figures that an increase in the thermal conductivity parameter (δ) enhances the momentum and thermal boundary layers. This agrees the general physical behavior of the thermal conductivity parameter.

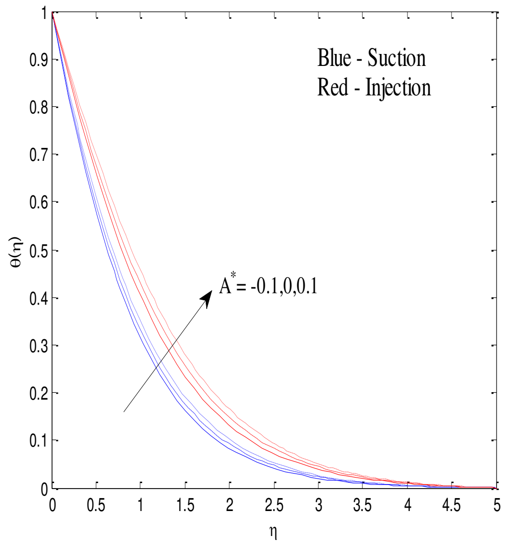

Fig. 5. Temperature profiles (PST) for different values of non-uniform heat source/sink parameter A^*.

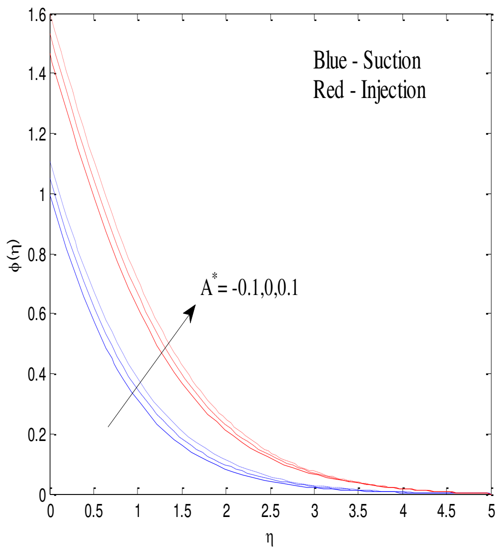

Fig. 6. Temperature profiles (PHF) for different values of non-uniform heat source/sink parameter A^*.

Figs. 12-14illustrate the influence of exponential parameter on velocity and temperature (both for PST and PHF) profiles for suction and injection cases. It is evident from figures that an increase in exponential parameter depreciates the velocity and temperature profiles. This is due to the fact that with the increase in exponential parameter reduces the velocity and thermal boundary layer thicknesses. Figs. 15-17display the effect of temperature distribution parameters a and b on temperature profiles. It is evident from the figures that the influence temperature distribution parameter b is more on prescribed heat flux case and an increase in the value of a depreciates the temperature profiles in PST case and enhances in PHF case for both suction and injection cases.

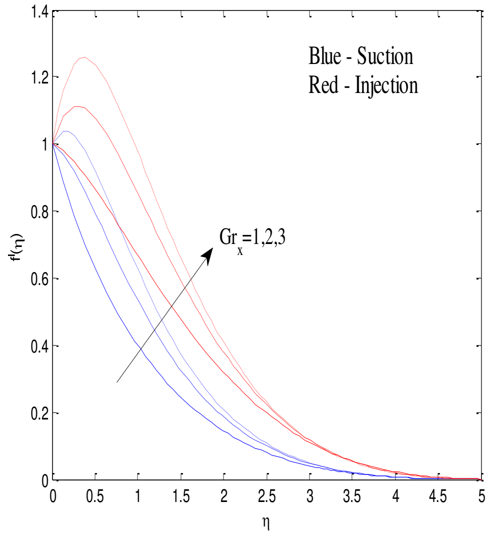

Fig. 7. Velocity profiles for different values of Grashof number Gr_x.

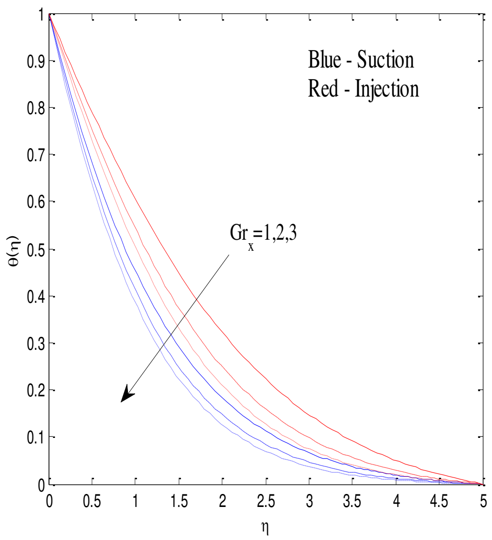

Fig. 8. Temperature profiles (PST) for different values of Grashof number Gr_x.

Table 1 shows the influence of non-dimensional governing parameters on friction factor and Nusselt number of PST and PHF cases for both suction/injection. It is evident from the table that increase in magnetic field parameter depreciates the friction factor and heat transfer rate in PST case but improves the rate of heat transfer in PHF case. Increase in radiation parameter, non-uniform heat source/sink parameters and thermal conductivity parameters enhances the skin friction coefficient, depreciate the heat transfer rate for PST case and improves the rate of heat transfer in PHF case. A raise in the value of Grashof number increases the friction factor and Nusselt number in PST case but deprecates the heat transfer rate in PHF case. Also, we observed that rate of heat transfer is more on injection case compared with suction case.

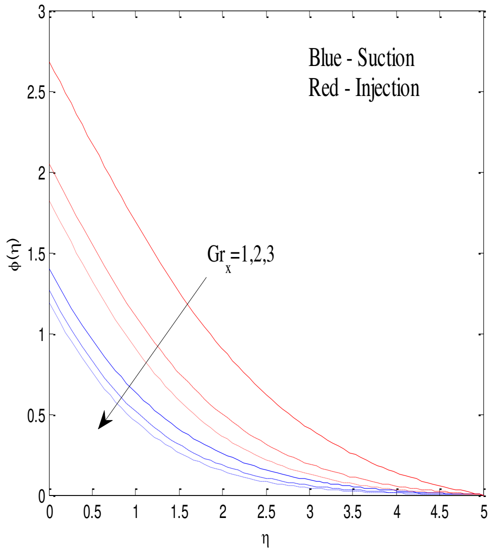

Fig. 9. Temperature profiles (PHF) for different values of Grashof number Gr_x .

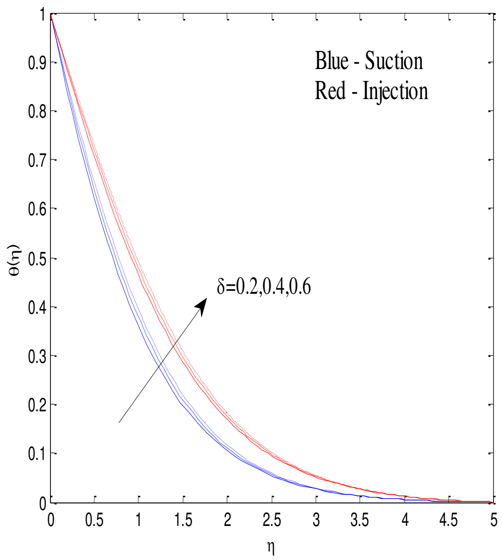

Fig. 10. Temperature profiles (PST) for different values of thermal conductivity parameter δ .

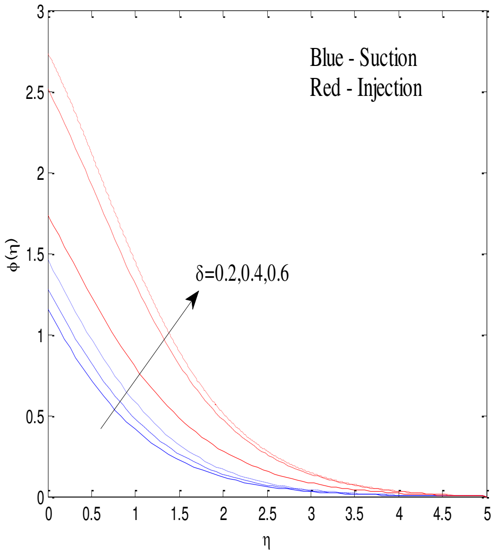

Fig. 11. Temperature profiles (PHF) for different values of thermal conductivity parameter δ .

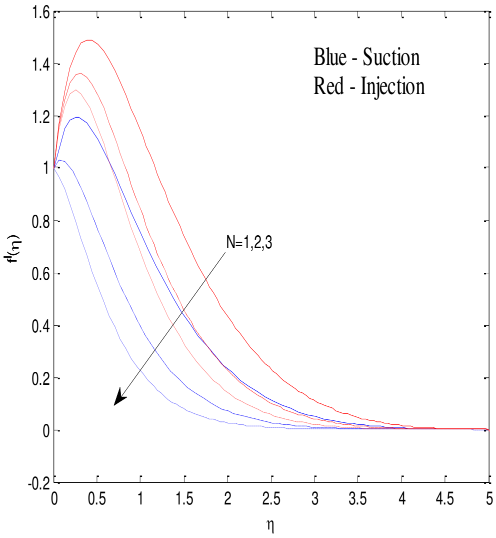

Fig. 12. Velocity profiles for different values of exponential parameter N .

4. CONCLUSIONS

In this study we analyzed the momentum and heat transfer characteristics of MHD boundary layer flow over an exponentially stretching surface in porous medium in the presence of radiation, non-uniform heat source/sink, external pressure and suction/injection. Dual solutions are presented for both suction and injection cases. The heat transfer analysis is carried out for both prescribed surface temperature (PST) and prescribed heat flux (PHF) cases. The impact of various non-dimensional governing parameters on velocity, temperature profiles for both PST and PHF cases, friction factor and rate of heat transfer are discussed and presented through graphs and tables. The conclusions are as follows:

- Non-uniform heat absorption is very useful for effective cooling of stretching surface.

- The Grashof number is considerable for the controlling of flow and enhances the heat transfer rate in prescribed temperature case.
- Exponential parameter has capability to reduce the thermal boundary layer thickness and enhance the heat transfer rate for both PST and PHF cases.
- Increase in thermal conductivity parameter enhances the flow along with temperature profiles of the flow.
- An increase in magnetic field parameter reduces the friction factor and increases the heat transfer rate in prescribed heat flux case.

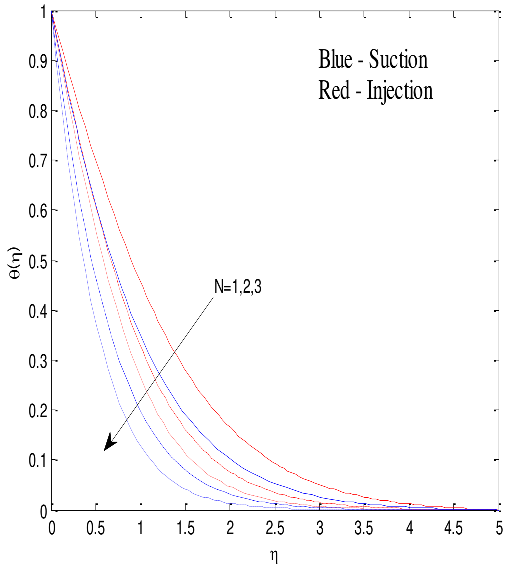

Fig. 13. Temperature profiles (PST) for different values of exponential parameter N.

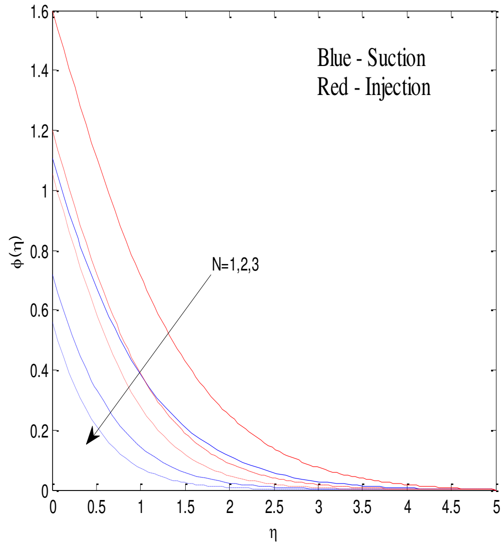

Fig. 14. Temperature profiles (PHF) for different values of exponential parameter N.

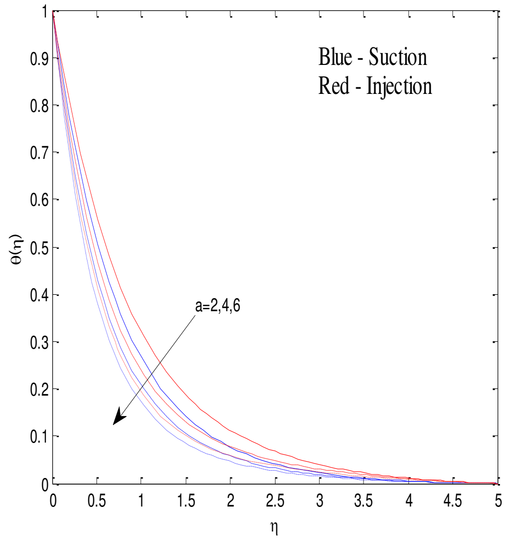

Fig. 15. Temperature profiles (PST) for different values of temperature distribution parameter a.

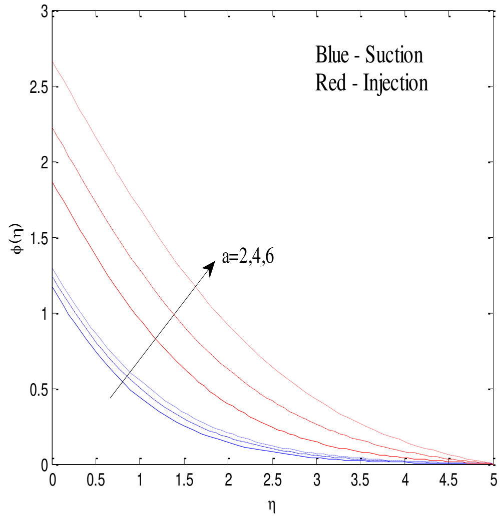

Fig. 16. Temperature profiles (PHF) for different values of temperature distribution parameter a.

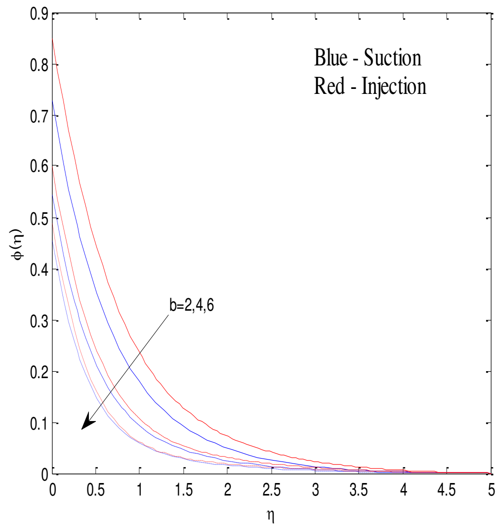

Fig. 17. Temperature profiles (PHF) for different values of temperature distribution parameter b.

Table 1 Numerical values of the Skin-friction coefficient $f''(0)$, Nusselt number for PST and PHF cases $-\theta'(0)$ and $\phi(0)$

S	M	R_a	A^*	B^*	Gr_x	δ	$f''(0)$	$-\theta'(0)$	$\phi(0)$
0.5	1	0.5	0.5	0.5	5	0.5	1.712858	0.898507	1.106171
	2	0.5	0.5	0.5	5	0.5	1.227323	0.862762	1.154092
	3	0.5	0.5	0.5	5	0.5	0.800261	0.832700	1.198591
-0.5	1	0.5	0.5	0.5	5	0.5	2.949828	0.620953	1.599865
	2	0.5	0.5	0.5	5	0.5	2.578169	0.587916	1.704681
	3	0.5	0.5	0.5	5	0.5	2.242130	0.559523	1.810476
0.5	1	1	0.5	0.5	5	0.5	2.479839	0.795864	1.249637
	1	2	0.5	0.5	5	0.5	3.632757	0.674179	1.475243
	1	3	0.5	0.5	5	0.5	4.490010	0.600762	1.655642
-0.5	1	1	0.5	0.5	5	0.5	3.473135	0.585029	1.698597
	1	2	0.5	0.5	5	0.5	4.288036	0.531201	1.871502
	1	3	0.5	0.5	5	0.5	4.910318	0.492257	2.020874
0.5	1	0.5	-0.1	0.5	5	0.5	1.215776	0.997939	0.995712
	1	0.5	0	0.5	5	0.5	1.452484	0.950486	1.049506
	1	0.5	0.1	0.5	5	0.5	1.712858	0.898507	1.106171
-0.5	1	0.5	-0.1	0.5	5	0.5	2.309866	0.725934	1.464988
	1	0.5	0	0.5	5	0.5	2.616623	0.676136	1.530458
	1	0.5	0.1	0.5	5	0.5	2.949828	0.620953	1.599865
0.5	1	0.5	0.5	-0.1	5	0.5	1.458645	0.962545	1.033715
	1	0.5	0.5	0	5	0.5	1.582975	0.930917	1.068331
	1	0.5	0.5	0.1	5	0.5	1.712858	0.898507	1.106171
-0.5	1	0.5	0.5	-0.1	5	0.5	2.670140	0.681959	1.458358
	1	0.5	0.5	0	5	0.5	2.807229	0.651805	1.524986
	1	0.5	0.5	0.1	5	0.5	2.949828	0.620953	1.599865
0.5	1	0.5	0.5	0.5	1	0.5	-0.971062	0.726630	1.397265
	1	0.5	0.5	0.5	2	0.5	-0.126879	0.793089	1.263297
	1	0.5	0.5	0.5	3	0.5	0.564062	0.837312	1.190913
-0.5	1	0.5	0.5	0.5	1	0.5	-0.097782	0.433411	2.683611
	1	0.5	0.5	0.5	2	0.5	0.877554	0.508168	2.046346
	1	0.5	0.5	0.5	3	0.5	1.661148	0.556051	1.819952
0.5	1	0.5	0.5	0.5	5	0.2	1.805478	0.862422	1.156952
	1	0.5	0.5	0.5	5	0.4	1.986194	0.801227	1.279211
	1	0.5	0.5	0.5	5	0.6	2.161110	0.751213	1.458170
-0.5	1	0.5	0.5	0.5	5	0.2	3.022968	0.603487	1.735443
	1	0.5	0.5	0.5	5	0.4	3.164653	0.573269	2.508847
	1	0.5	0.5	0.5	5	0.6	3.300716	0.547936	2.734577

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to the very competent anonymous referees for their valuable comments and suggestions. Authors from Gulbarga University acknowledge the UGC for financial support under the UGC Dr. D. S. Kothari Post-Doctoral Fellowship Scheme (No.F.4-2/2006 (BSR)/MA/13-14/0026).

REFERENCES

Abbas, Z., T.Javed, N. Ali andM. Sajid (2014). Flow and heat transfer of Maxwell fluid over an exponentially stretching sheet: a non-similar solution.*Heat Transfer Asian Research* 43(3), 233-242.

Abdul Hakeem, A.K.,R.Kalaivanan, N. Vishnu Ganesh and B. Ganga (2014).Effect of partial slip on hydromagnetic flow over a porous stretching sheet with non-uniform heat source/sink thermal radiation and wall mass transfer. *Ain Shams Engineering Journal*5, 913-922.

Ahmad, I., M.Sajid, W.Awan, M. Rafique, W. Aziz, M. Ahmed,A. Aamar and T. Moeen (2014).MHD flow of a viscous fluid over an exponentially stretching sheet in a porous medium.*Journal of Applied Mathematics* 8,256-761.

Ajay Kumar, S. (2008). Heat source and radiation effects on magneto-convection flow of a viscoelastic fluid past a stretching sheet: Analysis with Kummer's functions .*Int Communications in Heat and Mass Transfer* 35, 637-642.

Anwar, O.B., M. S. Khan,I. Karim, M. M. Alamand M. Ferdows(2014).Explicit numerical study of unsteady hydromagnetic mixed convective nanofluid flow from an exponentially stretching sheet in porous media.*AppliedNanoScience* 4,943-957.

Bachok, N., A. Ishakand I. Pop (2011). Stagnation point flow over a stretching/shrinking sheet in a nanofluid.*Nanoscale Research Letters* 6,623-631.

Bhargava, R. andM. Goyal(2014).MHD Non-Newtonian nanofluid flows over a permeable stretching sheet with heat generation and velocity slip.*Int J Mathematical Computational Physics and Quantum Engineering* 8(6), 906-912.

Brewster, M.Q. (1972). *Thermal radiative transfer properties*, John Wiley and Sons.

Chandramandal, I. and S. Mukhopadhyay (2013).Heat transfer analysis for fluid flow over an exponentially porous stretching sheet with surface heat flux in porous medium.*Ain shams Eng J 4, 103-110.*

Choudhury, R. and S. Kumar Das (2014).Visco-elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer.*J Applied Fluid Mechanics* 7(4), 603-609.

Cortell, R.(2014). MHD (magneto-hydrodynamic) flow and radiative nonlinear heat transfer of a visco- elastic fluid over a stretching sheet with heat generation/absorption.*Energy* 74,896-905.

Hajipour, M. andA. M. Dehkordi(2012).Analysis of nanofluid heat transfer in parallel-plate vertical channels partially filled with porous medium.*Int J Thermal Sci.* 55, 103-113.

Hussain, T., S. S.Ali, T. Hayat, A.Alsaedi,F. Al-SolamyandM. Ramzan(2014).Radiative hydro magnetic flow of Jeffrey nanofluid by an exponentially stretching sheet.*PLOS One*9(8), 103719 .

Madhavi, V., A. V. B. Reddy, K. G. Reddy andG. Madhavi (2012). A simple method for the determination of efficiency of stabilized FeOnano particles for detoxification of chromium (VI) in water.*J Chemical and Pharmaceutical research* 4(3), 1539-1545.

Mahentesh, M.N., A. M. SubhasandT. Jagadish(2010). Heat transfer in a Walter's liquid B fluid over an impermeable stretching sheet with non-uniform heat source/sink and elastic deformation.*Commun Nonlinear SciSimulat*15, 1791-1802.

Mahentesh, M.N., K. Vajravelu, M. Subhas Abel and N. g. Chiu-On(2011). Heat transfer over a nonlinearly stretching sheet with non-uniform heat source and variable wall temperature.*Int J Heat and Mass Transfer* 54,4960-4965.

Mohankrishna, P., V. Sugunamma and N. Sandeep(2014).Radiation and magneticfield effects on unsteady natural convection flow of a nanofluid past an infinite vertical plate with heat source.*Chemical and Process Engineering Research* 25, 39-52.

Murshed, F.K., R. Indranil, C. Arnaband N. Sandeep(2015).Radiation effect on boundary layer flow of a nanofluid over a nonlinearly permeable stretching sheet.*Adv.Physics theories and applications*40, 43-54.

Mustafa, M., J. AhmadKhan, T. Hayat and A. Alsaedi(2015).Analytical and numerical solutions for axisymmetric flow of nanofluid due to non-linearly stretching sheet.*Int J Non-linear Mechanics*71,22-29.

Nadeem, S., U. H. Rizwanand K. Z. Hayat(2014).Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet.*Alexandria Eng.J.*53(1),219-224.

Pal, D.and G. Mandal(2014).Influence of thermal radiation on mixed convection heat and mass transfer stagnation-point flow in nanofluids over stretching/shrinking sheet in a porous medium with chemical reaction. *Nuclear Eng and Design* 273,644-652.

Rashidi, M.M., B. Rostami,N. Freidoonimehrand S.Abbasbandy(2014).Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of theradiationand buoyancy effects.*Ain Shams Engineering Journal* 5, 901-912.

Sandeep, N., A.V. B. Reddy and V. Sugunamma(2012).Effect of radiation and chemical reaction on transient MHD free convective flow over a vertical plate through porousmedia.*Chemical and process engineering research*2,1-9.

Sandeep, N. andV. Sugunamma (2014). Radiation and inclined magnetic field effects on unsteady MHD convective flow past an impulsively moving vertical plate in a porous medium.*Journal of Applied Fluid Mechanics* 7(2),275-286.

Sandeep, N. and C.Sulochana(2015).Dual solutions of radiative MHD nanofluid flow over an exponentially xstretching sheet with heat generation or absorption. *ApplNanosci.*

Singh, V. andS. Agarwal(2014).MHD flow and heat transfer for Maxwell fluid over an exponentially stretching sheet with variable thermal conductivity in porous medium.*Thermal Sci*18, S599-S615.

Subhas Abel, M., P. G. Siddheswar and N. Mahesha(2009). Effects of thermal buoyancy and variable thermal conductivity on the MHD flow and heat transfer in a power-law fluid past a vertical sheet in the presence of non-uniform heat source. *Int J Nonlinear Mechanics*44,1-12.