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Research paper

Interval-Valued Intuitionistic Fuzzy INK-Ideals of INK-Algebra

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Abstract

Mathematical structures of Interim valued IF INK-ideal on INK-algebras are presented. We established that every IF- (INK)-ideal of an INK algebra A can be executed as an level ideal (INK) of an iv IF(INK) of U. As far as the idea of homomorphism, we talked about the cartesian result of i-v IF(INK)- ideal.

Keywords: *INK-algebra, INK-ideal, fuzzy INK-ideal, IF INK-ideal, Interval-valued fuzzy INK-ideal, Interval-valued intuitionistic fuzzy INK-ideal*

1. Introduction

Several researchers have developed algebraic structures. Imai and Iseki (1966) proposed two algebraic structure BCI and BCI-algebras. Then Hu and Li (1983) are expanding Algebra is called BCH-algebra, which is a generalization of BCK- and BCI-algebras. J. Neggers and H. S. Kim (1999) introduced d-algebras and studied the relationship flanked by d-algebras and BCK-algebras. Conceptual Fs and i-v Fs are then introduced by Zadeh (1982). Zadeh also used his i-v Fs to construct an near reasoning system. In addition, Attanssov [1986] introduced the concept of Intuitive Fuzzy Sets (IF_S) and Interval-valued $IVIF_S$ as a generalization of ordinary FS. Atanassov and Gargov [1989] show that IF_S and IVF_S are equal probability generalizations of F_S. In this paper, we first introduce an i-v IF INK-ideal INK-algebra. Then we prove that each intuitionistic fuzzy INK ideal of INK algebra A can be executed as the ideal INK ideal of intuitionistic fuzzy INK-U. In relation to the concept of homomorphism, we study the Cartesian product of the interval. Pay attention to intuitionistic fuzzy INK-ideal.

2. Preliminaries

Definition 2.1*:* *A INK-algebra (U, ·, 0) is a nonvoid set U with a non-varying value '0' and a binary operation ' · ' fulfilling the additional adages*

i) a · 0 = a

ii) (b · a) · (b · c) = a · c ∀a, b, c ∈ U.

In U we can regulate a binary association ≤ by a ≤ b *if and only if* a · b = 0.

The nonvoid subset T of INK-algebra (U, ·,0) is called sub-algebra of U, and if it is a · b ∈ T, ∀a, b ∈ T.

Let S be a nonvoid subset of U. Then S is called a INK-ideal of U if I- 1) 0 ∈ A I-2) (c · a) · (a· b) ∈ S and y ∈ S ⇔ x ∈ S, ∀a, b, c ∈ U.

Definition 2.4*:* *Let U be a universe set.* $\varkappa$ *be a Fs in U is a mapping* $\varkappa$*: U⟶ [0,1].*

Definition 2.2: *A Fs* $\varkappa$ *in a INK-algebra X is called a* fuzzy *subalgebra of U if,* $\varkappa$ (a · b) ≥ min { $\varkappa$ (a), $\varkappa$ (b)}, ∀ *a, b ∈ U.*

Definition 2.3: *A IFs* χ *is in the nonvoid U, in the form = {(U,* $\varkappa_\chi$ *(a),* σ_χ *(a)): x ∈ U} where* $\varkappa_\chi$*: U → [0, 1] and* λ_χ*: U → [0,1] means the degree of membership of each member x ∈ U (ie,* $\varkappa_\chi$ *(a)) and the degree of non-membership (in particular,* σ_χ *(a)),*$0 \le \varkappa_\chi(a) + \sigma_\chi(a) \le 1$*), ∀x ∈ U. We use the character* $\chi = (U, \varkappa_\chi, \sigma_\chi)$ *for IFs A = {(U,* $\varkappa_\chi$ *(a),* σ_χ *(a)) / a ∈ U}.*

Definition 2.4: *The IFs* $\chi = (U, \varkappa_\chi, \sigma_A)$ *in a INK-algebra U is called an IF INK-ideal of U, if*

1. $\varkappa_\chi(0) \ge \varkappa_\chi(a)$ *in addition* $\sigma_\chi(0) \le \sigma_\chi(a)$
2. $\varkappa_\chi(a) \ge \min\{\varkappa_\chi((c \cdot a) \cdot (a \cdot b)), \varkappa_\chi(b)\}$,
3. $\sigma_\chi(a) \le \max\{\sigma_\chi((c \cdot a) \cdot (a \cdot b)), \sigma_\chi(b)\}$,

∀a, b, c ∈U.

3. I-v IF_S -ideals (INK)

Definition 3.1: *An i-v IFs in INK-algebra U is called an i-v IF INK-ideal of U if it satisfies,*

1.$\bar{\varkappa}_\chi(0) \ge \bar{\varkappa}_\chi(a)$, $\bar{\sigma}_\chi(0) \le \bar{\sigma}_\chi(a)$,

2.$\bar{\varkappa}_\chi(a) \ge \text{rmin}\{\bar{\varkappa}_\chi((c \cdot a) \cdot (c \cdot b)), \bar{\varkappa}_\chi(b)\}$

3.$\bar{\sigma}_\chi(a) \le \text{rmax}\{\bar{\sigma}_\chi((c \cdot a) \cdot (c \cdot b)), \bar{\sigma}_\chi(b)\}$.

Example 3.2.

Discuss the following table of INK-Algebra: U = {0, p, q, r}.

*	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p

r	r	q	p	0

Let A be an *i-v* IF_S in U by

$$\bar{\varkappa}_A(x)=\begin{pmatrix} 0 & p & q & r \\ [0.6,0.8] & [0.4,0.5] & [0.3,0.4] & [0.3,0.4] \end{pmatrix}$$

$$\bar{\sigma}_A(x)=\begin{pmatrix} 0 & p & q & r \\ [0.3,0.4] & [0.5,0.7] & [0.5,0.7] & [0.4,0.6] \end{pmatrix}$$

Then routine calculations give that A is an i-v intuitionistic fuzzy INK-ideal of *U*.

Theorem 3.3. Let τ be i-v IF INK-ideal of U. If there exists a sequence $\{x_n\}$ in τ such that $\lim_{n\to\infty}\{xn\}$=[1,1], $\lim_{n\to\infty}\{xn\}$=[0,0]. Then $\bar{\varkappa}_\tau(0)$ = [1,1] and $\bar{\sigma}_\tau(0)$ = [0,0].

Proof. Since $\bar{\varkappa}_\tau(0) \geq \bar{\varkappa}_\tau(x)$ and $\bar{\sigma}_\tau(0) \leq \bar{\sigma}_\tau(x)$ $\forall x \in U$, We have $\bar{\varkappa}_\tau(0) \geq \bar{\varkappa}_\tau(x_n)$ and $\bar{\sigma}_\tau(0) \leq \bar{\sigma}_\tau(x_n)$, (n is positive integer) Note that

$[1,1] \geq \bar{\varkappa}_\tau(0) \geq \lim_{n\to\infty}\{xn\}$= [1,1]

$[0,0] \leq \bar{\sigma}_\tau(0) \leq \lim_{n\to\infty}\{xn\}$= [0,0].

Hence $\bar{\varkappa}_\tau(0)$ = [1,1] and $\bar{\sigma}_\tau(0)$ = [0,0].

Theorem 3.4. An i-v IF_s $\tau = [\langle \varkappa_\tau^L, \varkappa_\tau^U\rangle, \langle \sigma_\tau^L, \sigma_\tau^U\rangle]$ in U is an i-v intuitionistic fuzzy ideal (INK) of U if and just if $\langle \varkappa_\tau^L, \varkappa_\tau^U\rangle$and $\langle \sigma_\tau^L, \sigma_\tau^U\rangle$ are IF_s of U.

Proof. Since $\varkappa_A^L(x)$

$\varkappa_\tau^L(0) \geq \varkappa_\tau^L(x)$; $\varkappa_\tau^U(0) \geq \varkappa_\tau^U(x)$

$\sigma_\tau^L(0) \leq \sigma_\tau^L(x)$; $\sigma_\tau^U(0) \leq \sigma_\tau^U(x)$,

Therefore $\bar{\varkappa}_\tau(0) \geq \bar{\varkappa}_\tau(x)$; $\bar{\sigma}_\tau(0) \leq \bar{\sigma}_\tau(x)$.

$\langle \varkappa_\tau^L, \varkappa_\tau^U\rangle$ and $\langle \sigma_\tau^L, \sigma_\tau^U\rangle$ are IFI of U.

Let x, y ϵU, then

$\bar{\varkappa}_\tau(x) = [\varkappa_\tau^L(x), \varkappa_\tau^U(x)]$

$\geq[\min\{\varkappa_\tau^L(x\cdot y), \varkappa_\tau^L(y)\}, \min\{\varkappa_\tau^U(x\cdot y), \varkappa_\tau^U(y)\}]$

$= \text{rmin}\{[\varkappa_\tau^L(x\cdot y), \varkappa_\tau^U(x\cdot y)], [\varkappa_\tau^L(y), \varkappa_\tau^L(y)]\}$

$= \text{rmin}\{\bar{\varkappa}_\tau(x\cdot y), \bar{\varkappa}_\tau(y)\}$and

$\bar{\sigma}_\tau(x) = [\sigma_\tau^L(x), \sigma_\tau^U(x)]$

$\leq [\max\{\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)\}, \max\{\sigma_\tau^U(x\cdot y), \sigma_\tau^U(y)\}]$

$= \text{rmax}\{[\sigma_\tau^L(x\cdot y), \sigma_\tau^U(x\cdot y)], [\sigma_\tau^L(y), \sigma_\tau^U(y)]\}$

$= \text{rmax}\{\bar{\sigma}_\tau(x\cdot y), \bar{\sigma}_\tau(y)\}$.

Along these lines , τ is an i-v IFI of U. TUYT87T7Y89698689

Similarly, expect that A is an i-v IFI of U.

$[\varkappa_\tau^L(x), \varkappa_\tau^U(x)] = \bar{\varkappa}_\tau(x)$

$\geq \text{rmin}\{\bar{\varkappa}_\tau(x\cdot y), \bar{\varkappa}_\tau(y)\}$

$= \text{rmin}\{[\varkappa_\tau^L(x\cdot y), \varkappa_\tau^U(x\cdot y)], [\varkappa_\tau^L(y), \varkappa_\tau^U(y)]\}$

$=[\min\{\varkappa_\tau^L(x\cdot y), \varkappa_\tau^L(y)\}, \min\{\varkappa_\tau^U(x\cdot y), \varkappa_\tau^U(y)\}]$

what's more $[\sigma_\tau^L(x), \sigma_\tau^L(x)] = \bar{\sigma}_\tau(x) \leq \text{rmax}\{\bar{\sigma}_\tau(x\cdot y), \bar{\sigma}_\tau(y)\}$

$= \text{rmax}\{[\sigma_\tau^L(x\cdot y), \sigma_\tau^U(x\cdot y)], [\sigma_\tau^L(y), \sigma_\tau^U(y)]\}$

$= [\max\{\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)\}, \min\{\sigma_\tau^U(x\cdot y), \sigma_\tau^U(y)\}]$.

It pursues that,

$\varkappa_\tau^L(x) \geq \min\{\varkappa_\tau^L(x\cdot y), \varkappa_\tau^L(y)\}$,

$\sigma_\tau^L(x) \leq \max\{\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)\}$

and

$\varkappa_\tau^U(x) \geq \min\{\varkappa_\tau^U(x\cdot y), \varkappa_\tau^U(y)\}$,

$\sigma_\tau^L(x) \leq \max\{\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)\}$.

Hence $\langle\varkappa_\tau^L, \varkappa_\tau^U\rangle$ and $\langle \sigma_\tau^L, \sigma_\tau^U\rangle$ are IFI of U.

Proposition 3.5. Every i-v IF INK-ideal of a INK-algebra U is an i-v IFI.

Definition 3.6. *An IF_s τ in U is called an interval-valued intuitionistic fuzzy INK-sub algebra of U if*

i) $\bar{\varkappa}_\tau(x\cdot y) \geq \text{rmin}\{\bar{\varkappa}_\tau(x), \bar{\varkappa}_\tau(y)\}$ *and*

ii) $\bar{\sigma}_\tau(x\cdot y) \leq \text{rmax}\{\bar{\sigma}_\tau(x), \bar{\sigma}_\tau(y)\}$, $\forall x, y \in U$.

Proposition 3.7. Every i-v IF INK-ideal of a INK-algebra U is i-v IF sub algebra of U.

4. Product of i-v intuitionistic fuzzy INK-ideal

Definition 4.1. *An IF_r ύ on any set is a IF subset A with a membership function*

HVHVHJVHJVHJVH ΩRA: U×U→ [0, 1]

and non-membership function XHCHFT8698-=5654

ψ_A: U×U→ [0, 1].

Definition 4.2. *Let* $ύ = [\langle\varkappa_ύ^L, \varkappa_ύ^U\rangle, \langle \upsilon_ύ^L, \upsilon_{ύύ}^U\rangle]$ *and* $ώ = [\varkappa_ύ^L, \varkappa_ύ^U\rangle, \langle \upsilon_ώ^L, \upsilon_ώ^U\rangle]$ *be two i-v IFS in a set U. The product of ύ × ώ is defined by 6TTUI90L;*

$ύ \times ώ = \{((x, y), (\bar{\varkappa}_ύ \times \bar{\varkappa}_ώ), (\bar{\sigma}_ύ \times \bar{\sigma}_ώ)); \forall x, y \in U\times U\}$,

where ύ × ώ: U×U→D [0,1].

Theorem 4.3. Let $A = [\langle\varkappa_ύ^L, \varkappa_ύ^U\rangle, \langle \upsilon_ύ^L, \upsilon_ύ^U\rangle]$ and $B = [\langle\varkappa_ώ^L, \varkappa_ώ^U\rangle, \langle \upsilon_ώ^L, \upsilon_ώ^U\rangle]$ be two i-v on the off chance that subsets in a set U, ύ × ώ is an I-v IF INK-ideal of U×U.

Proof. Let (x, y) ϵU×U, at that point by definition

$(\bar{\varkappa}_ύ\times\bar{\varkappa}_ώ)$, (0,0) = r min $\{(\bar{\varkappa}_ύ(0), \bar{\varkappa}_ύ(0)\}$

$= \text{r min}\{[\varkappa_ύ^L(0), \varkappa_ύ^U(0)],[\varkappa_ώ^L(0), \varkappa_ώ^U(0)]\}$

$= [\min\{\varkappa_ύ^L(0), \varkappa_ώ^L(0)\},\min\{\varkappa_A^U(0), \varkappa_ώ^U(0)\}]$

$\geq [\min\{\varkappa_ύ^L(x), \varkappa_ώ^L(y)\}, \min\{\varkappa_ύ^U(x), \varkappa_ώ^U(y)\}]$

$= \text{rmin}\{[\varkappa_ύ^L(x), \varkappa_ύ^U(x)], [\varkappa_ώ^L(y), \varkappa_ώ^U(y)]\}$

$= \text{rmin}\{(\bar{\varkappa}_ύ(x), \bar{\varkappa}_ώ(y)\}$

$= (\bar{\varkappa}_ύ\times\bar{\varkappa}_ώ)$ (x, y)

in addition

$(\bar{\sigma}_ύ\times\bar{\sigma}_B)(0,0) = \text{rmax}\{(\bar{\sigma}_ύ(0), \bar{\sigma}_ύ(0)\}$

$= \text{rmax}\{[\sigma_ύ^L(0), \sigma_ύ^U(0)], [\sigma_ώ^L(0), \sigma_ώ^U(0)]\}$

$= [\max\{\sigma_ύ^L(0), \sigma_ώ^L(0)\}, \max\{\sigma_ύ^U(0), \sigma_ώ^U(0)\}]$

$\leq [\max\{\sigma_ύ^L(x), \sigma_ώ^L(y)\}, \max\{\sigma_ύ^U(x), \sigma_ώ^U(y)\}]$

$= \text{rmax}\{[\sigma_ύ^L(x), \sigma_ύ^U(x)], [\sigma_ώ^L(y), \sigma_ώ^U(y)]\}$

$= \text{rmax}\{(\bar{\sigma}_ύ(x), \bar{\sigma}_ύ(y)\}$

$= (\bar{\sigma}_ύ\times\bar{\sigma}_ώ)$ (x, y)

Thusly (FI_2) holds.

$(\bar{\varkappa}_ύ\times\bar{\varkappa}_ώ)((x, x')) = \text{rmin}\{\varkappa_ύ(x), \varkappa_ώ(x')\}$

$\text{r min}\{\text{rmin}(\bar{\varkappa}_ύ((z'\cdot x)\cdot(z'\cdot y)),\bar{\varkappa}_ύ(y)\}$,

$\text{rmin}\{\bar{\varkappa}_ώ((z'\cdot x')\cdot(z'\cdot y')), \bar{\varkappa}_ώ(y')\}\}$

$= \text{r min}\{\{\min\{\varkappa_ύ^L((z\cdot x)\cdot(z\cdot y)), \varkappa_ύ^L(y)\},\min\{\varkappa_ύ^U((z\cdot x)\cdot(z\cdot y)), \varkappa_ύ^U(y)\}\},\min\{\varkappa_ώ^L((z'\cdot x')\cdot(z'\cdot y')),\varkappa_ώ^L(y')\},\min\{\varkappa_ώ^U(((z'\cdot x')$.QWN $(z'\cdot y')), \varkappa_ώ^U(y')\}\}\}$.

$=\{\min\{\min\{\varkappa_ύ^L((z\cdot x)\cdot(z\cdot y)), \varkappa_ώ^L((z'\cdot x')\cdot(z'\cdot y'))\},\min\{\varkappa_ύ^L(y), \varkappa_ώ^L(y')\},\min\{\min\{\varkappa_ύ^U((z\cdot x)\cdot(z\cdot y)), \varkappa_ώ^U(((z'\cdot x')\cdot(z'\cdot y'))\}, \min\{\varkappa_ύ^U(y), \varkappa_ώ^U(y')\}\}\}$

$= \text{rmin}\{(\bar{\varkappa}_ύ\times\bar{\varkappa}_ώ)(((z\cdot x)\cdot(z\cdot y)),((z'\cdot x')\cdot(z'\cdot y'))),(\bar{\varkappa}_ύ\times\bar{\varkappa}_ώ)(y, y')\}$.

furthermore, $(\bar{\sigma}_ύ\times\bar{\sigma}_ώ)((x, x')) = \text{r max}\{\bar{\sigma}_ύ(x), \bar{\sigma}_ώ(x')\}$

$\leq \text{rmax}\{\text{rmax}\{\bar{\sigma}_ύ((z\cdot x)\cdot(z\cdot y))\bar{\sigma}_ύ(y)\},\text{rmax}\{\bar{\sigma}_ώ((z^1\cdot x^1)\cdot(z^1\cdot y^1)), \bar{\sigma}_ώ(y^1)\}\}$

$= \text{r max}\{\{\min\{\sigma_ύ^L((z\cdot x)\cdot(z\cdot y)), \sigma_ύ^L(y)\},\max\{\sigma_ύ^U((z\cdot x)\cdot(z\cdot y)), \sigma_ύ^U(y)\}\}, \max\{\sigma_ώ^L((z'\cdot x')\cdot(z'\cdot y)), \sigma_ώ^L(y')\}, \max\{\sigma_ώ^U(((z'\cdot x')(z'\cdot y')), \sigma_ώ^U(y')\}\}\}$.

$= \{\max\{\max\{\sigma_ύ^L((z\cdot x)\cdot(z\cdot y)), \sigma_ώ^L((z'\cdot x')\cdot(z'\cdot y'))\},\max\{\sigma_ύ^L(y), \sigma_ώ^L(y^1)\}, \max\{\max\{\sigma_ύ^U((z\cdot x)\cdot(z\cdot y)), \sigma_ύ^U(((z'\cdot x')\cdot(z'\cdot y'))\}, \max\{\sigma_ύ^U(y), \sigma_ύ^U(y')\}\}\}$

$= \text{rmax}\{(\bar{\sigma}_ύ\times\bar{\sigma}_ώ)(((z\cdot x)\cdot(z\cdot y)),((z'\cdot x')\cdot(z'\cdot y'))), (\bar{\sigma}_ύ\times\bar{\sigma}_ώ)(y,y')\}$.

Henceforth total the evidence.

Definition 4.4. *Let* $\overline{\kappa}_{\acute{\omega}}$ *and* $\overline{o}_{\acute{\omega}}$ *respectively be an i-v membership and non-membership function of every element* $x \in U$ *to the set* $\acute{\omega}$*. Then strongest i-v IFS relation on U, that is a membership function relation* $\overline{\kappa}_{\acute{\omega}}$ *on* $\overline{o}_{\acute{\omega}}$ *and* $\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}, \overline{o}_{\acute{\upsilon}_{\acute{\omega}}}$ *whose i-v membership and non-membership function of every element (x, y)* $\in U \times U$ *and defined by*

i) $\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(x,y) = \text{rmin}\{\overline{\kappa}_{\acute{\omega}}(x), \overline{\kappa}_{\acute{\omega}}(y)\}$ and

ii) $\overline{o}_{\acute{\upsilon}_{\acute{\omega}}}(x,y) = \text{rmax}\{\overline{o}_{\acute{\omega}}(x), \overline{o}_{\acute{\omega}}(y)\}$.

Definition 4.5. *Let* $\acute{\omega} = [\langle \kappa_{\acute{\omega}}^{L}, \kappa_{\acute{\omega}}^{U}\rangle, \langle \upsilon_{\acute{\omega}}^{L}, \upsilon_{\acute{\omega}}^{U}\rangle]$ *be an i-v subset in a set U, then the strongest i-v IF relation on U that is a i-v A on* $\acute{\omega}$ *is* $\acute{\upsilon}_{\acute{\omega}}$ *and defined by*

$\acute{\upsilon}_{\acute{\omega}} = [\langle \kappa_{\acute{\upsilon}_{\acute{\omega}}}^{L}, \mu_{\acute{\upsilon}_{\acute{\omega}}}^{U}\rangle, \langle \upsilon_{\acute{\upsilon}_{\acute{\omega}}}^{L}, \upsilon_{\acute{\upsilon}_{\acute{\omega}}}^{U}\rangle]$.

Theorem 4.6. Let $\acute{\omega} = [\langle \mu_{\acute{\upsilon}_{\acute{\omega}}}^{L}, \mu_{\acute{\upsilon}_{\acute{\omega}}}^{U}\rangle, \langle \upsilon_{\acute{\upsilon}_{\acute{\omega}}}^{L}, \upsilon_{\acute{\upsilon}_{\acute{\omega}}}^{U}\rangle]$ be an i-v subset in a set U and be the strongest i-v IF relation on U. at that point $\acute{\omega}$ is an i-v IF INK-ideal of U if and just if $\acute{\upsilon}_{\acute{\omega}}$ is an i-v IF INK-ideal of U×U.

Proof. Let B be an i-v IF INK-ideal of U.

$\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(0,0) = \text{rmin}\{\overline{\kappa}_{\acute{\omega}}(0), \overline{\kappa}_{\acute{\omega}}(0)\}$
$\geq \text{rmin}\{\overline{\kappa}_{\acute{\omega}}(x), \overline{\kappa}_{\acute{\omega}}(y)\}$
$= \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(x, y)$
and
$\overline{o}_{\acute{\upsilon}_{\acute{\omega}}}(0,0) = \text{rmax}\{\overline{o}_{\acute{\omega}}(0), \overline{o}_{\acute{\omega}}(0)\}$
$\leq \text{rmax}\{\overline{o}_{\acute{\omega}}(x), \overline{o}_{\acute{\omega}}(y)\}$
$= \overline{o}_{\acute{\upsilon}_{\acute{\omega}}}(x, y)$
On the other hand,
$\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(x, x') = \text{rmin}\{\overline{\kappa}_{\acute{\omega}}(x), \overline{\kappa}_{\acute{\omega}}(x')\}$
$\geq \text{rmin}\{\text{rmin}\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\omega}}(y)\},$
$\text{rmin}\{\overline{\kappa}_{\acute{\omega}}((z'\cdot x')\cdot(z'\cdot y')), \overline{\kappa}_{\acute{\omega}}(y^1)\}\}$
$= \text{rmin}\{\text{rmin}\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)),$
$\overline{\kappa}_{\acute{\omega}}((z'\cdot x')\cdot(z'\cdot y'))\},$
$\text{r min}\{\overline{\kappa}_{\acute{\omega}}(y), \overline{\kappa}_{\acute{\omega}}(y')\}\}$
$= \text{r min}\{\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), ((z'\cdot x')\cdot(z'\cdot y')), \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(y, y')\}$
$= \text{r min}\{\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(((z, z')\cdot(y, y'))\cdot((z, z')\cdot(x, x'))), \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(y, y')\}$
Also,
$\overline{o}_{\acute{\upsilon}_{\acute{\omega}}}(x, x') = \text{rmax}\{\overline{o}_{\acute{\omega}}(x), \overline{o}_{\acute{\omega}}(x')\}$
$\leq \text{rmax}\{\text{rmax}\{\overline{o}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{o}_{\acute{\omega}}(y)\},$
$\text{rmax}\{\overline{o}_{\acute{\omega}}((z'\cdot x')\cdot(z'\cdot y')), \overline{o}_{\acute{\omega}}(y')\}\}$
$= \text{rmax}\{\text{rmax}\{\overline{o}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y))\ \overline{o}_{\acute{\omega}}((z'\cdot x')\cdot(z'\cdot y'))\},$
$\text{r max}\{\overline{o}_{\acute{\omega}}(y), \overline{o}_{\acute{\omega}}(y^1)\}\}$
$= \text{rmax}\{\overline{o}_{\acute{\upsilon}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), ((z'\cdot x')\cdot(z'\cdot y')),$
$\overline{o}_{\acute{\upsilon}_{\acute{\omega}}}(y, y^1)\}.$
$= \text{rmax}\{\overline{o}_{\acute{\upsilon}_{\acute{\omega}}}(((z, z')\cdot(y, y'))\cdot((z, z')\cdot(x, x'))),$
$\overline{o}_{\acute{\upsilon}_{\acute{\omega}}}(y, y^1)\},$
for all (x, x'), (y, y'), (z, z') in U × U.

Equally, let $\acute{\upsilon}_{\acute{\omega}}$ be an i-v IF-ideal (INK) of U×U.
$(x, x) \in U\times U$.
$\text{rmin}\{\overline{\kappa}_{\acute{\omega}}(0), \overline{\kappa}_{\acute{\omega}}(0)\} = \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(0,0)$
$\geq \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(x, x)$
$\text{rmin}\{\overline{\kappa}_{\acute{\omega}}(0), \overline{\kappa}_{\acute{\omega}}(0)\} = \text{r min}\{\overline{\kappa}_{\acute{\omega}}(x), \overline{\kappa}_{\acute{\omega}}(x)\}$
furthermore, $\text{rmax}\{\overline{\kappa}_{\acute{\omega}}(0), \overline{\kappa}_{\acute{\omega}}(0)\} = \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(0,0)$
$\leq \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(x, x)$
$\text{rmax}\{\overline{\kappa}_{\acute{\omega}}(0), \overline{\kappa}_{\acute{\omega}}(0)\} = \text{rmax}\{\overline{\kappa}_{\acute{\omega}}(x), \overline{\kappa}_{\acute{\omega}}(x)\}$.
$(x, x^1), (y, y^1), (z, z^1)$ in U × U.
$\text{rmin}\{\overline{\kappa}_{\acute{\omega}}(x, x')\} = \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}\{(x, x')\}$
$\geq \text{rmin}\{\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(((z, z')\cdot(y, y'))\cdot((z, z')\cdot(x, x'))), \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(y, y')\}$
$= \text{rmin}\{\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), ((z'\cdot x')\cdot(z'\cdot y')), \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(y, y')\}$
$= \text{rmin}\{\text{rmin}\{\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}((z'\cdot x')\cdot(z'\cdot y'))\},$
$\text{r min}\{\overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(y), \overline{\kappa}_{\acute{\upsilon}_{\acute{\omega}}}(y')\}\}$
$= \text{r min}\{\text{r min}\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\omega}}(y)\},$
$\text{r min}\{\overline{\kappa}_{\acute{\omega}}((z^1\cdot x^1)\cdot(z^1\cdot y^1)), \overline{\kappa}_{\acute{\omega}}(y^1)\}\}$
If $x^1=y^1=z^1=0$, then
$\text{rmin}\{\overline{\kappa}_{\acute{\omega}}(x), \overline{\kappa}_{\acute{\omega}}(0)\}$
$= \text{rmin}\{\text{r min}\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\omega}}(y)\}, \overline{\kappa}_{\acute{\omega}}(0)\}$
$\overline{\kappa}_{\acute{\omega}}(x) \geq \text{rmin}\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\omega}}(y)\}$, essentially
$\overline{o}_{\acute{\omega}}(x) \leq \text{rmax}\{\overline{o}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{o}_{\acute{\omega}}(y)\}$.
Therefore, B is i-v IF INK-ideal of U.

Theorem 4.8. If $\overline{\kappa}_{\acute{\upsilon}}$ is a i-v IF INK-ideal of INK-algebra U, then $\overline{\mu}_{\square\square}$ is also i-v IF INK-ideal of INK-algebra U.

Proof.
$\overline{\kappa}_{\acute{\upsilon}}(0) \geq \overline{\kappa}_{\acute{\upsilon}}(x)$
$[\overline{\kappa}_{\acute{\upsilon}}(0)] \geq [\overline{\kappa}_{\acute{\upsilon}}(x)]$
$[\overline{\kappa}_{\acute{\upsilon}}(0)]^{m} \geq [\overline{\kappa}_{\acute{\upsilon}}(x)]^{m}$
$\overline{\kappa}_{\acute{\upsilon}}(0)^{m} \geq \overline{\kappa}_{\acute{\upsilon}}(x)^{m}$
$\overline{\kappa}_{\acute{\upsilon}}^{m}(0) \geq \overline{\kappa}_{\acute{\upsilon}}^{m}(x)$
$\overline{\kappa}_{\acute{\upsilon}}(x) \geq \text{r min}\{\overline{\kappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\upsilon}}(y)\}$
$[\overline{\kappa}_{\acute{\upsilon}}(x)]^{m} \geq [\text{r min}\{\overline{\kappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\upsilon}}(y)\}]^{m}$
$\overline{\kappa}_{\acute{\upsilon}}(x)^{m} \geq \text{r min}\{\overline{\kappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y))^{m}, \overline{\kappa}_{\acute{\upsilon}}(y)^{m}\}$
$\overline{\kappa}_{\acute{\upsilon}}^{n}(x) \geq \text{r min}\{\overline{\kappa}_{\acute{\upsilon}}^{m}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\upsilon}}^{m}(y)\}$
and
$\overline{o}_{\acute{\upsilon}}(0) \leq \overline{o}_{\acute{\upsilon}}(x)$
$[\overline{o}_{\acute{\upsilon}}(0)] \leq [\overline{o}_{\acute{\upsilon}}(x)]$
$[\overline{o}_{\acute{\upsilon}}(0)]^{m} \leq [\overline{o}_{\acute{\upsilon}}(x)]^{m}$
$\overline{o}_{\acute{\upsilon}}(0)^{m} \leq \overline{o}_{\acute{\upsilon}}(x)^{m}$
$\overline{o}_{\acute{\upsilon}}^{m}(0) \leq \overline{o}_{\acute{\upsilon}}^{m}(x)$
$\overline{o}_{\acute{\upsilon}}(x) \leq \text{r min}\{\overline{o}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{o}_{\acute{\upsilon}}(y)\}$
$[\overline{o}_{\acute{\upsilon}}(x)]^{m} \leq [\text{r min}\{\overline{o}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{o}_{\acute{\upsilon}}(y)\}]^{m}$
$\overline{o}_{\acute{\upsilon}}(x)^{m} \leq \text{r min}\{\overline{o}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y))^{m}, \overline{o}_{\acute{\upsilon}}(y)^{m}\}$
$\overline{o}_{\acute{\upsilon}}^{m}(x) \leq \text{r min}\{\overline{o}_{\acute{\upsilon}}^{m}((z\cdot x)\cdot(z\cdot y)), \overline{o}_{\acute{\upsilon}}^{m}(y)\}$.
Hence complete the proof.

Theorem 4.10. If $\overline{\kappa}_{\acute{\upsilon}}$ is also a i-v intuitionistic fuzzy INK-ideal of INK-algebra U, then $\overline{\kappa}_{\acute{\upsilon}\cap\acute{\omega}}$ is also a INK-ideal of INK-algebra U.

Proof. For all x, y, z ∈ U
$\overline{\kappa}_{\acute{\upsilon}}(0) \geq \overline{\kappa}_{\acute{\upsilon}}(x)$
$\min\{\overline{\kappa}_{\acute{\upsilon}}(0), \overline{\kappa}_{\acute{\omega}}(0)\} \geq \min\{\overline{\kappa}_{\acute{\upsilon}}(x), \overline{\kappa}_{\acute{\omega}}(x)\}$
$\overline{\kappa}_{\acute{\upsilon}\cap\acute{\omega}}(0) \geq \overline{\kappa}_{\acute{\upsilon}\cap\acute{\omega}}(x)$
$\overline{\kappa}_{\acute{\upsilon}}(x) \geq \text{r min}\{\overline{\kappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\upsilon}}(y)\}$
$\overline{\kappa}_{\acute{\omega}}(x) \geq \text{r min}\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\omega}}(y)\}$
$\{\overline{\kappa}_{\acute{\upsilon}}(x), \overline{\kappa}_{\acute{\omega}}(x)\} \geq \{\text{r min}\{\overline{\kappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\upsilon}}(y)\},$
$\text{r min}\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\omega}}(y)\}\}$
$\min\{\overline{\kappa}_{\acute{\upsilon}}(x), \overline{\kappa}_{\acute{\omega}}(x)\} \geq \{\text{r min}\{\overline{\kappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\upsilon}}(y)\},$

$r\min\{\overline{\varkappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\omega}}(y)\}\}$
$\geq \min\{r\min\{\overline{\varkappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y))\},$
$r\min\{\overline{\varkappa}_{\acute{\upsilon}}(y), \overline{\varkappa}_{\acute{\omega}}(y)\}\}$
$\overline{\varkappa}_{\acute{\upsilon}\cap\acute{\omega}}(x) = r\min\{\overline{\varkappa}_{\acute{\upsilon}\cap\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\upsilon}\cap\acute{\omega}}(y)\}.$
and
$\bar{\sigma}_{\acute{\upsilon}}(0) \leq \bar{\sigma}_{A}(x)$
$\max\{\bar{\sigma}_{\acute{\upsilon}}(0), \bar{\sigma}_{\acute{\omega}}(0)\} \leq \max\{\bar{\sigma}_{\acute{\upsilon}}(x), \bar{\sigma}_{\acute{\omega}}(x)\}$
$\bar{\sigma}_{\acute{\upsilon}\cap\acute{\omega}}(0) \leq \bar{\sigma}_{\acute{\upsilon}\cap\acute{\omega}}(x)$
$\bar{\sigma}_{\acute{\upsilon}}(x) \leq r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}}(y)\}$
$\bar{\sigma}_{\acute{\omega}}(x) \leq r\max\{\bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}(y)\}$
$\{\bar{\sigma}_{\acute{\upsilon}}(x), \bar{\sigma}_{\acute{\omega}}(x)\} \leq \{r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}}(y)\},$
$r\max\{\bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}(y)\}\}$
$\max\{\bar{\sigma}_{\acute{\upsilon}}(x), \bar{\sigma}_{\acute{\omega}}(x)\} \leq \{r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}}(y)\},$
$r\max\{\bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}(y)\}\}$
$\leq \max\{r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y))\},$
$r\max\{\bar{\sigma}_{\acute{\upsilon}}(y), \bar{\sigma}_{\acute{\omega}}(y)\}\}$
$\bar{\sigma}_{\acute{\upsilon}\cap\acute{\omega}}(x) = r\max\{\bar{\sigma}_{\acute{\upsilon}\cap\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}\cap\acute{\omega}}(y)\}.$
Complete the proof.

Theorem 4.9. If $\overline{\varkappa}_{\acute{\upsilon}}$ is also a i-v IF INK-ideal of U, then $\overline{\varkappa}_{\acute{\upsilon}\cup\acute{\omega}}$ is also a INK-ideal of U.

Proof. For all x, y, z ∈ U.
$\overline{\varkappa}_{\acute{\upsilon}}(0) \geq \overline{\varkappa}_{\acute{\upsilon}}(x)$
$\min\{\overline{\varkappa}_{\acute{\upsilon}}(0), \overline{\varkappa}_{\acute{\omega}}(0)\} \geq \min\{\overline{\varkappa}_{\acute{\upsilon}}(x), \overline{\varkappa}_{\acute{\omega}}(x)\}$
$\overline{\varkappa}_{\acute{\upsilon}\cup\acute{\omega}}(0) \geq \overline{\varkappa}_{\acute{\upsilon}\cup\acute{\omega}}(x)$
$\overline{\varkappa}_{\acute{\upsilon}}(x) \geq r\min\{\overline{\varkappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\upsilon}}(y)\}$
$\overline{\varkappa}_{\acute{\omega}}(x) \geq r\min\{\overline{\varkappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\omega}}(y)\}$
$\{\overline{\varkappa}_{\acute{\upsilon}}(x), \overline{\varkappa}_{\acute{\omega}}(x)\} \geq \{r\min\{\overline{\varkappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\upsilon}}(y)\},$
$r\min\{\overline{\kappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\acute{\omega}}(y)\}\}$
$\min\{\overline{\varkappa}_{\acute{\upsilon}}(x), \overline{\varkappa}_{\acute{\omega}}(x)\} \geq \{r\min\{\overline{\varkappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{A}(y)\},$
$r\min\{\overline{\varkappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\omega}}(y)\}\}$
$\geq \min\{r\min\{\overline{\varkappa}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y))\},$
$r\min\{\overline{\varkappa}_{\acute{\upsilon}}(y), \overline{\varkappa}_{\acute{\omega}}(y)\}\}$
$\overline{\varkappa}_{\acute{\upsilon}\cup\acute{\omega}}(x) = r\min\{\overline{\varkappa}_{\acute{\upsilon}\cup\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\acute{\upsilon}\cup\acute{\omega}}(y)\}.$
and
$\bar{\sigma}_{\acute{\upsilon}}(0) \leq \bar{\sigma}_{\acute{\upsilon}}(x)$
$\max\{\bar{\sigma}_{\acute{\upsilon}}(0), \bar{\sigma}_{\acute{\omega}}(0)\} \leq \max\{\bar{\sigma}_{\acute{\upsilon}}(x), \bar{\sigma}_{\acute{\omega}}(x)\}$
$\bar{\sigma}_{AUB}(0) \leq \bar{\sigma}_{AUB}(x)$
$\bar{\sigma}_{\acute{\upsilon}}(x) \leq r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}}(y)\}$
$\bar{\sigma}_{\acute{\omega}}(x) \leq r\max\{\bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}(y)\}$
$\{\bar{\sigma}_{\acute{\upsilon}}(x), \bar{\sigma}_{\acute{\omega}}(x)\} \leq \{r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}}(y)\},$
$r\max\{\bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}(y)\}\}$
$\max\{\bar{\sigma}_{\acute{\upsilon}}(x), \bar{\sigma}_{\acute{\omega}}(x)\} \leq \{r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}}(y)\},$
$r\max\{\bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}(y)\}\}$
$\leq \max\{r\max\{\bar{\sigma}_{\acute{\upsilon}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y))\},$
$r\max\{\bar{\sigma}_{\acute{\upsilon}}(y), \bar{\sigma}_{\acute{\omega}}(y)\}\}$
$\bar{\sigma}_{\acute{\upsilon}\cup\acute{\omega}}(x) \leq r\max\{\bar{\sigma}_{\acute{\upsilon}\cup\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\acute{\upsilon}\cup\acute{\omega}}(y)\}.$
Hence complete the proof.

5. Homomorphism of i-v Intuitionistic fuzzy INK-Algebra

Definition 5.1. *Let (U, ·, 0) and (V, .', 0') be INK-algebras. A function φ: U→V' is called a homomorphism if*
φ (x · y) = φ (x) · φ (y), ∀ x, y ∈ U.
For any interval-valued IF_S $\chi = (V, \overline{\varkappa}_{\chi}, \bar{\sigma}_{\chi})$ in V we define a new i-v φ B = (U, φ $\overline{\varkappa}_{\chi}$, φ $\bar{\sigma}_{\chi}$) in U, by
$\varphi\overline{\varkappa}_{\chi} = \overline{\varkappa}_{\chi}(\varphi(x))$ and $\varphi\bar{\sigma}\,\chi = \bar{\sigma}\,\chi(\varphi(x))$.

Theorem 5.2. Let (U, ·, 0) and (V, ·', 0') be INK-algebras. An onto homomorphic image of an i-v IF INK-ideal of U is also an i-v IF INK-ideal of V.

Proof. Let φ: U →V be an onto homomorphism of INK-algebras. Suppose $\chi = (V, \overline{\varkappa}_{\chi}, \bar{\sigma}_{\chi})$ is the image of an i-v IF INK-ideal $\varphi_{\chi} = (U, \varphi\overline{\varkappa}_{\chi}, \varphi\bar{\sigma}_{\chi})$ of U. We have to prove that $A = (V, \overline{\varkappa}_{\chi}', \bar{\sigma}_{\chi}')$ is an i-v IF INK-ideal of V. Since h: U →V is onto, then x', y', z∈ V there exist x, y, z ∈ U such that φ (x) = x' and φ (y) = y^1 also we study φ (0) = 0^1.
Then
$\overline{\varkappa}_{\chi}(0') = \overline{\varkappa}_{\chi}(\varphi(0))$
$= \varphi\overline{\varkappa}_{\chi}(0)$
$\geq \varphi\overline{\varkappa}_{\chi}(x)$
$= \overline{\varkappa}_{\chi}(\varphi(x))$
$\overline{\varkappa}_{\chi}(0') = \overline{\varkappa}_{\chi}(x')$
and
$\bar{\sigma}_{\chi}(0') = \bar{\sigma}_{\chi}(\varphi(0))$
$= \varphi\bar{\sigma}_{\chi}(0)$
$\leq \varphi\bar{\sigma}_{\chi}(x)$
$= \bar{\sigma}_{A}(\varphi(x))$
$\bar{\sigma}_{\chi}(0') = \bar{\sigma}_{\chi}(x').$
Also
$\overline{\varkappa}_{\chi}(x^1) = \overline{\varkappa}_{\chi}(\varphi(x))$
$= \varphi\overline{\varkappa}_{\chi}(x)$
$\geq \min\{\varphi\overline{\varkappa}_{\chi}((z\cdot x)\cdot(z\cdot y)), \varphi\overline{\varkappa}_{\chi}(y)\}$
$= \min\{\overline{\varkappa}_{\chi}(\varphi((z\cdot x)\cdot(z\cdot y)), \overline{\varkappa}_{\chi}(\varphi(y))\}$
$= \min\{\overline{\varkappa}_{\chi}(\varphi(z\cdot x)\cdot'\varphi(z\cdot y)), \overline{\varkappa}_{\chi}(\varphi(y))\}$
$= \min\{\overline{\varkappa}_{\chi}((\varphi(z)\cdot'\varphi(x))\cdot'(\varphi(z)\cdot'\varphi(y)), \overline{\varkappa}_{\chi}(\varphi(y))\}$
$\overline{\varkappa}_{\chi}(x') = \min\{\overline{\varkappa}_{\chi}((z'\cdot' x')\cdot'(z'\cdot' y')), \overline{\varkappa}_{\chi}(y')\}$
and
$\bar{\sigma}_{\chi}(x') = \bar{\sigma}_{\chi}(\varphi(x))$
$= \varphi\bar{\sigma}_{\chi}(x)$
$\leq \max\{\varphi\bar{\sigma}_{\chi}((z\cdot x)\cdot(z\cdot y)), \varphi\bar{\sigma}_{\chi}(y)\}$
$= \max\{\bar{\sigma}_{\chi}(\varphi((z\cdot x)\cdot(z\cdot y)), \bar{\sigma}_{\chi}(\varphi(y))\}$
$= \max\{\bar{\sigma}_{\chi}(\varphi(z\cdot x)\cdot'\varphi(z\cdot y)), \bar{\sigma}_{\chi}(\varphi(y))\}$
$= \max\{\bar{\sigma}_{\chi}((\varphi(z)\cdot'\varphi(x))\cdot'(\varphi(z)\cdot'\varphi(y)), \bar{\sigma}_{\chi}(\varphi(y))\}$
$\bar{\sigma}_{\chi}(x^1) = \max\{\bar{\sigma}_{\chi}((z'\cdot' x')\cdot'(z'\cdot' y')), \bar{\sigma}_{\chi}(y')\}$
Hence $A = (V, \overline{\varkappa}_{\chi}, \bar{\sigma}_{\chi})$ is an i-v IF INK-ideal of U.

Theorem 5.3. Let (X, ·,0) and (Y, ·', 0') be INK-algebras. An onto homomorphic inverse image of an i-v IF INK-ideal of V is also an i-v IF INK-ideal of U.

Proof. Let ψ: U → V be an onto homomorphism of INK-algebras. Suppose $\psi_{\chi} = (X, \psi\overline{\varkappa}_{\chi}, \psi\bar{\sigma}_{\chi})$ is the inverse image of an i-v IF INK-ideal $\chi = (Y, \overline{\varkappa}_{\chi}, \bar{\sigma}_{\chi})$ of V.
$\psi_{\chi} = (X, \psi\overline{\varkappa}_{\chi}', \psi\bar{\sigma}_{\chi})$ is an i-v INK-ideal of U.
For any x', y', z∈ V there exist x, y, z ∈ U.
such that ψ (x) = x' and ψ (y) = y' and ψ (0) = 0'.
$\psi\overline{\varkappa}_{\chi}(0) = \overline{\varkappa}_{\chi}(\psi(0))$
$= \overline{\varkappa}_{\chi}(0^1)$
$\overline{\varkappa}_{\chi}(x')$
$= \overline{\varkappa}_{\chi}(\psi(x))$
implies that $\psi\overline{\varkappa}_{\chi}(0) = \psi\overline{\varkappa}_{\chi}(x)$ and
$\psi\bar{\sigma}_{\chi}(0) = \bar{\sigma}_{\chi}(\psi(0))$
$= \bar{\sigma}_{\chi}(0')$

$\leq \bar{\sigma}_{\chi}(x')$
$= \bar{\sigma}_{\chi}(\psi(x))$
Implies that $\psi\bar{\sigma}_{\chi}(0) = \psi\bar{\sigma}_{\chi}(x)$.
$\psi\bar{\varkappa}_{\chi}(x) = \bar{\varkappa}_{\chi}(\psi(x))$
$= \bar{\varkappa}_{\chi}(x^1)$
$\psi\bar{\varkappa}_{\chi}(x) = \min\{\bar{\varkappa}_{\chi}((z' \cdot x') \cdot (z' \cdot y')), \bar{\varkappa}_{\chi}(y')\}$
$= \min\{\bar{\varkappa}_{\chi}((\psi(z) \cdot \psi(x)) \cdot (\psi(z) \cdot \psi(y)), \bar{\varkappa}_{\chi}(\psi(y))\}$
$= \min\{\bar{\varkappa}_{\chi}(\psi(z \cdot x) \cdot \psi(z \cdot y)), \bar{\varkappa}_{\chi}(\psi(y))\}$
$= \min\{\bar{\varkappa}_{\chi}(\psi((z \cdot x) \cdot (z \cdot y)), \bar{\varkappa}_{\chi}(\psi(y))\}$
$\geq \min\{\psi\bar{\varkappa}_{\chi}((z \cdot x) \cdot (z \cdot y)), \psi\bar{\varkappa}_{\chi}(y)\}$
and
$\psi\bar{\sigma}_{\chi}(x) = \bar{\sigma}_{\chi}(\psi(x))$
$= \bar{\sigma}_{\chi}(x')$
$\psi\bar{\sigma}_{\chi}(x) = \max\{\bar{\sigma}_{\chi}((z' \cdot x') \cdot (z' \cdot y')), \bar{\sigma}_{\chi}(y')\}$
$= \max\{\bar{\sigma}_{\chi}((\psi(z) \cdot \psi(x)) \cdot (\psi(z) \cdot \psi(y)), \bar{\sigma}_{\chi}(\psi(y))\}$
$= \max\{\bar{\sigma}_{\chi}(\psi(z \cdot x) \cdot \psi(z \cdot y)), \bar{\sigma}_{\chi}(\psi(y))\}$
$= \max\{\bar{\sigma}_{\chi}(\psi((z \cdot x) \cdot (z \cdot y)), \bar{\sigma}_{\chi}(\psi(y))\}$
$\psi\bar{\sigma}_{\chi}(x) \leq \max\{\psi\bar{\sigma}_{\chi}((z \cdot x) \cdot (z \cdot y)), \psi\bar{\sigma}_{\chi}(y)\}$

Therefore, complete the proof.

6. Conclusions

In this paper, we INK-algebra between the current value of the member and non-member functions of IF-INK classical with introducing the concept, and to study their properties.

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