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Research paper

Kamenev-Type Oscillation Criteria for Generalized Second Order Sublinear α-Difference Equations

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Abstract

By means of Riccati transformation techniques, authors establish some new oscillation criteria for generalized second order nonlinear α -difference equation $\Delta_{\alpha(\ell)}(p(k\ell+j)(\Delta_{\alpha(\ell)}(u(k\ell+j)))^{\gamma})+q(k\ell+j)u^{\beta}((k-\sigma)\ell+j)=0$, when $0<\beta<1$ and γ are quotient of odd positive integers.

Keywords: *Delay; Sublinear; Superlinear.*

1. Introduction

Difference equations represent a fascinating mathematical area on its own as well as a rich field of the applications in such diverse disciplines. For general background as difference equations with many examples from diverse fields, one can refer to [1].

The theory of difference equations is based on the operator Δ defined as $\Delta u(k)=u(k+1)-u(k),\ k\in\mathrm{N}=\{0,1,2,\cdots\}$. Even though some authors [1] have suggested the definition of Δ as

$$\Delta u(k)=u(k+\ell)-u(k),\quad \ell\in(0,\infty), \tag{E}$$

no significant progress took place on this line. Jerzy Popenda, et.al., [16] defined Δ_{α} as $\Delta_{\alpha}u(k)=u(k+1)-\alpha u(k)$. Authors in [14], considering the operator Δ defined by (E) as Δ_{ℓ} many interesting results in number theory were obtained [14]. In [15], they generalized the definition of Δ_{α} by $\Delta_{\alpha(\ell)}$ defined as $\Delta_{\alpha(\ell)}u(k)=u(k+\ell)-\alpha u(k)$ for the real valued function $u(k)$ and $\ell\in(0,\infty)$ and also obtained the solutions of certain types of generalized α -difference equations.

In recent years, the asymptotic behaviour of second order difference equations has been the subject of investigations by many authors [2-12,17-19,21-22].

In this paper, we will be concerned with a class of generalized second order sublinear delay difference equations of the form

$$\Delta_{\alpha(\ell)}(p(k\ell+j)(\Delta_{\alpha(\ell)}(u(k\ell+j)))^{\gamma}) + q(k\ell+j)u^{\beta}((k-\sigma)\ell+j)=0, \tag{1}$$

where $\Delta_{\alpha(\ell)}$ denotes the forward difference operator for any real valued function $u(k\ell+j)$, $k\in(0,\infty)$, $\ell\in(0,\infty)$, $j=k-\left[\frac{k}{\ell}\right]\ell$, γ is quotient of odd positive integers, $0<\beta<1$ is quotient of odd positive integers, σ is a fixed nonnegative integer, $p(k\ell+j)>0$, and $q(k\ell+j)\geq 0$ are real valued functions, and for some $k_0>0$,

$$\sum_{k=k_0}^{\infty}\left(\frac{1}{p(k\ell+j)}\right)^{1/\gamma}=\infty, \tag{2}$$

By a solution of (1) we mean a nontrivial real valued function $u(k)$ defined for $k\geq-\sigma$, and satisfies equation (1) for $k\in(0,\infty)$. Clearly, if

$$u(k\ell+j)=A(k\ell+j)\text{ for }k\in[-\sigma,0] \tag{3}$$

are given, then equation (1) has a unique solution satisfying the initial condition (3).

2. Main Results

Theorem 1 *Assume that (2) holds. Furthermore, assume that there exist a positive real valued functions $\rho(k\ell+j)$ such that for every $\eta\geq 1$ and positive number M.*

$$\limsup_{k\to\infty}\sum_{r=0}^{k}\Bigg[\rho(r\ell+j)q(r\ell+j)-\alpha(p((r-\sigma)\ell+j))^{1/\gamma} \times\frac{\eta^{1-\beta}((r-\sigma+1)\ell+j)^{1-\beta}(\Delta_{\ell}\rho(r\ell+j))^{2}}{4\beta(M)^{(\gamma-1)/\gamma}\rho(r\ell+j)}\Bigg] \tag{4}$$

Then every solution of equation (1) oscillates.

Proof. Suppose to the contrary that $u(k\ell+j)$ is an eventually nonoscillatory solution of (1) such that $u((k-\sigma)\ell+j)>0$ for all $k \ge k_0 > 0$. We shall consider only this case, since the substitution $v(k\ell+j) = -u(k\ell+j)$ transforms equation (1) into an equation of the same form. From equation (1) we have for $k \ge k_0$

$$\Delta_{\alpha(\ell)}(p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma}) = -q(k\ell+j)\, u^{\beta}((k-\sigma)\ell+j) \le 0, \tag{5}$$

and so $p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma}$ is an eventually nonincreasing sequence. We first show that $p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma} \ge 0$ for $k \ge k_0$. In fact, if there exists an real $k_1 \ge k_0$ such that $p(k_1\ell+j)(\Delta_{\alpha(\ell)} u(k_1\ell+j))^{\gamma} = c < 0$, then (5) implies that $p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma} \le c$ for $k \ge k_1$ that is $\Delta_{\alpha(\ell)} u(k\ell+j) \le \left(c/p(k\ell+j)\right)^{1/\gamma}$ hence as $k \to \infty$ $u(k\ell+j) \le \alpha u(k_1\ell+j)$

$$+\sum_{r=k_1}^{k-1}\left[(\alpha-1)\,u(r\ell+j) + \left(\frac{1}{p(r\ell+j)}\right)^{1/\gamma}\right] \to -\infty \tag{6}$$

which contradicts the fact that $u(k\ell+j) > 0$ for $k \ge k_0$, then $p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma} \ge 0.$ Also we claim that $\Delta^2_{\alpha(\ell)} u(k\ell+j) \le 0.$ If not there exists $k_1 \ge k_0$ such that $\Delta^2_{\alpha(\ell)} u(k\ell+j) > 0$ for $k \ge k_1$ and this implies that $\Delta_{\alpha(\ell)} u((k+1)\ell+j) > \Delta_{\alpha(\ell)} u(k\ell+j),$ so that since $\Delta_{\alpha(\ell)} p(k\ell+j) \ge 0,$ $p((k+1)\ell+j)(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma}$ $> p((k+1)\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma}$ $\ge p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma}$ and this contradicts the fact that $p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma}$ is nonincreasing sequence, then $\Delta^2_{\alpha(\ell)} u(k\ell+j) \le 0,$ and therefore we have for $k \ge k_0$

$$u(k\ell+j) > 0, \Delta_{\alpha(\ell)} u(k\ell+j) \text{ and } \Delta^2_{\alpha(\ell)} u(k\ell+j) \le 0. \tag{7}$$

Define the sequence $z(k\ell+j)$ by

$$z(k\ell+j) = \alpha^{\left[\frac{k}{\ell}\right]-1} \frac{\rho(k\ell+j)\, p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma}}{u^{\beta}((k-\sigma)\ell+j)} \tag{8}$$

then $z(k\ell+j) > 0,$ and

$$\Delta_{\alpha(\ell)} z(k\ell+j) = \frac{\alpha^{\left[\frac{k}{\ell}\right]}\rho(k\ell+j)\,\Delta_{\ell}(p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma})}{u^{\beta}((k-\sigma)\ell+j)}$$

$$+\, p((k+1)\ell+j)(\Delta_{\alpha(\ell)} u((k+1)\ell+j))\Delta_{\alpha(\ell)}\left(\frac{\alpha^{\left[\frac{k}{\ell}\right]-1}\rho(k\ell+j)}{u^{\beta}((k-\sigma)\ell+j)}\right) \tag{9}$$

From (1) and (9), we have

$$\Delta_{\ell} z(k\ell+j) = -\rho(k\ell+j)q(k\ell+j) + \frac{\Delta_{\ell}\rho(k\ell+j)z((k+1)\ell+j)}{\rho((k+1)\ell+j)}$$

$$-\frac{\alpha^{\left[\frac{k}{\ell}\right]}\rho(k\ell+j)\,p((k+1)\ell+j)(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma}}{u^{\beta}((k-\sigma+1)\ell+j)} \tag{10}$$

$$\times\frac{\Delta_{\ell} u^{\beta}((k-\sigma)\ell+j)}{u^{\beta}((k-\sigma)\ell+j)}$$

From (5) and (7), we get

$$p((k-\sigma)\ell+j)(\Delta_{\ell} u((k-\sigma)\ell+j))^{\gamma}$$

$$\ge p((k+1)\ell+j)(\Delta_{\ell} u((k+1)\ell+j))^{\gamma}$$

$$\text{and } u((k+1-\sigma)\ell+j) \ge u((k-\sigma)\ell+j) \tag{11}$$

and then from (10) and (11), we have

$$\Delta_{\alpha(\ell)} z(k\ell+j) \le -\rho(k\ell+j)\,q(k\ell+j)$$

$$+\frac{\Delta_{\ell}\rho(k\ell+j)\,z((k+1)\ell+j)}{\rho((k+1)\ell+j)}$$

$$-\frac{\rho(k\ell+j)\,p((k+1)\ell+j)(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma}}{\left(u^{\beta}((k-\sigma+1)\ell+j)\right)^2} \tag{12}$$

$$\times\Delta_{\alpha(\ell)}\alpha^{\left[\frac{k}{\ell}\right]-1} u^{\beta}((k-\sigma)\ell+j)$$

Now, by using the inequality in [2], for all $u \ne v > 0$ and for $0 < \beta \le 1$, $u^{\beta} - v^{\beta} \ge \beta u^{\beta-1}(u-v)$. Then, we have

$$\Delta_{\alpha(\ell)}\alpha^{\left[\frac{k}{\ell}\right]-1} u^{\beta}((k-\sigma)\ell+j)$$

$$= \beta(u((k-\sigma+1)\ell+j))^{\beta-1}\Delta_{\alpha(\ell)}\alpha^{\left[\frac{k}{\ell}\right]-1} u((k-\sigma)\ell+j). \tag{13}$$

Substitute from (13) in (12), we have

$$\Delta_{\alpha(\ell)} z(k\ell+j) \le -\rho(k\ell+j)\,q(k\ell+j)$$

$$+\frac{\Delta_{\ell}\rho(k\ell+j)\,z((k+1)\ell+j)}{\rho((k+1)\ell+j)}$$

$$-\left(\frac{\rho(k\ell+j)\,p((k+1)\ell+j)\,\beta(u((k+1-\sigma)\ell+j))^{\beta-1}}{\left(u^{\beta}((k-\sigma+1)\ell+j)\right)^2}\right)$$

$$\times\Delta_{\alpha(\ell)} u((k-\sigma)\ell+j)(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma} \tag{14}$$

From (11) and (14), we have

$$\Delta_{\alpha(\ell)} z(k\ell+j) \le -\rho(k\ell+j)\,q(k\ell+j)$$

$$+\frac{\Delta_{\ell}\rho(k\ell+j)\,z((k+1)\ell+j)}{\rho((k+1)\ell+j)}$$

$$-\frac{\beta\rho(k\ell+j)(p((k+1)\ell+j))^{1/\gamma}\,p((k+1)\ell+j)}{(p((k-\sigma)\ell+j))^{1/\gamma}\left(u((k-\sigma+1)\ell+j)\right)^{1-\beta}}$$

$$\times\frac{(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma+1}}{\left(u^{\beta}((k-\sigma+1)\ell+j)\right)}$$

$$\Delta_{\alpha(\ell)} z(k\ell+j) \le -\rho(k\ell+j)\,q(k\ell+j)$$

$$+\frac{\Delta_{\ell}\rho(k\ell+j)\,z((k+1)\ell+j)}{\rho((k+1)\ell+j)}$$

$$-\left(\frac{\beta\rho(k\ell+j)(p((k+1)\ell+j))^{(1/\gamma)-1}(p((k+1)\ell+j))^2}{(p((k+1)\ell+j))^2(p((k-\sigma)\ell+j))^{1/\gamma}\left(u((k-\sigma+1)\ell+j)\right)^{1-\beta}}\right.$$

$$\times \frac{(\rho((k+1)\ell+j))^2(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{2\gamma}}{\left(u^{\beta}((k-\sigma+1)\ell+j)\right)^2(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma-1}}\Bigg) \quad (15)$$

From (7), we conclude that
$u(k\ell+j) \le \alpha u(k_0\ell+j)+\Delta_{\alpha(\ell)} u(k_0\ell+j)((k-k_0)\ell+j), k \ge k_0$
and consequently there exists a $k_1 \ge k_0$ and appropriate constant $\eta \ge 1$ such that $u(k\ell+j) \le \eta(k\ell+j)$ for $k \ge k_1$ and this implies that $u((k-\sigma v+1v)\ell v+j) \le \eta((k-\sigma+1)\ell+j)$ for $k \ge k_2 = k_1+\sigma-1$ and, hence

$$\frac{1}{(u((k-\sigma+1)\ell+j))^{1-\beta}} \ge \frac{1}{(\eta((k-\sigma+1)\ell+j))^{1-\beta}}. \quad (16)$$

Since $p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma}$ is a positive and increasing function, there exists a $k_2 \ge k_1$ sufficiently large such that $p(k\ell+j)(\Delta_{\alpha(\ell)} u(k\ell+j))^{\gamma} \le \frac{1}{M}$ for some positive constant M and $k \ge k_2$, and hence by (5) we have $p((k+1)\ell+j)(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma} \le \frac{1}{M}$, so that

$$\frac{1}{(\Delta_{\alpha(\ell)} u((k+1)\ell+j))^{\gamma-1}} \ge (Mp((k+1)\ell+j))^{(\gamma-1)/\gamma} \quad (17)$$

Then from (8), (15), (16) and (17) we have

$$\Delta_{\alpha(\ell)} z(k\ell+j) \le -\rho(k\ell+j)q(k\ell+j) + \frac{\Delta_{\ell}\rho(k\ell+j)z((k+1)\ell+j)}{\rho((k+1)\ell+j)} - \frac{\beta\rho(k\ell+j)M^{(\gamma-1)/\gamma}(z((k+1)\ell+j))^2}{(p((k+1)\ell+j))^2(p((k-\sigma)\ell+j))^{1/\gamma}\eta^{1-\beta}\left((k-\sigma+1)\ell+j\right)^{1-\beta}} \quad (18)$$

$$\Delta_{\alpha(\ell)} z(k\ell+j) \le -\rho(k\ell+j)q(k\ell+j) + \frac{(p((k-\sigma)\ell+j))^{1/\gamma}\eta^{1-\beta}((k-\sigma+1)\ell+j)^{1-\beta}(\Delta_{\ell}\rho(k\ell+j))^2}{4\beta(M)^{(\gamma-1)/\gamma}\rho(k\ell+j)}$$
$$-\Bigg[\frac{\sqrt{\beta(M)^{(\gamma-1)/\gamma}\rho(k\ell+j)}\,z((k+1)\ell+j)}{\rho((k+1)\ell+j)\sqrt{\eta((k-\sigma+1)\ell+j)^{1-\beta}p((k-\sigma)\ell+j)}}$$
$$\frac{\sqrt{\eta^{1-\beta}((k-\sigma+1)\ell+j)^{1-\beta}(p((k-\sigma)\ell+j))^{1/\gamma}}\,\Delta_{\ell}\rho(k\ell+j)}{2\sqrt{\beta(M)^{(\gamma-1)/\gamma}\rho(k\ell+j)}}\Bigg]^2$$
$$< -\Big[\rho(k\ell+j)q(k\ell+j) - \frac{(p((k-\sigma)\ell+j))^{1/\gamma}\eta^{1-\beta}((k-\sigma+1)\ell+j)^{1-\beta}(\Delta_{\ell}\rho(k\ell+j))^2}{4\beta(M)^{(\gamma-1)/\gamma}\rho(k\ell+j)}\Big]$$

$$\sum_{r=0}^{k}\Big[\rho(r\ell+j)q(r\ell+j) - \frac{(p((r-\sigma)\ell+j))^{1/\gamma}\eta^{1-\beta}((r-\sigma+1)\ell+j)^{1-\beta}(\Delta_{\ell}\rho(r\ell+j))^2}{4\beta(M)^{(\gamma-1)/\gamma}\rho(r\ell+j)}\Big] < c_1$$

for all large k, and this is contrary to (4). The proof is complete.

Theorem 2 *Assume that (2) holds. Let* $\rho(k\ell+j)$ *be a real valued function. Furthermore, we assume that there exists a double function* $H(m,k\ell+j); m \ge k \ge 0$ *such that* (*i*) $H(m,m)=0$ *for* $m \ge 0$, (*ii*) $H(m,k\ell+j)>0$ *for* $m > k\ell+j \ge 0$, (*iii*) $\Delta_{2\alpha(\ell)}H(m,k\ell+j) = H(m,(k+1)\ell+j)-\alpha H(m,k\ell+j) \le 0$ *for* $m \ge k\ell+j \ge 0$. *If*

$$\limsup_{m\to\infty}\frac{1}{H(m,0)}\sum_{k=k_0}^{m-1}\Big[H(m,k\ell+j)\rho(k\ell+j)q(k\ell+j) - \frac{(\rho((k+1)\ell+j))^2}{\overline{\rho(k\ell+j)}}\times\left(h(m,k\ell+j)-\frac{\Delta_{\ell}\rho(k\ell+j)}{\rho((k+1)\ell+j)}\sqrt{H(m,k\ell+j)}\right)^2\Big]=\infty,$$

where $h(m,k\ell+j) = -\frac{\Delta_{2(\ell)}H(m,k\ell+j)}{\sqrt{H(m,k\ell+j)}}$,

$$\overline{\rho(k\ell+j)} = \frac{\beta\rho(k\ell+j)M^{(\gamma-1)/\gamma}}{(p((k-\sigma)\ell+j))^{1/\gamma}\eta^{1-\beta}\left((k-\sigma+1)\ell+j\right)^{1-\beta}}$$

Then every solution of equation (1) oscillates.

Proof. We proceed as in Theorem 1, we assume that equation (1) has a nonoscillatory solution, say $u((k-\sigma)\ell+j)>0$ for all $k \ge k_0$. From (18) we have for $k \ge k_2$,

$$\rho(k\ell+j)q(k\ell+j) \le -\Delta_{\alpha(\ell)}z(k\ell+j) + \frac{\Delta_{\ell}\rho(k\ell+j)z((k+1)\ell+j)}{\rho((k+1)\ell+j)} - \frac{\overline{\rho(k\ell+j)}}{(\rho((k+1)\ell+j))^2}w^2(k+\ell).$$

$$\sum_{k=n}^{m-1}H(m,k\ell+j)\rho(k\ell+j)q(k\ell+j) \le H(m,k\ell+j)z(k\ell+j) + \sum_{k=n}^{m-1}z((k+1)\ell+j)\Delta_{2(\ell)}H(m,k\ell+j)$$
$$+\sum_{k=n}^{m-1}H(m,k\ell+j)\frac{\Delta_{\ell}\rho(k\ell+j)z((k+1)\ell+j)}{\rho((k+1)\ell+j)} - \sum_{k=n}^{m-1}\frac{H(m,k\ell+j)\overline{\rho(k\ell+j)}z^2((k+1)\ell+j)}{(\rho((k+1)\ell+j))^2}$$
$$= H(m,k\ell+j)z(k\ell+j) - \sum_{k=n}^{m-1}\Big[z((k+1)\ell+j) + \frac{\rho((k+1)\ell+j)}{2\sqrt{H(m,k\ell+j)\overline{\rho(k\ell+j)}}}$$
$$\times\left(h(m,k\ell+j)\sqrt{H(m,k\ell+j)} - \frac{\Delta_{\ell}\rho(k\ell+j)H(m,k\ell+j)}{\rho((k+1)\ell+j)}\right)\Big]^2$$
$$+\frac{1}{4}\sum_{k=n}^{m-1}\frac{(\rho((k+1)\ell+j))^2}{\overline{\rho(k\ell+j)}}\times\left(h(m,k\ell+j)-\frac{\Delta_{\ell}\rho(k\ell+j)\sqrt{H(m,k\ell+j)}}{\rho((k+1)\ell+j)}\right)^2.$$

$$\limsup_{m\to\infty}\frac{1}{H(m,0)}\sum_{k=0}^{m-1}\Big[H(m,k\ell+j)\rho(k\ell+j)q(k\ell+j) - \frac{(\rho((k+1)\ell+j))^2}{\overline{\rho(k\ell+j)}}\times\left(h(m,k\ell+j)-\frac{\Delta_{\ell}\rho(k\ell+j)\sqrt{H(m,k\ell+j)}}{\rho((k+1)\ell+j)}\right)^2\Big]<0,$$

which contradicts to our assumption. Hence the proof.

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