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Duality for semi-infinite programming problems involving (H_p, r)-invex functions

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Abstract
The present paper is framed to study weak, strong and strict converse duality relations for a semi-infinite programming problem and its Wolfe and Mond-Weir-type dual programs under generalized (H_p, r)-invexity.
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1 Introduction

The root of optimization theory is penetrating into other branch of applied sciences at a rapid pace. Semi-infinite programming is a special case of bilevel programs (multilevel programming) in which lower-level variables do not participate in the objective function. In 1962, the theory of semi-infinite programming (SIP) was developed by Charnes *et al.* [1]. There are many practical as well as theoretical problems in which constraint depend on time and space and thus can be formulated as semi-infinite programming problems. In recent past, semi-infinite programming has became one of the most interesting research topic in the field of operation research as it has wide variety of applications in control of robots [2], transportation theory [3], eigenvalue computations [4], wavelet filter design [5, 6], statistical design [7], *etc.* Duality of semi-infinite programming arises in the theory of systems of linear inequalities, in the theory of uniform approximations of functions and in the classical theory of moments.

In the course of generalization of convex functions, Avriel [8] first introduced the definition of r-convex functions and established some characterizations and the relations between r-convexity and other generalization of convexity. Antczak [9] introduced the concept of a class of r-preinvex functions, which is a generalization of r-convex functions and preinvex functions, and obtained some optimality results under r-preinvexity assumption for constrained optimization problems. Lee and Ho [10] established necessary and sufficient conditions for efficiency of multiobjective fractional programming problems involving r-invex functions and investigated the parametric, Wolfe and Mond-Weir-type dual for multiobjective fractional programming problems concerning r-invexity. In order to generalize the notion of invex and pre-invex functions, Antczak [11] introduced p-invex sets and (p, r)-invex functions and derived sufficient optimality conditions for a nonlinear programming problem involving (p, r)-invex functions. Gupta and Kailey [12] introduced a new pair of second-order multiobjective symmetric dual programs over arbitrary cones and derived appropriate duality theorems under $K - \eta$-bonvexity assumptions.

Many practical and real situations give rise to logarithmic and exponential functions. Keeping this point of view, Yuan *et al.* [13] introduced locally (H_p, r, α)-preinvex functions and locally H_p-invex sets and derived necessary and sufficient optimality conditions for nonlinear programming problems. One of the major step is taken by Liu *et al.* [14] in the direction of obtaining sufficient optimality conditions for multiple objective programming problem and multiobjective fractional programming problem involving (H_p, r)-invex functions.

Taking into account the importance of duality results in optimization theory (see [9, 11, 15–17]), we generalize the notion of (H_p, r)-invex functions introduced by Yuan *et al.* [13] to (strict) (H_p, r)-pseudoinvex and (H_p, r)-quasiinvex functions and derive duality results for semi-infinite programming problems.

The rest of the paper is organized as follows: In Section 2, we focus on some notation and definitions. In Sections 3 and 4, weak, strong and strict converse duality theorems are established for Wolfe and Mond-Weir-type dual programs under generalized (H_p, r)-invexity. Conclusion and future works are given in Section 5.

2 Notation and preliminaries

Throughout the paper, let R^n be the n-dimensional Euclidean space, $R^n_+ = \{x \in R^n \mid x \geq 0\}$ and $\dot{R}^n_+ = \{x \in R^n \mid x > 0\}$.

Definition 2.1 [11] The weighted r-mean of a_1 and a_2 ($a_1, a_2 > 0$) is given by

$$M_r(a_1, a_2; \lambda) = \begin{cases} (\lambda {a_1}^r + (1-\lambda) a_2^r)^{\frac{1}{r}}, & \text{for } r \neq 0, \\ a_1^\lambda a_2^{(1-\lambda)}, & \text{for } r = 0, \end{cases}$$

where $\lambda \in (0,1)$ and $r \in R$.

Definition 2.2 A subset $X \subset R^n$ is said to be H_p-invex set, if for any $x, u \in X$, there exists a vector function $H_p : X \times X \times [0,1] \to R^n$, such that

$$H_p(x, u; 0) = e^u, \quad H_p(x, u; \lambda) \in \dot{R}^n_+,$$
$$\ln\bigl(H_p(x, u; \lambda)\bigr) \in X, \quad \forall \lambda \in [0,1], p \in R.$$

Remark 2.1 It is understood that the logarithm and the exponentials appearing in the above definitions are taken to be componentwise.

Throughout the paper, we take X to be a H_p-invex set unless otherwise specified, H_p right differentiable at 0 with respect to the variable λ for each given pair $x, u \in X$, and $f : X \to R$ is differentiable function on X. The symbol $H'_p(x, u; 0+) \triangleq (H'_{p1}(x, u; 0+), \ldots, H'_{pn}(x, u; 0+))^T$ denotes the right derivative of H_p at 0 with respect to the variable λ for each given pair $x, u \in X$; $\nabla f(x) \triangleq (\nabla_1 f(x), \ldots, \nabla_n f(x))^T$ denotes the differential of f at x, and so $\frac{\nabla f(u)}{e^u}$ denotes $(\frac{\nabla_1 f(u)}{e^{u_1}}, \ldots, \frac{\nabla_n f(u)}{e^{u_n}})^T$.

Remark 2.2 All the theorems in the subsequent parts of this paper will be proved only in the case when $r \neq 0$. The proofs in other cases are easier than this since only changes arise from form of inequality. Moreover, without loss of generality, we shall assume that $r > 0$ (in the case when $r < 0$, the direction some of the inequalities in the proof of the theorems should be changed to the opposite one).

Definition 2.3 [14] A differentiable function $f : X \to R$ is said to be (strictly) (H_p, r)-invex at $u \in X$, if for all $x \in X$, one of the relations

$$\frac{1}{r}\big[e^{r(f(x)-f(u))} - 1\big] \geq \frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) \quad (>) \text{ for } r \neq 0,$$
$$f(x) - f(u) \geq \frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) \quad (>) \text{ for } r = 0,$$

hold.

If the above inequalities are satisfied at any point $u \in X$ then f is said to be (H_p, r)-invex (strictly (H_p, r)-invex) on X.

Now, we introduce the generalized (H_p, r)-invex function as follows.

Definition 2.4 A differentiable function $f : X \to R$ is said to be (H_p, r)-pseudoinvex at $u \in X$, if for all $x \in X$, the relations

$$\frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) \geq 0 \quad \Rightarrow \quad \frac{1}{r}\big[e^{r(f(x)-f(u))} - 1\big] \geq 0, \quad \text{for } r \neq 0,$$
$$\frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) \geq 0 \quad \Rightarrow \quad f(x) - f(u) \geq 0, \quad \text{for } r = 0,$$

hold.

If the above inequalities are satisfied at any point $u \in X$ then f is said to be (H_p, r)-pseudoinvex on X.

Definition 2.5 A differentiable function $f : X \to R$ is said to be strict (H_p, r)-pseudoinvex at $u \in X$, if for all $x \in X$, the relations

$$\frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) \geq 0 \quad \Rightarrow \quad \frac{1}{r}\big[e^{r(f(x)-f(u))} - 1\big] > 0, \quad \text{for } r \neq 0,$$
$$\frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) \geq 0 \quad \Rightarrow \quad f(x) - f(u) > 0, \quad \text{for } r = 0,$$

hold.

If the above inequalities are satisfied at any point $u \in X$ then f is said to be strict (H_p, r)-pseudoinvex on X.

Definition 2.6 A differentiable function $f : X \to R$ is said to be (H_p, r)-quasiinvex at $u \in X$, if for all $x \in X$, the relations

$$\frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) > 0 \quad \Rightarrow \quad \frac{1}{r}\big[e^{r(f(x)-f(u))} - 1\big] > 0, \quad \text{for } r \neq 0,$$
$$\frac{\nabla f(u)^T}{e^u} H_p'(x, u; 0+) > 0 \quad \Rightarrow \quad f(x) - f(u) > 0, \quad \text{for } r = 0,$$

hold.

If the above inequalities are satisfied at any point $u \in X$ then f is said to be (H_p, r)-quasiinvex on X.

Let us consider the following semi-infinite programming (SIP) problems:

$$\text{(SIP)} \quad \min_{x \in R^n} f(x)$$
$$\text{subject to}$$
$$g_i(x) \leq 0, \quad i \in I,$$

where I is an index set which is possibly infinite, f and g_i, $i \in I$ are differentiable functions from R^n to $R \cup \{+\infty\}$.

3 First duality model

In this section, we consider the following Wolfe-type dual to (SIP):

$$\text{(WSID)} \quad \text{Maximize} \quad f(u) + \sum_{i \in I} \lambda_i g_i(u)$$
$$\text{subject to}$$
$$\nabla f(u) + \sum_{i \in I} \lambda_i \nabla g_i(u) = 0, \tag{1}$$

where $\lambda_i \geq 0$ and $\lambda_i \neq 0$ for finitely many $i \in I$.

Theorem 3.1 (Weak duality) *Let x and (u, λ), $\lambda = (\lambda_i)$, $i \in I$, be feasible solution to* (SIP) *and* (WSID), *respectively. Assume that $f(\cdot) + \sum_{i \in I} \lambda_i g_i(\cdot)$ be (H_p, r)-invex at u. Then the following cannot hold*:

$$f(x) < f(u) + \sum_{i \in I} \lambda_i g_i(u).$$

Proof Suppose contrary to the result, *i.e.*,

$$f(x) < f(u) + \sum_{i \in I} \lambda_i g_i(u),$$

which together with the feasibility of x to (SIP) gives

$$f(x) - f(u) + \sum_{i \in I} \lambda_i g_i(x) - \sum_{i \in I} \lambda_i g_i(u) < 0.$$

Since $r > 0$, using the fundamental properties of exponential function, the above inequality yields

$$\frac{1}{r}\left[e^{r(f(x) - f(u) + \sum_{i \in I} \lambda_i g_i(x) - \sum_{i \in I} \lambda_i g_i(u))} - 1\right] < 0.$$

The above inequality together with the assumption that $f(\cdot) + \sum_{i \in I} \lambda_i g_i(\cdot)$ is (H_p, r)-invex at u, we obtain

$$\frac{[\nabla f(u) + \sum_{i \in I} \lambda_i \nabla g_i(u)]^T}{e^u} H'_p(x, u, 0+) < 0,$$

which contradicts (1). This completes the proof. □

The proof of the following theorem is similar to Theorem 3.1, and hence being omitted.

Theorem 3.2 (Weak duality) *Let x and (u,λ), $\lambda = (\lambda_i)$, $i \in I$, be feasible solution to* (SIP) *and* (WSID), *respectively. Assume that $f(\cdot) + \sum_{i\in I} \lambda_i g_i(\cdot)$ be (H_p, r)-pseudoinvex at u. Then the following cannot hold*:

$$f(x) < f(u) + \sum_{i\in I} \lambda_i g_i(u).$$

Theorem 3.3 (Strong duality) *Let $\bar{x}$ be an optimal solution for* (SIP) *and $\bar{x}$ satisfies a suitable constraints qualification for* (SIP). *Then there exists $\bar{\lambda} = (\bar{\lambda}_i)$, $i \in I$ such that $(\bar{x}, \bar{\lambda})$ is feasible for* (WSID). *If any of the weak duality in Theorems* 3.1 *or* 3.2 *also holds, then $(\bar{x}, \bar{\lambda})$ is an optimal solution for* (WSID).

Proof Since $\bar{x}$ is optimal solution for (SIP) and satisfy the suitable constraint qualification for (SIP), then from Kuhn-Tucker necessary optimality condition there exists $\bar{\lambda} = (\bar{\lambda}_i)$, $i \in I$ such that

$$\nabla f(\bar{x}) + \sum_{i\in I} \bar{\lambda}_i \nabla g_i(\bar{x}) = 0, \qquad \bar{\lambda}_i g_i(\bar{x}) = 0,$$

which gives that the $(\bar{x}, \bar{\lambda})$ is feasible for (WSID). The optimality of $(\bar{x}, \bar{\lambda})$ for (WSID) follows from weak duality theorems. This completes the proof. □

Theorem 3.4 (Strict converse duality) *Let $\bar{x}$ and $(\bar{y}, \bar{\lambda})$ be feasible solutions to* (SIP) *and* (WSID), *respectively. Assume that $f(\cdot) + \sum_{i\in I} \bar{\lambda}_i g_i(\cdot)$ is strictly (H_p, r)-invex at $\bar{y}$. Further assume that*

$$f(\bar{x}) \leq f(\bar{y}) + \sum_{i\in I} \bar{\lambda}_i g_i(\bar{y}).$$

Then $\bar{x} = \bar{y}$.

Proof Let $\bar{x}$ be feasible solution to (SIP) and $(\bar{y}, \bar{\lambda})$ be feasible to (WSID). Then

$$\nabla f(\bar{y}) + \sum_{i\in I} \bar{\lambda}_i \nabla g_i(\bar{y}) = 0. \tag{2}$$

Now, we assume that $\bar{x} \neq \bar{y}$ and exhibit a contradiction.

From the assumption that $f(\cdot) + \sum_{i\in I} \bar{\lambda}_i g_i(\cdot)$ is strictly (H_p, r)-invex at $\bar{y}$, we have

$$\frac{1}{r}\left[e^{r(f(\bar{x})+\sum_{i\in I}\bar{\lambda}_i g_i(\bar{x})-f(\bar{y})-\sum_{i\in I}\bar{\lambda}_i g_i(\bar{y}))} - 1\right] > \frac{\nabla f(\bar{y}) + \sum_{i\in I} \bar{\lambda}_i \nabla g_i(\bar{y})}{e^{\bar{y}}}^T H_p'(\bar{x}, \bar{y}; 0+),$$

which by the virtue of (2) becomes

$$\frac{1}{r}\left[e^{r(f(\bar{x})+\sum_{i\in I}\bar{\lambda}_i g_i(\bar{x})-f(\bar{y})-\sum_{i\in I}\bar{\lambda}_i g_i(\bar{y}))} - 1\right] > 0.$$

As $r > 0$, using the fundamental properties of exponential functions, we get

$$f(\bar{x}) + \sum_{i\in I} \bar{\lambda}_i g_i(\bar{x}) - f(\bar{y}) - \sum_{i\in I} \bar{\lambda}_i g_i(\bar{y}) > 0.$$

From the feasibility of x to (SIP), the above inequality yields

$$f(\bar{x}) > f(\bar{y}) + \sum_{i\in I} \bar{\lambda}_i g_i(\bar{y}),$$

which contradicts the assumption that $f(\bar{x}) \leq f(\bar{y}) + \sum_{i\in I} \bar{\lambda}_i g_i(\bar{y})$. Hence $\bar{x} = \bar{y}$. This completes the proof. □

We now prove the duality relations for the following Mond-Weir-type dual problem.

4 Second duality model

$$\text{(MWSID)} \quad \text{Maximize } f(u)$$

subject to

$$\nabla f(u) + \sum_{i\in I} \lambda_i \nabla g_i(u) = 0, \tag{3}$$

$$\sum_{i\in I} \lambda_i g_i(u) \geq 0, \tag{4}$$

where $\lambda_i \geq 0$ and $\lambda_i \neq 0$ for finitely many $i \in I$.

Theorem 4.1 (Weak duality) *Let x and (u, λ), $\lambda = (\lambda_i)$, $i \in I$, be feasible solution to* (SIP) *and* (MWSID), *respectively. Assume that $f(\cdot)$ and $\sum_{i\in I} \lambda_i g_i(\cdot)$ be (H_p, r)-invex at u. Then the following cannot hold*:

$$f(x) < f(u).$$

Proof Suppose contrary to the result, *i.e.*,

$$f(x) < f(u).$$

As $r > 0$, the above inequality along with the fundamental properties of exponential function yields

$$\frac{1}{r}\left[e^{r(f(x)-f(u))} - 1\right] < 0.$$

The above inequality together with the assumption that $f(\cdot)$ is (H_p, r)-invex at u, gives

$$\frac{\nabla f(u)}{e^u}^T H_p'(x, u; 0+) < 0. \tag{5}$$

Since $\lambda_i \geq 0$, and $\lambda_i \neq 0$ for finitely many $i \in I$, from the feasibility of x and (u, λ) to (SIP) and (MWSID), respectively, we obtain

$$\sum_{i\in I} \lambda_i g_i(x) \leq 0 \leq \sum_{i\in I} \lambda_i g_i(u).$$

As $r > 0$, using the fundamental properties of exponential functions, we get

$$\frac{1}{r}\left[e^{r(\sum_{i\in I}\lambda_i g_i(x)-\sum_{i\in I}\lambda_i g_i(u))}-1\right]\leq 0,$$

which by the virtue of (H_p, r)-invexity of $\sum_{i\in I}\lambda_i g_i(\cdot)$ at u, gives

$$\frac{\sum_{i\in I}\lambda_i\nabla g_i(u)}{e^u}^T H_p'(x,u;0+)\leq 0. \tag{6}$$

On adding (5) and (6) gives

$$\frac{[\nabla f(u)+\sum_{i\in I}\lambda_i\nabla g_i(u)]}{e^u}^T H_p'(x,u;0+)<0,$$

which contradicts (3). This completes the proof. □

The proof of the following theorem is similar to Theorem 4.1, and hence being omitted.

Theorem 4.2 (Weak duality) *Let x and (u,λ), $\lambda=(\lambda_i)$, $i\in I$, be feasible solution to* (SIP) *and* (MWSID), *respectively. Assume that $f(\cdot)$ is (H_p,r)-pseudoinvex and $\sum_{i\in I}\lambda_i g_i(\cdot)$ is (H_p,r)-quasiinvex at u. Then the following cannot hold*:

$$f(x)<f(u).$$

Theorem 4.3 (Strong duality) *Let $\bar{x}$ be an optimal solution for* (SIP) *and $\bar{x}$ satisfies a suitable constraints qualification for* (SIP). *Then there exists $\bar{\lambda}=(\bar{\lambda}_i)$, $i\in I$ such that $(\bar{x},\bar{\lambda})$ is feasible for* (MWSID). *If any of the weak duality in Theorems* 4.1 *or* 4.2 *also holds, then $(\bar{x},\bar{\lambda})$ is an optimal solution for* (MWSID).

Proof Since $\bar{x}$ is optimal solution for (SIP) and satisfy the suitable constraint qualification for (SIP), then from Kuhn-Tucker necessary optimality condition there exists $\bar{\lambda}=(\bar{\lambda}_i)$, $i\in I$ such that

$$\nabla f(\bar{x})+\sum_{i\in I}\bar{\lambda}_i\nabla g_i(\bar{x})=0,\qquad \bar{\lambda}_i g_i(\bar{x})=0,$$

which gives that the $(\bar{x},\bar{\lambda})$ is feasible for (MWSID). The optimality of $(\bar{x},\bar{\lambda})$ for (MWSID) follows from weak duality theorems. This completes the proof. □

Theorem 4.4 (Strict converse duality) *Let $\bar{x}$ and $(\bar{y},\bar{\lambda})$ be feasible solutions to* (SIP) *and* (MWSID), *respectively. Assume that $f(\cdot)$ is strictly (H_p,r)-pseudoinvex and $\sum_{i\in I}\bar{\lambda}_i g_i(\cdot)$ is (H_p,r)-quasiinvex at $\bar{y}$. Further assume that*

$$f(\bar{x})\leq f(\bar{y}).$$

Then $\bar{x}=\bar{y}$.

Proof Let $\bar{x}$ be feasible solution to (SIP) and $(\bar{y},\bar{\lambda})$ be feasible to (MWSID). Then

$$\nabla f(\bar{y}) + \sum_{i\in I}\bar{\lambda}_i\nabla g_i(\bar{y}) = 0. \tag{7}$$

Now, we assume that $\bar{x} \neq \bar{y}$ and exhibit a contradiction.

Since $\lambda_i \geq 0$, and $\lambda_i \neq 0$ for finitely many $i \in I$, from the feasibility of $\bar{x}$ and $(\bar{y},\bar{\lambda})$ to (SIP) and (MWSID), respectively, we obtain

$$\sum_{i\in I}\bar{\lambda}_i g_i(\bar{x}) \leq 0 \leq \sum_{i\in I}\bar{\lambda}_i g_i(\bar{y}).$$

As $r > 0$, using the fundamental properties of exponential functions, we get

$$\frac{1}{r}\left[e^{r(\sum_{i\in I}\bar{\lambda}_i g_i(\bar{x})-\sum_{i\in I}\bar{\lambda}_i g_i(\bar{y}))} - 1\right] \leq 0,$$

which by the virtue of (H_p,r)-quasiinvexity of $\sum_{i\in I}\bar{\lambda}_i g_i(\cdot)$ at $\bar{y}$, gives

$$\frac{\sum_{i\in I}\bar{\lambda}_i\nabla g_i(\bar{y})}{e^{\bar{y}}}^T H_p'(\bar{x},\bar{y};0+) \leq 0,$$

which along with (7) gives

$$\frac{\nabla f(\bar{y})}{e^{\bar{y}}}H_p'(\bar{x},\bar{y};0+) \geq 0.$$

From the above inequality together with the assumption that $f(\cdot)$ is strictly (H_p,r)-pseudoinvex at $\bar{y}$, we obtain

$$\frac{1}{r}\left[e^{r(f(\bar{x})-f(\bar{y}))} - 1\right] > 0,$$

which by the fundamental properties of exponential functions, yields

$$f(\bar{x}) > f(\bar{y}),$$

which contradicts the fact that $f(\bar{x}) \leq f(\bar{y})$. Hence, $\bar{x} = \bar{y}$. This completes the proof. □

5 Conclusions

In the present paper, we introduced generalized (H_p,r)-invex functions and consider two types of dual program for a class of semi-infinite programming problem to establish the weak, strong and strict converse duality theorems assuming the functions involved to be generalized (H_p,r)-invex functions. In fact, a lot of efforts have been taken to extend some known results for generalized invex functions, for example, see [9, 11, 14, 15]. That is why we conclude that this paper enriched optimization theory as far as mathematics is concerned. Although there are some difficulties (like constructing the suitable examples or counter examples to show the existence), the semi-infinite programming problems involving the generalized invex functions are very interesting. As a future scope, the authors would like to extend the results to fractional semi-infinite programming problem.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

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References

1. Charnes, A, Cooper, WW, Kortanek, K: A duality theory for convex programs with convex constraints. Bull. Am. Math. Soc. **68**, 605-608 (1962)
2. Demeulenaere, B, Schutter, JD, Swevers, J: Robust high-order repetitive control: optimal performance trade-offs. Automatica **44**, 2628-2634 (2008)
3. Kortanek, K, Yamasaki, M: Semi-infinite transportation problems. J. Math. Anal. Appl. **88**, 555-565 (1982)
4. Hettich, R, Zencke, P: Two Case-Studies in Parametric Semi-Infinite Programming. In: Systems and Optimization. Lecture Notes in Control and Information Sciences, vol. 66, pp. 132-155 (1985)
5. Kirac, A: Theory and design of optimum FIR compaction filters. IEEE Trans. Signal Process. **46**, 903-917 (1998)
6. Kortanek, K, Moulin, P: Semi-infinite programming in orthogonal wavelet filter design. In: Reemtsen, R, Rückmann, J-J (eds.) Semi-Infinite Programming. Nonconvex Optim. Appl., pp. 323-357. Kluwer Academic, Dordrecht (1998)
7. Gürtuna, F: Duality of ellipsoidal approximations via semi-infinite programming. SIAM J. Optim. **20**, 1421-1438 (2010)
8. Avriel, M: r-Convex functions. Math. Program. **2**, 309-323 (1972)
9. Antczak, T: r-Preinvexity and r-invexity in mathematical programming. Comput. Math. Appl. **50**, 551-566 (2005)
10. Lee, JC, Ho, SC: Optimality and duality for multiobjective fractional problems with r-invexity. Taiwan. J. Math. **12**, 719-740 (2008)
11. Antczak, T: (p, r)-Invex sets and functions. J. Math. Anal. Appl. **263**, 355-379 (2001)
12. Gupta, SK, Kailey, N: Second-order multiobjective symmetric duality involving cone-bonvex functions. J. Glob. Optim. **55**, 125-140 (2013)
13. Yuan, DH, Liu, XL, Yang, SY, Nyamsuren, D, Altannar, C: Optimality conditions and duality for nonlinear programming problems involving locally (H_p, r, α)-pre-invex functions and H_p-invex sets. Int. J. Pure Appl. Math. **41**, 561-576 (2007)
14. Liu, X, Yuan, D, Yang, S, Lai, G: Multiple objective programming involving differentiable (H_p, r)-invex functions. CUBO **13**, 125-136 (2011)
15. Ahmad, I, Gupta, SK, Kailey, N, Agarwal, RP: Duality in nondifferentiable minimax fractional programming with B-(p, r)-invexity. J. Inequal. Appl. **2011**, 75 (2011)
16. Ahmad, I, Husain, Z: Second order (F, α, ρ, d)-convexity and duality in multiobjective programming. Inf. Sci. **176**, 3094-3103 (2006)
17. Soleimani-damaneh, M, Sarabi, ME: Sufficient conditions for nonsmooth r-invexity. Numer. Funct. Anal. Optim. **29**, 674-686 (2008)

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