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General bulk service queueing system with N-policy, multiplevacations, setup time and server breakdown without interruption

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Abstract. In this paper, we have considered an $M^X/G(a,b)/1$ queueing system with server breakdown without interruption, multiple vacations, setup times and N-policy. After a batch of service, if the size of the queue is $\xi\ (<a)$, then the server immediately takes a vacation. Upon returns from a vacation, if the queue is less than N, then the server takes another vacation. This process continues until the server finds atleast N customers in the queue. After a vacation, if the server finds at least N customers waiting for service, then the server needs a setup time to start the service. After a batch of service, if the amount of waiting customers in the queue is $\xi(\geq a)$, then the server serves a batch of $min(\xi,b)$ customers, where $b \geq a$. We derived the probability generating function of queue length at arbitrary time epoch. Further, we obtained some important performance measures.

1. Introduction

A comprehensive survey on server vacation models can be found in Doshi [1]. Ke et al. [14], Chang and Ke [17] have examined bulk arrival queueing system with atmost J vacations. Lee et al. [7, 8] have analyzed thenon-Markovian queueing system with N-policy, single and multiple vacations. Single server queueing model with bulk serviceand multiple vacations considered by numerous researchers such as Arumuganathan and Jayakumar [3, 4], Arumuganathan and Ramaswami [16], Jayakumar and Senthilnathan [6, 13], Krishna Reddy et al. [2], Sasikala and Indhira [19]. Sikdar and Gupta [9] have considered bulk service queueing systems with single vacation. Recently, Singh et al. [18] have considered thenon-Markovian queueing system Bernoulli vacation.Ke [10] have examined an $M^X/G/1$ queueing system with server break downs, startup and closedowntimes. Chaudhry [12] have analyzed $M^X/G/1$queueing system with setup time. Krishnamoorthy et al. [15] have analyzed an extensive study on queues with interruption. A detailed work on bulk queues can be found in Chaudhry and Templeton [11], Sasikala and Indhira [20].

This paper summarized as follows: In Section 2. We have given a brief description of the proposed queueing system. In Section 3.We derived the steady state differential-difference equations. Section 4 contains queue size distributions. In Section 5. We derived the probability generating function (PGF) of queue size. In Section 6.We have given some important performance measures.

2. Model description and system equations

We consider a gear and shafts manufacturing industry to specify an example for a bulk queueing system with server breakdown without interruption, multiple vacations, N-policy and setup time. A gear and shaft manufacturing industry makes different types of shafts andgears with different dimensions. Before starting of CNC machine, the operator requires pre-alignment steps. After every idle period, the operator starts to provide service only when the number of castings in thequeue is atleast N . If a breakdown occurs at any point, the service of the batch which is currently under service will be completed with some alternate technical arrangements.

Customers arrive in batches with the rate λ according to compound Poisson processes. The server provides service with minimum of a and maximum of b customers in a batch. After a service, if the server finds the queue is atleast a , then the server precedes its service for next batch. After a service, if the server finds the queue is less than a , then the server leaves for a vacation. On completion of a vacation, if the queue is less than N , then the server takes another vacation and so on. This process continues until the size of queue reaches atleast N . On completion of a vacation, if the queue is atleast N , then the server requires a setup time to start the service. If the server is break down at any point with probability π , then renovation period is considered.

We assume that,
X - be the batch size random variable.
$X(z)$ - be the probability generating function.
$S(x), V(x), R(x)$ and $H(x)$ represent the cumulative distribution functions of service time, vacation time, renovation time and setup time.
$s(x), v(x), r(x)$ and $h(x)$ represent the corresponding probability density functions of service time, vacation time, renovation time and setup time.
$S^{-}(t), V^{-}(t), R^{-}(t)$ and $H^{-}(t)$ represent the remaining service time, vacation time, renovation time and setup time.
$S^{*}(\theta), V^{*}(\theta), R^{*}(\theta)$ and $H^{*}(\theta)$ represent the Laplace-Stieltjes transforms of service time, vacation time, renovation time and setup time.
$\varphi(t) = j$ denotes the server is on j^{th} vacation.
$O_q(t)$ bethe number of customers in queue.
$O_s(t)$ bethe number of customers in service.
$M(t) = (1)[2]\{3\}(4)$, if server is on (busy)[secondary job]{renovation}(setup job)

$$\Pi_{i,j}(x,t)dt = \Pr\left\{O_s(t) = i, O_q(t) = j, x < S^{-}(t) \le x + dx, M(t) = 1\right\},\ j \ge 0,\ a \le i \le b$$

$$\Omega_{i,j}(x,t)dt = \Pr\left\{O_q(t) = n,\ x < V^{-}(t) \le x + dx,\ M(t) = 2,\ \varphi(t) = j\right\},\ n \ge 0,\ j \ge 1$$

$$\Phi_n(x,t)dt = \Pr\left\{O_q(t) = n, x < R^{-}(t) \le x + dx, M(t) = 3\right\},\ n \ge 0$$

$$\Psi_n(x,t)dt = \Pr\left\{O_q(t) = n, x < H^{-}(t) \le x + dx, M(t) = 4\right\},\ n \ge N$$

3. System equations

The following steady state differential-difference equations are derived by using supplementary variable technique (see Cox [5]).

$$-\frac{d\Pi_{i0}(x)}{dx} = -\lambda \Pi_{i0}(x) + (1-\pi)\sum_{m=a}^{b}\Pi_{mi}(0)s(x) + \Phi_i(0)s(x),\ a \le i \le b \quad (1)$$

$$-\frac{d\Pi_{ij}(x)}{dx} = -\lambda \Pi_{ij}(x) + \sum_{k=1}^{j}\Pi_{i,j-k}(x)\lambda g_k,\ a \le i \le b-1,\ j \ge 1 \quad (2)$$

$$-\frac{d\prod_{bj}(x)}{dx}=-\lambda\prod_{bj}(x)+(1-\pi)\sum_{m=a}^{b}\prod_{m,b+j}(0)s(x)+\sum_{k=1}^{j}\prod_{b,j-k}(x)\lambda g_k+\Phi_{b+j}(0)s(x) \quad 1\le j\le N-b-1 \tag{3}$$

$$-\frac{d\prod_{bj}(x)}{dx}=-\lambda\prod_{bj}(x)+(1-\pi)\sum_{m=a}^{b}\prod_{m,b+j}(0)s(x)+\sum_{k=1}^{j}\prod_{b,j-k}(x)\lambda g_k+\Psi_{b+j}(0)s(x),\ j\ge N-b \tag{4}$$

$$-\frac{d\Omega_{10}(x)}{dx}=-\lambda\Omega_{10}(x)+(1-\pi)\sum_{m=a}^{b}\prod_{m0}(0)v(x)+\Phi_0(0)v(x) \tag{5}$$

$$-\frac{d\Omega_{1n}(x)}{dx}=-\lambda\Omega_{1n}(x)+(1-\pi)\sum_{m=a}^{b}\prod_{mn}(0)v(x)+\sum_{k=1}^{n}\Omega_{1,n-k}(x)\lambda g_k+\Phi_n(0)v(x), n=1,2,...a-1 \tag{6}$$

$$-\frac{d\Omega_{1n}(x)}{dx}=-\lambda\Omega_{1n}(x)+\sum_{k=1}^{n}\Omega_{1,n-k}(x)\lambda g_k,\ n\ge a \tag{7}$$

$$-\frac{d\Omega_{j0}(x)}{dx}=-\lambda\Omega_{j0}(x)+\Omega_{j-1,0}(0)v(x),\ j\ge 2 \tag{8}$$

$$-\frac{d\Omega_{jn}(x)}{dx}=-\lambda\Omega_{jn}(x)+\Omega_{j-1,n}(0)v(x)+\sum_{k=1}^{n}\Omega_{1,n-k}(x)\lambda g_k,\ j\ge 2, 1\le n\le N-1 \tag{9}$$

$$-\frac{d\Omega_{jn}(x)}{dx}=-\lambda\Omega_{jn}(x)+\sum_{k=1}^{n}\Omega_{1,n-k}(x)\lambda g_k,\ j\ge 2,\ n\ge N \tag{10}$$

$$-\frac{d\Phi_0(x)}{dx}=-\lambda\Phi_0(x)+\pi\sum_{m=a}^{b}\prod_{m0}(0)r(x) \tag{11}$$

$$-\frac{d\Phi_n(x)}{dx}=-\lambda\Phi_n(x)+\pi\sum_{m=a}^{b}\prod_{mn}(0)r(x)+\sum_{k=1}^{n}\Phi_{n-k}(x)\lambda g_k,\ n\ge 1 \tag{12}$$

$$-\frac{d\Psi_n(x)}{dx}=-\lambda\Psi_n(x)+\sum_{l=1}^{\infty}\Omega_{ln}(0)h(x)+\sum_{k=1}^{n-N}\Psi_{n-k}(x)\lambda g_k,\ n\ge N \tag{13}$$

4. Queue size distribution

Let us assume the Laplace Stieltjes transforms (LST) of $\prod_{ij}(x)$, $\Omega_{jn}(x)$, $\Phi_n(x)$ and $\Psi_n(x)$ as

$$\begin{aligned}\prod{}^*_{ij}(\theta)&=\int_0^\infty e^{-\theta x}\prod_{ij}(x)dx,\ \Omega^*{}_{jn}(\theta)=\int_0^\infty e^{-\theta x}\Omega_{jn}(x)dx,\\ \Phi^*{}_n(\theta)&=\int_0^\infty e^{-\theta x}\Phi_n(x)dx,\ \Psi^*{}_n(\theta)=\int_0^\infty e^{-\theta x}\Psi_n(x)dx\end{aligned} \tag{14}$$

TakingLST of the Eqns. (1)-(13) and using Eqn. (14), we obtain,

$$\theta\prod{}^*_{i0}(\theta)-\prod_{i0}(0)=\lambda\prod{}^*_{i0}(\theta)-(1-\pi)\sum_{m=a}^{b}\prod_{mi}(0)S^*(\theta)-\Phi_i(0)S^*(\theta) \tag{15}$$

$$\theta\prod{}^*_{ij}(\theta)-\prod_{ij}(0)=\lambda\prod{}^*_{ij}(\theta)-\sum_{k=1}^{j}\prod{}^*_{i,j-k}(\theta)\lambda g_k,\ a\le i\le b-1 \tag{16}$$

$$\theta\prod{}^*_{bj}(\theta)-\prod_{bj}(0)=\lambda\prod{}^*_{bj}(\theta)-(1-\pi)\sum_{m=a}^{b}\prod_{m,b+j}(0)S^*(\theta)-\sum_{k=1}^{j}\prod{}^*_{b,j-k}(\theta)\lambda g_k-\Phi_{b+j}(0)S^*(\theta) \tag{17}$$

$$\theta \Pi^*_{bj}(\theta) - \Pi_{bj}(0) = \lambda \Pi^*_{bj}(\theta) - (1-\pi)\sum_{m=a}^{b} \Pi_{m,b+j}(0) S^*(\theta) - \sum_{k=1}^{j} \Pi^*_{b,j-k}(\theta)\lambda g_k - \Psi_{b+j}(0) S^*(\theta) \quad (18)$$

$$\theta \Omega^*_{10}(\theta) - \Omega_{10}(0) = \lambda \Omega^*_{10}(\theta) - (1-\pi)\sum_{m=a}^{b} \Omega_{m0}(0) V^*(\theta) - \Phi_0(0) V^*(\theta) \quad (19)$$

$$\theta \Omega^*_{1n}(\theta) - \Omega_{1n}(0) = \lambda \Omega^*_{1n}(\theta) - (1-\pi)\sum_{m=a}^{b} \Pi_{mn}(0) V^*(\theta) - \sum_{k=1}^{n} \Omega^*_{1,n-k}(\theta)\lambda g_k - \Phi_n(0) V^*(\theta) \quad (20)$$

$$\theta \Omega^*_{1n}(\theta) - \Omega_{1n}(0) = \lambda \Omega^*_{1n}(\theta) - \sum_{k=1}^{n} \Omega^*_{1,n-k}(\theta)\lambda g_k \quad (21)$$

$$\theta \Omega^*_{j0}(\theta) - \Omega_{j0}(0) = \lambda \Omega^*_{j0}(\theta) - \Omega_{j-1,0}(0) V^*(\theta) \quad (22)$$

$$\theta \Omega^*_{jn}(\theta) - \Omega_{jn}(0) = \lambda \Omega^*_{jn}(\theta) - \Omega_{j-1,\mathrm{n}}(0) V^*(\theta) - \sum_{k=1}^{n} \Omega^*_{j,n-k}(\theta)\lambda g_k \quad (23)$$

$$\theta \Omega^*_{jn}(\theta) - \Omega_{jn}(0) = \lambda \Omega^*_{jn}(\theta) - \sum_{k-1}^{n} \Omega^*_{j,n-k}(\theta)\lambda g_k \quad (24)$$

$$\theta \Phi^*_0(\theta) - \Phi_0(0) = \lambda \Phi^*_0(\theta) - \pi \sum_{m=a}^{b} \Pi_{m0}(0) R^*(\theta) \quad (25)$$

$$\theta \Phi^*_n(\theta) - \Phi_n(0) = \lambda \Phi^*_n(\theta) - \pi \sum_{m=a}^{b} \Pi_{mn}(0) R^*(\theta) - \sum_{k=1}^{n} \Phi^*_{n-k}(\theta)\lambda g_k \quad (26)$$

$$\theta \Psi^*_n(\theta) - \Psi_n(0) = \lambda \Psi^*_n(\theta) - \sum_{l=1}^{\infty} \Omega_{\ln}(0) H^*(\theta) - \sum_{k=1}^{n-N} \Psi^*_{n-k}(\theta)\lambda g_k \quad (27)$$

To find the system size distribution, we assume probability generating functions as follows,

$$\begin{aligned}
&\Pi^*_i(z,\theta) = \sum_{j=0}^{\infty} \Pi^*_{ij}(\theta) z^j,\ \Pi_i(z,0) = \sum_{j=0}^{\infty} \Pi_{ij}(0) z^j \\
&\Omega^*_j(z,\theta) = \sum_{l=1}^{\infty} \Omega^*_{lj}(\theta) z^j,\ \Omega_j(z,0) = \sum_{l=1}^{\infty} \Omega_{lj}(0) z^j \\
&\Phi^*(z,\theta) = \sum_{n=0}^{\infty} \Phi^*_n(\theta) z^n,\ \Phi(z,0) = \sum_{n=0}^{\infty} \Phi_n(0) z^n \\
&\Psi^*(z,\theta) = \sum_{n=N}^{\infty} \Psi^*_n(\theta) z^n,\ \Psi(z,0) = \sum_{n=N}^{\infty} \Psi_n(0) z^n
\end{aligned} \quad (28)$$

Multiply z^n with both sides of the Eqns. (15)-(27), taking summation over n and using the Eq. (28). Let $\Theta = \theta - \lambda + \lambda X(z)$, we get

$$\Theta \Pi^*_i(z,\theta) = \Pi_i(z,0) - S^*(\theta)\left[(1-\pi)\sum_{m=a}^{b} \Pi_{mi}(0) + \Phi_i(0)\right] \quad (29)$$

$$\Theta \Pi^*_b(z,\theta) = \Pi_b(z,0) - \frac{S^*(\theta)}{z^b}\left[\begin{aligned}&(1-\pi)\sum_{m=a}^{b}\left(\Pi_m(z,0) - \sum_{j=0}^{b-1} \Pi_{mj}(0) z^j\right) + \\ &\Psi(z,0) + \sum_{j=1}^{N-b-1} \Phi_{b+j} z^{b+j}\end{aligned}\right] \quad (30)$$

$$\Theta\Omega^*{}_1(z,\theta)=\Omega_1(z,0)-V^*(\theta)\left[(1-\pi)\sum_{n=0}^{a-1}\prod_{mn}(0)z^n+\sum_{n=0}^{a-1}\Phi_n(0)z^n\right] \quad (31)$$

$$\Theta\Omega^*{}_j(z,\theta)=\Omega_j(z,0)-V^*(\theta)\left[\sum_{n=0}^{N-1}\Omega_{j-1,n}(0)z^n\right] \quad (32)$$

$$\Theta\Phi^*(z,\theta)=\Phi(z,0)-\pi R^*(\theta)\prod_m(z,0) \quad (33)$$

$$\Theta\Psi^*(z,\theta)=\Psi(z,0)-H^*(\theta)\sum_{l=1}^{\infty}\Omega_{\text{ln}}(0)z^n \quad (34)$$

Put $\theta=\lambda-\lambda X(z)$ in the Eqns. (29)-(34), we get

$$\prod_i(z,0)=S^*(\lambda-\lambda X(z))\left[(1-\pi)\sum_{i=a}^{b-1}\prod_{mi}(0)+\Phi_i(0)\right] \quad (35)$$

$$\prod_b(z,0)=\frac{S^*(\lambda-\lambda X(z))}{z^b}\left[\left(1-\pi\right)\sum_{m=a}^{b}\left(\prod_m(z,0)-\sum_{j=0}^{b-1}\prod_{mj}(0)z^j\right)+\Psi(z,0)+\sum_{j=1}^{N-b-1}\Phi_{b+j}z^{b+j}\right] \quad (36)$$

$$\Omega_1(z,0)=V^*\left(\lambda-\lambda X(z)\right)\left[(1-\pi)\sum_{n=0}^{a-1}\prod_{mn}(0)z^n+\sum_{n=0}^{a-1}\Phi_n(0)z^n\right] \quad (37)$$

$$\Omega_j(z,0)=V^*(\lambda-\lambda X(z))\sum_{n=0}^{N-1}\Omega_{j-1,n}(0)z^n \quad (38)$$

$$\Phi(z,0)=\pi R^*(\lambda-\lambda X(z))\prod_m(z,0) \quad (39)$$

$$\Psi(z,0)=H^*(\lambda-\lambda X(z))\sum_{l=1}^{\infty}\Omega_{\text{ln}}(0)z^n \quad (40)$$

Substitute the Eqns. (35)-(40) in the Eqns. (29)-(34), we obtained as

$$\prod{}^*_i(z,\theta)=\frac{\left(S^*(\lambda-\lambda X(z))-S^*(\theta)\right)\left[(1-\pi)\sum_{i=a}^{b-1}\prod_{mi}(0)+\Phi_i(0)\right]}{\Theta} \quad (41)$$

$$\Omega^*{}_1(z,\theta)=\frac{\left(V^*(\lambda-\lambda X(z))-V^*(\theta)\right)\left[(1-\pi)\sum_{i=0}^{a-1}\prod_{mi}(0)z^i+\sum_{n=0}^{a-1}\Phi_n(0)z^n\right]}{\Theta} \quad (42)$$

$$\Omega^*{}_j(z,\theta)=\frac{\left(V^*(\lambda-\lambda X(z))-V^*(\theta)\right)\sum_{n=0}^{N-1}\Omega_{j-1,n}(0)z^n}{\Theta} \quad (43)$$

$$\Phi^*(z,\theta)=\frac{(\mathrm{R}^*(\lambda-\lambda X(z))-R^*(\theta))[\pi\prod_m(z,0)]}{\Theta} \quad (44)$$

$$\Psi^*(z,\theta)=\frac{(\mathrm{H}^*(\lambda-\lambda X(z))-H^*(\theta))[\sum_{l=1}^{\infty}\Omega_{ln}(0)z^n]}{\Theta} \quad (45)$$

$$\prod{}^*(z,\theta)=\frac{(S^*(\lambda-\lambda X(z))-S^*(\theta))f(z)}{(z^b-(1-\pi)S^*(\lambda-\lambda X(z)))\Theta} \quad (46)$$

$$f(z)=(1-\pi)\left[\sum_{i=a}^{b-1}\prod_{i}(z,0)-\sum_{j=0}^{b-1}\prod_{mj}(0)z^{j}\right]+\Psi(z,0)+\sum_{j=1}^{N-b-1}\Phi_{b+j}(0)z^{b+j}$$

5. Probability generating function

We can attain probability generating functions of the queue size at different completion epochs by substituting $\theta=0$ in the Eqns. (41)-(46). Let $P(z)$ be the probability generating function of the expected queue length at an arbitrary time epoch, then

$$P(z)=\sum_{i=a}^{b-1}\prod_{i}(z)+\prod_{b}(z)+\sum_{l=1}^{\infty}\Omega_{l}(z)+\Phi(z)+\Psi(z) \tag{47}$$

$$=\frac{\left[\begin{array}{l}(1-\pi)S^{*}(\lambda+\lambda X(z))\sum_{i=0}^{a-1}d_{i}z^{i}+\left(S^{*}(\lambda+\lambda X(z))-1\right)G_{i}+H^{*}(\lambda+\lambda X(z))\left(V^{*}(\lambda+\lambda X(z))-1\right)\\ \left(\pi R^{*}(\lambda+\lambda X(z))S^{*}(\lambda+\lambda X(z))+z^{b}-1\right)\sum_{i=0}^{N-1}q_{i}z^{i}+\pi S^{*}(\lambda+\lambda X(z))\left(R^{*}(\lambda+\lambda X(z))-1\right)G_{i}+\\ \left(H^{*}(\lambda+\lambda X(z))V^{*}(\lambda+\lambda X(z))-1\right)\sum_{i=0}^{a-1}d_{i}z^{i+b}+\\ H^{*}(\lambda+\lambda X(z))V^{*}(\lambda+\lambda X(z))\left(\pi R^{*}(\lambda+\lambda X(z))S^{*}(\lambda+\lambda X(z))-1\right)\sum_{i=0}^{a-1}d_{i}z^{i}\end{array}\right]}{\left(-\lambda+\lambda X(z)\right)\left(z^{b}-(1-\pi)S^{*}(\lambda+\lambda X(z))\right)}$$

$$p_{i}=\sum_{m=a}^{b}\prod_{mi}(0),\ q_{i}=\sum_{l=1}^{\infty}\Omega_{\ln}(0),\ R_{i}=\Phi_{i}(0),\ d_{i}=(1-\pi)p_{i}+R_{i}$$

$$G_{i}=\sum_{i=a}^{b-1}d_{i}z^{b}-(1-\pi)\sum_{i=0}^{b-1}p_{i}z^{i}+\sum_{i=1}^{N-b-1}R_{i+b}z^{i+b}$$

Remark 1: The probability generating function has to satisfy $P(1)=1$.

$$\lambda E(X)\Big[(E(S)+\pi E(R))\Big(\sum_{i=0}^{b-1}d_{i}-(1-\pi)\sum_{i=0}^{b-1}p_{i}+\sum_{i=0}^{N-b-1}R_{i+b}\Big)+\pi E(V)\Big(\sum_{i=0}^{a-1}d_{i}+\sum_{i=0}^{N-1}q_{i}\Big)+\pi E(H)\sum_{i=0}^{a-1}d_{i}=\lambda\pi E(X)$$

Here q_i, p_i are probabilities, so the left side of the above Eqn. must be positive.Thus $\lambda\pi E(X)>0$, if $\rho=\lambda\pi E(X)$, then $\rho<1$ is the case to be fulfilled for the existence of steady state under consideration.

6. Some important performance measures

6.1. Expected queue length

The expected queue size L_q is obtained by differentiating *P(z)* at *z = 1* and it is given below,

$$L_q = \frac{\begin{bmatrix} f_1\left(\sum_{i=a}^{b-1} d_i - (1-\pi)\sum_{i=0}^{b-1} p_i + \sum_{i=1}^{N-b-1} R_{i+b}\right) + f_2\sum_{i=0}^{a-1} d_i + f_3\sum_{i=0}^{a-1} id_i + \\ f_4\left(\sum_{i=a}^{b-1} bd_i - (1-\pi)\sum_{i=0}^{b-1} ip_i + \sum_{i=1}^{N-b-1}(i+b)R_{i+b}\right) + \\ f_5\left[\sum_{i=0}^{a-1} d_i + \sum_{i=0}^{N-1} q_i\right] + f_6\left[\sum_{i=0}^{a-1} bd_i + \sum_{i=0}^{N-1} bq_i\right] + f_7\left[\sum_{i=0}^{a-1} id_i + \sum_{i=0}^{N-1} iq_i\right] + f_8\left[\sum_{i=0}^{a-1} bd_i + \sum_{i=0}^{a-1} id_i\right] \end{bmatrix}}{2T_1^2} \quad (48)$$

where,

$$f_1 = R_2T_1^2 - 2\pi bR_1 - 2\lambda\pi^2 R_1 + 2\pi R_1S_1T_1 + 2\pi S_1R_1 + \lambda E(X)S_2T_1 + S_3T_1 + 2S_1T_1 + 2S_1^2(1-\pi) - 2bS_1 - S_1T_2$$

$$f_2 = R_2T_1^2 - 2b\pi R_1 + 2\pi R_1S_1(1-\pi+T_1) + \lambda E(X)S_2T_1 - 2bS_1 + 2(1-\pi)S_1^2 + 2\pi R_1H_1T_1 + 2\pi S_1H_1(1-\pi+T_1) + H_2T_1^2 - 2b\pi H_1$$

$$f_3 = 2\pi R_1T_1 + 2S_1,\ f_4 = 2\pi R_1T_1$$

$$f_5 = V_2T_1^2 + \lambda\pi V_3T_1 - 2\pi bV_1 - \pi V_1T_2 + 2\pi V_1H_1T_1 + 2\pi R_1V_1T_1 + 2\pi S_1V_1(1-\pi+T_1)$$

$$f_6 = 2V_1T_1,\ f_7 = 2\pi V_1T_1,\ f_8 = 2\pi H_1T_1$$

and $T_1 = \lambda\pi E(X),\ T_2 = \lambda\pi E(X^2),\ S_1 = \lambda E(S)E(X),\ S_2 = \lambda E(S^2)E(X),\ S_3 = \lambda E(S)E(X^2)$

$R_1 = \lambda E(R)E(X),\ R_2 = \lambda E(R^2)E(X),\ H_1 = \lambda E(H)E(X),\ H_2 = \lambda E(H^2)E(X),\ V_1 = \lambda E(V)E(X)$

$V_2 = \lambda E(V^2)E(X),\ V_3 = \lambda E(V)E(X^2)$

6.2. Expectedwaiting time in the queue

$$E(W) = \frac{L_q}{\lambda E(X)}$$

Where L_q is given in Eqn. (48).

7. Conclusion

We analyzed the steady state behavior of $M^X/G(a,b)/1$ queue with server breakdown without interruption, multiple vacations, setup time and N-policy. We derived the steady state equations for the proposed queueing system. Also, we derived the probability generating functions for queue size. Further, we have presented some important performance measures.

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