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Estimating Cost Analysis using Goal Programming

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Abstract

Objectives: The purpose of this paper is to develop an application of goal programming for a mathematical economics problem. In economic optimization given a cost target, the producer conditionally maximizes output or revenue. **Methods/ Statistical Analysis:** The paper aims in estimating cost limited maximal output based on full and stochastic frontiers of Cobb-Douglas and variable returns to scale production structure. The estimation procedure constitutes primarily formulating goal programming problems, later solving them by mathematical programming. The cost limited output expressions are derived. **Findings:** The method developed was applied to six manufacturing sectors of all India and the expression derived for cost limited outputs showed that the cost limited maximal output is cost efficient. Empirical analysis carried out resulted in showing the states whose cost efficiency was underachieved, overachieved and one and the same. **Application/Improvements:** The model has practical application in manufacturing sectors and was applied six sectors of all India. In future fuzziness can be applied to analyse cost efficiency in the manufacturing sectors of different states of all India.

Keywords: Cost Efficiency and Stochastic Cost Frontier, Data Envelopment Analysis, Linear Goal Programming, Manufacturing Sector

1. Introduction

In management and production economics, it is often assumed that the producer is cost minimizer. However, these assumptions are not always realistic, especially in a competitive situation where different producers employ different techniques. For example, to produce a homogeneous product, the producer may have multiple goals instead of a single goal such as cost minimization. The various diversified goals may be, maintenance of stable profits and prices, improving market share and so on. Goal programming provides an objective function for each objective and consequently finds a solution that minimizes the weighted sum of deviations of these objective functions from their respective goals. One can come across three possible of goals. A lower one–sided goal that sets a lower limit which does not fall under; an upper on sided goal that sets an upper limit which is not allowed to exceed and a two sided goal that specifies a lower limit which does not fall under and an upper limit which is to not exceed.

A goal programming problem may be pre-emptive or non-pre-emptive. Further, it can be linear and non-linear. Goal programming has applications in the theory of production. It is also used to estimate frontier full and stochastic cost functions of a production unit in a competitive environment. To understand estimation of stochastic cost function one can refer into[1,2]. It is hypothesized that

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all producers are not equally efficient, since different procedures employ different techniques, though they employ same techniques, they may differ in terms of managerial efficiency. Thus, Data Envelopment Analysis (DEA) is in efficient as observed in[3–6] which shows that the input sets when production technology is piece wise linear. A production function is a technological relationship between inputs and outputs. In[7,8] generalized non–homogeneous production function returns to scale. In[9] has done estimation for full frontier cost functions. In[10] estimates stochastic frontier cost function using data envelopment analysis. Further[10] used linear programming approach to estimate the stochastic production functions. Recently[11] did a case study for Tehran PAK Diary Company using data envelopment analysis.

Goal programming has wide applications. It is applied in discriminant analysis, portfolio management and so on[12,13]. Extensions on goal programming were done by[14-17] and many others. In[18] developed goal programming approach for cross efficiency evaluation. Recently[19], adopted goal programming method to solve data envelopment analysis with judgments. Theoretical background of goal programming was developed with a detailed review by[20]. Of late many authors have studied incorporating fuzziness in goal programming (look into[21,22]). The proposed work is an application of goal programming. In economic optimization given a cost target the producer conditionally maximizes output or revenue. Two popular production functions are considered firstly the Cobb-Douglas production frontier and then the Variable Returns to Scale (VRTS) Production Frontier. A producer in the process of expansion, targets cost and requires knowing the potential output and revenue. Such an output is called cost limited maximal output. The layout of the paper is as follows: Modelling of the system is done in section 2. Empirical investigation is carried out for six manufacturing sectors in section 3. While the conclusion are laid in section 4 followed by references in the section 5.

2. Mathematical Modelling

Production function explains how inputs are combined to produce a scalar output subject to the underlying production technology. Thus, production function is an engineering relationship. It has nothing to do with vector prices. Production function is defined on input quantity space. Traditionally to each input combination the production function associates maximum output. But, in a competitive environment, it is likely that different production unit employ different production techniques. Consequently, some production units are more efficient than others. The 'best practice' production units determine the frontier production, while the outputs of inefficient units fall below the production frontier. In classical approach, which assumes that production is efficient, the structure of production can be examined by a study of isoquants of production function. However, if inefficiency into production is introduced, the structure of production can be studied by examining its input level sets $L(u)$ None the less the frontier production function can be viewed as an optimization problem defined on input level sets. $\phi(x) = \text{Max}\{u : x \in L(u)\}$. Associated with a suitably structured production function there exits its dual, called the factor minimal cost function, that is defined on input price space. $Q(u,p) = \text{Min}\{px : x \in L(u)\}$, $\phi(x)$ and $Q(u,p)$ are, respective the frontier production and cost functions respectively. In input quantity space two types of production inefficiencies are observed. These are pure technical and scale inefficiencies. The product of pure technical and scale efficiency is called over all technical efficiency. In input price two more inefficiencies are encountered, viz., allocate and cost inefficiencies.

Departure of observed cost from factor minimal cost leads to cost inefficiency. Cost efficiency of production unit is defined as, $\delta = CE = \dfrac{Q(u,p)}{px_0}$ Where $Q(u,p)$: minimal cost incurred to produce an output rate u and px_0 is observed cost. Most of the parametric production function commonly employed in many empirical analysis are homogeneous and /or homothetic. A production function $\phi(x)$ is said to be homogeneous of degree θ, if it satisfies the function, $\phi(\lambda x) = \lambda^{\theta}\phi(x)$. In addition, $\phi(x)$ is said to be linear homogeneous if and only if, $\phi(\lambda x) = \lambda\phi(x)$. Returns to scale implied by a linear homogeneous production frontier are constant.

$\lambda > 1; \theta = 1 \Rightarrow$ Returns to scale are constant
$\lambda > 1; \theta > 1 \Rightarrow$ Returns to scale are increasing
$\lambda > 1; \theta < 1 \Rightarrow$ Returns to scale are decreasing

An increasing transformation of a linear homogeneous production frontier is homothetic production frontier. $u = F[\phi(x)]$, is homothetic production function, satisfies the following properties.

- $F\phi(x) \geq 0, \forall x \in R_+^n$
- 2. $\phi(x)$ finite $\Rightarrow$ $F\phi(x)$ is finite

- 3. $F\phi(x)$ is non −decreasing; $x' \geq x \Rightarrow \phi(x') \geq \phi(x) \Rightarrow F\phi(x') \geq F\phi(x)$
- 4. $F(0)=0$, $x=0 \Rightarrow \phi(0)=0 \Rightarrow F\phi(0)=F(0)=0$ i.e. null input vector yields null output.
- 5. $\phi(x)$ continuous in $x \Rightarrow F\phi(x)$ is also continuous $u = F\phi(x)$

$\Rightarrow F^{-1}(u)=\phi(x)$; $f(u)=\phi(x)$ where $f(u)=F^{-1}(u)$ and $\phi(x)$ is linear homogeneous.

2.1 The Cobb-Douglas Production Function is not Only Homogeneous, also Homothetic

Production function: $u = A\prod x_i^{\alpha_i} = \left[A^{\frac{1}{\theta}} \prod x^{\alpha_i/\theta}\right]^{\theta}$ (1)

Where $\theta = \sum \alpha_i$ and $u = [\phi(x)]^{\theta} = F[\phi(x)]$. $\phi(x)$ is linear homogeneous, $F\phi(x)$ is non-decreasing, $F(0)=0$ and , $F\phi(x)$ is continuous function of $\mathbf{x}$.The variable returns to scale production frontier,

$$f(u) = A\prod x_i^{\alpha_i} \quad (2)$$

where $\sum \alpha_i = 1$,is homothetic production function. Also, it can be expressed as $f(u)=\phi(x)$ where $\phi(x)=A\prod x_i^{\alpha_i}$ and $f(u)=u^{\alpha}e^{\theta u}$. $\phi(x)$ is linear homogeneous, non-decreasing and function and continuous. The production function is input homothetic. Homothetic production structures have a special geometric property, expressible in turns of the isoquants of the production possibility sets. The Isoquant for any output rate $u \geq 0$ of a homothetic production structure may be obtained from that for unit output rate by scalar magnification from the origin in fixed ratio $\frac{f(u)}{f(1)}$.

An input set $L(u)$ may be defined in terms of input distance function as follows:

$$L(u) = \{x : D(u,x) \geq 1\} \quad (3)$$

If the underlying production frontier is $\phi(x)$ then the input sets induced by $\phi(x)$ are,

$$L_{\phi}(u) = \left\{x : \frac{\phi(x)}{u} \geq 1\right\} \quad (4)$$

The input sets induced by $f(u)=\phi(x), u=F(\phi(x))$, respectively are

$$L_{\phi}(f(u)) = \left\{x : \frac{\phi(x)}{f(u)} \geq 1\right\} \quad (5)$$

$$L_F(u) = \left\{x : \frac{F\phi(x)}{u} \geq 1\right\} \quad (6)$$

It can be shown that, $L_{\phi}[f(u)] = L_F(u)$ (7)

Consider the optimization problem:

$\text{Min}\{px + \lambda(1 - D(u,x))\}$, where λ is Lagrangian parameter

The first order conditions for minimum are $p_i = \lambda$

$$\frac{\partial D[u, X(u,p)]}{\partial X_i(u,p)} \quad (8)$$

$$\frac{\partial D[u, X(u,p)]}{\partial X_i(u,p)} = \frac{p_i}{\lambda} \quad (9)$$

Substitute equation (9) in input demand equation to obtain,

$$\sum \frac{p_i}{\lambda}\frac{\partial X_i(u,p)}{\partial p_i} = 0 \Rightarrow \sum p_i \frac{\partial X_i(u,p)}{\partial p_i} = 0 \quad (10)$$

Substituting equation (9) Shepherd's lemma we obtain

$$\frac{\partial X_i(u,p)}{\partial p_i} = X_i(u,p) \quad (11)$$

Replacing $Q(u,p)$ by $D(u,x)$, $X_i(u,p)$ by $p_i(u,X)$, p_i by X_i we get

$$Q(u,p) = f(u)\phi(p)$$

$$Q(u^*,p) = C$$

$$\Rightarrow f(u*) = \frac{C}{\phi(p)} \quad (12)$$

$$u^* = F\left(\frac{C}{\phi(p)}\right)$$

The expressions in equation (10) and equation (11) constitute Shepherd's lemma

2.2 Input Demand Equations

The Shephard's lemma connects primal input space with the dual input price space.

$\frac{\partial Q(u,p)}{\partial p_i} = X_i(u,p)$ gives input demand equation for i^{th} input.

2.2.1 Input Demand Equations of Cobb-Douglas Production Structure

It can be shown that $\lambda = Q(u,p)$

From the second part of Shephard's lemma

$$\frac{\partial D(u,x)}{\partial x_i} = p_i(u,x)$$

$$\sum x_i p_i(u,x) = \sum x_i \frac{\partial D(u,x)}{\partial x_i}$$

$$D(u,x)=\sum x_i \frac{\partial D(u,x)}{\partial x_i}$$

Since x belongs to Isoquant $L(u)$, then

$$D(u,x)=\sum x_i \frac{\partial D(u,x)}{\partial x_i}=1$$

From equation (8) we obtain

$$\sum p_i x_i(u,p)=\lambda \sum x_i(u,p)\frac{\partial D[u,x(u,p)]}{\partial x_i(u,p)}$$

$$=\lambda D[u,x(u,p)]=\lambda$$

$$\lambda=\sum p_i x_i(u,p)=Q(u,p)$$

$$\Rightarrow \quad \lambda=Q(u,p)$$

Thus the following theorems can be dealt with and their proofs are very straight forward.

Theorem 1: For homothetic production structure, the cost limited maximal output,

$$\Gamma(p)=F\left(\frac{C}{\phi(p)}\right)$$

Theorem 2: If target cost is observed cost and the production structure is homothetic $f[\Gamma(p)]=\delta^{-1}f(u)$ w here $\Gamma(p)$ is cost limited maximal output and δ is cost efficiency of production.

Theorem 3: If the underlying production process obeys Cobb-Douglas production structure retunes to scale are constant then the cost limited maximal output is the product of observed output and inverse of overall input productive efficiency.

Theorem 4: If production obeys Cobb-Douglas production structure, returns to scale are non-constant, then $\Gamma(p)=\delta^{-\theta}u$

Theorem 5: If production obeys Zellner-Revankar Variable Returns To Scale (VRTS) production structure which is input homothetic, then the cost limited maximal output can be obtained as solution of the following non–linear equation

$$[\Gamma(p)]^{\alpha} e^{\theta\Gamma(p)}=\delta^{-1}u_0^{\alpha}e^{\theta u_0}$$

Further, estimation of cost minimal output requires explicit specification of production structure that is explained by a production frontier, equivalently by its factor minimal cost function. Let $Q(u,p,\otimes)$ be the factor minimal cost, that depends upon output (u), input price vector (p) and the vector of parameters $\otimes$. Let $p_i x_i=C_i$ be the observed cost of i^{th} production unit. Cost constraint for i^{th} production unit, $Q(u,p_i,\otimes)\le C_i, i=1,2,\ldots\ldots.k$.

The factor minimal cost function may be estimated solving the following mathematical programming problem

$$\underset{\otimes}{\text{Max}} \quad Q(u,p,\otimes) \tag{13}$$

Subject to $Q(u,p,\otimes)\le C_1, i=1,2,\ldots\ldots,k$ and $\theta\ge 0$

where u_0 and p_0 are the output and price vector of the production unit whose efficiency is under evaluation. That is, cost efficiency is exactly achieved. A positive slack implies that cost efficiency is under achieved.

2.3 Cobb-Douglas Cost Frontier

The Cobb-Douglas cost frontier as derived may be expressed as,

$$Q(u,p,\otimes)=Bu^{\frac{1}{\theta}}\prod_i p_i^{\alpha_i/\theta} \text{ Where } \theta=\sum \alpha_i, 0\le\alpha_i\le 1 \text{ for} \tag{14}$$

Cost constraint: $Q(u,p,\otimes)\le C$

$$Bu^{\frac{1}{\theta}}\prod_i p_i^{\alpha_i/\theta}\le C$$

$$\ln B+\frac{1}{\theta}\ln u+\sum\frac{\alpha_i}{\theta}\ln p_i\le \ln C$$

$$b+\eta \ln u+\sum\beta_i \ln p_i\le d$$

$$b+\eta \ln u_j+\sum\beta_i \ln p_{ij}+\varepsilon_j=d_j$$

$$j=1,2,\ldots\ldots.k$$

Where $b=\ln B, \eta=\frac{1}{\theta}, \beta_i=\frac{\alpha_i}{\theta}\Rightarrow \alpha_i=\frac{\beta_i}{\eta}$

For j^{th} production unit the cost constraint takes the form, $b+\eta \ln u_j+\sum\beta_i \ln p_{ij}\le d_j$ $\eta>0$, $\beta_i\ge 0, \sum\beta_i=1$ and b is unrestricted for sign. This constraint is linear in parameters b, η, β_i which are currently unknown. Addition of the slack variable ε_j transforms the linear in equation into equation

$$b+\eta \ln u+\sum\beta_i \ln p_{ij}+\varepsilon_j=d_j \text{ for } j=1,2,\ldots\ldots.k$$

The complete linear programming problem in terms of slack is postulated as follows:

$$\text{Minimize } Z=\sum_{j=1}^{k}\varepsilon_j \tag{15}$$

Subject to $b+\eta \ln u_j+\sum\beta_i \ln p_{ij}+\varepsilon_j=d_j$ for $j=1,2,\ldots\ldots.k$, $0\le\beta_i\le 1, \sum\beta_i=1, \eta\ge 0$ and b is not unrestricted for sign.

For j^{th} production unit, $\varepsilon_j=d_j-\left[b+\eta \ln u_j+\sum\beta_j \ln p_{ij}\right]$

$$= ln\left[\frac{C_j}{Q(u_j,p_j,\otimes)}\right] \Rightarrow e^{\varepsilon_j} = \left[\frac{C_j}{Q(u_j,p_j,\otimes)}\right]$$

$$e^{-\varepsilon_j} = \left[\frac{Q(u_j,p_j,\otimes)}{C_j}\right] = \delta_j \quad (16)$$

Thus, δ_j is cost efficiency of j^{th} production unit. The solution of the linear programming problem equation (16) gives firm specific cost efficiencies.

$\varepsilon_j = 0 \Rightarrow \delta_j = 1 \Rightarrow$ cost efficiency is exactly achieved

$\varepsilon_j > 0 \Rightarrow \delta_j < 1 \Rightarrow$ cost efficiency is under achieved .

Thus, the linear programming problem in equation (15) can be viewed as a linear goal programming problem. The Cobb-Douglas cost frontier estimated by solving the linear goal programming problem equation (15) is full frontier, in the sense that the observed input costs of all the production units fall on or above the factor minimal cost frontier.

An alternative for full frontier is the stochastic cost frontier, which may be estimated solving the goal programming problem.

$$\text{Minimize}(Z) = \sum \varepsilon_j^+ + \sum \varepsilon_j^- \quad (17)$$

Subject to $b + \eta\, ln\, u_j + \sum \beta_i\, ln\, p_{ij} + \varepsilon_j^+ - \varepsilon_j^- = d_j$ for $j = 1,2,\ldots\ldots k$, $0 \le \beta_i \le 1$ and $\sum \beta_i = 1, \eta \ge 0$ b is not restricted for sign. $\varepsilon_j^+.\varepsilon_j^- = 0, \text{and} \varepsilon_j^+ \ge 0, \varepsilon_j^- \ge 0$

- $\varepsilon_j^+ > 0 \Rightarrow \varepsilon_j^- = 0$

$\Rightarrow b + \eta\, ln\, u_j + \sum \beta_i\, ln\, p_{ij} < d_j \Rightarrow$ Cost efficiency is under achieved.

- $\varepsilon_j^+ > 0 \Rightarrow \varepsilon_j^+ = 0 \ \Rightarrow b + \eta\, ln\, u_j + \sum \beta_i\, ln\, p_{ij} > d_j \ \Rightarrow$ Cost efficiency is over achieved.
- $\varepsilon_j^+ = \varepsilon_j^+ = 0 \Rightarrow b + \eta\, ln\, u_j + \sum \beta_i\, ln\, p_{ij} = d_j \Rightarrow$ Cost efficiency is exactly achieved.

2.4 Variable Returns to Scale Cost Frontier-Goal Programming

The VRTS cost frontier may be expressed as

$$Q(u,p,\otimes) = Bu^{\alpha}e^{\theta u}\Pi p_i^{\alpha_i} \quad (18)$$

where $\alpha, \theta \ge 0$ and $0 \le \alpha_i \le 1, \sum \alpha_i = 1$; cost constraint $Q(u,p,\otimes) \le C$ and $Bu^{\alpha}e^{\theta u}\Pi p_i^{\alpha_i} \le C$

$$\Rightarrow ln\, B + \alpha\, ln\, u + \theta u + \sum \alpha_i\, ln\, p_i \le ln\, C$$
$$b + \alpha\, ln\, u + \theta u + \sum \alpha_i\, ln\, p_i \le d$$

where $b = ln\, B, d = ln\, C$ and b is unrestricted for sign. The above constraint is in the parameters b , α , θ and α_i .

The following is the full frontier goal programming problem:

$$\text{Min}(Z) = \sum \varepsilon_j \quad (19)$$

Subject to $b + \alpha\, ln\, u + \theta u + \sum \alpha_i\, ln\, p_{ij} = d_j, j = 1,2,\ldots\ldots.k$; $\alpha, \theta > 0$ and $0 \le \alpha_i \le 1$, $\sum \alpha_i = 1$ where b is unrestricted for sign. For j^{th} production unit, $\varepsilon_j = 0$ implies that cost efficiency is exactly achieved. While $\varepsilon_j > 0 \Rightarrow$ cost efficiency is under achieved.

Stochastic cost frontier based goal programming problem may be postulated as,

$$\text{Min}(Z) = \sum \varepsilon_j^+ + \sum \varepsilon_j^- \quad (20)$$

Subject to $b + \alpha\, ln\, u_j + \theta u_j + \sum \alpha_i\, ln\, p_{ij} + \varepsilon_j^+ - \varepsilon_j^- = d_j$ and $\varepsilon_j^+.\varepsilon_j^- = 0, j = 1,2,\ldots\ldots\ldots.,k$

$\alpha, \theta > 0$ and $\theta \le \alpha_i \le 1$, $\sum \alpha_i = 1$; b is not restricted for sign and $\varepsilon_j^+, \varepsilon_j^- \ge 0$.

- $\varepsilon_j^+ > 0 \Rightarrow \varepsilon_j^- = 0 \Rightarrow$ Cost efficiency is under achieved
- $\varepsilon_j^- > 0 \Rightarrow \varepsilon_j^+ = 0 \Rightarrow$ Cost efficiency is over achieved.
- $\varepsilon_j^- = \varepsilon_j^+ = 0 \Rightarrow$ Cost efficiency is exactly achieved.

Further for the j^{th} production unit cost frontier is full VRTS frontier the cost limited maximal output can be obtained solving the following non-linear equation

$$\overline{(p)}^{a} e^{\theta \overline{(\beta)}} B \prod p_{ij}^{\alpha_i} = C_j$$

Also based on the analysis done in this section the set of equations are validated using the data collected in the further sections.

3. Empirical Illustrations

The cost limited maximal output is directly related to the cost maximal output is directly related to the cost efficiency which is defined as the ratio of factor minimal cost to observed cost. If the observed cost is target cost,

$$\text{Cost efficiency } CE = \frac{Q(u,p)}{C} \quad (21)$$

As the target cost varies factor minimal cost also changes, inducing a change in cost efficiency. Consequently, the cost limited maximal output increases depending upon an increase in target cost. If the producer is cost efficient c , consequently CE = 1.

Cost efficiency varies among the production units for two reasons:

- Different producers employ different production techniques as such technical inefficiency pervades in production environment.
- Failure to operate at cost minimizing inputs leads to allocative inefficiency.

It can be shown that cost efficiency is the cost efficiency is the product of technical and allocative efficiencies. CE=TE.AE. Given the target cost and input prices it is possible to estimate cost-limited maximal output. Of course, the estimates change depending upon the choice of cost frontier also. If a parametric production frontier is postulated to explain the underlying production process if he production frontier satisfies certain axioms, using the duality theory it is possible to derive the underlying cost frontier also. Such a cost frontier may be full or stochastic. To estimate the parameters of the stochastic cost frontier one may postulate suitable goal programming problems and solve them either by using linear or non-linear programming problems.

For Cobb-Douglas and variable returns to scale production frontiers the underlying cost function are first deduced to obtain the equation 2.The parameter A and a_i can be obtained directly estimating the production frontier or by estimating the cost frontier. The two sets of estimates of parameters possibly differ from each other, since the later cost of the parameters' estimates depend upon the assumption of cost minimization. Cost limited maximal output of C-D production structure $\overline{|(p)} = \left[\frac{C}{B\Pi p_i^{\beta_i}}\right]^{\theta}$ where $\theta = \sum a_i$, measures to scale and C is the target cost. As target cost C changes $\overline{|(p)}$ also changes. To estimate $\overline{|(p)}$ for each target cost C a knowledge of $\theta, B,$ and β_i is necessary.

3.1 Full Cost Frontier

The cost frontier $Q(u.p)$ is said to be a full cost frontier, if it satisfies the property, $Q(u_i, p^i) \le C_i,\ i = 1,2,3,\ldots,k$ where u_i is output of i^{th} production unit, p^i the price vector of the i^{th} production unit and C_i is observed output of i^{th} production unit. The full cost frontier may be estimated by solving the following linear programming problem given equation 13. Another version of estimating full frontier is solving the following linear programming problem is given in the equation 15. If the underlying cost structure is Cobb-Douglas the full frontier is obtained solving the equation 5 linear programming problem. An alternative for full cost frontier is the stochastic cost frontier, whose parameters can be estimated proposing and solving the following goal programming problem given in the equation 17. For empirical implementation the total manufacturing sectors of the South Indian states, viz., Andhra Pradesh, Karnataka, Kerala, Maharashtra, and Tamil Nadu are taken into consideration along with the total manufacturing sector of all India. The variables of study are,

- Value added (u)
- Number of persons employed (x_1)
- Fixed capital (x_2) and
- Total emoluments (w)

The data are secondary collected from Annual Survey Industries for the year (2000-2001) was used. Number of persons employed is proxy for the number of labourers and total emoluments is proxy for total wages. Unit price of labour is the ratio of the total wages to the number of persons employed. $p_1 = \frac{w}{x_1}$ A proxy for unit price of capital is _. The following linear programming problem is formulated and solved.

_ (22)

AUQ: equations are missing please add equations.

_ : All India; _: Andhra Pradesh_: Karnataka _: Kerala_: Maharashtra_=Tamil Nadu

The total manufacturing sector of all India is augmented along with the five south Indian states, in order to estimate structural cost efficiency. The solution of equation 22 is as follows:

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Table 1 indicates, Kerala and Maharashtra are cost efficient. Structural cost efficiency implied by the total manufacturing sector of all India is 0.7860. About 21% of inputs are lost due to cost inefficiency at the country level. In South India the most cost inefficient total manufacturing sector is that of Karnataka. 44% of its inputs are lost due to cost inefficiency. 40% input losses are experienced by the total manufacturing sector of Andhra Pradesh. About 28% percent input losses are due to cost inefficiency for the total manufacturing sector of Tamil Nadu.

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For Cobb-Douglas cost structure the cost limited maximal output is given by

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From Table 1, for Kerala and Maharashtra the cost limited maximal output and observed outputs are one and the same. To attain cost limited maximal output the total manufacturing sector of all India has to increase its output by 27% more than what produces currently. The total manufacturing sectors of Andhra Pradesh, Karnataka and Tamil Nadu have to produce their output by 66, 77 and 34 percent more than what they actually produce. The cost limited maximal outputs listed in the Table 2 are based upon full cost frontier. To estimate the stochastic cost frontier, solve the following goal programming problem.

_ (23)

Table 1. Estimation of cost efficiency for manufacturing sectors

S.No.	Total Manufacturing Sector	Cost efficiency _
1	All India	0.7860
2	Andhra Pradesh	0.6007
3	Karnataka	0.5617
4	Kerala	1.0000
5	Maharashtra	1.0000
6	Tamil Nadu	0.7154

Table 2. Cost limited maximal output for various manufacturing sectors

S.No.	Total Manufacturing Sector	Cost limited maximal output
1	All India	1.2694x15497442
2	Andhra Pradesh	1.6569x911042
3	Karnataka	1.7709x834737
4	Kerala	1.0000x362980
5	Maharashtra	1.0000x3458772
6	Tamil Nadu	1.3379x1479535

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The optimal solution of problem in equation 23 is

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For all India _ consequently _ cost efficiency is achieved exactly. For the total manufacturing sector of Andhra Pradesh _

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Cost efficiency is underachieved.

For the total manufacturing sectors of Karnataka and Tamil Nadu cost efficiency is exactly achieved. However, the total manufacturing sectors of Kerala and Maharashtra over achieved cost efficiency are shown in Table 3.

Table 3. Full cost frontier cost efficiency

S.No.	Total Manufacturing Sector	Cost Efficiency
1	All India	Exactly achieved
2	Andhra Pradesh	Under achieved
3	Karnataka	Exactly achieved
4	Kerala	Over achieved
5	Maharashtra	Over achieved
6	Tamil Nadu	Exactly achieved

Andhra Pradesh is the only state in South India for which cost efficiency is under achieved. For stochastic cost frontier cost function we have the following relationship:

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For stochastic cost frontier the goal programming estimates of _ is_. From Table 4 it is obvious that the total manufacturing sectors of All India, Karnataka and Tamil Nadu are cost efficient. Their current outputs shown in Table 5 are cost limited maximal outputs. About 16% of input losses are due to cost inefficiency, for the total manufacturing sector of Andhra Pradesh. To attain cost limited maximal output it has to promote its output by 21% more than what it is producing currently.

Table 4. Estimation of stochastic cost efficiency for manufacturing sectors

S.No.	Total Manufacturing Sector	Cost Efficiency
1	All India	1.0000
2	Andhra Pradesh	0.8369
3	Karnataka	1.0000
4	Kerala	1.3109
5	Maharashtra	1.5416
6	Tamil Nadu	1.0000

Table 5. Stochastic cost limited maximal output for various sectors

S.No.	Manufacturing Sector	Cost limited maximal output
1	All India	1x15497442
2	Andhra Pradesh	1.2116x911042
3	Karnataka	1x834737
4	Kerala	0.7469x362980
5	Maharashtra	0.6271x3458772
6	Tamil Nadu	1x1479535

For the total manufacturing sector of Kerala the cost limited maximal output is less by 24% than what it currently produces. On the other hand, the cost limited maximal output is 37% less than the current output produced by the total manufacturing sector of Maharashtra.

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In Table 6, the estimates implied by full cost frontier differ widely when compared with those of stochastic cost frontier. For full cost frontier the estimate of elasticity of output with respect to labour _ is zero, whereas the estimate of same elasticity is 0.4373 for the stochastic cost frontier.

Return to scale implied by full cost frontier are decreasing while those implied by the stochastic cost frontier are increasing.

Table 6. Comparison of costs between stochastic and full cost frontier

Parameter	Full cost frontier	Stochastic cost frontier
B	2.8301	6.6068
_	0.9908	6.6068
_	0.0000	0.4373
_	0.9908	0.6405

3.2 Variable Returns to Scale Production and Cost Structures

The Variable Returns To Scale (VRTS) production frontier is _ where _ is the output and _ is the _input. The dual cost function of VRTS is _, _elasticity of scale implied by the VRTS production frontier is, _ which is a function _.

Stochastic VRTS cost frontier may be fitted solving the following goal programming problem

$$\text{Minimze } Z=\sum_{i=1}^{6}\varepsilon_i^{+}+\sum_{i=1}^{6}\varepsilon_i^{-}$$

Subject to

__

The optimal solution of the above goal programming problem is as follows:__ The returns to scale function are given by __implies _ then the observed output _ is cost efficient output. Both the slacks vanish for the total manufacturing sectors of all India, Karnataka, Kerala and Tamil Nadu, consequently they attain cost efficiency exactly. For the production units which are cost efficient cost limited maximal output and observed output remains to be the same. If a production unit is cost inefficient, the cost limited maximal output exceeds observed output. The total manufacturing sector of Andhra Pradesh is cost inefficient which is reflected by, _. The cost limited maximal output for the total manufacturing sector is _. Thus from Table 7 it is observed that the total manufacturing sector of Andhra Pradesh the cost limited maximal output is worth 1176532 lakh rupees._implies that the total manufacturing sector of Maharashtra had over achieved cost efficiency. Consequently, its cost limited maximal output is less than its observed output. _

Table 7. Stochastic VRTS cost limited maximal output for manufacturing sectors

S.No.	Total manufacturing Sector	Cost limited maximal output
1	All India	15497442
2	Andhra Pradesh	1176532
3	Karnataka	834737
4	Kerala	362980
5	Maharashtra	1539476
6	Tamil Nadu	1479535

Elasticity of scale measures returns to scale of a production unit. If the underlying production structure is VRTS, elasticity of scale are measured by _ . Table 8 furnishes elasticity of scale for the total manufacturing sectors of all India, Andhra Pradesh, Karnataka, Kerala, Maharashtra and Tamil Nadu.

Returns to scale as implied by the elasticity of scale are increasing for the total manufacturing sector of all India, more or less constant for Maharashtra. In all other total manufacturing sectors returns to scale are decreasing.

Table 8. Elasticity and returns for scale of various sectors

S.No.	Manufacturing Sector	Elasticity of scale	Returns for scale
1	All India	1.6366	IRTS
2	Andhra Pradesh	0.9461	DRTS
3	Karnataka	0.9421	DRTS
4	Kerala	0.9291	DRTS
5	Maharashtra	1.0225	CRTS
6	Tamil Nadu	0.9607	DRTS

4. Conclusions

The work presented in this paper is an application of Goal programming for Mathematical Economics problem which involves properly structured production functions and their dual cost functions. Two popular production functions are considered. One the Cobb-Douglas production frontier and the other is variable Returns to Scale Production Frontier. For the Cobb-Douglas and VRTS production frontiers, the expression for cost limited outputs is derived. It is further shown that the cost limited maximal output is cost efficient. Further, one can visualize two cost frontiers, viz., the full and stochastic cost frontiers. Fitting of a full frontier is possible by postulating and solving suitable linear programming problems. The Cobb-Duglas and the VRTS cost frontiers can be expressed as linear in unknown parameters by taking logarithms.

The method developed is applied to six total manufacturing sectors of all India, Karnataka and Kerala achieved cost efficiency exactly. Consequently, their cost limited maximal output and the observed outputs are one and the same. The total manufacturing sector of Andhra Pradesh underachieved cost efficiency, so that its cost limited maximal output exceeds in the value of the observed output. The total manufacturing sector of Maharashtra overachieved cost efficiency for which the observed output is more than its cost limited maximal output.

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