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Estimation of Bending Moment Coefficients of Two Way Rc Slabs: Finite Element Approach

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Abstract

Objective: The main objective of this study is to find out the bending moment co-efficient of two way reinforced concrete (RC) slabs. **Methods/Analysis:** Linear and nonlinear static analysis of single panel two way RC slabs have been performed in finite element program ANSYS 12.0. The yielding behavior of slabs for various edge restrained conditions in both linear and non-linear analysis was studied. **Findings:** The yield line patterns of slabs with all boundary conditions were observed. From the yielding pattern, it was studied that the longer edges yielding first followed by the shorter edges and mid-span of slab. Based on the observation of yield line pattern, the bending moment coefficients were found. The obtained values were compared with IS: 456 recommendations. The comparison of analytical results show that the method adopted to obtain the moment coefficients of two way slabs is satisfactory. **Applications:** Using detailed observation of yielding of slabs, the bending moment coefficients shall be arrived for the use of design of slabs of any irregular shapes.

Keywords: Bending Moment Coefficient, Boundary Conditions, Non-Linear Analysis, Yield Line Pattern

1. Introduction

In reinforced concrete construction, slabs are used to provide flat surfaces. The span in the larger direction is denoted by L_y and that in the shorter direction by L_x. Slabs may be supported on two opposite sides only, in which case the structural action of the slab may be in one way i.e. the load carrying by the slab in the direction normal to the supporting beams. Also when the ratio of L_y/L_x is more than two, it can be defined as one way slab. Slabs which are supported on unyielding supports like walls and stiff beams on all four sides and if the ratio of L_y/L_x is less than or equal to two, then the action of slabs may be in two way. The boundaries of a two way slab can be simply supported or restrained at the edges. Slabs resting on walls are considered as simply supported slabs, wherein edges are free to rotate. Slab constructed monolithically with beams have some restrained against rotation. These slabs may be continuous or discontinuous. Slab design is predominantly done for bending (or flexure) behavior. Sagging moment (positive moment) and hogging moment (negative moment), depending on edge conditions, are to be obtained.For the help of designer,[1] has given following bending moment values in terms of coefficients and applied pressure for various values of aspect ratio (L_y/L_x),

$$M_x = \alpha_x W L_x^2 \quad (1)$$

$$M_y = \alpha_y W L_x^2 \quad (2)$$

Where, M_x = moment in x-direction, M_y = moment in y-direction, α_x = co-efficient in x-direction, α_y = co-efficient in y-direction, W = total design load per unit area, L_x = length of shorter span and L_y = length of longer span. These bending moment coefficients are based on yield line theory[2,3] presented bending moment coefficients for analysis and design of rectangular slabs supported on all sides and loaded uniformly. For preparing these coefficients, simple yield line theory ignoring the effect of corner levers has been considered[4]provided equation for calculating bending moment coefficients for simply supported and edge retrained slabs. Though the equations are based

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on aspect ratio (L_y/L_x), the equations for simply supported slabs are derived using Rankine-Garshoff theory and edge restrained slabs are based on number of discontinuities for larger span co-efficient and based on aspect ratio for shorter span coefficients.

[5]developed equations using virtual work method for trapezoidal shape reinforced concrete slabs and presented coefficients for all nine boundary conditions. He varied the values of positive and negative yield moments for different aspect ratio, calculated the coefficients and prepared charts and tables from which one can easily obtain the values for the corresponding trapezoidal slab. Many of the researches have been successfully carried out using non-linear finite element programming such as ANSYS, ABAQUS, etc. For example,[6]performed linear static and dynamic analysis of brick infill considering different types of opening using ANSYS.[7] did his research on application of optimization techniques for yield line pattern determination in slabs. He modeled few slab elements using ANSYS and obtained the proper pattern for each case study. He proposed that modeling with finer mesh leads to accurate results[8] presented moment coefficients for waffle slabs from linear and non-linear analysis[9] presented automated yield line procedure for identifying yield line pattern for irregular shapes[10] proposed new kinematic formulation for yield design of reinforced concrete slabs based on Nielson yield criterion[11]provided a safe, and efficient lower bound solution for the analysis of reinforced concrete slabs up to failure.

In this study, analysis of slabs is carried out using finite element method performing linear and non-linear analysis and yield line behavior was depicted. The bending moment values (positive and negative) are obtained and compared with coefficients of IS456:2000. To identify the effect of loading on these coefficients, various loading values are applied. Similarly the effect of beam depth on grid panel modeling, the effect of various L_x and L_y for same L_y/L_x ratio have been analyzed and presented. By close review of moments in every load increments, the coefficients are arrived till the material reaches to yielding state and the corresponding values of coefficients were obtained.

2. Modeling of two way Slab

2.1 Linear Analysis

A slab panel of size 4 m x 6 m with the mesh size of 0.2 m x 0.2 m is modeled with appropriate boundary condition is given in IS456 and linear analysis is carried out in SAP2000. Thickness of slab is 200 mm; Loading is 5 kN/m^2; Material type is Concrete; Grade of concrete is M25; Young's Modulus is 5000 $\sqrt{f_{ck}}$ and Poisson's ratio is 0.15. For applying boundary conditions, fixed support and hinged support boundary conditions are provided at nodes in the edges for continuous and discontinuous edges respectively. The loading applied in the form of pressure on top of the element. For all nine cases same loading and mesh sizes have been used. The bending moments developed at the positive center and negative edges are observed. It has been observed that out of the all boundary conditions, four edges discontinuous slab panel reaches the maximum positive bending moment for the same applied loading. The values of bending moment coefficients for the all nine cases of boundary conditions have been arrived and compared with IS456:2000 values as shown in Table 1.

4EC – four edges discontinuous, 1SED – One short edge discontinuous, 1LED – One long edge discontinuous, 2AED – Two adjacent edges discontinuous, 2SED – Two short edges discontinuous, 2LED – Two long edges discontinuous, 1LEC – One long edge continuous, 1SEC – One short edge continuous, 4ED – four edges discontinuous, Xsap – Bending moment coefficient in x direction obtained from SAP2000, Xis – Bending moment coefficient in x direction as per IS456: 2000.

On the contrary, the loading applied and material properties defined for all cases are same, the obtained values of coefficients of these modeling values are not equal with IS values. The positive values are lesser and negative values are larger than IS values. Therefore, it

Table 1. Comparison of bending moment coefficients – Linear Analysis

Case	Bending moment coefficient							
	Positive				Negative			
	Xsap	Xis	Ysap	Yis	Xsap	Xis	Ysap	Yis
4EC	0.037	0.041	0.025	0.024	0.075	0.053	0.050	0.032
1SED	0.039	0.044	0.026	0.028	0.079	0.057	0.053	0.037
1LED	0.047	0.051	0.031	0.028	0.094	0.067	0.063	0.037
2AED	0.053	0.056	0.035	0.035	0.103	0.075	0.069	0.047
2SED	0.041	0.045	0.027	0.035	0.081	0.06	-	-
2LED	0.055	0.068	0.037	0.035	-	-	0.040	0.045
1LEC	0.058	0.064	0.039	0.043	0.11	0.084	-	-
1SEC	0.066	0.076	0.044	0.043	-	-	0.056	0.057
4ED	0.078	0.089	0.052	0.056	-	-	-	-

was interested to perform nonlinear analysis to study the effect of material nonlinearity on bending moment coefficients.

2.2 Nonlinear Analysis

Since the results of linear analysis obtained from SAP2000 give variation in comparison with IS values, a commercially available finite element program ANSYS 12.0, was used in this study to perform the nonlinear analysis. A single panel of slab with all nine possible boundary conditions is modeled for non-linear analysis. The difference between linear and non-linear analysis in software oriented analysis is defining material properties and type of analysis. The element type assumed is Shell 91 and thickness of slab considered is 200mm. Panel size is 4 m x 6 m, inelastic material type considered is multilinear and grade of concrete is M25.

For defining nonlinear material properties, the stress strain formula proposed by Hognestad has been used (Figure 1) as follows,

For, $\varepsilon_c < \varepsilon_0$

$$f_c = f_{ck}\left(2\left(\frac{\varepsilon_c}{\varepsilon_0}\right) - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2\right) \quad (3)$$

Where, f_c= compressive stress, f_{ck}= characteristic compressive strength of cubes, ε_c= compressive strain, ε_0= strain corresponding to f_{ck}= 0.002 and ε_{cu}= ultimate compressive strain = 0.0035.The loading was increased slowly to collapse occurs.

For four edges discontinuous slab, initially the slab is stressed at mid-span. The linearity of the material during analysis had been verified by monitoring the stress at the particular sub step. At loading 162 kN/m², the stress reaches its maximum value of 25 N/mm² at centre.Similarly the respective elastic and plastic strain values also had been observed. The elastic strain

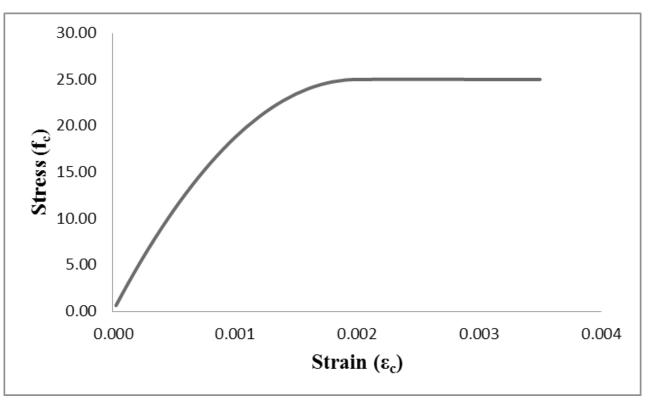

Figure 1. Stress strain model for concrete.

Table 2. Bending moment coefficients of 4EC slab - Nonlinear

Description	Sign		Coefficients		
			Nonlinear	Linear	IS456
Yielding at longer edge	Negative	x	0.055	0.075	0.053
Yielding at shorter edge	Negative	Y	0.037	0.050	0.032
Yielding at center	Positive	X	0.040	0.037	0.041
	Positive	Y	0.027	0.025	0.024

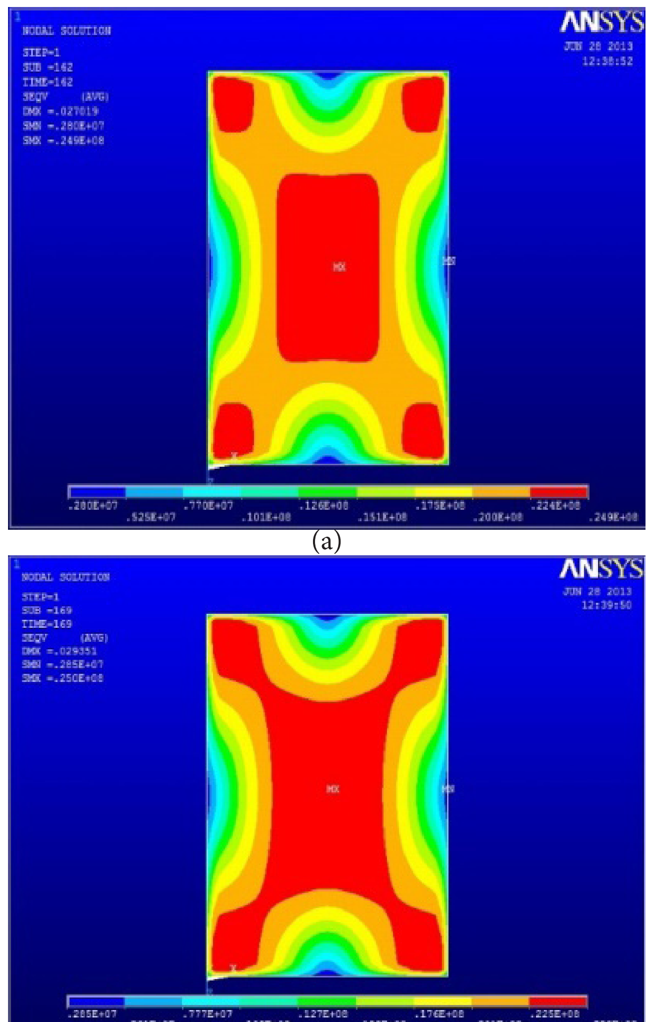

Figure 2. Formation of yield lines of 4ED two way slab panel.Formation of yield line (b) Complete yield line pattern

Table 3. Comparison of linear and nonlinear bending moment coefficients with IS456 values

Case	Bending moment coefficients											
	Positive X			Positive Y			Negative X			Negative Y		
	Linear	Non linear	IS456	Linear	Non linear	IS456	Linear	Non linear	IS456	Linear	Non linear	IS456
4EC	0.037	0.040	0.041	0.025	0.027	0.024	0.075	0.055	0.053	0.050	0.037	0.032
1SED	0.039	0.044	0.044	0.026	0.029	0.028	0.079	0.054	0.057	0.053	0.036	0.037
1LED	0.047	0.047	0.051	0.031	0.031	0.028	0.094	0.068	0.067	0.063	0.045	0.037
2AED	0.053	0.051	0.056	0.035	0.034	0.035	0.103	0.068	0.075	0.069	0.045	0.047
2SED	0.041	0.046	0.045	0.027	0.031	0.035	0.082	0.081	0.06	-	-	-
2LED	0.055	0.055	0.057	0.037	0.037	0.035	-	-	-	0.105	0.075	0.045
1LEC	0.058	0.055	0.064	0.039	0.037	0.043	0.110	0.083	0.084	-	-	-
1SEC	0.066	0.065	0.076	0.044	0.043	0.043	-	-	-	0.113	0.081	0.057
4ED	0.081	0.073	0.089	0.054	0.049	0.056	-	-	-	-	-	-

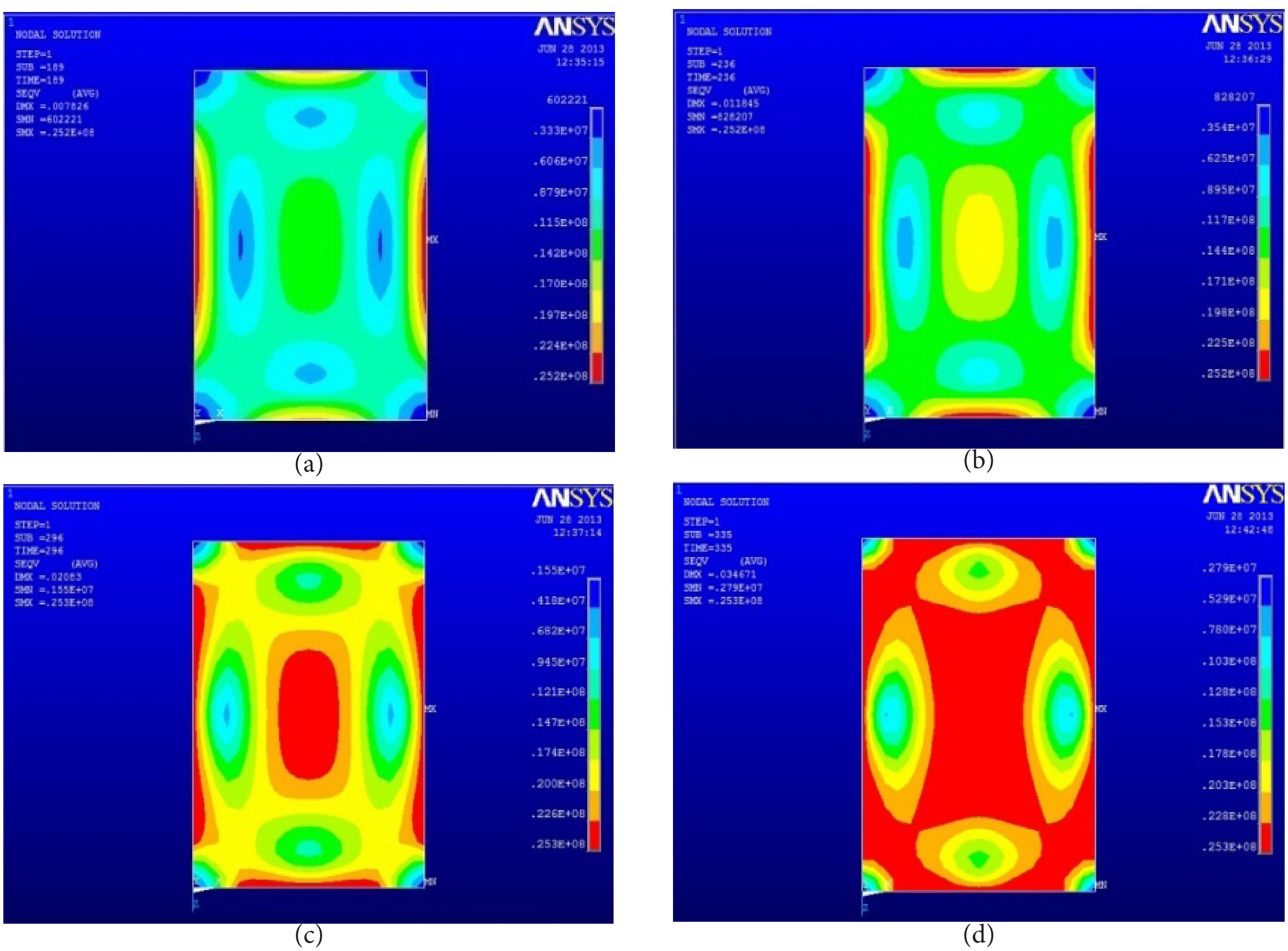

Figure 3. Step by step formation of yield line in 4EC two way slab panel.(a)Yield line at longer edges(b) At shorter edges(c) At mid-span (d) Complete formation of yield line

value reaches its maximum i.e. 0.001 and plastic strain reaches 0.001043.During this stage, the modulus has been calculated which became less than 25000 N/mm^2. Then the pattern slowly increases and reached into the corner at the loading of 168 kN/m^2 and forms complete yield line. The formation of yielding at mid-span and complete yield line pattern for four edges discontinuous slab (4ED) is observed and the bending moment coefficients are calculated.

3. Results and Discussion

It is aimed to find out the bending moment coefficients of single panel for all possible boundary conditions for the aspect ratio of 1.5. At negative edges the yielding will occur first and then at the positive edges. Therefore, for four edges continuous (4EC) case stresses are observed till yielding occurs. For calculating bending moment coefficients, the von mises stress values corresponding to the loads at yielding has been noted. Therefore, for 4EC slab, a coefficient corresponding to longer edge is negative in x direction for the loading of 189 kN/m^2 and for shorter edge is negative in y direction for the loading of 236kN/m^2. Similarly for positive coefficients yielding at mid-span at the loading of 296 kN/m^2. The values of coefficients arrived using yield line method of observation is given in Table 2. For negative edges von mises stress are required to be considered and for positive mid-span stresses corresponding to x and y direction are to be considered. Similarly for other possible boundary conditions which are specified in IS: 456 also modeled and the bending moment coefficients are calculated. The formation of yield line at shorter and longer edges and at the mid-span of the two ways slab for all possible boundary conditions are observed to calculate the bending moment coefficients. The step by step observation of yield lines at different loading levels for 4ED and 4EC two way slab elements are given in Figure 2 and Figure 3 respectively. Similarly, the linear and nonlinear values of coefficients are compared with IS: 456 values. From Table 3, it is observed that the values obtained from nonlinear analysis depict similar to codal values.

Therefore, it is evident from the nonlinear finite element analysis that, the designer shall be able to find out the bending moment coefficients for any irregular shape based on the detailed observation of formation of yield lines.

4. Summary andConclusion

The primary objective of this study is to understand the linear and nonlinear behavior of two way RC slabs and to estimate the bending moment coefficients. In order to find out the values, nonlinear analysis was performed and formation of yield lines at different parts of the slab panel at various stages of loading was observed. The obtained values are then compared with linear and codal values. The following observation are made from this study,

- Bending moment coefficients do not depend on loading and panel size in linear analysis.
- Using Non-linear analysis,
  - Yield line behavior of slab can be depicted.
  - Observing von-mises stress strain distribution, yield line pattern can be identified.
  - Design bending moment and bending moment coefficients can be obtained.
- Non-linearity spreads from edge to center in case of continuous edge slabs. Yielding first occurs at longer edge, then at shorter edge and finally at center, which gives the clear idea of behavior of two way slabs in non-linear analysis.
- Based on yielding of material at edges and mid span, bending moment coefficients shall be calculated.

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