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Joint two-echelon inventory model for optimality of cycle time and inventory decisions with finite production rate under credit period

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Abstract. In the universal competition of business environment, to face the global challenges around the business, it has turn into more traditional of giving trade credit by the vendor to the buyer as part of improving the sales revenue. The buyer need not to pay the vendor immediately whenever buyer received the goods, if a vendor offers credit period to the buyer. In this scenario focusing on the entire supply chain (SC) performance in terms of cycle time, inventory levels, number of shipment and the overall variable cost of the entire SC is essential. Mathematical models are developed for the joint two-echelon inventory system by assuming that production rate is finite. The present work aims to illustrate the optimality of cycle time, levels of inventory, number of shipments and the overall variable cost of the SC under the phenomena of credit period. Finally, numerical examples are solved to illustrate the theoretical results and also the sensitivity analysis is carried out.

1. Introduction

The goal of SCM is to increase the differential or aggressive benefit of the channels as a whole rather than to enlarge the improvement of any sole firm. Client value is produced through cooperation and teamwork to improve effectiveness (lower costs) and market value (added benefits) in ways that are most precious to key customers.

In this prospective, Goyal [1] is the pioneer of developing integrated inventory model. Saha and Goyal [2] proposed SC cooperation contracts where the demand is expressed in terms of retailer's price. They have expressed three coordinating contracts for coordination in two-echelon SC like combined repayment contract, price discount contract in wholesale and contract with cost sharing. Gallego et al [3] had presented a paper on pricing decision under coordination and stock replenishment policy for single merchant and single or extra geologically discrete buyers. In addition to the above, they derived the price-sensitive buyer order to analyze a SC with issues in coordination. A two-level SC model developed by Rahdar et al [4] with coordination methods for items of deterioration type. The authors considered a SC, comprising of a vendor and several buyers and made an assumption that the inventory items deteriorate over time and its inventory level reduces. Li [5] had made an attempt on a method of analytical for price analysis in multi-stage SC. A mathematical model for cost distribution analysis is proposed under stochastic environment for multi-stage SC network.

In this study, we made an attempt to develop a coordinated inventory model with a finite production rate for a two-echelon SC. Further, it is imagined that the supplier adopts credit period strategy to the buyer. Besides, the overall variable cost of SC is assumed to be a convex function of the cycle time. In addition, it is assumed that no



shortages are allowed. The objective of this present research is to show the optimal cycle time, inventory levels, and overall variable cost and number of shipments from the vendor to the buyer in order to minimize the total variable cost. The current model is illustrated with a numerical examples and sensitivity analysis is done with reference to the parameters of the model.

This research article is described as follows. Section 2 deals with the mathematical model development consisting of the assumptions and features, notations used. A numerical illustration is described in Section 3. Section 4 deals with the summary of the findings and further extension of the current work.

2. Mathematical Model Development

For the development of the present model, one product is shipped from one vendor to one buyer is considered for joint system.

2.1 Notations

λ	Buyer demand rate per annum (units/year)
C	Production rate (units)
ε_B	Ordering price of Buyer (in Rs. per order)
ε_v	Cost at the manufacturer per setup (in Rs. per batch)
β_B	Price of shipment incurred to buyer to receive goods from vendor (in Rs. per delivery)
β_v	Cost of shipment incurred to vendor to transport goods to buyer (in Rs. per shipment)
U_B	Buyer's cost per unit (in Rs./unit)
U_v	Vendor price per unit (in Rs./unit)
S_B	Buyer's selling price per unit (in Rs./unit)
W_e	Earned rate of interest (in Rs. per Re per year)
W_p	Paid rate of interest (in Rs./Re./year)
t	Allowable credit period
q	Replenishment quantity (in units)
η	Number of shipments (+ve integer, decision variable)
M	Cycle time length at buyer (in years, decision variable)
π_B	Overall variable cost of the buyer (in Rs.)
π_v	Overall variable cost of the vendor (in Rs.)
π_S	Overall variable cost of the SC (in Rs.)

2.2 Features and assumptions

- Deterministic rate of demand
- Replenishment rate is instantaneous
- Vendor's inventory level is numeral multiple of buyer's inventory level
- Trade credit is provided by vendor to the buyer
- Shortages are not considered
- Finite production rate

2.3 Model Formulation

2.3.1 Buyer Node

Case I: ($M \geq t$)

Annual variable cost of the buyer = Annual ordering cost + Annual carrying cost + Annual transportation cost + Interest amount paid per annum – Interest amount earned per annum.

$$\pi_B(\eta, M) = \frac{\varepsilon_B}{M} + \frac{\lambda M U_B W_p}{2\eta} + \frac{\beta_B \delta}{M} + \frac{U_B \lambda (M-t)^2 W_p}{2M} - \frac{S_B \lambda t^2 W_e}{2M} \quad (1)$$

Case II: ($M < t$)

Annual variable cost of the buyer = Annual ordering cost + Annual carrying cost + Annual transportation cost – Interest amount earned per annum.

$$\pi_B(\eta, M) = \frac{\varepsilon_B}{M} + \frac{\beta_B \delta}{M} + \frac{\lambda M U_B W_p}{2\eta} - S_B \lambda W_e \left(t - \frac{M}{2} \right) \quad (2)$$

2.3.2 Vendor Node

Annual variable cost of the vendor = Annual ordering/setup cost + Annual transportation cost+ Annual carrying cost

$$\pi_v(\eta, M) = \frac{\varepsilon_v}{M} + \frac{\beta_v \eta}{M} + \frac{\lambda M C_v W_p}{2} \left(1 - \frac{\lambda}{C} - \frac{1}{\eta} + \frac{2\lambda}{\eta C} \right) \quad (3)$$

2.3.3 Joint Supply Chain

Vendor & buyer jointly decided a combined policy for optimal inventory decision.

Case I: ($M \geq t$)

Annual variable cost of the SC = Annual variable cost of the buyer + annual variable cost of the vendor- Interest amount earned per annum. Upon simplification, the annual variable cost of the SC is expresses as:

$$\pi_S(\eta, M) = \frac{1}{M} (\varepsilon_B + \beta_B \eta + \varepsilon_v + \beta_v \eta) + \lambda W_p \left(\frac{U_B (M-t)^2}{2M} + \frac{M U_v}{2} \left(1 - \frac{\lambda}{P} - \frac{1}{\eta} + \frac{2\lambda}{\mu C} \right) + \frac{M U_B}{2\eta} \right) - \frac{S_B \lambda t^2 W_e}{2M} \quad (4)$$

Optimality Criteria:

For known value of η , π_S is convex in terms of M . The proof is as shown below.

$$\frac{\partial}{\partial M} (\pi_S(\eta, M)) = 0$$

$$\lambda W_p \left(U_B + U_v \left(1 - \frac{\lambda}{P} - \frac{1}{\eta} + \frac{2\lambda}{\eta C} \right) + \frac{U_B}{\eta} \right) = \frac{2}{M^2} \left\{ (\varepsilon_B + \beta_B \eta + \varepsilon_v + \beta_v \eta) + \left(\frac{\lambda t^2}{2} \right) (U_B W_p - S_B W_e) \right\} \quad (5)$$

$$\frac{\partial^2}{\partial M^2} (\pi_S(\eta, M)) = \frac{2}{M^3} (\varepsilon_B + \beta_B \eta + \varepsilon_v + \beta_v \eta) + \frac{\lambda t^2}{M^3} (U_B W_p - S_B W_e) \quad (6)$$

Since $\frac{\partial^2}{\partial M^2} (\pi_S(\eta, M)) > 0$ for all values of cycle time, rate of shipment and other model parameters, the cycle time becomes optimal. Optimal cycle time is derived from equation (5) as shown in equation (7).

$$M^* = \left(\left(2 \left(\varepsilon_B + \beta_B + \frac{\varepsilon_v}{\eta} + \beta_v \right) + 0.5 \lambda t^2 (U_B W_p - S_B W_e) \right) / \left(\lambda W_p \left(\left(U_B + U_v \left(1 - \frac{\lambda}{C} - \frac{1}{\eta} + \frac{2\lambda}{\eta P} \right) + \frac{U_B}{\eta} \right) \right) \right)^{0.5} \quad (7)$$

In the same way, for the known value of M , π_S is convex in terms of η . The inequality conditions for optimal η are derived and are expressed as shown below.

$$\pi_S(\eta^*) \leq \pi_S(\eta^* - 1) \quad \text{and} \quad \pi_S(\eta^*) \leq \pi_S(\eta^* + 1)$$

$$\eta^* (\eta^* - 1) \leq \left(\left(\lambda M^2 W_p \left(U_B + U_v \left(\frac{2\lambda}{C} - 1 \right) \right) \right) / (2(\beta_B + \beta_v)) \right) \leq \eta^* (\eta^* + 1) \quad (8)$$

Case II: ($M < t$)

Annual variable cost of the SC = Annual variable cost of the buyer + annual variable cost of the vendor- Interest amount earned per annum. Upon simplification, the annual variable cost of the SC is expresses as:

$$\pi_S(\eta, M) = \frac{1}{M} (\varepsilon_B + \beta_B \eta + \varepsilon_v + \beta_v \eta) + \frac{\lambda M W_p}{2} \left(\frac{U_B}{\eta} + U_v \left(1 - \frac{\lambda}{C} - \frac{1}{\eta} + \frac{2\lambda}{\eta P} \right) \right) - S_B \lambda W_e \left(t - \frac{M}{2} \right) \quad (9)$$

For known value of η , π_S is convex in terms of M . The proof is as shown below.

$$\frac{\partial}{\partial M} (\pi_S(\eta, M)) = 0$$

$$\frac{1}{M^2} (\varepsilon_B + \beta_B \eta + \varepsilon_v + \beta_v \eta) = \frac{\lambda W_p}{2} \left(\frac{U_B}{\mu} + U_v \left(1 - \frac{\lambda}{C} - \frac{1}{\eta} + \frac{2\lambda}{\eta C} \right) \right) + \left(\frac{S_B \lambda W_e}{2} \right) \quad (10)$$

$$\frac{\partial^2}{\partial M^2} (\pi_S(\eta, M)) = \frac{2}{M^3} (\varepsilon_B + \beta_B \eta + \varepsilon_v + \beta_v \eta)$$

Since $\frac{\partial^2}{\partial M^2}(\pi_S(\eta, M)) > 0$ for all values of cycle time, rate of shipment and other model parameters, the cycle time becomes optimal. Optimal cycle time is derived from equation (10) as shown in equation (11).

$$M^* = \left(\left(2 \left(\varepsilon_B + \beta_B + \frac{\varepsilon_v}{\eta} + \beta_v \right) \right) / \left(\lambda W_p \left(\frac{U_B}{\eta} + U_v \left(1 - \frac{\lambda}{C} - \frac{1}{\eta} + \frac{2\lambda}{\eta C} \right) \right) + S_B \lambda W_e \right) \right)^{0.5} \tag{11}$$

In the same way, for the known value of M , π_S is convex in terms of η . The inequality conditions for optimal η are derived and are expressed as shown below.

$$\pi_S(\eta^*) \leq \pi_S(\eta^* - 1) \quad \text{and} \quad \pi_S(\eta^*) \leq \pi_S(\eta^* + 1)$$

$$\eta^*(\eta^* - 1) \leq \left(\left(\lambda M^2 I_p \left(U_B + U_v \left(\frac{2\lambda}{C} - 1 \right) \right) \right) / \left(2(\beta_B + \beta_v) \right) \right) \leq \eta^*(\eta^* + 1) \tag{12}$$

Case III: ($t = M$)

If credit period and cycle time both are equal, then overall variable cost of the total SC is obtained as:

$$\pi_S(\eta, M) = \frac{1}{M}(\varepsilon_B + \beta_B \eta + \varepsilon_v + \beta_v \eta) + \frac{\lambda M W_p}{2} \left(\frac{U_B}{\eta} + U_v \left(1 - \frac{\lambda}{C} - \frac{1}{\eta} + \frac{2\lambda}{\eta C} \right) \right) - \left(\frac{S_B \lambda W_e M}{2} \right) \tag{13}$$

For known value of η , π_S is convex in terms of M . In the same way, for the known value of M , π_S is convex in terms of η . By using the similar procedure followed in Case I and Case II, the optimality criteria is derived for Case III.

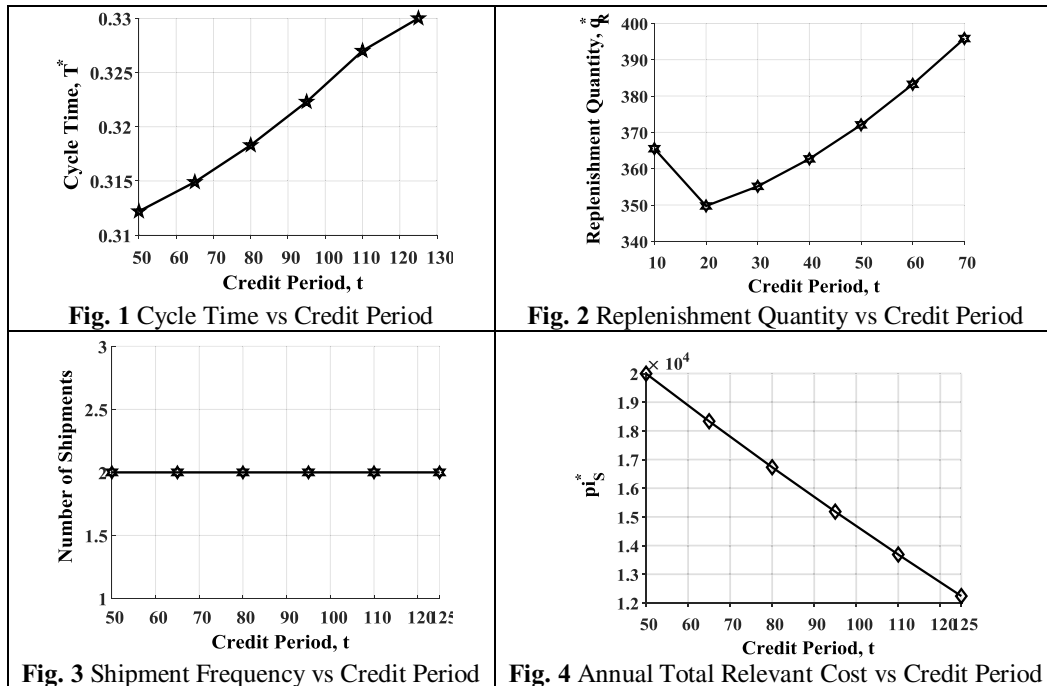
3. Numerical Investigation

Numerical data is set for illustrating the model and is solved using MATLAB program and the results are shown in Table 1. The values of various parameters for present inventory system are: $\varepsilon_v =$ INR 1600/setup, $\varepsilon_B =$ INR 400/order, $U_v =$ INR 100/unit, $U_B =$ INR 140 /unit, $S_B =$ INR 160 /unit, $\lambda =$ 1800 units /year, $C =$ 2000 units / year $W_p =$ 0.18 INR / INR /year, $W_e =$ 0.12 INR/ INR/year, $\beta_B =$ INR 750/shipment, $\beta_v =$ INR 250/shipment.

Table 1: Optimal Values of Decision Variables and Objective Function

Description	$t = 10$	$t = 20$	$t = 30$	$t = 40$	$t = 50$	$t = 60$
M^* (in Years)	0.1925	0.1943	0.1973	0.2015	0.2067	0.2129
q^* (in Units)	365.5	349.74	355.14	362.7	372.06	383.22
μ^* (Integer)	1.00	1.00	1.00	1.00	1.00	1.00
Q^* (in Units)	365.5	349.74	355.14	362.7	372.06	383.22
π_B^* (inRs.)	14136.02	12679.32	11440.55	10404.71	9553.14	8865.37
π_v^* (inRs.)	11253.71	11168.96	11033.31	10854.27	10640.91	10402.71
π_S^* (inRs.)	25389.73	23848.28	22473.86	21258.98	20194.05	19268.08

The optimal values of decision variables and objective function are shown in Table 1. From table 1, it is shown that under trade credit scenario, the buyer cost and vendor cost are decreased, with increase in the trade credit. The overall variable cost of the SC can be reduced with the effect of trade credit. The shipment quantities are not influenced with increment and decrement of trade credit. Table 1, also demonstrates the optimal cycle time, overall variable cost, replenishment quantities and rate of shipments from the vendor to the buyer. The cost of SC is decreased with increasing trade credit. The overall buyer cost is reduced with increasing trade credit. Similarly, the cost of vendor is also reduced by increasing trade credit. Finally, it is evident that the overall variable SC cost is reduced with increasing trade credit. From Figure1, the credit period is directly proportional to the cycle time. As the trade credit increases, the cycle time also increases. From Figure 2, the buyer replenishment quantity is suddenly decreased to certain period and then suddenly increases as credit period goes on increases. From Figure 3, as long as the credit period is increased or decreased, there will be no effect on the shipment quantity



Otherwise, simply the rate of shipment is remains constant with respect to credit period. Constant rate of shipment can be observed by increasing the credit period in this scenario. It is also observed that from Figure 4, the overall variable cost is reduced with increasing credit period. Further, the sensitivity analysis is carried out to analyze the variation in decision variables and objective function with respect to variation in model parameters. The results are tabulated in Table 2 and it is observed that variation in model parameters significantly influence the optimality of decision variables and objective function.

4. Conclusions

In this particular work, a mathematical model is developed for a two-echelon inventory system with a one kind of product supplying from one-vendor to the one-buyer under finite production rate with trade credit. In the development of the model, inventory related costs like ordering/setup costs, costs of carrying and transportation are considered. The optimality condition is established for the decision variables and objective function. MATLAB computer program is developed to solve the model, based upon on the optimality condition. From research findings, it is concluded that cycle time increases with increasing trade credit and no change in the shipment quantity. Also, it is concluded that the overall variable cost of the SC decreases with increasing credit period.

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