

PAPER • OPEN ACCESS

Entanglement of spin qubits involving pure DM interaction

To cite this article: Ankita Sahu *et al* 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **263** 022001

View the [article online](#) for updates and enhancements.

Related content

- [Stationary Entanglement of Two Charge Qubits Induced by a Damped and Driven Cavity](#)
Li Jun-Gang, Zou Jian, Cai Jin-Fang et al.
- [Entanglement between two qubits induced by thermal field](#)
E K Bashkurov and M S Mastuygin
- [Entanglement of Non-symmetric Two Charge Qubits Induced by a Damped Cavity](#)
Li Zhao-Xin, Li Jun-Gang, Li Juan-Juan et al.

Entanglement of spin qubits involving pure DM interaction

Ankita Sahu, Rupesh Singh and S Balakrishnan

Department of Physics, School of Advanced Sciences, VIT University, Vellore, Tamil Nadu 632014, India

Email: sbalakrishnan@vit.ac.in

Abstract. In this work, we study the system of three spin qubits exhibiting pure Dzialoshinskii- Moriya interaction, shortly DM interaction. The dynamics and the concurrence - the measure of the entanglement, are analyzed. It is found that higher the DM interaction strength, shorter will be the duration of two-qubit gate operations.

1. Introduction

Entanglement, a weird nature of quantum mechanics, is found to useful for information processing tasks [1]. The interaction between two particles can generate entanglement. Such two qubit operations become the two qubit gates which are necessary for the realization of quantum computers. Heisenberg magnetic spin interaction models are studied extensively for the realization of two qubit operations. For instance, SWAP^α gates can be implemented with isotropic Heisenberg exchange interaction of strength J [2].

Dzialoshinskii-Moriya (DM) interaction is a result of spin orbit coupling in the spin chain systems which leads to anisotropic antisymmetric exchange DM interaction. In most of the materials, DM coupling parameter D is very small compared to the exchange interaction strength J. However, many magnetic compounds with dominant DM interaction have been found and known as pure DM chain [3, 4]. Recently, the possibility of realizing SWAP^{±α} gates with DM interaction with much shorter time than using Heisenberg interaction is exhibited [5].

In this work, we study spin chain model of two spin particles coupled by an intermediate ancilla qubit with pure DM interaction [6]. Concurrence, a measure of entanglement is studied along with the dynamics of the system [7]. It is found that higher the DM interaction strength D, shorter will be the duration of gate operations. This is a desired feature of any gate operations from the perspective of decoherence time. We highlight the results in the conclusion of the paper.

2. DM interaction

We consider a system of three spins qubits such that two spin qubits are coupled through an intermediate ancilla qubit (see Fig. 1). The system is described by the Hamiltonian only with DM interaction:

$$H = D_1^z [\sigma_1^x \sigma_a^y - \sigma_a^y \sigma_1^x] + D_2^z [\sigma_a^x \sigma_2^y - \sigma_2^y \sigma_a^x] \quad (1)$$

where $\sigma_i^x, \sigma_i^y, i = 1, 2, a$ denotes Pauli matrices related with first, second and the ancilla qubit with exchange coupling strength D_1 and D_2 along z axis.



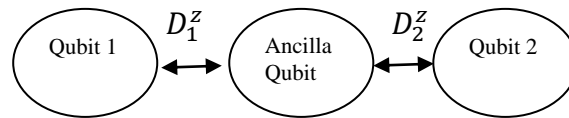


Figure 1. Two spin qubits coupled with the help of an intermediate ancilla qubit. The ancilla system allows the two qubits to evolve and helps to generate entanglement between them.

The Hamiltonian takes the following matrix form:

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2i(D_1 + D_2) & 0 \\ 0 & -2i(D_1 + D_2) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

The time evolution of a state under this Hamiltonian is given by $|\Psi(t)\rangle = U(t)|\Psi(0)\rangle$ with $U(t) = e^{-iHt/\hbar}$ and the initial state of the two particle system is represented in the computational basis as

$$|\Psi(0)\rangle = C_1(0)|00\rangle + C_2(0)|01\rangle + C_3(0)|10\rangle + C_4(0)|11\rangle \text{ with } \sum_i^4 |C_i|^2 = 1.$$

Therefore, the coefficients of time evolved state are

$$C_1(t) = C_1(0) \quad (3a)$$

$$C_2(t) = C_2(0) \cos(\lambda t) - iC_3(0) \sin(\lambda t) \quad (3b)$$

$$C_3(t) = C_3(0) \cos(\lambda t) - iC_2(0) \sin(\lambda t) \quad (3c)$$

$$C_4(t) = C_4(0) \quad (3d)$$

where $\lambda = 2(D_1^Z + D_2^Z)/\hbar$. Having known the coefficients C_i , we can study entanglement and dynamics of the states evolved under DM interaction.

3. Entanglement and dynamics

Entanglement, one of the nonlocal features of quantum mechanics, can be quantified by concurrence, which is defined as [7]

$$C(|\Psi(t)\rangle) = 2|C_1(t)C_4(t) - C_2(t)C_3(t)|. \quad (4)$$

It is known that determinant of reduced density matrix $\rho^A = \text{Tr}_B \rho^{AB}$ where $\rho^{AB} = |\Psi(t)\rangle\langle\Psi(t)|$, is related to concurrence in the following way [6]:

$$\text{Det}(\rho^A) = \frac{1}{4} [C(|\Psi(t)\rangle)]^2 \quad (5)$$

Thus the range of $\text{Det}(\rho^A)$ is between 0 and 1/4 as the range of concurrence is between 0 and 1. While the concurrence is zero for product states, the maximally entangled states reaches the maximum value of one. Hence, $\text{Det}(\rho^A)$ can also be thought of a measure of entanglement.

The concurrence of the time evolved state under the influence of the Hamiltonian H is

$$C(|\Psi(t)\rangle) = 2 \left| C_1(0)C_4(0) - C_2(0)C_3(0) \cos(2\lambda t) + \frac{i}{2} [C_3^2(0) + C_2^2(0)] \sin(2\lambda t) \right| \quad (6)$$

For the choice of initial state $|\Psi(0)\rangle = |01\rangle$, the concurrence takes the form

$$C(|\Psi(t)\rangle) = \sin(2\lambda t) \quad (7)$$

and the probability densities assume the form

$$|C_1(t)|^2 = |C_1(0)|^2 = 0 \quad (8a)$$

$$|C_2(t)|^2 = \cos^2(\lambda t) \quad (8b)$$

$$|C_3(t)|^2 = \sin^2(\lambda t) \quad (8c)$$

$$|C_4(t)|^2 = |C_4(0)|^2 = 0 \quad (8d)$$

Concurrence can take the maximum value of one, when $t = \frac{h}{16(D_1^2 + D_2^2)}$. Correspondingly, the state of the system becomes $|\Psi(t = \frac{h}{16(D_1^2 + D_2^2)})\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ which is a Bell state. Similarly, the initial state $|10\rangle$ reaches a Bell state at $t = \frac{h}{16(D_1^2 + D_2^2)}$.

For the initial state $|\Psi(0)\rangle = |01\rangle$, concurrence can take the minimum value of zero, when $t = \frac{h}{8(D_1^2 + D_2^2)}$. At this time, probabilities become

$$\left|C_1(t = \frac{h}{8(D_1^2 + D_2^2)})\right|^2 = |C_1(0)|^2 = 0 \quad (9a)$$

$$\left|C_2(t = \frac{h}{8(D_1^2 + D_2^2)})\right|^2 = 0 \quad (9b)$$

$$\left|C_3(t = \frac{h}{8(D_1^2 + D_2^2)})\right|^2 = 1 \quad (9c)$$

$$\left|C_4(t = \frac{h}{8(D_1^2 + D_2^2)})\right|^2 = |C_4(0)|^2 = 0 \quad (9d)$$

That is, the initial state $|01\rangle$ is swapped to $|10\rangle$. Similarly, the initial state $|10\rangle$ is swapped to $|01\rangle$ at $t = \frac{h}{8(D_1^2 + D_2^2)}$.

It is worth mentioning that SWAP ^{α} gates can be implemented with isotropic Heisenberg exchange interaction of strength J , in which $\alpha = 8Jt/h$ [6] as well with DM interaction with interaction of strength D , in which $\alpha = 8Dt/h$ [7] where $0 \leq \alpha \leq 1$. The present analysis indicates that

$$\alpha = 8(D_1 + D_2)t/h \quad (10)$$

This suggests that gate operations can be achieved in a much shorter duration.

4. Discussion

In this work, we have analyzed pure DM interaction in a system of two spin particles coupled by an intermediate ancilla qubit which facilitates the DM interaction between the two spin particles. It is found that ancilla system helps to achieve the gate operation much shorter time. In other words, higher the DM interaction strength, shorter will be the duration of two-qubit gate operations.

References

- [1] Nielsen M A and Chuang I L 2003 *Quantum Computation and Quantum Information* (Cambridge : Cambridge University Press)
- [2] Muthuganesan R, Balakrishnan S and Sankaranarayanan R 2015 *AIP Conf. Proc.* **1665**, 130050
- [3] McElearney J N and Merchant S 1999 *Phys. Rev. B* **18** 3612
- [4] Pires A S T and Gouva M E 2000 *J. Magn. Mater.* **212** 251
- [5] Balakrishnan S, Muthuganesan R and Sankaranarayanan R 2017 *Proc. of Quantum Information and Measurement OSA Technical Digest* (online) paper QT6A.26
- [6] Vahid Azimi Mousolou 2017 *Phys. Rev. A* **96** 012307
- [7] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022