



ORIGINAL ARTICLE

# Fusion of geometric and texture features for finger knuckle surface recognition



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## KEYWORDS

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**Abstract** Hand-based biometrics plays a significant role in establishing security for real-time environments involving human interaction and is found to be more successful in terms of high speed and accuracy. This paper investigates on an integrated approach for personal authentication using Finger Back Knuckle Surface (FBKS) based on two methodologies viz., Angular Geometric Analysis based Feature Extraction Method (AGFEM) and Contourlet Transform based Feature Extraction Method (CTFEM). Based on these methods, this personal authentication system simultaneously extracts shape oriented feature information and textural pattern information of FBKS for authenticating an individual. Furthermore, the proposed geometric and textural analysis methods extract feature information from both proximal phalanx and distal phalanx knuckle regions (FBKS), while the existing works of the literature concentrate only on the features of proximal phalanx knuckle region. The finger joint region found nearer to the tip of the finger is called distal phalanx region of FBKS, which is a unique feature and has greater potentiality toward identification. Extensive experiments conducted using newly created database with 5400 FBKS images and the obtained results infer that the integration of shape oriented features with texture feature information yields excellent accuracy rate of 99.12% with lowest equal error rate of 1.04%.

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## 1. Introduction

Personal recognition based on hand biometric traits has been widely used in most of the modern security applications due to its low cost in acquiring data, its reliability in identifying the individuals and its degree of acceptance by the user [1]. Most of the research works proposed in hand based biometric authentication used different modalities viz., fingerprint, palm

print, hand geometry, hand vein patterns, finger knuckle print and palm side finger knuckle print [2]. Among these biometric traits, fingerprint is considered to be the very old trait and known as the first modality used for personal identification. Apart from the various beneficial aspects, fingerprint also possesses some limitations such as its vulnerability toward intrusion of acquired image and its features such as minutiae, singular points, and delta points, are highly distracted by means of wounds and injuries [3] created on the finger surfaces.

On the other hand, palm print recognition system captures large area for identification, while it contains limited number of features such as principal lines, wrinkles [4]. In case of finger

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geometry and hand geometry, the features extracted are not distinctive enough to identify the individuals, when the number of users grows exponentially [5]. In hand vein system, the vein structures present in the dorsum area of the hand are captured by means of high cost devices [6].

In this paper, we contribute a new approach for personal recognition using Finger Back Knuckle Surface (FBKS) based on geometric analysis and texture analysis by considering both proximal phalanx and distal phalanx. Here, the proximal phalanx refers to the major bend surface of a finger and is found in the middle portion of the finger back region, whereas distal phalanx refers to minor bend surface found nearer to the tip of the finger back region. Even though, the distal phalanx is smaller in size, it has unique dermal patterns which when exploited along with proximal phalanx results in highly improved performance in finger knuckle print recognition. The rich set of patterns generated by each of these finger knuckle surfaces with lines, contours and creases is highly unique for distinctive identification of individuals, which could be easily acquirable in contact less manner. This FBKS biometric modality is highly accepted by the user, since it requires less cooperation from the subject and produces high speed authentication.

In the literature, the geometric based feature extraction methods implemented on hand-based biometric trait including finger knuckle print derive magnitude based feature information which has limited power of discrimination [7]. In contrast, this paper addresses this issue by recommending an Angular Geometric Analysis based Feature Extraction Method (AGFEM) capable of extracting angular based feature information from the finger back knuckle surface, which efficiently authenticates the individuals. For effective representation of texture features of FBKS images, the multi-resolution analysis is required since it could be able to handle distorted finger knuckle regions resulted due to scaling, rotation and transformation variant properties [8]. A multi-resolution transform known as Contourlet Transform which effectively represents the curved singularities than the wavelets is highly suitable for representing finger back knuckle surface texture feature since the texture pattern of FBKS images is lines, curves and contours [9]. Hence, we incorporate Contourlet Transform based Feature Extraction Method (CTFEM) to represent the texture features of the captured finger knuckle images. The extracted shape oriented and texture feature information is integrated to yield better accuracy results and makes it highly suitable for large scale personal authentication system.

The rest of the paper is organized as follows. Section 2 gives a brief survey on some of the feature extraction methodologies for hand biometric traits available in the literature. Section 3 demonstrates the proposed personal authentication system design using finger back knuckle surface. Section 4 presents the methods used for preprocessing and extraction of ROI from the acquired image. Section 5 introduces angular geometric based feature extraction methodology to extract angular feature information from the proximal knuckle and distal knuckle regions. Section 6 presents contourlet transform based feature extraction method to extract texture feature information from the proximal knuckle and distal knuckle regions. The various fusion rules related to matching score level fusion incorporated in this paper are illustrated in section 7. The thorough experimental analyses of the proposed methodologies are presented in Sections 8 and 9 concludes the paper.

## 2. Related work

In the literature, researchers have proposed various promising methods for hand based biometrics. These methods can be broadly classified into three categories viz., geometric based methods, texture based methods and statistical methods [10]. Generally, in geometrical based feature extraction methods, several edge detecting approaches were used for extracting features such as edge points, lines, creases, wrinkles, from various hand biometric traits [11]. The extracted edge information is either utilized directly or converted into the form of geometrical feature information to represent the feature vector for matching [12]. In texture based feature extraction methods, the Region of Interest (ROI) captured is categorized into blocks. From the ROI, the features extracted from the blocks or variations existing in different blocks are represented as feature information for matching [13].

### 2.1. Geometrical analysis based feature extraction method

Woodard and Flynn [14] were the first to propose the Finger Knuckle Print (FKP) as a biometric trait in 2005. In their work, the FKP image was acquired by means of a 3D sensor and the feature extraction process is done by means of geometrical analysis by exploiting the curvature shape features of FKP. The complexity toward 3D data processing which is computationally expensive is the main drawback of this scheme. Later in 2009, Kumar et al. have proposed number of techniques for personal authentication using hand biometric traits. In the first work, Kumar et al. [15] proposed a new personal authentication system using finger knuckle surface. The feature extraction from the finger knuckle surface was carried out by means of both texture and geometrical feature analysis methods. Finger length, finger width etc., were some of the geometrical features extracted from the finger knuckle surface by means of Finger Geometric Feature Extraction Method (FGFEM). The texture information of the finger knuckle surface is obtained by means of principal component analysis, independent component analysis and linear discriminant analysis. Scores are generated by means of computing Euclidean distance obtained from reference and input vectors.

Kumar et al. in the second work [16], introduced a new modality known as hand vein structure for personal authentication. In this biometric system, dorsum surface of the hand is captured using infra-red imaging. The captured image is subjected to histogram equalization for enhancement and the structure of the vein is studied using Key Point Triangulation Method (KPTM). This paper also focuses on incorporating the simultaneously extracted knuckle shape information to achieve better performance. Kumar et al. have further explored [17] the analysis of finger knuckle surface by incorporating the quality feature of the trait which highly depends on the capturing device. In this work, the entire hand image is acquired and feature extraction is done by means of palm print textural analysis, hand geometry analysis and finger knuckle print paternal analysis. The geometric method incorporated in this work is termed as Knuckle Geometric Feature Extraction Method (KGFEM). All the above described methods were taken as benchmark systems for comparing proposed AGFEM approach.

## 2.2. Texture analysis based feature extraction method

Feature extraction using texture based methods was first explored by Kumar and Ravikanth [14]. In their work, the finger knuckle surface was captured by means of 2D sensor device and then appearance based methods were used to extract feature information. Zhang et al. proposed a method personal authentication using finger knuckle print based on texture analysis. In this work authors incorporated band limited phase only correlation method for finger knuckle print matching [18]. Further, in 2012, Zhang and Li investigated a novel method to encode local image patterns using Riesz transform. Authors examine two coding mechanisms known as Rcode1 and Rcode2 by subjecting the obtained finger knuckle images to the first and second order Riesz transform. Hamming distance that exists between the Rcodes of registered and input images is estimated for matching finger knuckle images [19].

Furthermore, Gao et al. [20] recommend a new approach for processing scale, rotational and translational variant finger knuckle images by incorporating the method of reconstruction. Authors incorporated discovery learning process from which the reconstruction of rotated or distorted knuckle images was done. Gao et al. [21] in their recent work, investigated a new approach which combines multiple orientation coding and texture features for finger knuckle print matching.

Additionally, Shariatmadar et al. [22] proposed a new finger knuckle print recognition scheme in which ROI of the captured finger knuckle print image is categorized into blocks and each block is subjected to Gabor filters for generating binary patterns in the form of histograms. Bio hashing method is incorporated to perform matching process between the obtained fixed length feature vectors of registered image and the input image. Yu et al. [23] recommended an effective approach for personal authentication using FKP. The ROI of finger knuckle image is divided into sub-blocks and from these sub-blocks local feature information was derived and stored as local binary pattern for matching.

Some of the transform based texture based analysis methods which are taken as benchmark systems for comparing the proposed CTFEM are detailed below. In [24], the fusion of the palm print and knuckle print at matching score level is done to achieve high performance in authentication. The features of palm print and knuckle print are represented by means of Phase Correlation Function using Discrete Fourier Transform (PCFDFT). Linear phase shift in the frequency domain of palm print images and finger knuckle print images are derived by Discrete Fourier Transforms. The observations based on Phase Correlation Functions (PCF) are calculated by means of cross correlation and results were further analyzed by locating similarity between the reference images and input image.

Further, Saigaa et al. [25] proposed a new biometric system based on information extracted from the texture of finger back knuckle surfaces. The captured image is preprocessed by means of smoothing algorithms and candy edge detector algorithm for locating the region of interest. Extraction of knuckle texture feature in the ROI image was carried out by means of two dimensional block based discrete cosine transforms (DCTFEM). The obtained DC coefficient gives the average value of gray scale pixel whereas AC coefficient describes the

information about the lower frequencies and higher frequencies of the block. Furthermore, in [26] Hedge et al. contributed a real-time personal authentication using finger knuckle print. The features of the finger knuckle surface were extracted using three unique algorithms viz., (i) Random Transform, (ii) Gabor Wavelet Transform and (iii) Combining both the feature information of matching (RTGWT).

## 2.3. Extracts of the literature

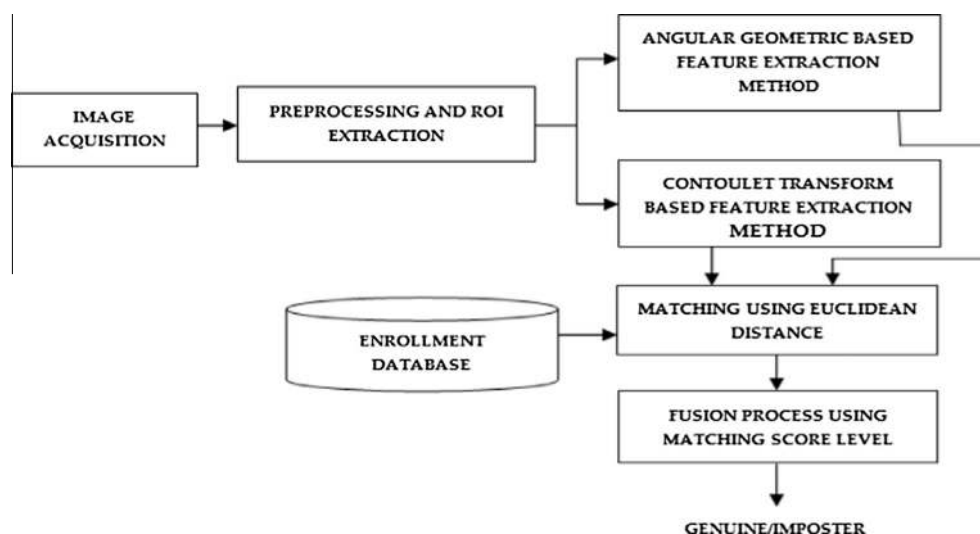
From the survey conducted, it has been inferred that the existing feature extraction methods have the following limitations.

- (a) The existing geometric analysis based feature extraction approach of hand traits extracts features information from finger such as, finger length, finger width, palm length, palm width, palm area and perimeter that possess lower degree of discrimination and leads to inaccurate authentication within the large population.
- (b) In sub-space based feature extraction methods such as PCA, LDA and ICA the difference in appearance is recorded to generate unique information. This may lead to performance degradation when the image captured is of poor quality.
- (c) The existing transform based texture analysis methods fail to handle multidimensional singularities, since they have a major limitation in selecting the orientation and scaling factors.
- (d) The existing statistical based texture analysis method extracts and analyses only local feature information which is highly dependent on the number of non-overlapping blocks of a captured image. This produces a trade-off between accuracy and computational complexity.
- (e) Moreover, there are no known attempts to integrate geometric/shape oriented features and texture features of finger knuckle surface (both proximal and distal phalanx patterns) in order to improve the performance in terms of accuracy.

Hence, we are motivated to implement an integrated approach based on angular geometric analysis and multi-resolution transform based texture analysis methods for extracting geometric and texture features respectively from both proximal and distal finger knuckle regions. The integration methodology is implemented through score level fusion.

## 3. The proposed system design

This paper proposes a new method for personal authentication using Finger Back Knuckle Surface (FBKS), as a biometric identifier. The block diagram of the proposed system is shown in Fig. 1. The image acquisition phase of the proposed biometric system captures FBKS using a low resolution digital camera kept in a consistently illuminated environment. The FBKS of a finger is captured by placing them individually in a straight position on a white surface provided in front of the digital camera in a contact less manner. This system captures FBKS images of four fingers except the thumb finger from a hand for processing. In the second phase, the captured FBKS images are preprocessed and ROI is extracted for fur-



**Figure 1** System design of the proposed personal authentication system using FBKS.

ther processing. In the third phase, knuckle feature points are identified and feature information is obtained from those points by means of angular geometrical analysis and also by means of texture analysis. The extracted shape and texture feature information of the registered finger images are recorded in the form of vectors separately. This recorded vector information is passed to the matching module which performs matching between the reference and input vector by computing weighted Euclidean distance. Finally, the generation of matching scores from different finger knuckles is fused based on matching score level fusion in order to conclude the decision on authentication.

The key contributions of the proposed work are discussed below.

- (a) This work attempts to extract dorsal knuckle surface and finds the possibility of integrating with proximal finger knuckle regions using matching score level fusion in order to achieve improved performance.
- (b) This work contributes simultaneous extraction of ROI of proximal and distal knuckle regions using construction of coordinate system method which results in rich set of texture patterns used for identifying an individual.
- (c) This work shows the usefulness of multi-direction and multi-resolution analysis in processing finger knuckle images in order to handle curved singularities for better identification of individual.
- (d) This work also attempts to integrate geometric and texture features of both proximal and distal finger regions in order to achieve higher recognition accuracy.
- (e) Finally, this work investigates the possibility of the thumb finger knuckle regions toward reliable identification of individual.

#### 4. Preprocessing and ROI extraction

The captured FBKS of four fingers is shown in Fig. 2. The physiological features of the finger back region shown in Fig. 3 are described as follows. Each finger back region of

the hand has three phalangeal joints. The joint that connects the finger with the hand surface is called as Metacarpophalangeal joint. The joint that is formed in the middle surface of the finger is called as Proximal Inter Phalangeal (PIP) joint. The presence of PIP joint in the finger back knuckle surface forms the flexion shrinks on the inner surface of the skin which creates unique dermal patterns. These dermal patterns are denoted as proximal knuckle patterns. Similarly, the joint present in the tip of the FBKS is called as distal inter phalangeal joint. This joint also forms unique dermal pattern which is denoted as distal knuckle patterns. In this paper, the inherent skin patterns of both proximal and distal knuckle are considered for processing.

##### 4.1. Preprocessing

The preprocessing and ROI extraction involve the process of mining the proximal and distal knuckle regions from the FBKS. Further, it does the process of aligning finger knuckle images captured in various scenarios through the acquisition process. As a first step in preprocessing, the acquired FBKS is subjected to binarization in which each pixel of a finger knuckle print image is converted into one bit information ( $B(x,y)$ ) obtained through (1)



**Figure 2** Acquired FBKS image from four fingers.

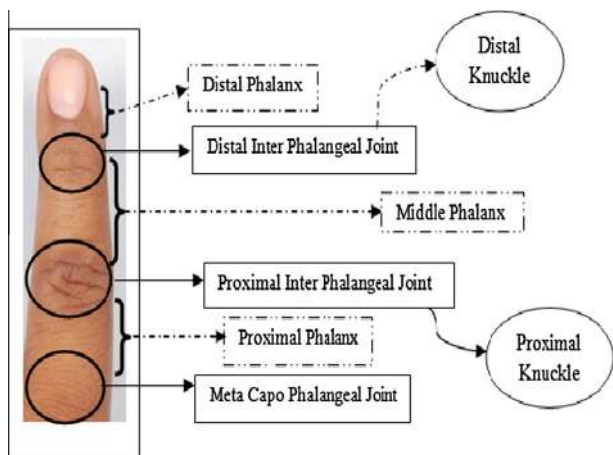


Figure 3 Physiological features of FBKS.

$$B(x, y) = \begin{cases} 0, & \text{if } F_k(x, y) \leq T_h(x, y) \\ 1, & \text{if } F_k(x, y) > T_h(x, y) \end{cases} \quad (1)$$

where  $F_k(x, y)$  represents the intensity of the pixel at location  $(x, y)$  of finger knuckle image and  $T_h(x, y)$  represents the threshold which is calculated for each pixel using Sauvola's technique [26] through (2)

$$T_h(x, y) = \bar{F}_k(x, y) \left[ 1 + k \left( \frac{\delta(x, y)}{M} - 1 \right) \right] \quad (2)$$

where

- $\bar{F}_k(x, y)$  – mean of pixel values,
- $\delta(x, y)$  – standard deviation of pixel values,
- $M$  – maximum value of standard deviation (here  $M = 128$ , since obtained finger knuckle image is converted to gray scale image) and
- $k$  – bias value (here  $k$  is considered as 0.3, since boundary of the image is distinctly identified at this point).

Further,  $\bar{F}_k(x, y)$  denotes mean value of pixels obtained through integral sum method. This integral sum of an input finger knuckle image  $F_k(x, y)$  with size  $I_1 \times I_2$  defines the intensities at pixel position  $s(x, y)$  is equal to the sum of the intensities of the pixels present above and toward left of its position in the original image. Using this integral sum  $s(x, y)$ , the mean value of pixels can be calculated as given in (3)

$$\bar{F}_k(x, y) = \frac{s(x, y)}{I_1 \times I_2} \quad (3)$$

From the obtained binarized image  $B(x, y)$ , the boundary of the image is extracted by means of contour tracing [27]. The exact boundary of the image is obtained by tracing the largest possible contour in the binarized finger knuckle image by eliminating the small contours. Fig. 4(a)–(e) shows the acquired finger back knuckle surface, the binarized image of the finger back knuckle surface and contour extracted images from the binarized image respectively. The boundary image obtained through contour tracing is called as a contour image of FBKS ( $I_{contour}$ ).

In the image  $I_{contour}$ , the stable baseline obtained from the left side boundary of the finger knuckle region is set as the X-axis of the coordinate system. From the same image, the contour pixels identified on the either side of the boundary according to the shape of the image are stored as contour set points in a vector. These contours set points are analyzed for estimating their inter distance values which can be as the distance between the corresponding contour set points identified from the boundaries of the FBKS. The contour set points that have maximum inter distance value are considered to be the boundary points of proximal knuckle. Similarly, the contour set points having the minimum inter distance value are the boundary points of the distal knuckle. The boundary points of the proximal and distal knuckle are joined independently to obtain their respective Y-axis [28,29].

Fig. 5(a)–(d) shows the construction of X-axis in the finger knuckle region, contour set point representation in the contour

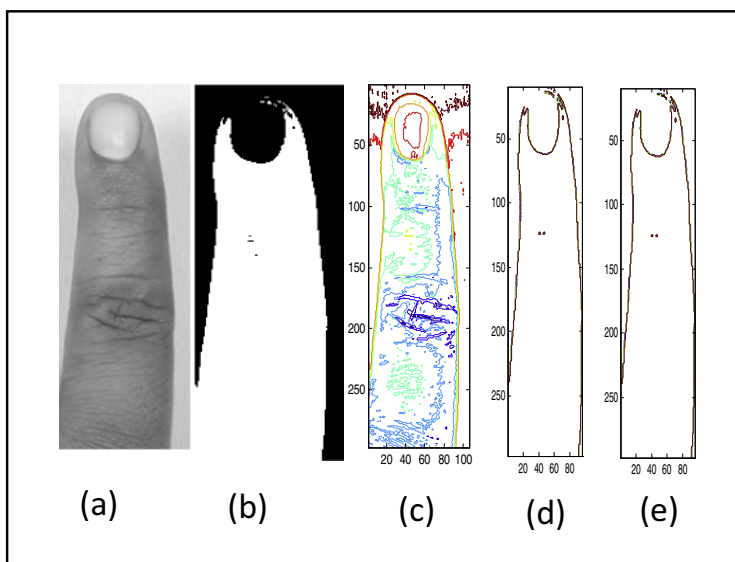
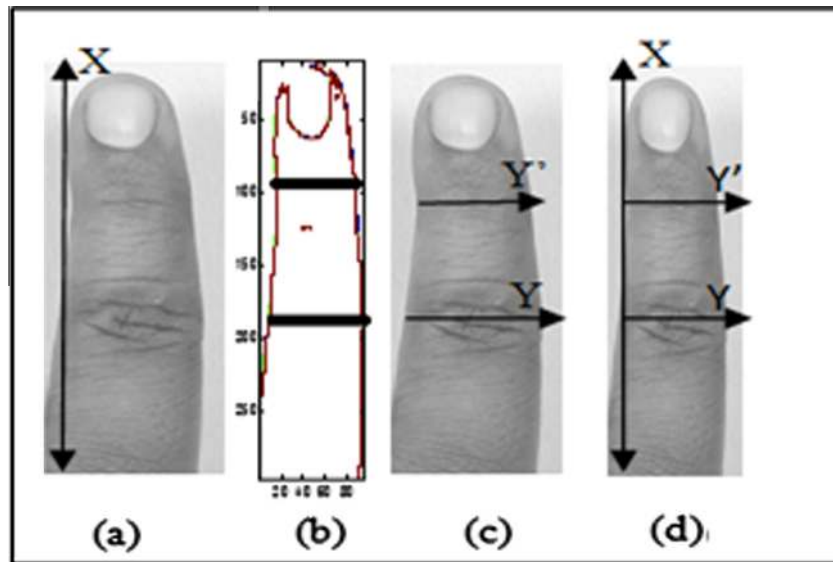


Figure 4 (a) Acquired FBKS image (b) binarized image of FBKS (c), (d) representation of contour extracted from binarized FBKS Image. (e) Contour image of FBKS and referred as  $I_{contour}$ .



**Figure 5** (a)  $X$ -axis construction in FBKS image, (b) contour set point representation in FBKS contour image, (c) construction of  $Y$ -axis in FBKS image and (d) construction of complete coordinate system in FBKS image and referred as  $I_{cord}$ .

image, construction of  $Y$ -axis for both proximal and distal knuckle regions of finger knuckle regions and finger back knuckle surface constructed with the complete coordinate system respectively. The captured finger knuckle region is constructed with the coordinate system named as  $I_{cord}$ .

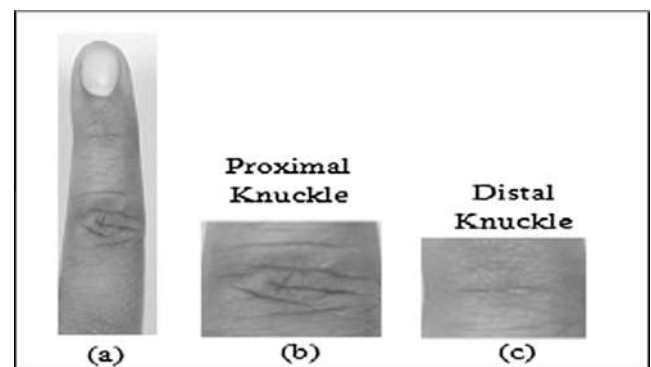
#### 4.2. ROI extraction

The ROI sub images of proximal and distal knuckle are obtained from  $I_{cord}$  knuckle image by extracting fixed size pixel value of  $110 \times 220$  and  $90 \times 180$  respectively. The extracted sub images of both proximal and distal knuckle regions produce rich set of patterns used for the identifying the individuals and  $I_{distal}$  respectively [31].

This ROI extraction effectively aligns and normalizes the different set of knuckle images to extract unique feature information. The extracted sub images of proximal and distal knuckle are named as  $I_{proximal}$ . Fig. 6(a)–(c) represents the gray scale image of captured finger back knuckle surface, extracted ROI of proximal knuckle surface and extracted ROI of distal knuckle surface respectively.

#### 4.3. Correctness of preprocessing and ROI extraction method

The proposed approach simultaneously extracts proximal and distal knuckle regions from the captured finger knuckle regions using coordinate axis method. The proposed method of preprocessing and ROI extraction is found to be robust against scaling, rotational and translational variations of finger knuckle surface. The proposed method produces accurate localization than the state of art methods since (i) they extract ROI knuckle surface which is not of fixed size i.e., it varies according to the length and width of the finger and (ii) also they does not depend on any empirically estimated value for locating distal and proximal knuckle regions; instead, they depend on contour set points of finger knuckle.



**Figure 6** (a) Gray scale image of FBKS, (b) ROI of proximal knuckle surface ( $I_{proximal}$ ) and (c) ROI of distal knuckle surface ( $I_{distal}$ ).

### 5. Angular geometric based feature extraction method (AGFEM)

The main objective of this study is to evaluate the improvement in performance induced by the integration of geometric and texture features of a finger knuckle biometric system. The geometric measurements are extracted from the ROI images of proximal and distal knuckle regions using angular geometric based feature extraction method (AGFEM) as discussed in [34]. As detailed in [34], the angular geometric analysis method extracts six geometric features from proximal knuckle and six from distal knuckle region. Hence totally, 12 geometric measurements were derived from a finger knuckle surface which includes, two finger knuckle length, six finger knuckle widths and four finger knuckle angular information [30]. The distance between the input finger knuckle geometric measures ( $fk_{s_i}$ ) and the registered feature vector ( $fk_{s_r}$ ) is computed through Weighted Euclidean Distance (WED) [32] rule, which is given by (4)

$$D(fks_i, fks_r) = \sqrt{\sum_{i=1}^k w_i (fks_i - fks_r)^2} \quad (4)$$

where

$fks_i$  – represents the geometric measurement vector of an input finger knuckle surface image.

$fks_r$  – represents the geometric measurement of registered finger knuckle surface image.

$w_i$  – corresponds to the weight which is assigned a lower value for lower variance between input and registered value and assigned higher value for higher variance between input and registered value.

The key significance of the proposed angular geometric analysis method is that it extracts angular-based feature information which is highly potential enough to distinguish the individuals.

### 6. Contourlet transform based feature extraction method (CTFEM)

#### 6.1. Contourlet transform

Contourlet transform is the directional multiresolutional transform for image representation. Contourlet establishes a multi-resolution frames oriented in various directions in multiple scales with dynamic aspect ratios that could be able to represent images effectively which has smooth regions separated by smooth contours [33]. The skin pattern of finger back knuckle surface consists of curved lines and contours which could be well represented by means of Contourlet transform. Contour transform represents images by means of two filter banks viz., (i) Laplacian Pyramid [LP] which effectively captures point discontinuities of an image and (ii) Directional Filter Bank (DFB) which characterizes linear structures of an image by means of basic functions and link point discontinuities [34]. Image representation by means of Contourlet transform can be given as follows.

Let the captured FBKS image be a real valued function which is represented as  $fbks(t)$  defined on the grid  $[2^l, 2^l]$  with integer values. The discrete contourlet transform with scale  $i$ , direction  $j$  and level  $l$  of  $fbks(t)$  [35,36] is given through (5)

$$\lambda_{i,j,l}(t) = \sum_{k=0}^3 \sum_{m \in \mathbb{Z}^2} d_j(m) \psi_{i,l}^{(k)} \quad (5)$$

where

$d_j(m)$  is the directional co-efficient.

$\psi_{i,l}^{(k)} = \sum_{m \in \mathbb{Z}^2} f_k(m) \phi_{i,l+m}(t)$  – represents the contourlet waveform.

In this work, the implementation of contourlet transform is carried out in three level Laplacian pyramidal decomposition and one orthogonal filter directions are applied in four directions and hence captured FBKS image is decomposed into 7 contourlet sub-band images. Each sub-band image contains a part of original FBKS which is called as ‘‘Knuckle Contours’’. The contourlet coefficient for each knuckle contourlet is derived and represented in the forms of vectors. The value attempted for scaling factor is  $i = 0.8 - 3$ , directional factor is  $j = 0.5 - 4$  and level  $l = 0, 2, 3$  and 4. The value of  $m$  denotes the number of coefficients retrieved for contourlet

sub-band analysis. The significance of aforementioned parameters in contourlet transform is it enables to perform multidirectional and multiresolutional analysis.

#### 6.2. Principal component analysis

Principal Component Analysis (PCA) is a linear dimensionality reduction technique based on mean square error. In this work, the obtained contourlet coefficients from Knuckle Contour images are subjected to PCA for the reduction in dimension since PCA has following advantages in representing image variations [37]. They are (i) reconstructs the image representation by considering the principle components without redundancy and (ii) minimizes squared reconstruction error since maximum of projected input vectors is chosen and small variations are eliminated automatically.

The PCA applied on contourlet coefficients of Knuckle Contours yields lower dimensions by identifying orthogonal linear combinations with greater variances exist among the coefficients. The following steps are involved to estimate the covariance matrix for the given finger knuckle sub-band images.

- (1) Calculate the algorithmic means of all the feature information vectors viz.,  $F_1, F_2, \dots, F_5$  containing contourlet coefficients through (6).

$$\bar{M} = \frac{1}{n} \sum_{j=1}^n F_j \quad (6)$$

- (2) The difference between each feature information vector and the calculated mean is computed through (7).

$$\delta_j = F_j - \bar{M} \quad (7)$$

- (3) The covariance matrix is computed through (11).

$$\psi = \frac{1}{n} \sum_{j=1}^n \delta_j \delta_j^T \quad (8)$$

- (4) The Eigen value viz.,  $\tau_1, \tau_2, \tau_3 \dots, \tau_n$  from the obtained covariance matrix is derived through (9).

$$\tau_j = \frac{1}{n} \sum_{j=1}^n (v_j^T \delta_j^T)^2 \quad (9)$$

The obtained Eigen values represent the feature information of finger knuckle images which can be stored in a feature vector. The matching process between the registered and input images is done by calculating weighted Euclidean distance among the feature vectors of those images (as discussed in Section 5).

### 7. Fusion process

The goal of this fusion process is to integrate the matching scores obtained through geometric and texture analysis methods from different finger knuckle regions in order to investigate the integrated performance of the proposed personal authentication system. The real-time data set used for experi-

mentation in this paper consists of four finger back knuckle regions of a person. All the four finger knuckle regions exhibit different dermal patterns which can be treated as different modality; hence, it is meaningful to combine the score information obtained from different finger knuckle regions using fusion process [38]. Fusion process can be defined as the process of combining matching scores obtained from each finger back knuckle regions using different fusion rules in order to obtain better performance in terms of accuracy. In this paper, three basic rules of matching score level fusion such as sum of matching scores (SOM) ( $\Sigma$ ), product of matching scores (POM) ( $\pi$ ) and weighted sum of matching scores (SWM) ( $\Sigma_w$ ) were incorporated to estimate the combined performance.

The final matching ( $F_M$ ) score obtained by fusing the obtained score using SOM rule is defined in (10)

$$F_M = \sum_{i=1}^n F_i \quad (10)$$

Similarly, the final matching ( $F_M$ ) score obtained by fusing the obtained score using POM rule is defined in (11)

$$F_M = \prod_{i=1}^n F_i \quad (11)$$

In addition to this, the final matching ( $F_M$ ) score obtained by fusing the obtained score using SWM rule is defined in (12)

$$F_M = \sum_{i=1}^n W_i \times F_i \quad (12)$$

In all the above equations,  $F_i$  defines the score obtained from the  $i^{th}$  classifier incorporated for finger knuckle print matching. Moreover, the value of  $W_i$  is obtained through (13),

$$w_i = \frac{\frac{1}{\sum_{j=1}^n \text{EER}_j}}{\text{EER}_i} \quad (13)$$

This infers that, the classifier result showing higher values of EER will be assigned lower weight values. In case of lower values of EER in classifier results, the value assigned to  $W_i$  will be higher.

## 8. Experimental analysis and results discussion

The performance of the proposed framework personal authentication system is evaluated using a real-time database of finger back knuckle surface region. The FBKS region consisting of distal and proximal finger knuckle patterns is captured by means of an acquisition setup as described in Section 3. The finger back knuckle surfaces were captured from 150 subjects including 80 males and 70 females in three different sessions with the time interval of 5–6 weeks. In each session three images of four finger knuckle regions viz., index finger, middle finger, ring finger and little finger are captured from either left or right hand. Therefore, 12 images were collected from each person in one session. Totally, 36 images were collected from a person in three sessions. The data set of the FBKS contains totally 5400 images which contains 600 different fingers back knuckle surfaces.

In all the experiments, images collected in the first session are considered for gallery set, while the set of images collected during second and third session is taken as probe set in order

to determine number of genuine and imposter matching scores. The genuine acceptance rate is computed for analyzing the number of genuine matches corresponding to the total number of matches done with the system. The equal error rate is the point at which the false acceptance rate and false rejection rate become equal. The decidability threshold (DT) defines the distribution of genuine and imposter matching scores which is calculated through (14)

$$DT = \frac{|\mu_1 - \mu_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}} \quad (14)$$

where

$\mu_1$  and  $\mu_2$  – defines the mean of genuine and imposter matching distances respectively.

$\sigma_1$  and  $\sigma_2$  – defines the standard deviation of genuine and imposter matching distances respectively.

The number of false acceptances and its corresponding genuine acceptance rates are obtained for all possible decidability threshold values and plotted as Receiver Operational Characteristics (ROC) curve. The obtained ROC curve will reflect the overall accuracy of the proposed personal authentication framework. Hence, in this paper the performance analysis of both AGFEM and CTFEM approaches is achieved by constructing ROC curves.

### 8.1. Experiment 1 – performance analysis of proposed AGFEM and CTFEM approaches

The main objective of this experiment is to analyze the performance of the proposed personal authentication system by evaluating the performance of each type of finger back knuckle surfaces separately. As discussed earlier, the real-time acquired database consists of four different finger knuckle images such as index finger, middle finger, ring finger and little finger, and the performance of proposed AGFEM and CTFEM approaches implemented on each FBKS images is evaluated separately. The number of images in the gallery set is taken as 150 different finger knuckle images captured in first session and the number of images in probe set is taken as 900 ( $150 \times 2 \times 3$ ) which are collected from 150 subjects in two sessions with three images per session for each finger. The number of genuine matches and imposter matches obtained during the evaluation of AGFEM is given as 2168 and 1,608,678. However, the number of genuine matches and imposter matches obtained during the evaluation of CTFEM is given as 2198 and 1,608,984.

The existing geometric analysis based recognition schemes such as Finger Geometric Feature Extraction Method (FGFEM), Finger Knuckle Feature Extraction Method (FKFEM) and Key Point Triangulation Method (KPTM) are taken as benchmark systems for analyzing the performance of proposed AGFEM and hence these methods were evaluated using the real-time FBKS database presented in this paper. Similarly, existing transform based texture analysis methods such as Random transform and Gabor Wavelet Transform based Feature Extraction Method (RTGWT), DCT based feature extraction method (DCTFEM) and Phase Correlation Function using Discrete Fourier Transform (PCFDFT) are taken as benchmark systems for analyzing the performance of CTFEM and these methods were also evaluated using the



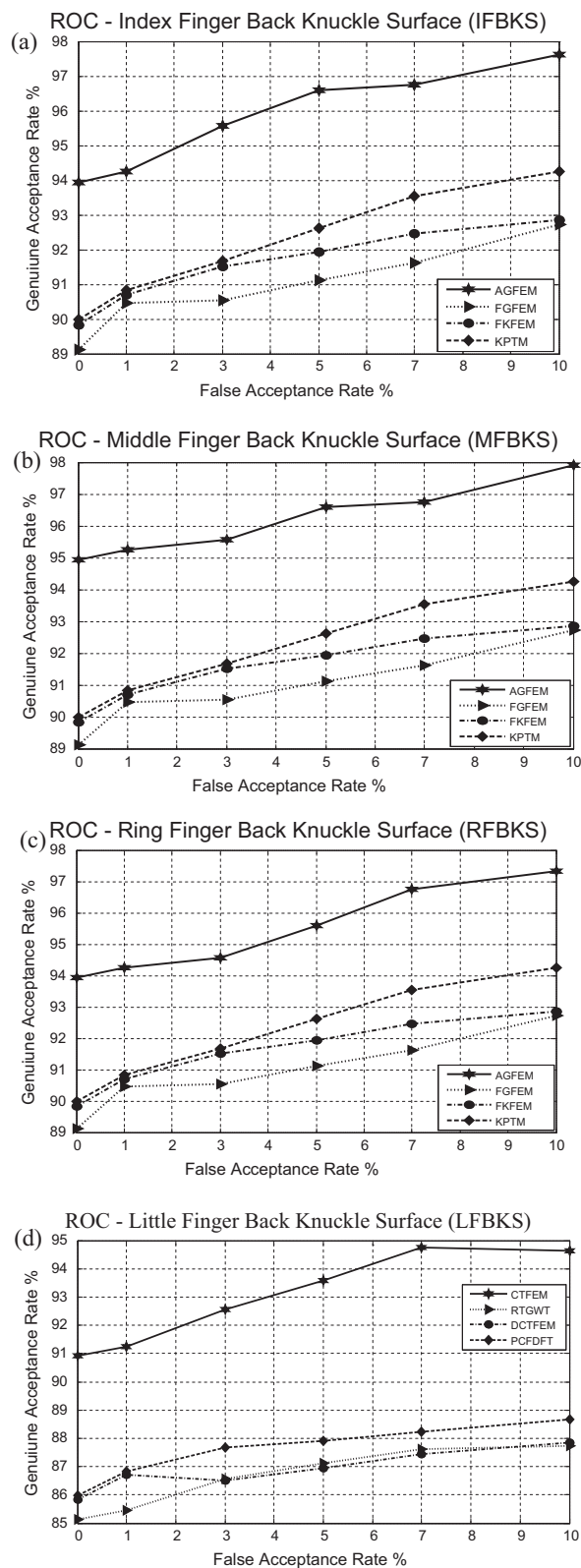
real-time FBKS data set for the sake of comparison. Figs. 7 and 8 show ROC curves obtained through evaluation of various methods on finger knuckle regions. Fig. 7(a) and (b) shows ROC plot for Index and middle finger knuckle regions obtained through the implementation of proposed AGFEM approach and various existing recognition approaches as mentioned earlier. Similarly, Fig. 7(c) and (d) shows the ROC plot for ring and little finger knuckle regions for various recognition approaches respectively. In addition to this, Fig. 8(a)–(d) illustrates ROC curves for index, middle, ring and little finger knuckle regions obtained through proposed CTFEM approach and also through various existing texture analysis based feature extraction methods.

The experimental results indicate that the proposed AGFEM gives higher GAR value of 97.15%, 97.78% and 97.86% when it is implemented on index, middle and ring finger knuckle regions, which differs from 0.67% to 1.23% over FGFEM, from 1.23% to 1.895% over FKFEM and from 2.56% to 4.34% over KPTM. However, the proposed AGFEM approach yields only 91.12% of GAR value when it is implemented with little finger knuckle region. This performance degradation is due to unclear texture patterns exhibited by the little finger which makes the derivation of shape orientation features more complex, whereas, the proposed CTFEM approach yields a considerably high GAR value of 94.35% when it is implemented on little finger knuckle region. This remarkable variation in the performance is obtained due to the derivation of Contourlet coefficients from the little finger knuckle surface and these coefficients are further subjected to principal component analysis which derives most prominent distinct knuckle feature information for reliable identification.

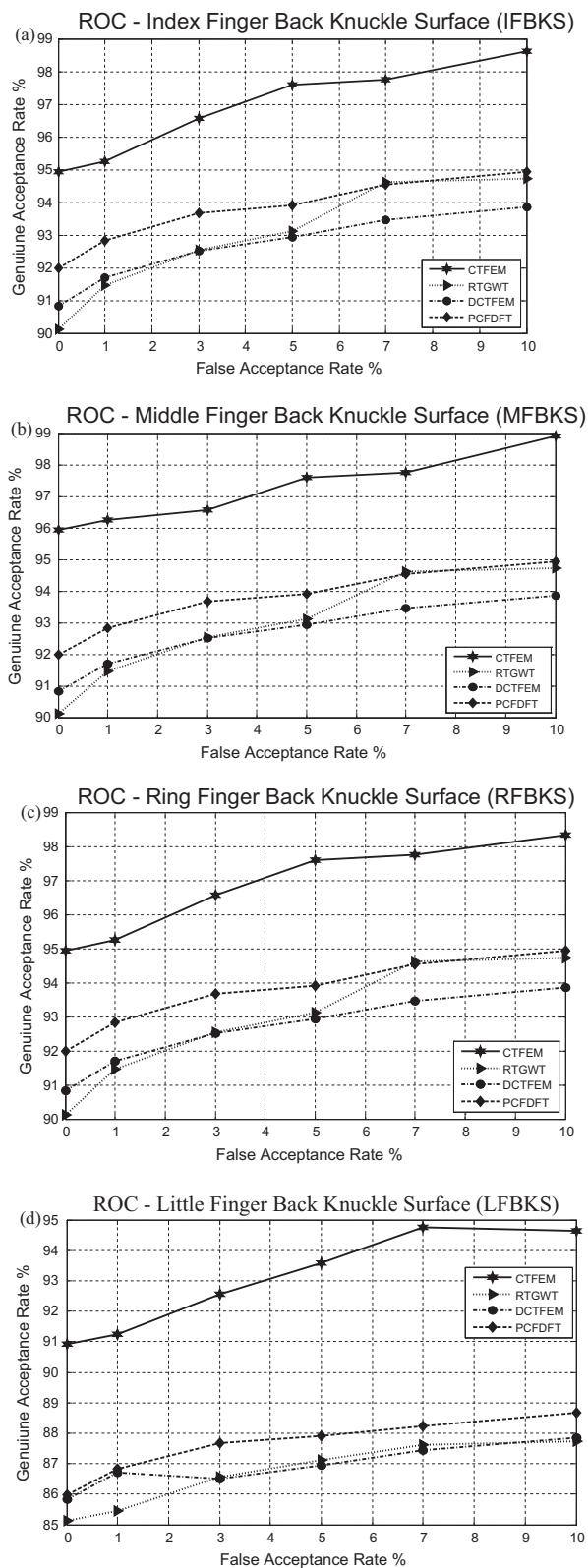
Similarly, the experimental results of proposed CTFEM show higher GAR values of 98.45%, 98.12% and 98.79% for index, middle and ring finger knuckle regions which differs from 1.75% to 3.35% over DCTFEM, 2.23% to 3.67% over RTGWT and 2.29% to 4.01% over PCFDFT (as shown in Fig. 8(a)–(c)). The results shown for CTFEM by setting the parameters are  $i = 2.5, j = 0.5, l = 4$  and the values of  $m$  are set maximum. From the tabulated results of Tables 1 and 2, it is clear that the proposed AGFEM and CTFEM yield the lowest error rate of 1.45% and 1.22% respectively which indicates that these methods increase the accuracy rate to an average value of 47% and 59% respectively, when compared to the exiting shape oriented and texture analysis based approaches.

## 8.2. Experiment 2 – performance analysis of score level fusion process

The main objective of this experiment is to investigate the performance of the personal authentication when we incorporate fusion process (as describe in Section 8) of score information obtained through AGFEM and CTFEM approaches from two or more finger knuckle regions of an individual. As discussed in Section 8, the basic fusion rules such as SOM ( $\Sigma$ ), POM ( $\pi$ ), and SWM ( $\Sigma_w$ ) are incorporated through investigation of proposed recognition methods. In addition to that, hybrid fusion rule is also incorporated which is defined as the combination of two basic fusion strategies. For example, matching score obtained from finger knuckle regions either through AGFEM or CTFEM approach is combined using weighted sum rule and further integration with all other finger



**Figure 7** ROC curves for (a) index finger back knuckle surface, (b) middle finger back knuckle surface, (c) ring finger back knuckle surface and (d) little finger back knuckle surface, using various geometric analysis approaches.



**Figure 8** ROC curves for (a) index finger back knuckle surface, (b) middle finger back knuckle surface, (c) ring finger back knuckle surface and (d) little finger back knuckle surface, using texture analysis approaches.

**Table 1** EER (%) values for various geometric analysis based feature extraction methods.

Finger knuckle type	FGFEM	FKFEM	KPTM	AGFEM (the proposed method)
Index finger back knuckle region (IFBKS)	3.42	3.89	2.98	0.96
Middle finger back knuckle region (MFBKS)	3.98	3.26	3.01	1.01
Ring finger back knuckle region (RFBKS)	4.10	3.72	3.21	0.87
Little finger back knuckle region (LFBKS)	5.61	6.72	5.98	1.45

**Table 2** EER (%) values for various texture analysis based feature extraction methods.

Finger knuckle type	DCTFEM	RTGWT	PCFDFT	CTFEM (the proposed method)
Index finger back knuckle region (IFBKS)	2.96	2.72	2.98	0.725
Middle finger back knuckle region (MFBKS)	2.17	2.12	2.38	0.67
Ring finger back knuckle region (RFBKS)	1.98	2.01	2.14	0.73
Little finger back knuckle region (LFBKS)	3.72	3.98	3.67	1.22

knuckle regions is done by means of POM fusion rule. Thus the combinations of any two fusion strategies are defined as hybrid rule. The following Table 3 illustrates experimental results in terms of EER (%) values obtained by the implementation of all the three basic fusion rules and the hybrid fusion rule in various combinations of matching scores corresponding to the AGFEM and CTFEM approaches. The value of the 'k' specified in Table 3 can be taken as k = 1, 2, 3 and 4 which corresponds to index finger back knuckle surface, middle finger back knuckle surface, ring finger back knuckle surface and little finger back knuckle surface respectively.

The tabulated experimental results indicate that combining matching scores yields better results when compared to the individual performance of the system. Moreover, among all combinations, the hybrid fusion rule combination reported in 16th serial number of the table

**Table 3** EER (%) values obtained for various combinations of matching scores.

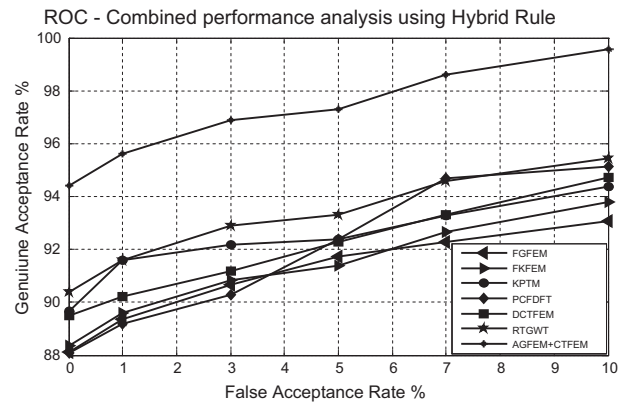
Sl. no	Combinations	ERR (%)	DT
1	$\sum_w(AGFEM_k)$	1.98	4.9823
2	$\prod(AGFEM_k)$	1.72	4.8141
3	$\sum_w AGFEM_k$	1.67	4.7231
4	$\sum_w(CTFEM_k)$	1.62	4.7014
5	$\prod(CTFEM_k)$	1.58	4.7008
6	$\sum_w CTFEM_k$	1.49	4.6198
7	$\sum_w(AGFEM_k, CTFEM_k)$	1.32	4.5121
8	$\prod(AGFEM_k, CTFEM_k)$	1.21	4.5216
9	$\sum_w(\sum_w(AGFEM_k), \sum_w(CTFEM_k))$	1.14	4.3121
10	$\prod(\prod(AGFEM_k), \prod(CTFEM_k))$	0.98	4.4123
11	$\sum_w(\sum_w(\sum_w(AGFEM_k), \sum_w(CTFEM_k)))$	0.86	4.3261
12	$\prod(\prod(\sum_w(AGFEM_k), \sum_w(CTFEM_k)))$	0.82	4.4721
13	$\sum_w(\prod(AGFEM_k), \prod(CTFEM_k))$	0.75	4.3762
14	$\prod(\sum_w(AGFEM_k), \sum_w(CTFEM_k))$	0.62	4.3141
15	$\prod(\sum_w(AGFEM_k), \sum_w(CTFEM_k))$	0.44	4.3258

$(\prod(\sum_w(AGFEM_k), \sum_w(CTFEM_k)))$  shows the lowest value of EER of 0.45%, this is because, the weighted sum rule derives more information from the gallery data set than that of the other basic fusion rules and also the matching scores obtained through the AGFEM and CTFEM from all the finger knuckle regions are fused using product rule.

8.3. Experiment 3 – performance analysis of proposed integrated approach

In the third experiment all the types of finger knuckle surfaces were involved. The main objective of this experiment is to analyze the performance of the system when the matching scores obtained through AGFEM and CTFEM are combined using hybrid rule. The obtained results through integrated performance are compared with various existing geometric and texture analysis based approaches in the literature such as FGFEM, FKFEM, KPTM, RTGWT, DCTFEM and PCFDFT (as stated in experiment 1). The gallery set for the experiments is taken as 600 (150 × 4), whereas the number of images in the probe set is taken as 1800 images (600 × 3). The total number of genuine and imposter matched obtained in this experiment is 9000 and 6,947,672 respectively. Fig. 9 shows the ROC curves plotted for various feature recognition methods. The experimental results illustrate that the proposed AGFEM and CTFEM perform well when they are implemented individually and also yield high GAR value of 99.56% when the matching scores obtained from these methods are integrated using hybrid fusion rule.

The experimental results in terms of EER (%) for various recognition methods are tabulated in Table 4. The integrated performance of proposed AGFEM and CTFEM yields lowest error rate of 0.44%, and this is due to the following reasons: (i) the proposed AGFEM derives angle oriented feature information which has potentiality in discriminating the individuals and (ii) the proposed CTFEM derives multi-resolution and multiple orientation texture information from the finger knuckle surface which could definitely yield high accuracy rate than that of the existing methods which possess only any one of the property.



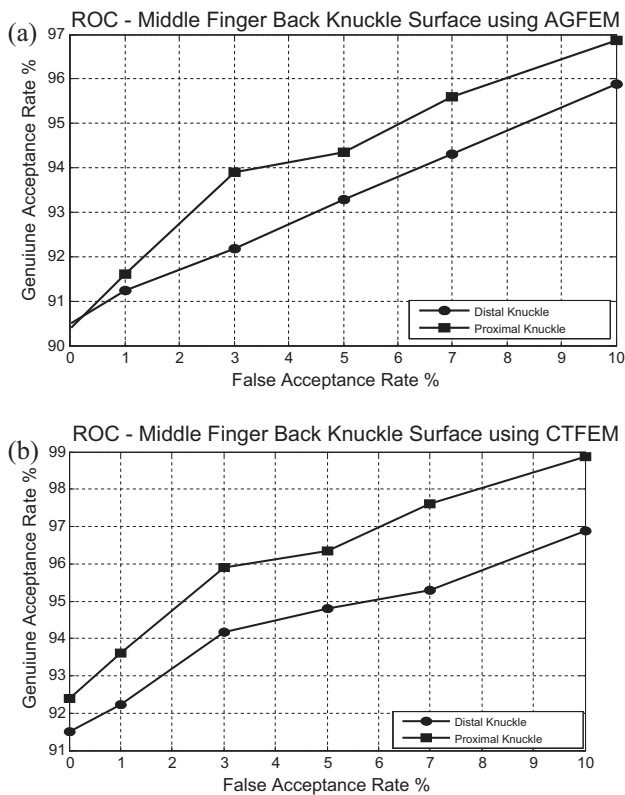
**Figure 9** ROC curves from seven different feature recognition methods in experiment 3.

**Table 4** EER (%) values obtained for various feature recognition methods.

Recognition methods	EER (%)	DT	FRR (%) (when FAR = $1 \times 10^{-3}$ )
FGFEM	4.53	4.5684	8.5964
FKFEM	5.28	4.8694	7.6522
KPTM	4.23	4.6512	8.5673
AGFEM	1.67	4.7231	3.8974
PCFDFT	3.56	4.5685	5.4567
DCTFEM	3.12	4.2354	5.4568
RTGWT	2.45	4.2135	5.6478
CTFEM	1.49	4.6198	3.4256
AGFEM + CTFEM	0.44	4.3258	0.9587

8.4. Experiment 4 – performance analysis of proximal and distal knuckle surface

This experiment is conducted to analyze the significance of distal finger knuckle region toward personal authentication. For this experiment, the database of 150 middle finger back knuckle surface images obtained during the first session of image acquisition was utilized as gallery set. The probe set for this experiment consists of 900 (150 × 6) middle finger back knuckle surface images obtained during the second and third session of image acquisition. This experiment is conducted in two folds: (i) performance evaluation AGFEM approach implemented on distal and proximal finger knuckle regions separately and analyzing their combined performance using basic fusion rules and (ii) performance evaluation of CTFEM approach implemented on distal and proximal finger knuckle regions separately and analyzing their combined performance using basic fusion rules (SOM, POM, and SWM). Fig. 10 (a) and (b) illustrates the ROC plot derived from genuine and imposter matching scores obtained through the implementation of AGFEM and CTFEM approaches on both distal and proximal knuckle regions of middle finger. Fig. 11(a) shows the ROC plots obtained by fusing the matching scores derived



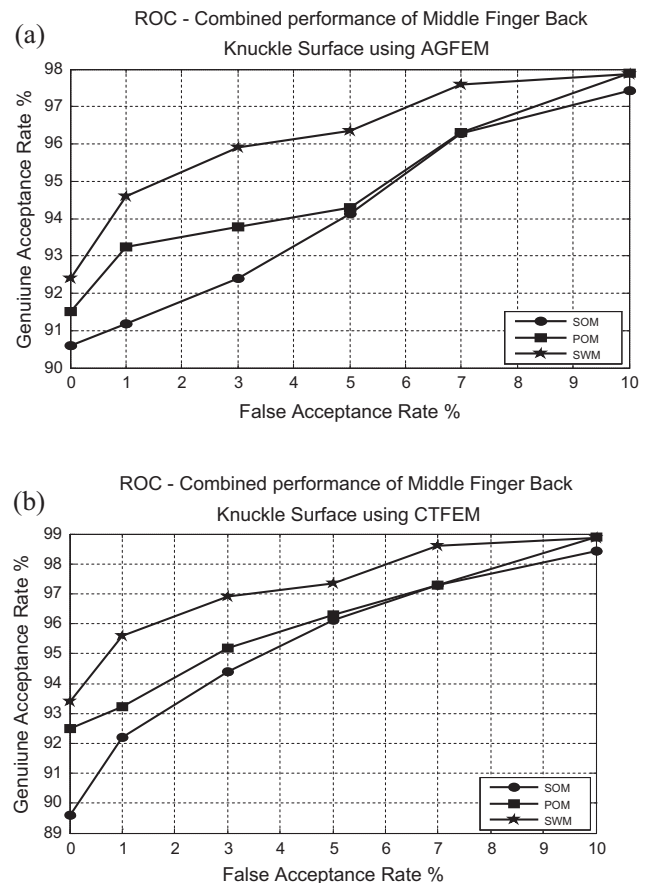
**Figure 10** ROC obtained from (a) proximal and distal knuckle regions of Middle finger using AGFEM and (b) proximal and distal knuckle regions of Middle finger using CTFEM.

through AGFEM approach from distal and proximal knuckle regions using three basic fusion rules and similarly matching scores derived using CTFEM approach are shown in Fig. 11 (c).

From the experimental results it is observed that, integrating distal finger knuckle region with proximal finger knuckle region significantly improves the performance of the personal authentication system. The individual performance of distal and proximal finger knuckle obtained through CTFEM (Fig. 10(b)) shows higher GAR values than the values obtained through AGFEM approach. This considerable variation in performance is due to CTFEM approach potentially discriminates intra class and inter class knuckle sub-image values.

#### 8.5. Experiment 5 – performance analysis of thumb finger knuckle surface

This experiment is conducted using thumb finger knuckle biometric samples. The thumb finger images are newly captured using the acquisition setup in Section 3. The following Fig. 12 shows sample image of the thumb finger and the extracted knuckle region from thumb finger using the ROI segmentation algorithm discussed in Section 3. The methods used extract the features from thumb finger knuckle regions are same as described in Section 4. The database of 100 subjects was taken in which one-third of the images were taken as gallery set and remaining images were taken as probe set. The matching scores were generated as discussed in Sections 5



**Figure 11** ROC obtained from (a) fusing match score obtained through AGFEM from distal and proximal phalanx using three basic fusion rules. (b) Fusing match score obtained through CTFEM from distal and proximal phalanx using three basic fusion rules.

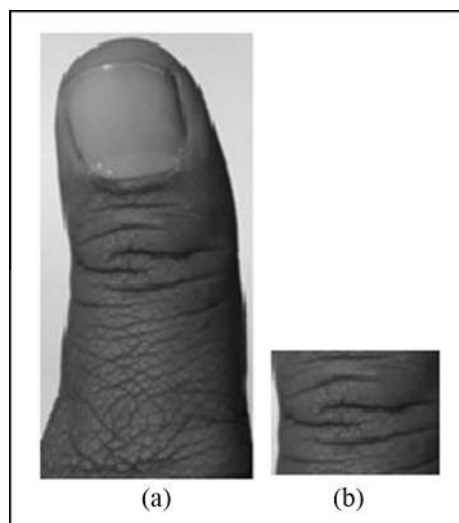
and 6. The ROC plots for the probe set data are depicted in Fig. 13.

The results show that, genuiune acceptance rate of 95.53% was obtained from testing samples using AGFEM method and 96.35% was obtained using CTFEM approach. The GAR value resulted using hybrid fusion methodology is 98.73%, which is considerable high accuracy rate even though the thumb finger has only the proximal knuckle pattern. The experiment results of the thumb finger analysis show significant performance improvement when it is integrated with other finger knuckle regions. However, capturing the thumb finger knuckle region is performed only by inducing pegs, which reduces the level of user acceptability.

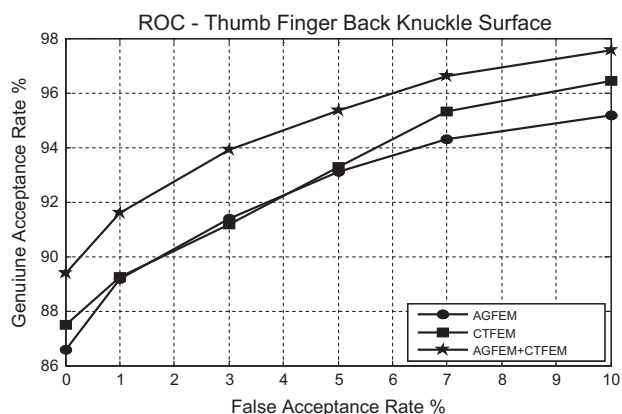
#### 8.6. Experiment 6 – computational complexity analysis of proposed personal recognition system

The computational complexity of the system is evaluated by estimating the time taken in millisecond (msecs) to execute each step of personal recognition process as shown below in Table 5.

The proposed system is implemented in VC++ with the system configuration of Intel core i3 CPU with 5 GHz proces-



**Figure 12** (a) Thumb finger knuckle image and (b) ROI extracted from Thumb finger knuckle region.



**Figure 13** ROC curves from Thumb finger knuckle region.

sor, 4 GB RAM and compiled using GNU compiler with the support of open CV library. Table 5 shows the time taken for each processing step of the proposed personal authentication system. The total time taken for the entire authentication process to complete is 4.67 s. This analysis shows that the pre-processing and ROI extraction are a crucial processing step of the proposed personal authentication since it is computationally costlier when compared to other processing steps. Similarly, texture analysis method is found to be more complex when compared with geometrical analysis method. On the whole, the overall complexity is found to be appreciable and it can be deployed in real-time security applications.

### 8.7. Discussions

The performance of the proposed approaches such as AGFEM and CTFEM is found to be superior to the existing methods. The discussions on the arrived results through AGFEM, CTFEM and their integration are given below.

**Table 5** Computational complexity analysis of proposed personal recognition system.

Key process	Time (msecs)
Image loading	135
Image processing and ROI extraction	220
Geometric analysis based shape oriented feature extraction	1.78
Texture analysis based feature extraction	155
Score generation	45

- (i) AGFEM achieves better performance than the existing geometric analysis methods by efficiently representing shape oriented features in terms of angular information. The angle oriented features are more discriminant than the magnitude oriented features extracted by state-of-art geometrical methods.
- (ii) Experimental results obtained using CTFEM show significant GAR values than the existing transform based texture analysis methods and also found to be superior when compared with proposed AGFEM approach. This significant improvement in performance is due to the unique characteristics of contourlet transform which could able to represent the captured finger knuckle images in multi-scale, multi-directional and multi-resolution analysis that allows sparse representation of texture feature information.
- (iii) In addition, fusion of AGFEM and CTFEM approaches using weighted SUM rule significantly improves the performance of the entire system by 0.86–1.92% in terms of accuracy due to its flexibility in assigning score weights according to the result of single method. Also the performance of the system in terms of speed is also appreciable and it can be deployed in real-time environment.

## 9. Conclusions

This paper introduces a new intra-model personal authentication system by integrating the shape and texture features of finger back knuckle surface which includes the dermal patterns of both proximal and distal phalanx. The proposed method of distal and proximal phalanx region segmentation has found to be very effective in achieving higher performance. The experimental results of experiment 1 proved the significance of the shape oriented features (knuckle length, width, mid-point angle and knuckle area) obtained through AGFEM approach is efficiently used to improve the performance of the finger knuckle recognition. Similarly, the proposed texture feature extraction from finger knuckle surface using Contourlet Transform (CTFEM) also yields promising results (as discussed in experiment 2). Further, the results of experiment 3 suggested that the integration of knuckle shape features with its textural pattern information is found to be very effective and aids in yielding well promising results. Furthermore, the importance of distal phalanx finger knuckle patterns which can be effectively integrated with proximal phalanx (matching

score level fusion) aids in achieving the high accuracy rate of 99.12% with lowest value of equal error rate as 0.44%. Moreover, the obtained results from all the four experiments indicate that the integration of knuckle shape orientation and texture feature information for personal recognition by considering both distal and proximal phalanx is the considerable addition to the current state-of-art of personal recognition using hand traits.

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