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Dynamics of a two prey and one predator system with time interruption and random fluctuations

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ABSTRACT

The present work focuses on the dynamics of a two prey and one predator model where the preys are subjected to the logistic growth law and the predator is subjected to the mortality rate and intra specific competition. The effects of time delay and random environmental fluctuations on the stability of the model around the interior equilibrium point are analytically tested using classical mathematical tools. The stable, periodic and chaotic behaviours of the model for different sets of chosen parameters are explored in numerical simulations.

1. Introduction

There has been a large amount of research during the last few decades regarding the dynamical features of inhabitant structures [1–6]. Among these, hunter–victim systems play a vital role in inhabitant dynamics. The dynamical association between hunters and their victims has been a significant issue in theoretical environmental science since the renowned Lotka–Volterra equations. Theoretical environmental science has not addressed the surprising variety of the dynamic performances of three species structures for a long time. Freedman and Waltman [7] evaluated three level food webs – a single predator feeding on two conflicting prey species. They determined conditions for the system to be persistent. Kar and Chaudhuri [8] deliberated a two-prey one-predator harvesting structure with interference. The structure was founded on Lotka-Volterra dynamics with two conflicting species, which are pretentious not only by harvesting but also by the presence of a predator. The possibility of the existence of a bionomic stability and ideal harvesting plan is discussed. Dubey and Upadhyay [9] proposed a two predator one prey structure with a ratio reliant on the predator evolution rate. The conditions for the native stability and overall stability of the interior equilibria were achieved. They also discussed the enduring co-existence of the three species. Braza [10] considered a two predator and one prey structure in which one predator significantly interferes with the other examined predator. Zhang et al. [11] studied the steadiness of three species inhabitant structure that consists of an endemic prey, alien prey and alien predator. The utmost decisive portions in predator–prey structures are the ‘interaction with delay’ and ‘effect of noise on the stability of the system’. Numerous models of the inhabitant's growth were also studied with time delays [12–17]. Some other authors [18,19] studied the dynamics of prey predator models by including attribute-like stochastics.

2. Mathematical equations

The two preys and one predator model with time delay is represented by the following system of three nonlinear delay differential equations:

$$x'(t) = r_1 x\left(1 - \frac{x}{k_1}\right) - \alpha_{13}xz \tag{2.1}$$

$$y'(t) = r_2 y\left(1 - \frac{y}{k_2}\right) - \alpha_{23}yz \tag{2.2}$$

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$$z'(t) = -dz - \alpha_{33}z^2 + \alpha_{31}x(t-\tau)z(t-\tau) + \alpha_{32}y(t-\tau)z(t-\tau) \quad (2.3)$$

where $x(t)$, $y(t)$ and $z(t)$ represent the population density of prey1, prey2 and predator species, respectively. r_1 and r_2 represent the intrinsic growth rates of prey1 and prey2, respectively. k_1 and k_2 represent the carrying capacities of prey1 and prey2, respectively. α_{13} and α_{23} represent the decrease rates of prey1 and prey2, respectively, due to predation. α_{31} and α_{32} represent the gain rates of the predator due to the predation of prey1 and prey2, respectively. d denotes the mortality rate of the predator. α_{33} denotes the decreased rate of the predator due to intra-specific competition. τ represents the time delay parameter. All of the model parameters are assumed to be positive.

3. Analysis of steady states

The interior equilibrium point $E^*(x^*, y^*, z^*)$ of the system is given by

$$x^* = \frac{\Delta_1}{\Delta}, y^* = \frac{\Delta_2}{\Delta}, z^* = \frac{\Delta_3}{\Delta}$$

where

$$\Delta = -\frac{r_1}{k_1}\left(\frac{r_2}{k_2}\alpha_{33} + \alpha_{23}\alpha_{32}\right) - \alpha_{13}\alpha_{31}\frac{r_2}{k_2},$$

$$\Delta_1 = -r_1\left(\frac{r_2}{k_2}\alpha_{33} + \alpha_{23}\alpha_{32}\right) + \alpha_{13}\left(r_2\alpha_{32} - d\frac{r_2}{k_2}\right)$$

$$\Delta_2 = -\frac{r_1}{k_1}(r_2\alpha_{33} + d\alpha_{23}) + r_1\alpha_{31}\alpha_{23} - r_2\alpha_{13}\alpha_{31},$$

$$\Delta_3 = \frac{r_1}{k_1}\left(d\frac{r_2}{k_2} - r_2\alpha_{32}\right) - r_1\alpha_{31}\frac{r_2}{k_2}$$

3.1. Boundedness and analysis of the limit cycle

Theorem (3.1.1): The solution of the system (2.1)–(2.3) in R^3_+ for $t \geq 0$ is bounded.

Proof. Let $w(t) = x(t) + y(t) + z(t)$ and $\eta > 0$ be a constant. Then, $w'(t) + \eta w = x'(t) + y'(t) + z'(t) + \eta w$

$$w'(t) + \eta w = (r_1 + \eta)x - \frac{r_1}{k_1}x^2 - \alpha_{13}xz + (r_2 + \eta)y - \frac{r_2}{k_2}y^2 - \alpha_{23}yz + (r_3 + \eta)z - \frac{r_3}{k_3}z^2 + \alpha_{31}xz + \alpha_{32}yz$$

If $\alpha_{13} \geq \alpha_{31}$, $\alpha_{23} \geq \alpha_{32}$,
$w'(t) + \eta w \leq (r_1 + \eta)x - \frac{r_1}{k_1}x^2 + (r_2 + \eta)\, y - \frac{r_2}{k_2}y^2 + (r_3 + \eta)z - \frac{r_3}{k_3}z^2$

$$w'(t) + \eta w \leq -\frac{r_1}{k_1}\left(x - \frac{k_1(r_1 + \eta)}{2r_1}\right)^2 + \frac{k_1(r_1 + \eta)^2}{6r_1} - \frac{r_2}{k_2}\left(y - \frac{k_2(r_2 + \eta)}{2r_2}\right)^2 + \frac{k_2(r_2 + \eta)^2}{6r_2} - \frac{r_3}{k_3}\left(z - \frac{k_3(r_3 + \eta)}{2r_3}\right)^2 + \frac{k_3(r_3 + \eta)^2}{6r_3}$$

$$w'(t) + \eta w \leq \frac{k_1(r_1 + \eta)^2}{6r_1} + \frac{k_2(r_2 + \eta)^2}{6r_2} + \frac{k_3(r_3 + \eta)^2}{6r_3} = \mu(say)$$

By using the differential inequality, we obtain $w'(t) + \eta w = \mu$, and its solution is $w = \frac{\mu}{\eta} + ce^{-\eta t}$. By $t = 0$, we obtain $w(x(0), y(0)) = \mu/\eta + c$, and then, $c = w(x(0), y(0)) - \mu/\eta$. Hence, $w(x(t), y(t)) = \frac{\mu}{\eta}(1 - e^{-\eta t}) + (w(x(0), y(0)))e^{-\eta t}$ and where $0 < w(x(t), y(t)) \leq \frac{\mu}{\eta}(1 - e^{-\eta t}) + (w(x(0), y(0)))e^{-\eta t}$. By taking the limit $t \to \infty$, we have $0 < w(t) \leq \mu/\eta$. This proves the theorem.

3.2. Analysis of limit cycle

Theorem (3.2.1): (a) The system (2.1)–(2.3) cannot have any periodic solution in the interior of the quadrant of the xy-plane. (b) The system (2.1)–(2.3) cannot have any periodic solution in the interior of the quadrant of the yz-plane. (c) The system (2.1)–(2.3) cannot have any periodic solution in the interior of the quadrant of the zx-plane.

Proof of Theorem (3.2.1): (a) $H_1(x, y) = \frac{1}{xy}$, $h_1(x, y) = r_1x\left(1 - \frac{x}{k_1}\right)$, $h_2(x, y) = r_2y\left(1 - \frac{y}{k_2}\right)$

$$\Delta(x, y) = \frac{\partial}{\partial x}(h_1H_1) + \frac{\partial}{\partial y}(h_2H_1) = \frac{\partial}{\partial x}\left(\frac{r_1}{y}\left(1 - \frac{x}{k_1}\right)\right) + \frac{\partial}{\partial y}\left(\frac{r_2}{x}\left(1 - \frac{y}{k_2}\right)\right) = -\frac{r_1}{k_1y} - \frac{r_2}{k_2x} < 0$$

(b) $H_2(y, z) = \frac{1}{yz}, h_3(y, z) = r_2y\left(1 - \frac{y}{k_2}\right) - \alpha_{23}yz, h_4(y, z) = r_3z\left(1 - \frac{z}{k_3}\right) + \alpha_{32}yz$

$$\Delta(y, z) = \frac{\partial}{\partial y}(h_3H_2) + \frac{\partial}{\partial z}(h_4H_2) = \frac{\partial}{\partial y}\left(\frac{r_2}{z}\left(1 - \frac{y}{k_2}\right) - \alpha_{23}\right) + \frac{\partial}{\partial z}\left(\frac{r_3}{y}\left(1 - \frac{z}{k_3}\right) + \alpha_{32}\right) = -\frac{r_2}{k_2z} - \frac{r_3}{k_3y} < 0$$

(c) $H_3(x, z) = \frac{1}{xz}$, $h_5(x, z) = r_1x\left(1 - \frac{x}{k_1}\right) - \alpha_{13}xz$, $h_6(x, z) = r_3z\left(1 - \frac{z}{k_3}\right) + \alpha_{32}yz$

$$\Delta(x, z) = \frac{\partial}{\partial x}(h_5H_3) + \frac{\partial}{\partial z}(h_6H_3) = \frac{\partial}{\partial x}\left(\frac{r_1}{z}\left(1 - \frac{x}{k_1}\right) - \alpha_{13}\right) + \frac{\partial}{\partial z}\left(\frac{r_3}{x}\left(1 - \frac{z}{k_3}\right) + \alpha_{31}\right) = -\frac{r_1}{k_1z} - \frac{r_3}{k_3x} < 0.$$

From the above equation, we note that $\Delta(x, y)$ does not change sign and is not identically zero in the interior of the positive quadrant of the xy, yz, and zx-planes.

4. Discussion of local and global stability without delay

The variational matrix of the system (2.1)–(2.3) with $\tau = 0$ evaluated at the interior equilibrium point is

$$J = \begin{bmatrix} -\frac{r_1}{k_1}x^* & 0 & -\alpha_{13}x^* \\ 0 & -\frac{r_2}{k_2}y^* & -\alpha_{23}y^* \\ \alpha_{31}z^* & \alpha_{32}z^* & -\alpha_{33}z^* \end{bmatrix}$$

and the characteristic equation of J is $\lambda^3 + \eta_1\lambda^2 + \eta_2\lambda + \eta_3 = 0$ where

$$\eta_1 = \frac{r_1}{k_1}x^* + \frac{r_2}{k_2}y^* + \alpha_{33}z^*$$

$$\eta_2 = \frac{r_1}{k_1}\frac{r_2}{k_2}x^*y^* + \left(\frac{r_1}{k_1}\alpha_{33} + \alpha_{13}\alpha_{31}\right)x^*z^* + \left(\frac{r_2}{k_2}\alpha_{33} + \alpha_{23}\alpha_{32}\right)y^*z^*$$

$$\eta_3 = \left(\frac{r_1}{k_1}\frac{r_2}{k_2}\alpha_{33} + \frac{r_1}{k_1}\alpha_{23}\alpha_{32} + \frac{r_2}{k_2}\alpha_{13}\alpha_{31}\right)x^*y^*z^*$$

According to the Routh-Hurwitz criteria, the system is locally asymptotically stable.

Let us now consider the following Lyapunov function to verify the global asymptotic stable behaviour of the interior equilibrium point.

$$V(x,y) = x - x^* - x^*\ln\left(\frac{x}{x^*}\right) + l_1\left(y - y^* - y^*\ln\left(\frac{y}{y^*}\right)\right) + l_2\left(z - z^* - z^*\ln\left(\frac{z}{z^*}\right)\right)$$

$$V'(t) = (x-x^*)\left[r_1\left(1-\frac{x}{k_1}\right) - \alpha_{13}z\right] + l_1(y-y^*)\left[r_2\left(1-\frac{y}{k_2}\right) - \alpha_{23}z\right] + l_2(z-z^*)(-d-\alpha_{33}z+\alpha_{31}x+\alpha_{32}y)$$

If $l_1 = \frac{\alpha_{32}}{\alpha_{23}}\frac{\alpha_{13}}{\alpha_{31}}, l_2 = \frac{\alpha_{13}}{\alpha_{31}}$, then we have

$$V'(t) = -\frac{r_1}{k_1}(x-x^*)^2 - \frac{r_2}{k_2}\frac{\alpha_{32}}{\alpha_{23}}\frac{\alpha_{13}}{\alpha_{31}}(y-y^*)^2 - \alpha_{33}\frac{\alpha_{13}}{\alpha_{31}}(z-z^*)^2 < 0$$

Hence, the system is globally asymptotically stable near E^*.

5. Analysis of stability with delay

The characteristic equation of the delayed model (2.1)–(2.3) evaluated at the interior equilibrium point E^* is

$$X(\lambda) + e^{-\lambda\tau}Y(\lambda) = 0 \tag{5.1}$$

where

$$X(\lambda) = \lambda^3 + \lambda^2 x_1 + \lambda x_2 + \lambda\ ;\ Y(\lambda) = \lambda^2 y_1 + \lambda y_2 + y_3$$

$$x_1 = \frac{r_1}{k_1}x + \frac{r_2}{k_2}y + \alpha_{31}x + \alpha_{32}y + \alpha_{33}z;$$

$$x_2 = \frac{r_1 r_2}{k_1 k_2}xy + \frac{r_1}{k_1}\alpha_{33}xz + \frac{r_1}{k_1}\alpha_{31}x^2 + \frac{r_1}{k_1}\alpha_{32}xy + \frac{r_2}{k_2}\alpha_{33}yz + \frac{r_2}{k_2}\alpha_{31}xy + \frac{r_2}{k_2}\alpha_{32}y^2$$

$$x_3 = \frac{r_1 r_2}{k_1 k_2}\alpha_{33}xyz + \frac{r_1 r_2}{k_1 k_2}\alpha_{31}x^2 y + \frac{r_1 r_2}{k_1 k_2}\alpha_{32}xy^2;$$

$$y_1 = -\alpha_{31}x - \alpha_{32}y;$$

$$y_2 = -\frac{r_1}{k_1}\alpha_{31}x^2 - \frac{r_1}{k_1}\alpha_{32}xy - \frac{r_2}{k_2}\alpha_{31}xy - \frac{r_2}{k_2}\alpha_{32}y^2 + \alpha_{23}\alpha_{32}yz + \alpha_{13}\alpha_{31}xz;$$

$$y_3 = -\frac{r_1 r_2}{k_1 k_2}\alpha_{31}x^2 y - \frac{r_1 r_2}{k_1 k_2}\alpha_{32}xy^2 + \frac{r_1}{k_1}\alpha_{23}\alpha_{32}xyz + \frac{r_2}{k_2}\alpha_{13}\alpha_{31}xyz$$

Let $\lambda = i\omega$ be a root of (5.1), where ω is a real number. Placing $\lambda = i\omega$ into (5.1) and separating the real and imaginary parts, we obtain

$$\left(x_3 - \omega^2 x_1\right) = \left(y_1\omega^2 - y_3\right)\cos\omega\tau - \omega y_2 \sin\omega\tau \tag{5.2}$$

$$\left(\omega x_2 - \omega^3\right) = \left(y_3 - y_1\omega^2\right)\sin\omega\tau - \omega y_2\cos\omega\tau \tag{5.3}$$

Squaring and adding (5.2) and (5.3), we get

$$\omega^6 + \omega^4 B_1 + \omega^2 B_2 + B_3 = 0 \tag{5.4}$$

where
$B_1 = x_1^2 - 2x_2 - y_1^2 > 0; B_2 = x_2^2 - 2x_1x_3 + 2y_1y_3 - y_2^2; B_3 = x_3^2 - y_3^2$.

By the Descartes rule, if $B_3 < 0$, then (5.4) has an unique positive root, ω_0^2, and then, the equation (5.1) has a pair of imaginary roots $\pm i\omega_0$.

From (5.2) and (5.3), we obtain

$$\cos\omega\tau = \frac{(x_3 - \omega^2 x_1)(y_3 - \omega^2 y_1) + \omega y_2(\omega^3 - \omega x_2)}{(y_3 - \omega^2 y_1)^2 + (\omega y_2)^2}$$

Then, τ_k, corresponding to $\omega = \omega_0$, is given by

$$\tau_k = \frac{1}{\omega_0}\cos^{-1}\left[\frac{(x_3 - \omega^2 x_1)(y_3 - \omega^2 y_1) + \omega y_2(\omega^3 - \omega x_2)}{(y_3 - \omega^2 y_1)^2 + (\omega y_2)^2}\right] + \frac{2k\pi}{\omega_0}, k = 0,1,2,3,\ldots \tag{5.5}$$

By Butler's lemma, the model (2.1)–(2.3) is stable around E^* for $\tau < \tau_0$.

To check the condition for Hopf-bifurcation, we differentiate (5.1) with respect to τ,

$$X'(\lambda)\frac{d\lambda}{d\tau} + e^{-\lambda\tau}Y'(\lambda)\frac{d\lambda}{d\tau} + Y(\lambda)e^{-\lambda\tau}\left(-\lambda - \tau\frac{d\lambda}{d\tau}\right) = 0 \tag{5.6}$$

Therefore, $\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{X'(\lambda)}{-\lambda X(\lambda)} + \frac{Y'(\lambda)}{\lambda Y(\lambda)} - \frac{\tau}{\lambda} = \frac{2\lambda^3 + \lambda^2 x_1 - x_3}{-\lambda^2(\lambda^3 + \lambda^2 x_1 + \lambda x_2 + x_3)} + \frac{\lambda^2 y_1 - y_3}{\lambda^2(\lambda^2 y_1 + \lambda y_2 + y_3)} - \frac{\tau}{\lambda}$

$$\left[\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\lambda = i\omega_0} = \frac{-(x_3 + \omega_0^2 x_1 + 2i\omega_0^3)}{\omega_0^2(x_3 - \omega_0^2 x_1 + i\omega_0 x_2 - i\omega_0^3)} + \frac{(y_3 + \omega_0^2 y_1)}{\omega_0^2(y_3 - \omega_0^2 y_1 + i\omega_0 y_2)} - \frac{\tau}{i\omega_0}$$

$$Re\left[\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\lambda = i\omega_0} = \frac{\omega_0^6 - \omega_0^2 B_2 - 2B_3}{\xi^2\omega_0^2}$$

where $\xi^2 = (x_3 - \omega_0^2 x_1)^2 + (\omega_0 x_2 - \omega_0^3)^2 = (y_3 - \omega_0^2 y_1)^2 + (\omega_0 y_2)^2$, by using (5.4).

If $B_2 < 0$ and $B_3 < 0$, then $Re\left[\left(\frac{d\lambda}{d\tau}\right)^{-1}\right] > 0$; hence, $d/d\tau(Re\lambda) > 0$.

Therefore, the condition for Hopf-bifurcation holds and the system undergoes periodic oscillations at $\tau = \tau_0$.

6. Analysis of random fluctuations with white noise

In this section, we allow stochastic perturbations of the variables x, y and z around E^* in the case when it is locally asymptotically stable. We consider the white noise stochastic perturbations, which are proportional to the distances of x, y and z from x^*, y^*, z^*. As a result, the stochastically perturbed system (2.1)–(2.3) with $\tau = 0$ is given by

$$dx = \left(r_1 x\left(1 - \frac{x}{k_1}\right) - \alpha_{13}xz\right)dt + \sigma_1(x - x^*)d\xi_t^1 \tag{6.1}$$

$$dy = \left(r_2 y\left(1 - \frac{y}{k_2}\right) - \alpha_{23}yz\right)dt + \sigma_2(y - y^*)d\xi_t^2 \tag{6.2}$$

$$dz = \Big(-dz - \alpha_{33}z^2 + \alpha_{31}xz + \alpha_{32}yz \Big)dt + \sigma_3(z - z^*)d\xi_t^2 \qquad (6.3)$$

where $\sigma_i, i = 1,2,3$ are real constants and $\xi_t^i = \xi_i(t), i = 1,2,3$ are independent standard Wiener processes [20]. To analyse the stochastic stability of E^*, we consider the linear system of (6.1)–(6.3) around E^* as follows:

$$du(t) = f(u(t))dt + g(u(t))d\xi(t) \qquad (6.4)$$

where $u(t) = col(u_1(t),u_2(t),u_3(t)); f(u(t)) = Ju(t); g(u) = \begin{bmatrix} \sigma_1 u_1 & 0 & 0 \\ 0 & \sigma_2 u_2 & 0 \\ 0 & 0 & \sigma_3 u_3 \end{bmatrix}$;

$d\xi(t) = col(\xi_1(t), \xi_2(t)); u_1 = x - x^*; u_2 = y - y^*, u_3 = z - z^*.$

Let $U = \{(t \geq t_0) \times R^n, t_0 \in R^+\}$. Hence, $V \in C_2^0(U)$ is a continuous function with respect to t and a twice continuously differentiable function w.r.t to u. With reference to [21], we have

$$LV(t,u) = \frac{\partial V(t,u)}{\partial t} + f^T(u)\frac{\partial V(t,u)}{\partial u} + \frac{1}{2}Tr\left(g^T(u)\frac{\partial^2 V(t,u)}{\partial u^2}g(u)\right) \qquad (6.5)$$

where $\frac{\partial V}{\partial u} = Col\left(\frac{\partial V}{\partial u_1}, \frac{\partial V}{\partial u_1}\right)$; $\frac{\partial^2 V(t,u)}{\partial u^2} = \frac{\partial^2 V}{\partial u_j \partial u_i}$; $i,j = 1,2$ and T means transposition.

With reference to [18], the following theorem holds.

Theorem (6.1): If there is a function $V(u,t) \in C_2^0(U)$ satisfying the following

$$K_1|u|^p \leq V(t,u) \leq K_2|u|^p; LV(t,u) \leq -K_3|u|^p, K_i > 0, p > 0, \qquad (6.6)$$

the trivial solution of (6.4) is exponentially p-stable for $t \geq 0$.

Note that if in (6.6) p = 2, then the trivial solution of (6.4) is globally asymptotically stable.

Theorem (6.2): Suppose that $\left(\frac{r_1}{k_1}x^* - \frac{1}{2}\sigma_1^2\right) > 0$, $\left(\frac{r_2}{k_2}y^* - \frac{1}{2}\sigma_2^2\right) > 0$ and $\left(\alpha_{33}z^* - \frac{1}{2}\sigma_3^2\right) > 0$, the zero solution of (6.4) is asymptotically mean square stable.

Proof: Let us consider the Lyapunov function

$$V(u) = \frac{1}{2}\left(w_1u_1^2 + w_2u_2^2 + w_3u_3^2\right), w_i > 0 \in R \qquad (6.7)$$

The inequalities in (6.6) are true when p = 2, and we have

$$\begin{aligned} LV(u) = & w_1\left(\left(-\frac{r_1}{k_1}x^*\right)u_1 - \alpha_{13}x^*u_3\right)u_1 \\ & + w_2\left(-\frac{r_2}{k_2}y^*u_2 - \alpha_{23}y^*u_3\right)u_2 + w_3(\alpha_{31}z^*u_1 \\ & + \alpha_{32}z^*u_2 - \alpha_{33}z^*u_3)u_2 + \frac{1}{2}Tr\left[g^T(u)\frac{\partial^2 V}{\partial u^2}g(u)\right] \end{aligned} \qquad (6.8)$$

We can easily observe that $\frac{\partial^2 V}{\partial u^2} = \begin{bmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{bmatrix}$, and hence, $g^T(u)\frac{\partial^2 V}{\partial u^2}g(u) = \begin{bmatrix} w_1\sigma_1^2 u_1 & 0 & 0 \\ 0 & w_2\sigma_2^2 u_2 & 0 \\ 0 & 0 & w_3\sigma_3^2 u_3 \end{bmatrix}$, with

$$\frac{1}{2}Tr\left[g^T(u)\frac{\partial^2 V}{\partial u^2}g(u)\right] = \frac{1}{2}\left[w_1\sigma_1^2u_1^2 + w_2\sigma_2^2u_2^2 + w_3\sigma_3^2u_3^2\right] \qquad (6.9)$$

If in (6.8), we choose $w_1\alpha_{13}x^* = w_3\alpha_{31}z^*$, $w_2\alpha_{23}y^* = w_3\alpha_{32}z^*$, and then, from (6.9), we have

$$\begin{aligned} LV(u) = & -w_1\left(\frac{r_1}{k_1}x^* - \frac{1}{2}\sigma_1^2\right)u_1^2 - w_2\left(\frac{r_2}{k_2}y^* - \frac{1}{2}\sigma_2^2\right)u_2^2 \\ & - w_3\left(\alpha_{33}z^* - \frac{1}{2}\sigma_3^2\right)u_3^2 \end{aligned}$$

According to ***Theorem (6.1)***, the proof is completed.

7. Computer simulations

In this section, we checked the conditions, especially the stability and impact of white noise, which are performed in the above sections by randomly choosing appropriate and suitable sets of attributes (mentioned in examples). Here, we provide simulations in view of the stability using Matlab, as follows.

Example 1: $r_1 = 0.625$, $k_1 = 10$, $\alpha_{13} = 2.036$, $r_2 = 1.228$, $k_2 = 10$, $\alpha_{23} = 4$, $d = 1.5$, $\alpha_{33} = 0.15$, $\alpha_{31} = 0.112$, $\alpha_{32} = 2.02$, $\tau = 0.04$, $x(0) = 0.5$, $y(0) = 0.5$, $z(0) = 0.5$

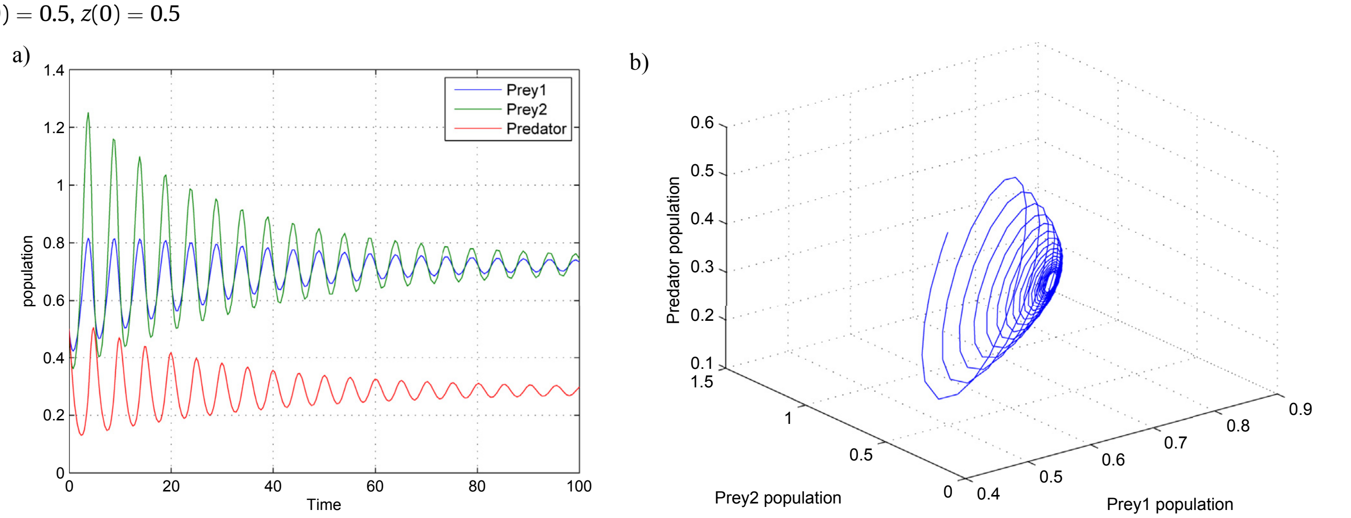

Fig. 1. (a) shows that the time series evaluation of the deterministic system with a stable equilibrium point (0.7229, 0.7720, and 0.2848). (b) shows the stable spiral in behaviour between prey1, prey2 and predator.

Example 2: $r_1 = 0.625$, $k_1 = 10$, $\alpha_{13} = 2.036$, $r_2 = 1.228$, $k_2 = 10$, $\alpha_{23} = 4$, $d = 1.5$, $\alpha_{33} = 0.15$, $\alpha_{31} = 0.112$, $\alpha_{32} = 2.02$, $x(0) = 0.5$, $y(0) = 0.5$, $z(0) = 0.5$, $\tau = 0.05$

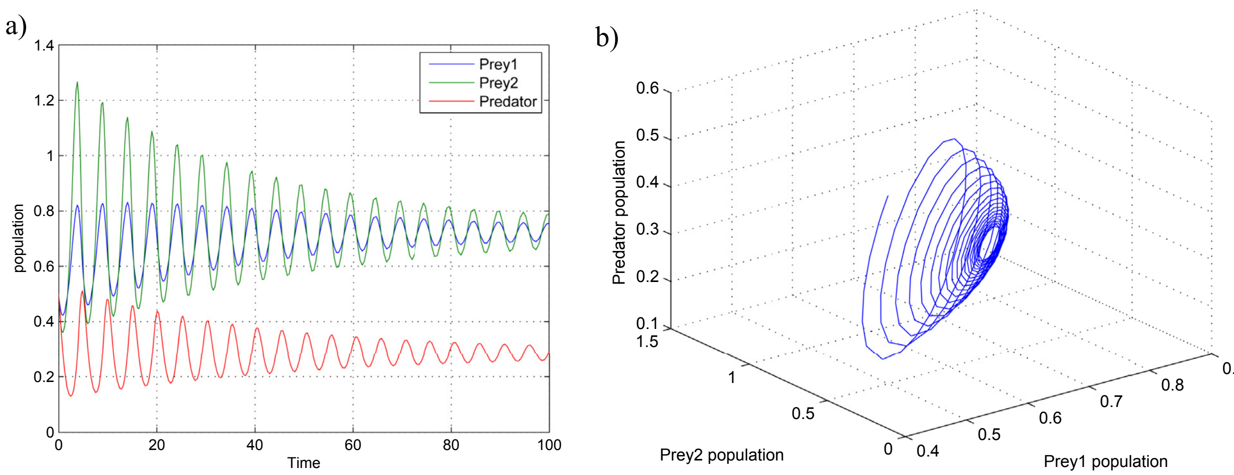

Fig. 2. (a) shows that the time series evaluation of the deterministic system with $\tau = 0.05$ around equilibrium point (0.7229, 0.7720, and 0.2848). (b) shows the space-phase delay dynamics between prey1, prey2 and predator.

Example 3: $r_1 = 0.625$, $k_1 = 10$, $\alpha_{13} = 2.036$, $r_2 = 1.228$, $k_2 = 10$, $\alpha_{23} = 4$, $d = 1.5$, $\alpha_{33} = 0.15$, $\alpha_{31} = 0.112$, $\alpha_{32} = 2.02$, $x(0) = 0.5$, $y(0) = 0.5$, $z(0) = 0.5$, $\tau = 0.2$

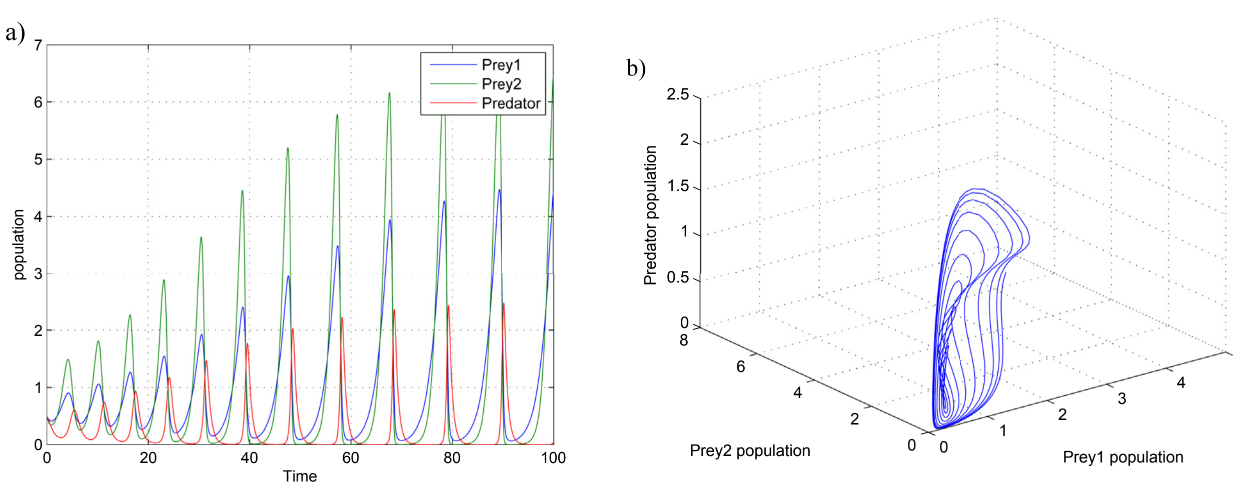

Fig. 3. (a) shows the time series evaluation of the deterministic system with $\tau = 0.2$ around equilibrium point (0.7229, 0.7720, and 0.2848). (b) shows the space-phase delay dynamics between prey1, prey2 and predator.

Example 4: $r_1 = 0.625$, $k_1 = 10$, $\alpha_{13} = 2.036$, $r_2 = 1.228$, $k_2 = 10$, $\alpha_{23} = 4$, $d = 1.5$, $\alpha_{33} = 0.15$, $\alpha_{31} = 0.112$, $\alpha_{32} = 2.02$, $\tau = 0$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.02$

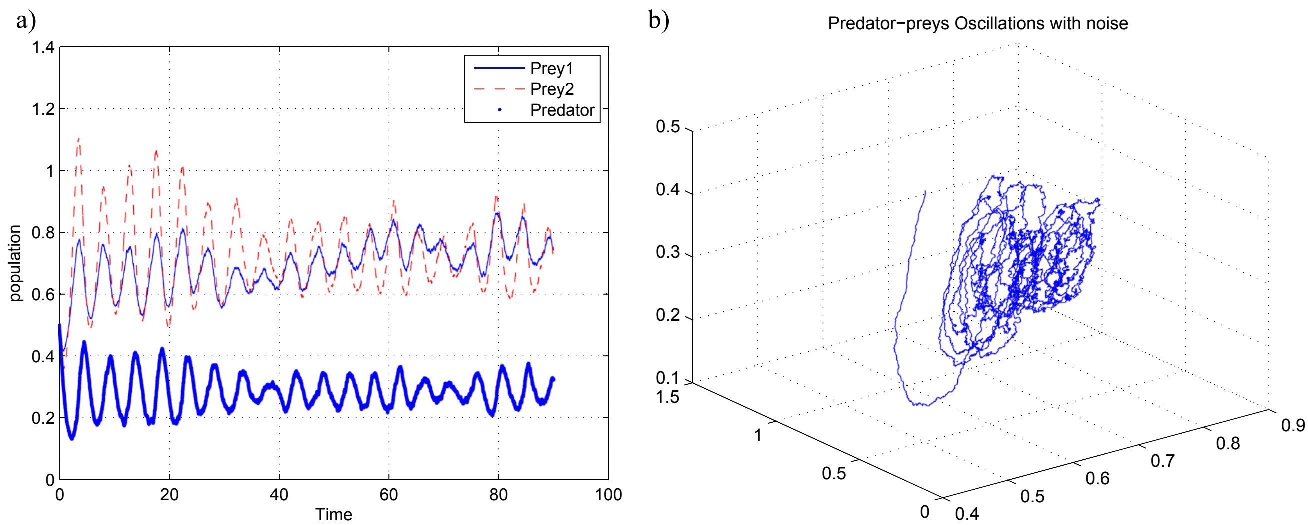

Fig. 4. (a) shows the time series evaluation of the deterministic system with random fluctuations around the equilibrium point (0.7229, 0.7720, and 0.2848). (b) shows the space-phase stochastic dynamics between prey1, prey2 and predator.

Example 5: $r_1 = 2.58$, $k_1 = 6.68$, $\alpha_{13} = 3.326$, $r_2 = 1.076$, $k_2 = 8.01$, $\alpha_{23} = 1.372$, $d = 1.5$, $\alpha_{33} = 0.15$, $\alpha_{31} = 0.112$, $\alpha_{32} = 2.02$, $\tau = 0$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.02$

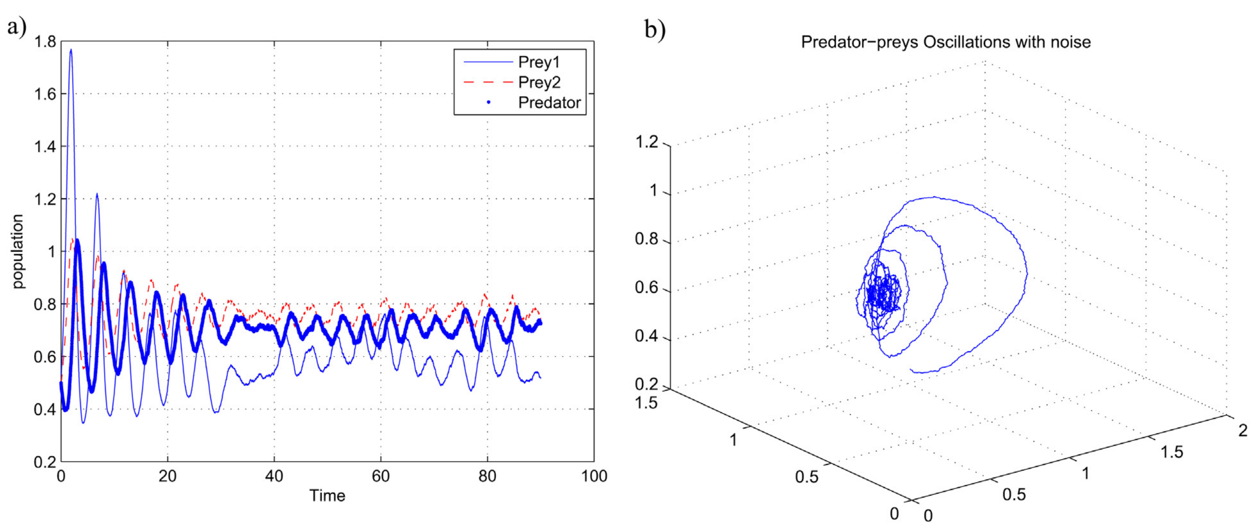

Fig. 5. (a) shows the time series evaluation of the deterministic system with random fluctuations around the equilibrium point (0.5702, 0.7636, and 0.7095). (b) shows the space-phase stochastic dynamics between prey1, prey2 and predator.

8. Concluding remarks

In this paper, we considered an ecological system consisting of two preys and one predator model where the preys are subjected to logistic growth and the predator is subjected to the mortality rate and intra specific competition. The boundedness and existence of the limit cycle of the system are also verified. The effects of time delay and random environmental fluctuations on the stability of the model around the interior steady state point are analytically confirmed using tackles, such as butler's lemma, Lyapunov function, and so on. The stable, periodic and chaotic performances of the model for various sets of suitable attributes are reconnoitred in the numerical replications in terms of Matlab Figs. 1(a)–5(b).

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