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Entropy Analysis on Planar Anamorphic Images

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Abstract

Anamorphosis is an art of drawing, which creates illusion effect over the drawn image plane, and the impact of the illusion is nullified when the specific viewing position is used to view the drawn image. In digital imaging domain, the effect of anamorphosis is analyzed quantitatively by the amount of distortion present in the anamorphized image. This study on the anamorphized image suggested that the optimal combination of parameters can create the better-anamorphized image.

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1. Introduction

Anamorphic images are unique images which, when seen from the correct perspective reveals the intended image. Viewing from the other angles shows a distorted version. Some anamorphic images need the right attitude as well as some individual devices to view the image [1-4]. The former images are a planar anamorphic image, and the latter is known as mirror anamorphic images. Figure.1 shows the different types of anamorphic images. Mirrors could be conical, cylindrical, or pyramid. Traditionally, anamorphic images are part of the arts. European art and architecture introduced an advanced linear perspective projection in the form of anamorphosis [5]. Moreover, the anamorphized images are used to hide the secret details into it. "The Ambassador" image of the Hans Holbein the Younger, National Gallery, London, 1533 is the best example for anamorphic image with secret detail, which hides the skull image into it. The skull image is visible only when "The Ambassador" image is viewed from the particular direction

[6]. Jean Francois Niceron [7] created the anamorphic image with the help of grid construction. Hunt et al. [8-9] derived the equation to generate planar and mirror anamorphic images and analyzed the generated images with the conventional methods of anamorphosis in the digital era. Anamorphic image generation over the complex surface is a complicated process. Paola et al. [10] suggested the simplest way of creating the anamorphic projection over the complex surface to overcome the problems associated along with the anamorphic image generation. Above studies of anamorphic image generation highly depends on the anamorphic drawing in the real-world environment. There is a limited amount of work related to digital anamorphic image generation and analysis.

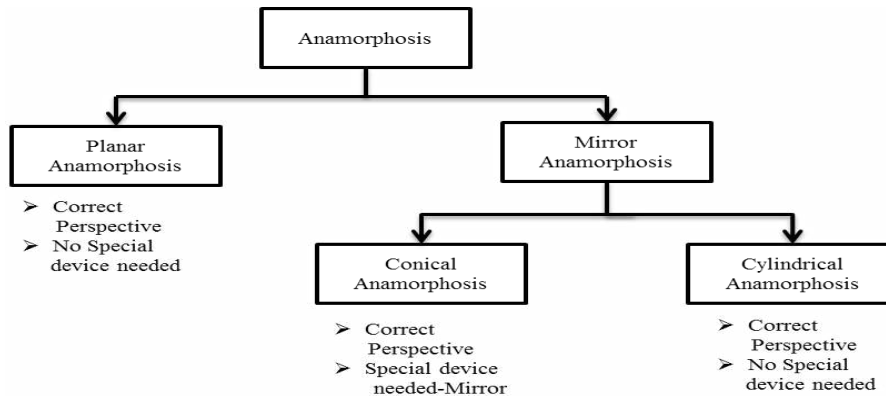


Fig. 1. Type of Anamorphosis

In this work, we have automated the process of planar anamorphization using the computers, i.e., given an ordinary image, and a user-specified the perspective, the algorithm generates an anamorphic image automatically. Here, the light rays equations are used to generate the anamorphic images. A given image can be anamorphized in various ways depending upon the perspectives. A perspective is characterized by three parameters, (i) viewing angle, (ii) viewing distance and, (iii) viewing height.

Since the anamorphized image tools distorted from the incorrect perspectives, we would like to maximize the distortion. We have addressed "What is the best way to anamorphize the given image?" or in other words, "How to maximize the distortion?". This depends on the above stated three parameters and can be called as optimum anamorphic parameters. In this work, the set of parameters maximize the anamorphic distortion that is sought by comparing the entropies and relative entropies of the given and anamorphized images.

The organization of this paper is given as follows: Section 2 explains the anamorphic image generation process. The entropy of the anamorphic images is analyzed in Section 3, and Section 4 gives the conclusion of this work.

2. Anamorphic Image Generation

The anamorphic image generation process in the digital imaging field transforms the pixel of the two dimensional image matrix into another image matrix using the Eq(1-2) [6].

$$y' = \frac{y}{\sin(\alpha)} \cdot \frac{1}{\left[1 - \left(\frac{y}{h}\right) \cos(\alpha)\right]} \quad (1)$$

$$x' = \frac{x}{\sqrt{[h^2 + d^2 + y^2]}} \cdot \sqrt{[h^2 + (d + y)^2]} \quad (2)$$

The relation between the observer height (h), distance (d) from the image and viewing angle (α) is expressed in Eq. (3).

$$\tan(\alpha) = \frac{h}{d} \quad (3)$$

While transforming the pixel from the original image matrix to anamorphic image matrix, it creates gaps in anamorphic images. These gaps are filled using the three different interpolation methods named as sample and hold, nearest neighbor, and linear interpolation methods [11-12]. The flow diagram of anamorphic image generation process is given in Fig.2.

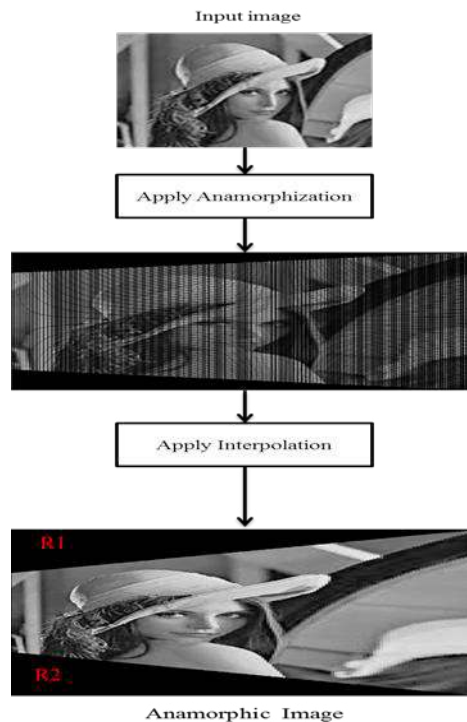


Fig. 2. Anamorphic image generation

3. Entropy Analysis

In this section, the anamorphic images are analyzed using their entropies. Scaling up/super-resolution of the images does not change the entropy value, i.e., both the original and the scaled-up images have the same entropy values. However, this is not true for the anamorphic images, i.e., the original image and the anamorphized image can have different entropies. In other words, the information contained in the scaled-up images does not change, whereas the anamorphic transformation tries to reduce the information content. The entropy reflects the amount of information present in an image. We have used Shannon and Renyi entropies [13-15] to analyze the anamorphic images generated from section 2. Eqs. (4) and (5) give the Shannon and Renyi entropy formulae.

$$\text{Shannon's Entropy} = - \sum_k p_k \times \log_2(p_k) \tag{4}$$

The Renyi entropy is the extended form of Shannon's entropy which is given below,

$$\text{Renyi Entropy} = \frac{1}{1-q} \log_2 \sum_k p_k^q \tag{5}$$

where, the index k runs from 0 to 255, and p_k is the probability associated with a particular gray level which can be computed from the image histogram. In Renyi entropy, the value of q is defined as greater than 1 and when the value of q is equal to 1, Renyi entropy turns into Shannon entropy.

The anamorphic transform changes the rectangular/square matrix into a trapezoidal shape. The pixels in regions 1 (R1) and 2 (R2) in Fig. 2 are not considered for entropy calculations. Entropy analysis takes anamorphic images from different combinations of d, h, and α . Figure. 3 depict the anamorphic image generation process for the different values of d, h, and α .

3.1. Impact of different interpolation schemes on the entropy of the anamorphic images

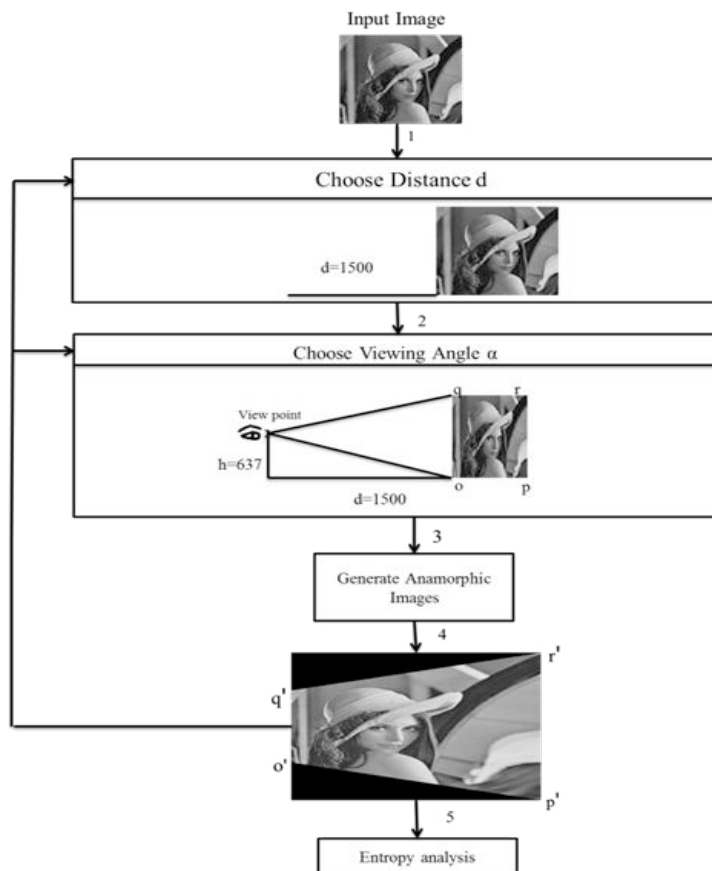


Fig. 3. Entropy analysis on anamorphic images

First, the effect of different interpolation schemes on the entropy of the anamorphic images is studied. Figure. 4 shows the Shannon and Renyi entropy of the anamorphic image (512 x 512-Lena image) as a function of α , for three different interpolation schemes. In Fig. 4, d is fixed, and the corresponding h is computed through Eq. (3), for various α . Fixing d at 1500, for $\alpha = \{20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ\}$, the corresponding heights are, $h = \{546, 693, 755, 953, 1212, 1785, 3402\}$. The Fig.4 tells us that as α decreases the entropy decreases, for all the three interpolation schemes. We have already discussed that it is more difficult to recognize the original image in the anamorphic image generated with lower α . This means that entropy decreases at α values (since the entropy measures the information content of the image). Fig 4 depict both Shannon and Renyi ($q=5$) entropy, and the trends are the same in both cases. It can also be observed from Fig. 4 that the plots show the same behavior for all interpolations. Further, results in the paper have used linear interpolation.

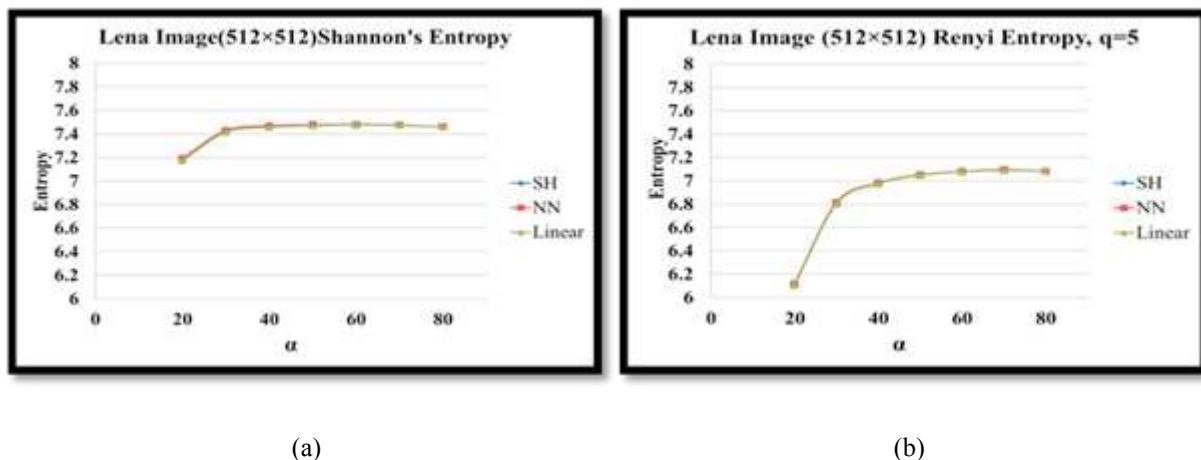


Fig. 4. Shannon and Renyi ($q=5$.) entropy of Sample and Hold (SH), Nearest Neighbor (NN), Linear interpolation techniques used in Lena anamorphic image generation (a-b).

3.2. Effect of α , d and h on the entropy of the anamorphic images

The effect of α , d , and h on entropy is explored. Eq. (3) tells us that the same α can be obtained for various combinations of d and h . Different combinations of d and h can produce the same α , which is clearly shown in Fig. 5 (i.e., 3D plot of Eq. (3)). For an example, $\alpha = 20^\circ$ is obtained for four different combinations of $(d, h) = (1500, 546); (1750, 637); (2000, 728); (2500, 909)$.

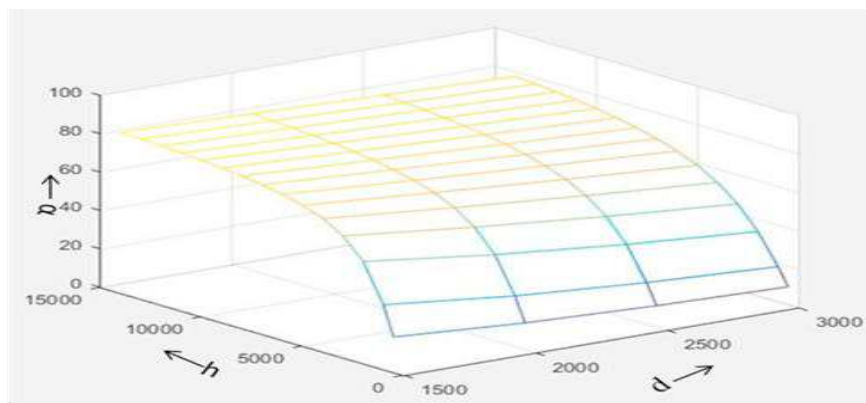
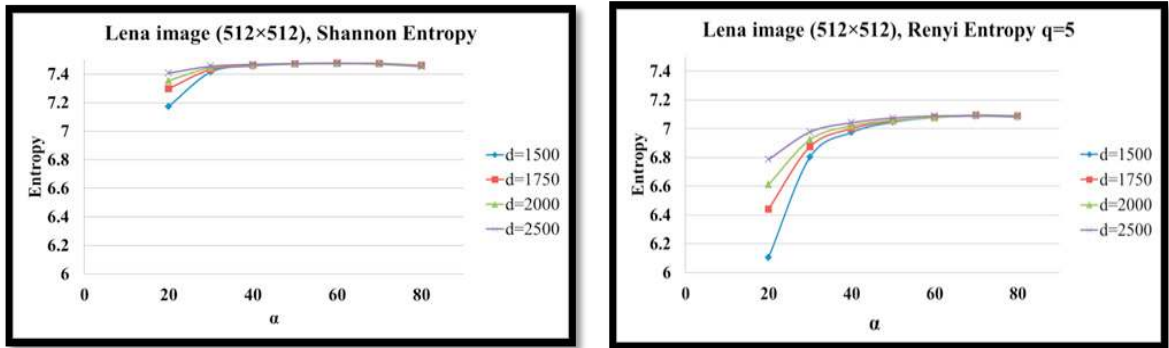


Fig. 5. 3D plot of Eq. (3) for various α , d and h

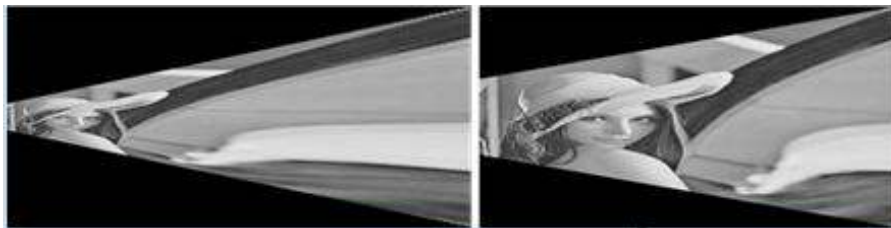
Figure 6 shows the Shannon and Renyi entropy of the anamorphic image (512 x 512-Lena image) as a function of α , with the corresponding d and h values. We can see that at lower α values entropy decreases, and we can also note that the lower d and h values for the given α , decrease the entropy, for both Shannon and Renyi entropies. The variation is more clear in Renyi entropy with $q=5$. Since we desire lower entropy to get the better anamorphic effect, the image rotations/orientations result in lower entropy is of our interest. To conclude, lower α , lower d , and lower h decrease the entropy, i.e., the information content is difficult to recognize the anamorphic images generated with lower α , lower d , and lower h which is depicted in Fig.7.



(a)

(b)

Fig. 6. Anamorphic images entropy (Shannon, Renyi Entropy $q=5$) analyses based on distance 1500, 1750, 2000, 2500 over target Lena image.

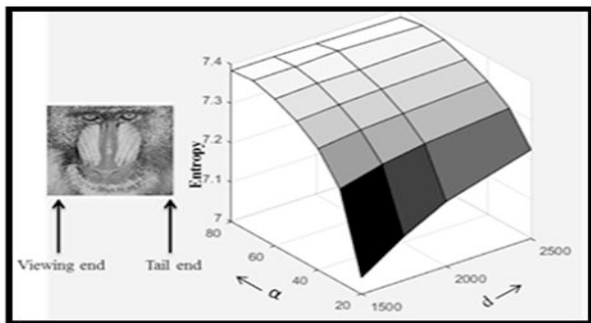


(a)

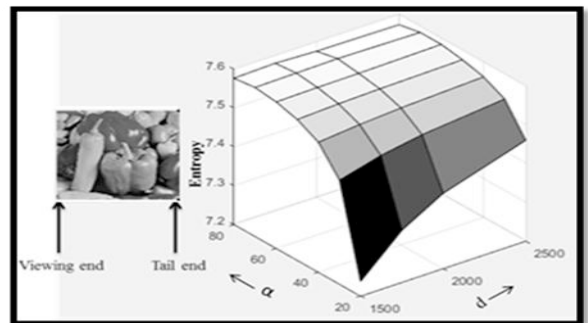
(b)

Fig. 7. Different combination of α , d and h (a) $\alpha=20$, $h=546$ and $d=1500$ (b) $\alpha=20$, $h=728$ and $d=2000$ involved in anamorphic image generation.

The same analysis is taken place on the other two images, which also results in the same. Figure 8 depicts the Renyi entropy values as a function of α and d in a 3D plot. Even though the h is not shown in Fig. 8, it should be kept in mind that h changes along with α and d .



(a)



(b)

Fig. 8. Entropy analysis of the anamorphic images

4. Conclusion

This work has analyzed the anamorphic images using Shannon and Renyi entropies, for various viewing angle, distance, and height. It was hypothesized that the application of anamorphic transformation reduces the entropy, at appropriate rotations of the image on which the transform is being applied. The hypothesis was checked through systematic simulations, and the results have exposed that the entropy reduces at lower viewing angle, distance, and height. We have also found that Renyi entropy is more suitable for analyzing the anamorphic images.

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