

PAPER • OPEN ACCESS

## EOQ model for perishable products with price-dependent demand, pre and post discounted selling price

To cite this article: G Santhi and K Karthikeyan 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **263** 042133

View the [article online](#) for updates and enhancements.

### Related content

- [An EOQ model for weibull distribution deterioration with time-dependent cubic demand and backloging](#)  
G Santhi and K Karthikeyan
- [Analysis of production-inventory decisions in a decentralized supply chain with price-dependent demand](#)  
N.A. Kurdhi, S.T. Irsanianto and Sutanto
- [Unraveling the histories of pre planetary nebulae](#)  
Bruce Balick

# EOQ model for perishable products with price-dependent demand, pre and post discounted selling price

**G Santhi and K Karthikeyan**

Department of Mathematics, School of Advanced Sciences, VIT University, Vellore-632014, India

E-mail: kkarthikeyan67@yahoo.co.in

**Abstract.** In this article we introduce an economic order quantity model for perishable products like vegetables, fruits, milk, flowers, meat, etc., with price-dependent demand, pre and post discounted selling price. Here we consider the demand is depending on selling price and deterioration rate is constant. Here we developed mathematical model to determine optimal discount on the unit selling price to maximize total profit. Numerical examples are given for illustrated.

## 1. Introduction

Deterioration is defined as decay, dryness, spoilage, obsolescence, damage, pilferage, evaporation. Ghare and Schrader [2] was originally established an inventory model with exponential decay items and deterministic demand. Ardalan [1] presented temporary price discounts and optimal policies are used. Wee and Yu [13] developed a temporary price discount, here consider two models for the exponentially deteriorating items. Nahmias [7], Raafat [10], Goyal and Giri [3] proposed survey papers of the published in inventory literature considering deteriorating inventory models. Khouja [5] analyzed an optimal ordering policy, discounting and the single-period problem. Papachristos and Skouri [8] studied an inventory model with a quantity discount, partial backlogging. Lin and Lin [6] represented an inventory model with time-varying deterioration, partial backlogging and time discounting. Hsu and Yu [4] analyzed an EOQ model with one-time only discounts are allowed. Sana [11] analyzed an inventory model for perishable items with stock level dependent demand rate and price discounts are used. Uthayakumar and Rameswari [12] presented time varying demand with time discounting. Panda et al. [9] developed an economic order quantity model with stock dependent demand and price discount is used. In the present paper, an economic order quantity model for deterioration rate is constant and demand rate is a linearly decreasing function of the unit selling price, linear holding cost. Here we consider with and without price discount selling price is allowed.

## 2. Assumptions and notations

### 2.1 Assumptions

- 1) Inventory system is included only one item.
- 2) Deterioration rate  $\theta$  is constant and ( $0 \leq \theta \leq 1$ ).
- 3) Replenishment rate is infinite.



- 4)  $\alpha_1 = (1 - r_1)^{-n_1}$ ,  $n_1 \in R$  is the effect of pre-deterioration discount on demand.  
 $\alpha_2 = (1 - r_2)^{-n_2}$ , ( $n_2 \in R$ , set of real numbers), is the effect of discounted selling price.

## 2.2 Notations

$D(p)$  Demand rate as a function of the selling price

$a$  Demand rate constant,  $a > 0$

$b$  Price-dependent demand rate,  $b > 0$

$\theta$  Constant deterioration rate

$HC$  Holding cost per unit time,  $H(t) = h + \gamma t$ ,  $h > 0$ ,  $\gamma > 0$

$C$  Unit purchasing cost

$p$  Unit selling price, where  $p \geq C$

$r_1$  Discount rate per unit before deterioration

$r_2$  Discount rate per unit after deterioration

$A$  Ordering cost

$I(t)$  Inventory level at time  $t$ , where  $t \in [0, T]$

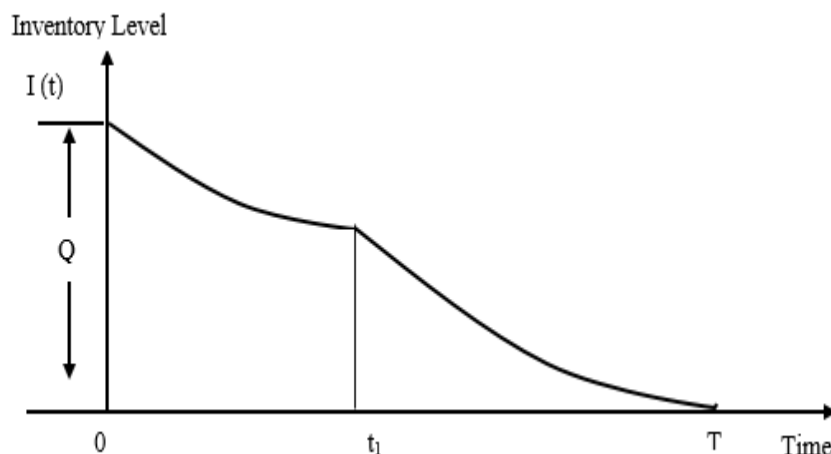
$Q$  Order quantity

$T$  Cycle length of inventory

$TP$  Total profit of an inventory system

## 3. Model formulation

In this inventory model we introduce pre and post deterioration discounts on selling price. During the interval  $[0, t_1]$ ,  $r_1$  % of pre deterioration discounts on unit selling price and during the interval  $[t_1, T]$ ,  $r_2$  % post deterioration discounts on unit selling price the following Figure-1 depicts the model.



**Figure1.** The graphical representation of inventory system

### 3.1 EOQ model with Pre-deterioration discount model

In this case, we consider the model pre-deterioration discount on selling price only. So  $t_1 = T$ ,  $r_2 = 0$ .

$$\frac{dI(t)}{dt} = -\alpha_1(a - bp), \quad 0 \leq t \leq T \quad (1)$$

the boundary conditions  $I(0) = Q$  and  $I(T) = 0$

From (1) we have

$$I(t) = -\alpha_1(a - bp)(T - t) \quad 0 \leq t \leq T \quad (2)$$

The optimum order quantity is given by  $Q = \alpha_1(a - bp)T$  (3)

Ordering cost  $OC = A$

$$\text{Holding cost } HC = \int_0^T (h + \gamma t)I(t)dt$$

$$HC = \alpha_1(a - bp) \left[ \frac{3hT^2 + \gamma T^3}{6} \right]$$

Purchase cost  $PC = C.Q$

$$PC = C[\alpha_1(a - bp)T]$$

$$\text{Total sales revenue } SR = p \left[ \alpha_1(1 - r_1) \int_0^T (a - bp)dt \right]$$

$$SR = p[\alpha_1(1 - r_1)(a - bp)T]$$

Thus total profit per unit time is given by

$$TP = \frac{1}{T} [SR - HC - PC - OC]$$

$$TP = \frac{1}{T} \left[ p[\alpha_1(1 - r_1)(a - bp)T] - \alpha_1(a - bp) \left[ \frac{3hT^2 + \gamma T^3}{6} \right] - C[\alpha_1(a - bp)T] - A \right] \quad (4)$$

The maximize the total profit we have

$$\frac{dTP}{dT} = 0 \quad (5)$$

$$\text{and } \frac{d^2TP}{dT^2} < 0$$

### 3.2 EOQ model with post-deterioration discount model

In this case, we consider the model post-deterioration discount on selling price only. So  $t_1 = 0$ ,  $r_1 = 0$ .

$$\frac{dI(t)}{dt} + \theta I(t) = -\alpha_2(a - bp) \quad 0 \leq t \leq T \quad (6)$$

With boundary conditions  $I(0) = Q$  and  $I(T) = 0$

From (6) we have

$$I(t) = \alpha_2(a - bp) \left[ (T - t) + \frac{\theta(T^2 - t^2)}{2} \right] \quad 0 \leq t \leq T \quad (7)$$

$$\text{The optimum order quantity is given by } Q = \alpha_2(a - bp) \left[ T + \frac{\theta T^2}{2} \right] \quad 0 \leq t \leq T \quad (8)$$

$$\text{Holding cost } HC = \int_0^T (h + \gamma t)I(t)dt$$

$$HC = \alpha_2(a - bp) \left[ \frac{hT^2}{2} + \frac{\theta hT^3}{3} + \frac{\gamma T^3}{6} + \frac{\theta \gamma T^4}{8} \right]$$

$$\text{Purchase cost } PC = C.Q$$

$$PC = C \left[ \alpha_2(a - bp) \left( T + \frac{\theta T^2}{2} \right) \right]$$

$$\text{Deterioration cost } DC = C_d \left[ Q - \int_0^T (a - bp)dt \right]$$

$$DC = C_d \left[ (a - bp)T \left\{ \alpha_2 \left( 1 + \frac{\theta T}{2} \right) - 1 \right\} \right]$$

$$\text{Total sales revenue } SR = p \left[ \alpha_2(1 - r_2) \int_0^T (a - bp)dt \right]$$

$$SR = p[\alpha_2(1 - r_2)(a - bp)T]$$

$$TP = \frac{1}{T} [SR - HC - PC - DC - OC]$$

$$TP = \frac{1}{T} \left[ p[\alpha_2(1 - r_2)(a - bp)T] - \alpha_2(a - bp) \left[ \frac{hT^2}{2} + \frac{\theta hT^3}{3} + \frac{\gamma T^3}{6} + \frac{\theta \gamma T^4}{8} \right] - C \left[ \alpha_2(a - bp) \left( T + \frac{\theta T^2}{2} \right) \right] - C_d \left[ (a - bp)T \left\{ \alpha_2 \left( 1 + \frac{\theta T}{2} \right) - 1 \right\} \right] - A \right] \quad (9)$$

The maximize the total profit we have

$$\frac{dTP}{dT} = 0 \quad (10)$$

$$\text{and } \frac{d^2TP}{dT^2} < 0$$

### 3.3 Without discount model for Pre-deterioration

In this case, we consider the model without discount on the unit selling price only. Substituting  $t_1 = T$ ,  $r_1 = 0$ . we have from Eq. (3) the initial inventory level as

$$\text{The optimum order quantity is given by } Q = (a - bp) \left[ T + \frac{\theta T^2}{2} \right] \quad (11)$$

and from Eq. (4) total profit per unit time becomes,

$$TP = \left[ p[(a - bp)T] - (a - bp) \left[ \frac{hT^2}{2} + \frac{\theta hT^3}{3} + \frac{\gamma T^3}{6} + \frac{\theta \gamma T^4}{8} \right] - C \left[ (a - bp) \left( T + \frac{\theta T^2}{2} \right) \right] - A \right] \quad (12)$$

The maximize the total profit we have

$$\frac{dTP}{dT} = 0 \quad (13)$$

$$\text{and } \frac{d^2TP}{dT^2} < 0$$

### 3.4 Without discount model for Post-deterioration

Here we consider the model without discount on the unit selling price only. Substituting  $t_1 = 0, r_2 = 0$  we have from Eq. (8) the initial inventory level as

The optimum order quantity is given by

$$Q = (a - bp) \left[ T + \frac{\theta T^2}{2} \right] \quad (14)$$

and from Eq. (9) total profit per unit time becomes,

$$TP = \frac{1}{T} \left[ p[(a - bp)T] - (a - bp) \left[ \frac{hT^2}{2} + \frac{\theta hT^3}{3} + \frac{\gamma T^3}{6} + \frac{\theta \gamma T^4}{8} \right] - C \left[ (a - bp) \left( T + \frac{\theta T^2}{2} \right) \right] - C_d \left[ (a - bp)T \left\{ \left( 1 + \frac{\theta T}{2} \right) - 1 \right\} \right] - A \right] \quad (15)$$

The maximize the total profit we have

$$\frac{dTP}{dT} = 0 \quad (16)$$

$$\text{and } \frac{d^2TP}{dT^2} < 0$$

## 4. Numerical examples

### Example 1. Pre-deterioration discount model

Let  $A = 420, a = 50, b = 3, p = 10, C = 5, h = 0.9, \gamma = 0.4, n_1 = 3.0, r_1 = 0.02$ . The optimal solutions are  $T = 0.9276, Q = 196.063$  and  $TP = 489.62$ .

### Example 2. Without discount model for Pre-deterioration

Let  $A = 420, a = 50, b = 3, p = 10, C = 5, h = 0.9, \gamma = 0.4$ . The optimal solutions are  $T = 1.1048, Q = 169.4$  and  $TP = 429.99$ .

### Example 3. Post-deterioration discount model

Let  $A = 420, a = 50, b = 3, p = 10, C = 5, h = 0.9, \gamma = 0.4, n_2 = 3.0, r_2 = 0.05, \theta = 0.3, C_d = 0.04$ . The optimal solutions are  $T = 1.3369, Q = 168.289$  and  $TP = 411.394$ .

### Example 4. Without discount model for Post-deterioration

Let  $A = 420, a = 50, b = 3, p = 10, C = 5, h = 0.9, \gamma = 0.4, \theta = 0.03$ . The optimal solutions are  $T = 1.4859, Q = 115.056$  and  $TP = 388.6$ .

**Table 1.** Various optimal solutions of the examples 1, 2, 3 and 4

Model	Cycle length	Order quantity	Total profit
Pre-deterioration discount	0.9276	196.063	489.62
without discount	1.1048	169.4	429.99

Post deterioration discount	1.3369	168.289	411.394
without discount	1.4859	115.056	388.6

## 5. Results and Discussion

This research work is mainly based on with and without discount for pre deterioration and post deterioration of the items. Here we consider 2% discount for pre deterioration and 5% for post deterioration and calculated the values of Cycle length, Order quantity and Total profit with and without specified discount. It is clear from the above table that the order quantity and total profit are more for pre deterioration discount than post deterioration discount and no discount for the two cases.

## 6. Conclusion

In this article, we proposed an inventory model with the demand rate is a linearly decreasing function of the selling price and deterioration rate is constant, holding costs as a linear function of time. Here the Pre and post deterioration discounts are provided on the unit selling price of the product. It is clear from the numerical illustration that the total profit on Pre deterioration discount is more compare with the Post deterioration discount. The frame work of the model presented in this article guides business sector on various discounts of selling price so that the total profit will be maximized. This research work further can be extended in many directions like the inflation and time value of money, stock dependent demand, etc. in future.

## References

- [1] Ardalan A 1994 *European Journal of Operational Research* **72** 52-61
- [2] Ghare P M and Schrader G F 1963 *J. Ind. Eng* **14** 238–243
- [3] Goyal S K and Giri, B C 2001 *Eur. J. Oper. Res* **134** 1-16
- [4] Hsu W K and Yu H F 2009 *Omega* **37** 1018-1026
- [5] Khouja M 2000 *Int. J. Prod. Econ* **65** 201-216
- [6] Lin Y and Lin C 2006 *Int. J. Adv. Manuf. Technol* **27** 816–823
- [7] Nahmias P 1982 *Operations Research* **30** 680-708
- [8] Papachristos S and Skouri K 2003 *Int. J. Prod. Econ* **83** 247-256
- [9] Panda S, Saha S and Basu M 2009 *CEJOR* **17** 31-53
- [10] Raafat E 1991 *J oper Res Soc* **42** 27–37
- [11] Sana S 2010 *American Journal of Mathematical and Management Sciences* **30** 299-316
- [12] Uthayakumar R and Rameswari M 2012 *Int. J. Adv. Manuf. Technol* **58** 817–840
- [13] Wee H M and Yu J 1997 *Int. J. Production Economics* **53** 81-90