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ORIGINAL ARTICLE

# MHD Casson fluid in a suspension of convective conditions and cross diffusion across a surface of paraboloid of revolution

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**KEYWORDS**

Convective conditions;
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Magnetic field parameter;
Internal heat source parameter

**Abstract** In this paper, we analyzed the mathematical model of the convective conditions on Casson fluid across a Paraboloid of Revolution (PR) in the presence of pertinent parameters like magnetic field parameter, Soret, and Dufour. We used R-K-Felhberg-integration scheme based shooting technique to solve the modified governing equations and discussed the appearance of curious parameters on the profiles velocity, temperature and concentration profiles in three cases $n = 0$, $n = 0.5$ and $n = 1$ with the aid of plots. We found that the profiles of velocity, temperature, and concentration are non-uniform for all these three cases. Also, we have taken the help of tables to explore the skin friction coefficient, local Nusselt and Sherwood numbers, which are useful for the purpose of engineering interest. It is noticed that the cross diffusion helps to control the thermal, diffusion and momentum boundary layers for all the three cases. The mixed convective conditions are useful in improving the heat and mass transport phenomena. We also validated the present methodology with already existing methodologies under some limited cases.



## 1. Introduction

In general, some industries like paint, biological, chemical, food, polymer and pharmaceutical, handling flow behavior is not usual when compared to ordinary Newtonian fluids. We can call that behavior as non-Newtonian fluid. The performance of non-Newtonian flow is a well-known concept to wide-ranging fields in science and technology. For instance, we can observe the non-Newtonian behavior in the oil industry applications like hydraulic fracturing to enhance the production of oil from reservoirs with a low natural permeability. Initially, Rivlin [1] described the hydrodynamics non-Newtonian fluid by considering the steady-state laminar flow. Later, Gee and Lyon [2] suggested a mathematical model for the non-isothermal non-Newtonian fluid across circular channels. The Casson fluid is also one type a non-Newtonian fluid developed by the suspension of pigment oil in the base liquid. The

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**Nomenclature**

| | |
|---|---|
| $A, b$ | Coefficients related to stretching sheet |
| $n$ | Velocity power index parameter |
| $U_w$ | Velocity of the surface |
| $T_w$ | Temperature of the surface |
| $C_w$ | Concentration of the surface |
| $u, v$ | Velocity components in $x$ and $y$ directions |
| $x$ | Direction along the surface |
| $y$ | Direction normal to the surface |
| $g$ | Acceleration due to gravity |
| $B(x)$ | Dimensional magnetic field parameter |
| $T$ | Temperature of the fluid |
| $k$ | Thermal conductivity |
| $C_p$ | Specific heat capacity at constant pressure |
| $m$ | Intensity of the internal heat generation parameter |
| $D_m$ | Molecular diffusivity of the species concentration |
| $k_T$ | Thermal diffusion ratio |
| $C_s$ | Concentration susceptibility |
| $T_m$ | mean fluid temperature |
| $f$ | Dimensionless velocity |
| $Gr$ | Dimensionless buoyancy parameter |
| $M$ | Magnetic field parameter |
| $Bi_1, Bi_2$ | Biot numbers |
| $Ec$ | Eckert number |
| Pr | Prandtl number |
| $Du$ | Dufour number |
| $Sc$ | Schmidt number |
| $Sr$ | Soret number |
| $C$ | Concentration of the fluid |
| $T_\infty$ | Temperature of the fluid in the free stream |
| $C_\infty$ | Concentration of the fluid in the free stream |
| $C_{fx}$ | Local skin friction coefficient |
| $Nu_x$ | Local Nusselt number |
| $Sh_x$ | Local Sherwood number |
| $Re_x$ | Local Reynolds number |

*Greek symbols*

| | |
|---|---|
| $\beta_T$ | Volumetric coefficient of thermal expansion |
| $\sigma$ | Electrical conductivity of the fluid |
| $\rho$ | Density of the fluid |
| $h$ | Heat transfer coefficient |
| $\phi$ | Dimensionless concentration |
| $\zeta$ | Similarity variable |
| $\gamma$ | Space dependent internal heat source parameter |
| $\theta$ | Dimensionless temperature |
| $\mu$ | Dynamic viscosity |
| $\upsilon$ | Kinematic viscosity |

detailed description about Casson fluid is given by Annimasun [3]. Later on, Raju et al. [4] analyzed the comparative study of unsteady three-dimensional Casson and Carreau fluid flows. In this study, they highlighted that Casson has higher heat transfer rate compared to Carreau fluid. Afterwords, [5–10] considered the Casson fluid characteristics on various geometries and under various flow configurations. Raju et al. [11], Nadeem et al. [12] and Jayachandrababu et al. [13] analyzed the heat transfer in the non-Newtonian fluid across the stretching sheet by viewing parameters like Brownian motion and thermophoresis parameters. They observed that the decrement in the temperature distribution increases the thermophoresis and Brownian motion lessens the rate of heat transfer performance. With the help of homotopy analysis method Xu et al. [14] explained the stagnation point flow of the non-Newtonian fluids. Later on, Sajid et al. [15] made the relative study between HAM and HPM methods for the non-Newtonian fluid flow over a thin film and found that HAM is the better and simple method to guarantee the convergence of the solution series. Later, few others [16–18] discussed the non-Newtonian fluid flow over different geometries like pipe and rotating cone/plate (Figs. 1a and 1b).

In 1873, Dufour defined the diffusion-thermo or the Dufour effect as the energy flux induced by the species gradient. Later, Soret proposed the Soret effect which is named with his name and also called as a thermal-diffusion effect. This is defined as the mass flux induced by the temperature gradient. These two effects are found in several applications like chemical engineering, geosciences, and hydrology. Hayat et al. [19] described the flows in the presence of cross diffusion. Some of their findings are (a) Dufour number reduces the thickness of the concentration boundary layer (b) Soret number reduces

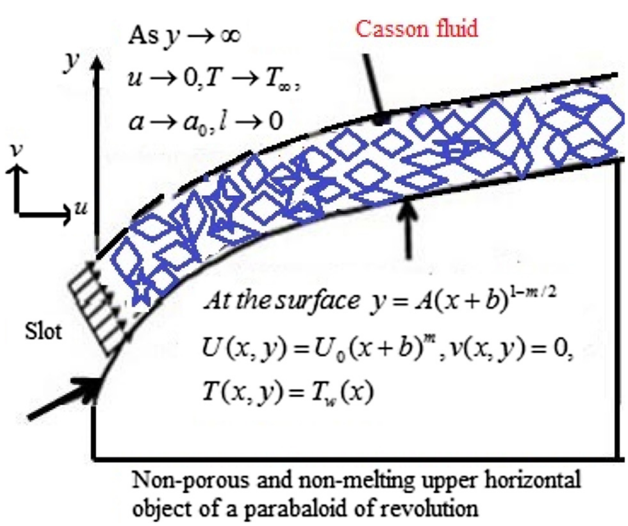


**Fig. 1a** The coordinate system of Casson fluid.

the velocity. Later on, by viewing into this the same parameters explained Pal and Mandal [20]. Hayat et al. [21] examined the mixed convective flows across the stretching sheet which is immersed in a porous medium. They observed that (a) Soret parameter raise the mass transfer (b) chemical reaction parameter reduce both the velocity and concentration (c) Schmidt number reduces the concentration. Later, Zheng et al. [22] provided HAM solution to the mathematical model which is formulated for the unsteady boundary layer flow across an oscillatory stretching surface. They found that the unsteadiness

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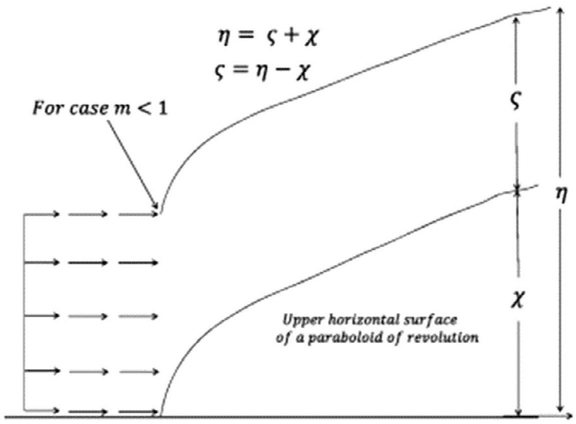


**Fig. 1b** Graphical design of fluid domain and change of the domain from $[\varsigma, \infty)$to$[0, \infty)$.

parameter lessens the velocity. After that, some authors [23–27] analyzed different flows over different domains which are immersed in a porous medium

The variable thickness with the upper half face of an object can be termed as a paraboloid of revolution. In 1972, Davis and Werle [28] derived a new technique to solve the laminar flow over a paraboloid of revolution. Recently, Makine and Animasaun [29] and Animasaun [30] made a contribution to the work of nanofluid over a paraboloid of revolution in the presence of some parameters like thermophoresis, Brownian motion, and quartic autocatalysis chemical reaction. Some of their findings are (a) velocity index parameter enhance the local skin friction coefficient and lessen the heat transfer rate (b) Brownian motion and thermophoresis parameters showed opposite behaviors on concentration (c) internal space dependent heat source parameter raise the velocity and temperature. Later on, Animasaun and Koriko [31] proposed new similarities for UHSPR with the existence of quadratic autocatalytic chemical reaction. The extended [31] is studied by Koriko et al. [32]. In this study, they incorporated Nano properties and analyzed the boundary layer theory. Very recently, Raju et al. [33] to improve the mass transfer profile consider the thermally radiated Casson fluid filled with gyrotactic organisms due to moving wedge. The magnetohydrodynamic also has most significant in industrial and biological processes such as activating the cell inside the body, material preparation processes, radiology treatment etc. By considering this application, the authors [34–37] studied the magnetohydrodynamic flow over various geometries such as a cone, plate, cylinder, and sheet. With their studies they highlighted that the magnetic field useful for controlling the boundary layer flow and heat transport phenomena.

The present study is an extension of the work of Makinde and Animasaun [23]. We have studied the convective conditions on Casson fluid flows across a paraboloid of revolution by considering the parameters like buoyancy, Soret and Dufour parameters. The derived non-linear ordinary differential equations (NODEs) are solved with the help of Runge-Kutta Felhberg integration scheme with help of shooting technique. The effects of aforesaid parameters along with other parameters (Casson fluid parameter, biot number and grashof number) on velocity, temperature, and concentration profiles are discussed with the aid of plots. The physical quantities of the local Nusselt and Sherwood numbers with skin friction coefficient are summarized in the tabular form for the same parameters.

## 2. Mathematical formulation

We considered a steady, laminar, two-dimensional and Magnetohydrodynamic flow across an upper paraboloid of revolution with the assumption that $y = A(x+b)^{(1-n)*0.5}$. Throughout this study, we consider the value of $n$ as less than 1 and $T_w > T_\infty$ to indicate that the flow across an upper horizontal surface (UHS) of a paraboloid of revolution. Also, we assumed that velocity, temperature and concentration of the surface as, $T_w(x) = A(x+b)^{(1-n)*0.5}$ and $C_w(x) = A(x+b)^{(1-n)*0.5}$.

The governing equations for the physical model (from [29;30]) are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \left(g\beta_T x\frac{n+1}{2}\right)(T - T_\infty) - \frac{\sigma(B(x))^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \left(1+\frac{1}{\beta}\right)\frac{\mu}{\rho c_p}\frac{\partial u}{\partial y}\frac{\partial u}{\partial y} + \frac{Q_0(T_w - T_\infty)}{\rho C_p}\exp\left(-my\sqrt{\frac{(n+1)U_0}{2\upsilon}}(x+b)^{(n-1)*0.5}\right) + \frac{D_m k_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} - D_m\frac{\partial^2 C}{\partial y^2} = \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

and the corresponding boundary conditions are

$$\left.\begin{array}{l} u = U_w, v = 0, k\frac{\partial T}{\partial y} = -h(T - T_w), \\ k\frac{\partial C}{\partial y} = -h\left(C - C_w\right) \text{ at } y = (x+b)^{0.5*(1-n)}A \\ u \to 0,\ T \to T_\infty\ ,\ C \to C_\infty \text{ as } y \to \infty. \end{array}\right\}. \tag{5}$$

where $\beta$ is Casson fluid parameter, ($\beta \to \infty$ is referred as a Newtonian fluid and $\beta \neq \infty$ is referred as non-Newtonian fluid) and $B(x) = B_0(x+b)^{(n-1)*0.5}$. With the aid of the below transformations, the Eqs. (2)-(4) transformed as the set of non-linear ordinary differential equations:

$$\left.\begin{array}{c} \zeta = y\sqrt{\frac{n+1}{2}}\sqrt{\frac{U_0}{\upsilon}}(x+b)^{0.5*(n-1)}, u = U_0\frac{\partial f}{\partial \zeta}(x+b)^n, \phi = \frac{C - C_\infty}{C_w(x) - C_\infty} \\ v = \left[-\sqrt{\frac{U_0\upsilon(n+1)}{2}}(x+b)^{0.5*(n-1)}f - yU_0(x+b)^{(n-1)}\left(\frac{n-1}{2}\right)\frac{\partial f}{\partial \zeta}\right], \theta = \frac{T - T_\infty}{T_w(x) - T_\infty}, \end{array}\right\} \tag{6}$$

With the help of (6), Eqs. (2)-(4) transmuted as the following equations:

$$\left(1+\frac{1}{\beta}\right)\frac{d^3 f}{d\zeta^3} + f\frac{d^2 f}{d\zeta^2} - \left(\frac{2n}{n+1}\right)\left(\frac{df}{d\zeta}\right) + Gr\theta - \left(\frac{2}{n+1}\right)M\frac{df}{d\zeta} = 0, \tag{7}$$

$$\frac{d^2\theta}{d\zeta^2}+\Pr f\frac{d\theta}{d\zeta}+\Pr Ec\frac{n+1}{2}\frac{d^2f}{d\zeta^2}\frac{d^2f}{d\zeta^2}\left(1+\frac{1}{\beta}\right)-\left(\frac{1-n}{n+1}\right)\Pr\frac{df}{d\zeta}\theta+\left(\frac{2}{n+1}\right)\Pr\delta\exp(-m\varsigma)+\Pr Du\frac{d^2\phi}{d\zeta^2}=0, \quad (8)$$

$$\frac{d^2\phi}{d\zeta^2}-Sc\left(\frac{1-n}{n+1}\frac{df}{d\zeta}\phi-f\frac{d\phi}{d\zeta}-Sr\frac{d^2\theta}{d\zeta^2}\right)=0, \quad (9)$$

and the corresponding boundary conditions (4) are transformed to

$$\left.\begin{array}{l} f(0)=\zeta\left(\frac{1-n}{1+n}\right),\left.\frac{df}{d\zeta}\right|_{\zeta=0}=1,\frac{d\theta}{d\zeta}(0)=-Bi_1(1-\theta(0)),\\ \frac{d\phi}{d\zeta}(0)=-Bi_2(1-\phi(0)),\\ f(\infty)=0,\theta(\infty)=0,\phi(\infty)=0 \end{array}\right\} \quad (10)$$

where $Gr$, $M$, $Bi_1$, $Bi_2$, Pr, $\delta$, $E_c$, $Du$, $Sc$ and $Sr$ are defined as Grashof number, magnetic field parameter, Biot numbers, Prandtl number, heat source or sink parameter, Eckert number, Dufour number, Schmidt number and Soret numbers respectively.

$$\left.\begin{array}{l} Gr=\frac{g\beta(T_w-T_\infty)}{U_0^2(x+b)^{2n-1}},M=\frac{\sigma B_0^2}{\rho U_0},\Pr=\frac{\mu C_p}{k},\\ Bi_1=\frac{h}{k}\sqrt{\frac{2\upsilon}{(m+1)U_0}},Sr=\frac{D_m k_T(T_w-T_\infty)}{\upsilon T_m(C_w-C_\infty)}\\ \delta=\frac{Q_0}{\rho C_p U_0},Du=\frac{D_m k_T(C_w-C_\infty)}{\upsilon C_s C_p(T_w-T_\infty)},Bi_2=\frac{h}{k}\sqrt{\frac{2\upsilon}{(m+1)U_0}},\\ E_c=\frac{U_0{}^2}{c_p(T_w-T_\infty)},Sc=\frac{\upsilon}{D_m}, \end{array}\right\} \quad (11)$$

The skin friction coefficient $C_{fx}$, the local Nusselt number $Nu_x$ and the local Sherwood number $\mathrm{Sh}_x$are specified as (after non-dimensionalization) the following:

$$\left.\begin{array}{l} C_{fx}=\left(1/\sqrt{Re_x}\right)\left.\frac{\partial^2 f}{\partial\zeta^2}\right|_{\zeta=0},\ Nu_x=-\left(\sqrt{Re_x}\right)\left.\frac{\partial\theta}{\partial\zeta}\right|_{\zeta=0}\\ \mathrm{Sh}_x=-\left(\sqrt{Re_x}\right)\left.\frac{\partial\phi}{\partial\zeta}\right|_{\zeta=0} \end{array}\right\} \quad (12)$$

## 3. Results and discussion

We solved the set of converted non-linear ordinary differential equations (NODEs) (7)-(9) with respect to the conditions (10) by using Runge-Kutta (R-K) - Felhberg method with shooting technique. We analyzed the effects of several parameters on velocity, temperature and concentration profiles with the considerations as $Sc=2,\Pr=2,n=0.3,M=0.5,Du=0.2,Sr=0.1,$ $\gamma=0.2,Bi_1=Bi_2=0.5,\zeta=10,Gr=2,\beta=0.2,Ec=0.3$. Plots are used to discuss the impact of various parameters like Casson fluid parameter, Dufour and Soret, Magnetic field, thermal Grashof number, heat source parameter, thermal and diffusion Biot numbers. The values of the skin friction coefficient $C_{fx}\left(1/\sqrt{Re_x}\right)$, local Nusselt and Sherwood numbers are summarized in the tabular form for the aforementioned parameters. The present study solid, dotted and dashed lines respectively indicate the $n=0, n=1$ and $n=0.5$ cases over paraboloid of revolution. Figs. 2 and 3 displays the effect of Casson fluid parameter on velocity $f'(\zeta)$ and temperature fields $\theta(\zeta)$ for $n=0, n=1$ and $n=0.5$ cases over a paraboloid of revolution. This result agrees well with the earlier works of Animasaun et al. [23]. The reason for this trend is when we incorporate the Casson fluid the higher viscous forces are functioning on the flow; this can support to depreciate the velocity

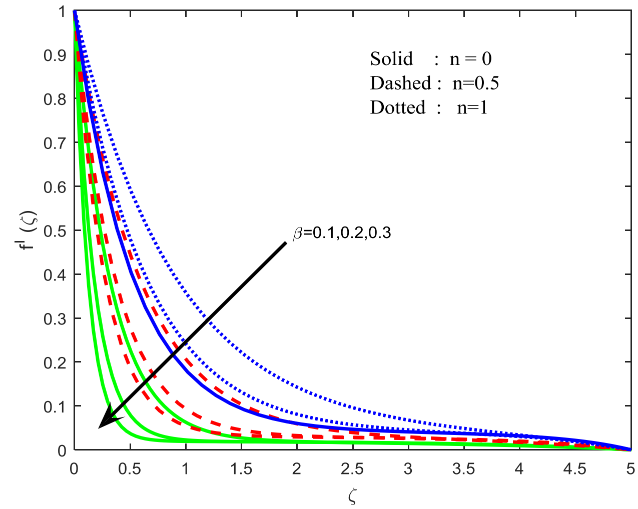


**Fig. 2** Velocity field for several values of $\beta$.

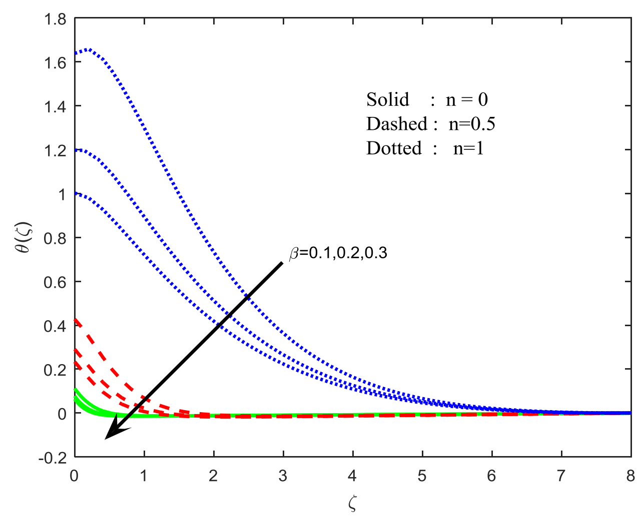


**Fig. 3** Temperature distribution for different values of $\beta$.

$f'(\zeta)$ as well as temperature field $\theta(\zeta)$. With the growing values of Eckert number boosting the temperature field for three cases $n=0$, $n=1$ and $n=0.5$ over paraboloid of revolution are plotted in Fig. 4. Generally, the boosting values of viscous dissipation parameter enhance the diffusion of the particles in the flow. This helps to encourage the temperature field.

Fig. 5 reported that the magnetic field parameter M decreases the velocity field for $n=0, n=1$ *and* $n=0.5$ cases over paraboloid of revolution. The reason behind this is the Lorentz force which is defined as the force exerted on a moving electric charge by a magnetic field. Figs. 6 and 7 represent the opposite behavior of buoyancy parameter $Gr$ on velocity and temperature profiles. It boosts up the velocity as well as the temperature. Generally, an increasing in buoyancy parameter enhances the domination of buoyancy forces in the flow, which can help us to enhance the more particle interaction, this leads to increases the temperature and velocity fields. The Figs. 8 and 9 show the effects of $Bi_1$ and $Bi_2$ on temperature field for $n=0$, $n=1$ and $n=0.5$ over paraboloid of revolution. Due to the domination of mixed convection, we saw enhancement

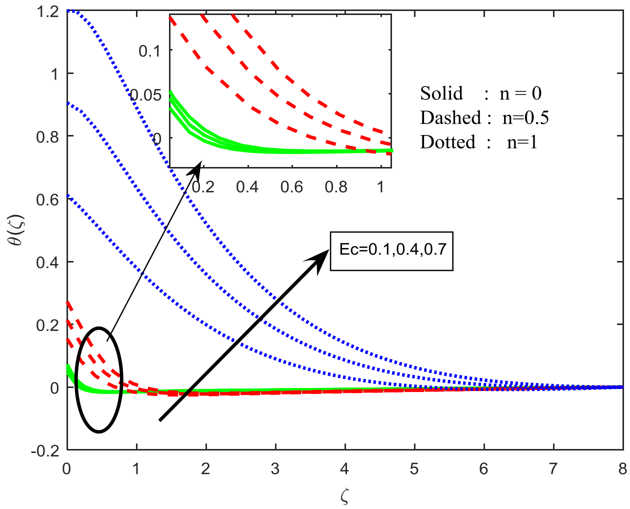


**Fig. 4** Temperature distribution for different values of *Ec*.

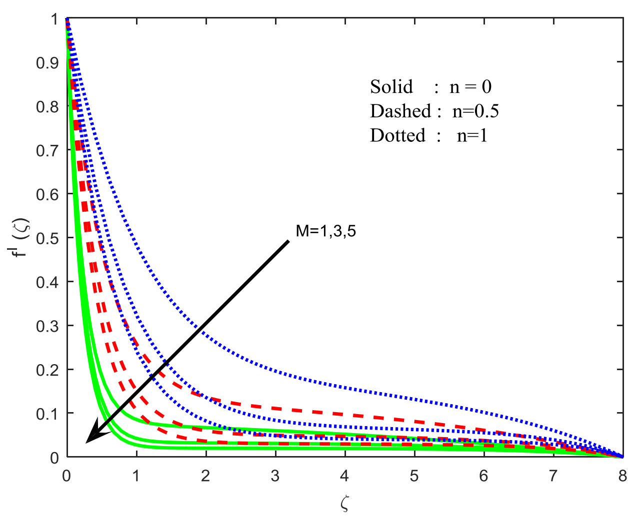


**Fig. 5** Velocity field for various values of *M*.

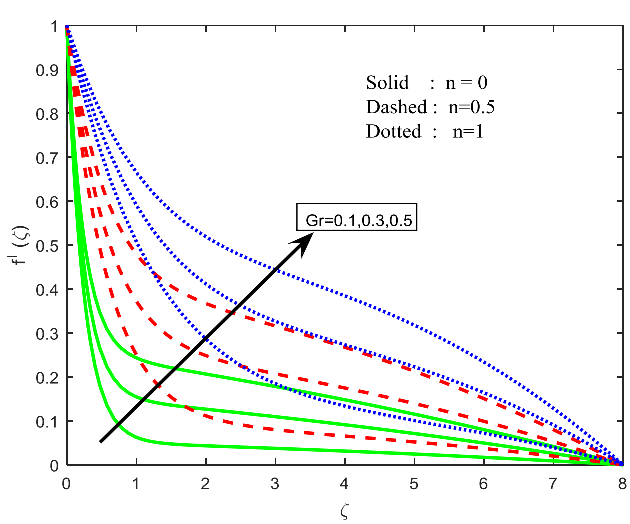


**Fig. 6** Velocity field for different values of *Gr*.

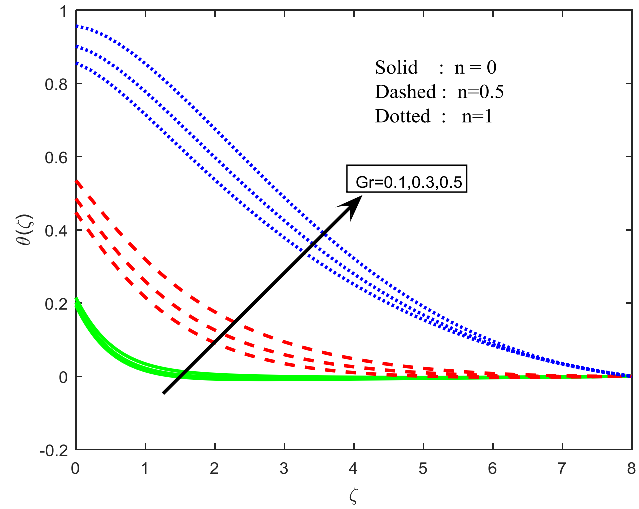


**Fig. 7** Temperature distribution for various values of *Gr*.

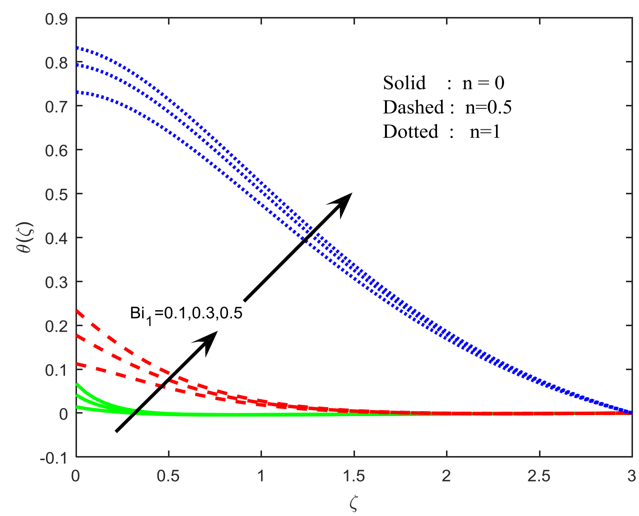


**Fig. 8** Temperature distribution for various values of $Bi_1$.

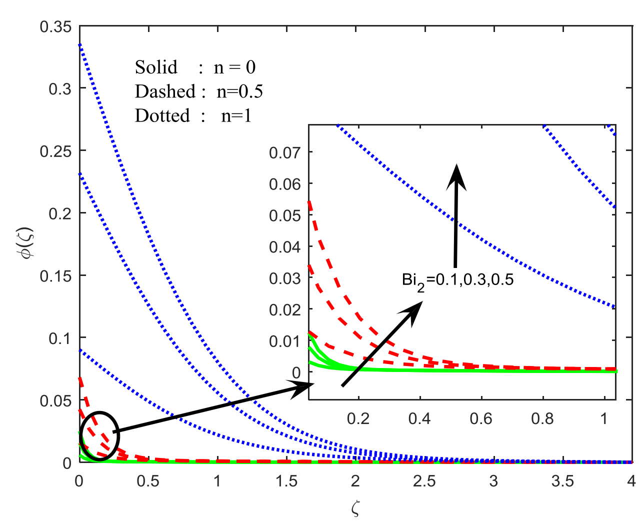


**Fig. 9** Temperature distribution for various values of $Bi_2$.

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in temperature field. We know that the boundary layer produces the heat energy with the rising values of internal heat source parameter. The dominance of mixed convection we have seen opposite sense in the flow. We can observe that behavior in Fig. 10. When Soret number $Sr$ improves the wideness of the temperature and decreases the concentration for $n = 0$, $n = 1$ and $n = 0.5$ over paraboloid of revolution. This result is pictorially shown in Figs. 11 and 12. It interesting to mention that the rising values velocity power-law index improves the pressure gradient on the flow due to this we have seen higher boundary in $n = 1$ case when compared with other two cases. Fig. 13 showed that temperature increase with the rise in the Dufour number $Du$. In fact, Dufour number raises the thickness of both the momentum and the thermal boundary layers.

We used Table 1 to explain the effects of aforesaid parameters on the local Nusselt $-Nu_x/(\sqrt{Re_x})$ and Sherwood numbers $-Sh_x/(\sqrt{Re_x})$ in three cases $n = 0$, $n = 1$ and $n = 0.5$ over paraboloid of revolution. In all the cases, parameters performed alike. The Casson fluid, magnetic field, thermal Biot number and heat source parameters are increases the local Nusselt number and depreciate the mass transfer rate. The thermal Grashof number, Eckert and Dufour numbers improve the mass transfer rate and reduce the heat transfer rate. It is found that the $n = 1$ case has lower mass transfer rate as compared with other two cases $n = 0$ and $n = 0.5$. But, the local Nusselt and Sherwood numbers are improved with rising values of diffusion Biot number. It is found from that $n = 0$ situation has greater heat transfer rate when equated with $n = 1$ and $n = 0.5$ case.

Table 2 in order to confirm the correctness of the current solution, the solutions of Runge-Kutta Felhberg method together with shooting technique is compared with that of the MATLAB default solver bvp5c solution under limited case when $Sc = 2, \Pr = 2, n = 0.3, M = 0.5, Du = 0.2, Sr = 0.1, \gamma = 0.2, Bi_1 = Bi_2 = 0.5, \zeta = 10, Gr = 2, \beta = 0.2, Ec = 0.3$ at various values of Eckert number $0 \leqslant Ec \leqslant 1$. As displayed in Table 2 the comparison of the aforesaid mentioned case is found to be in worthy agreement. This agreement is an inspiration for further analysis.

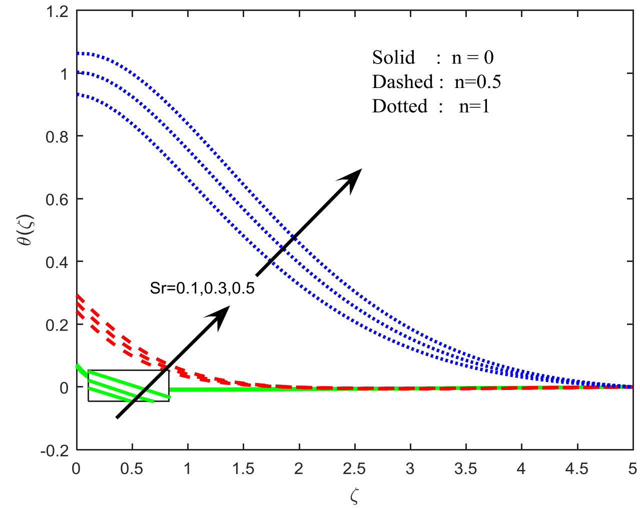


**Fig. 11** Temperature distribution for various values of $Sr$.

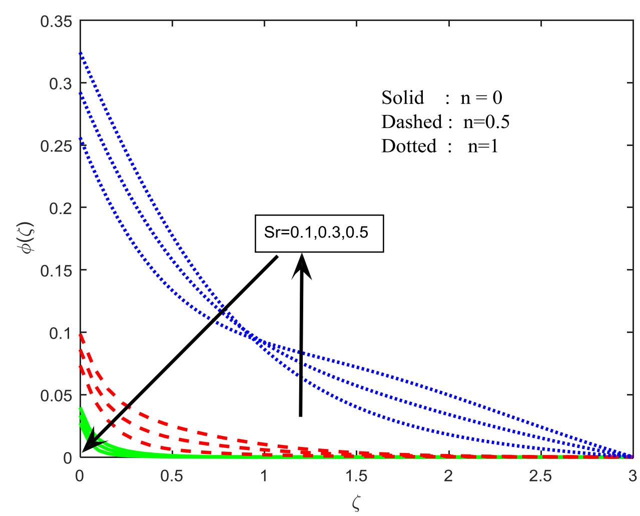


**Fig. 12** Concentration distribution for various values of $Sr$.

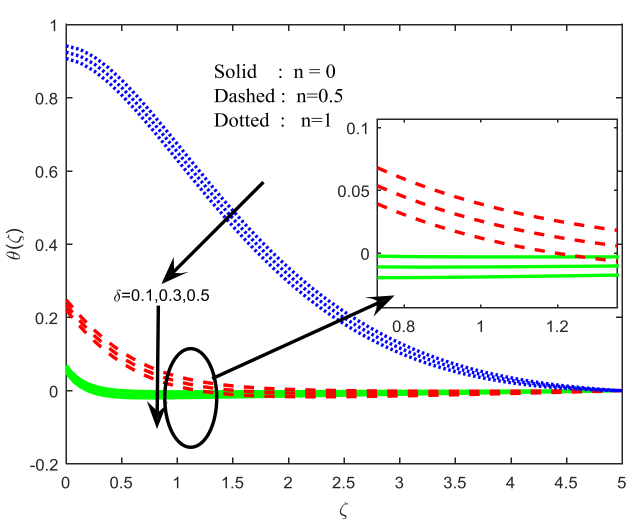


**Fig. 10** Temperature distribution for various values of $\delta$.

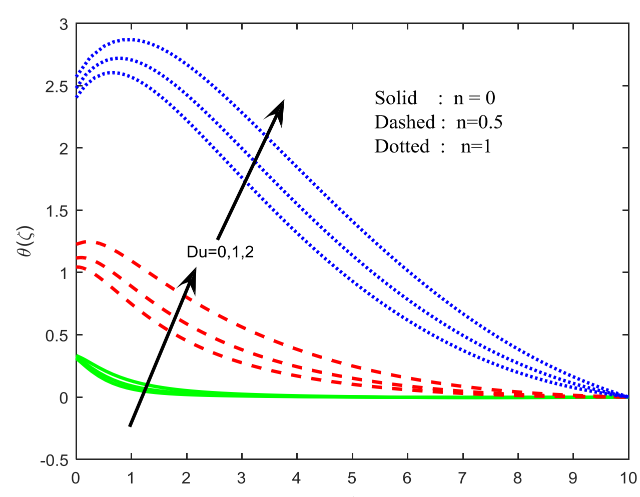


**Fig. 13** Temperature distribution for various values of $Du$.

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**Table 1** The rate of heat and mass transfer for various values of non-dimensional governing parameters.

| $\beta$ | $Du$ | $Ec$ | $M$ | $Gr$ | $Bi_2$ | $Bi_1$ | $Sr$ | $\delta$ | Reduced Nusselt number | | | Reduced Sherwood number | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | *n = 0 case* | *n = 0.5 case* | *n = 1 case* | *n = 0 case* | *n = 0.5 case* | *n = 1 case* |
| 0.1 | | | | | | | | | 0.426946 | 0.201778 | −0.842668 | 0.48750 | 0.469303 | 0.436284 |
| 0.2 | | | | | | | | | 0.460912 | 0.353020 | −0.231069 | 0.48726 | 0.466210 | 0.356760 |
| 0.3 | | | | | | | | | 0.468275 | 0.389647 | −0.061552 | 0.48720 | 0.465396 | 0.327765 |
| | 0 | | | | | | | | 0.344689 | −0.021882 | −0.702229 | 0.49442 | 0.486314 | 0.411944 |
| | 1 | | | | | | | | 0.340238 | −0.058032 | −0.739963 | 0.49442 | 0.486472 | 0.411973 |
| | 2 | | | | | | | | 0.332715 | −0.113046 | −0.785453 | 0.49444 | 0.486728 | 0.412121 |
| | | 0.1 | | | | | | | 0.475731 | 0.423125 | 0.193964 | 0.40106 | 0.298812 | 0.139059 |
| | | 0.4 | | | | | | | 0.469763 | 0.393485 | 0.047226 | 0.40107 | 0.298989 | 0.140838 |
| | | 0.7 | | | | | | | 0.463795 | 0.363845 | −0.099512 | 0.40109 | 0.299166 | 0.142617 |
| | | | 1 | | | | | | 0.402138 | 0.274532 | 0.074260 | 0.48752 | 0.465929 | 0.330025 |
| | | | 3 | | | | | | 0.403693 | 0.284193 | 0.092671 | 0.48751 | 0.465801 | 0.319009 |
| | | | 5 | | | | | | 0.404284 | 0.287370 | 0.097861 | 0.48751 | 0.465732 | 0.310430 |
| | | | | 0.1 | | | | | 0.402818 | 0.276158 | 0.071936 | 0.48752 | 0.465929 | 0.331541 |
| | | | | 0.3 | | | | | 0.398904 | 0.257416 | 0.049044 | 0.48753 | 0.466057 | 0.335279 |
| | | | | 0.5 | | | | | 0.392541 | 0.232539 | 0.021606 | 0.48754 | 0.466191 | 0.338228 |
| | | | | | 0.1 | | | | 0.396600 | 0.273364 | 0.117179 | 0.09943 | 0.098465 | 0.090985 |
| | | | | | 0.3 | | | | 0.398178 | 0.276333 | 0.119660 | 0.29537 | 0.287269 | 0.230439 |
| | | | | | 0.5 | | | | 0.399726 | 0.279143 | 0.121473 | 0.48752 | 0.465965 | 0.332303 |
| | | | | | | 0.1 | | | 0.098578 | 0.088777 | 0.026861 | 0.48757 | 0.465845 | 0.338870 |
| | | | | | | 0.3 | | | 0.287556 | 0.246763 | 0.061924 | 0.48693 | 0.464475 | 0.338232 |
| | | | | | | 0.5 | | | 0.466364 | 0.383125 | 0.083804 | 0.48634 | 0.463293 | 0.337834 |
| | | | | | | | 0.1 | | 0.466364 | 0.383125 | 0.083804 | 0.48634 | 0.463293 | 0.337834 |
| | | | | | | | 0.3 | | 0.465708 | 0.379502 | 0.076009 | 0.48322 | 0.456891 | 0.353817 |
| | | | | | | | 0.5 | | 0.464966 | 0.375407 | 0.067678 | 0.48012 | 0.450674 | 0.371965 |
| | | | | | | | | 0.1 | 0.465216 | 0.375704 | 0.029131 | 0.48635 | 0.463410 | 0.339019 |
| | | | | | | | | 0.3 | 0.469870 | 0.382647 | 0.037727 | 0.48633 | 0.463321 | 0.338606 |
| | | | | | | | | 0.5 | 0.474525 | 0.389589 | 0.046324 | 0.48630 | 0.463231 | 0.338193 |

**Table 2** Confirmation of numerical method: Comparison between the solution of Classical Runge-Kutta-Felhberg method (RKFSM) and MATLAB solver bvp5c under some the limiting case ($Sr = Du = \varsigma = 0$).

| $Ec$ | $-\theta'(0)$ | | $-\theta'(0)$ | | $-\theta'(0)$ | |
|---|---|---|---|---|---|---|
| | *n = 0* | | *n = 0.5* | | *n = 1* | |
| | RKFSM($\varsigma = 0$) | bvp5c ($\varsigma = 0$) | RKFSM($\varsigma = 0$) | bvp5c ($\varsigma = 0$) | RKFSM($\varsigma = 0$) | bvp5c ($\varsigma = 0$) |
| 0.1 | 0.885768 | 0.885771 | 0.420154 | 0.420155 | 0.346891 | 0.34681002 |
| 0.4 | 0.273739 | 0.273750 | 0.115342 | 0.115360 | 0.102068 | 0.1020801 |
| 0.7 | −0.338290 | −0.33821 | −0.189470 | −0.189450 | −0.142754 | −0.142864 |

## 4. Conclusions

In this study we considered a boundary layer investigation of convective conditions on Casson fluid across the paraboloid of revolution. We presented the solutions for important parameters like Biot number, magnetic field parameter, Soret and Dufour numbers and also we explained the physical effect of these parameters skin friction coefficient, local Nusselt and Sherwood numbers with aid of plots and tables. The main list of the findings are the following:

- $M$ and $\beta$ are minimizes the momentum boundary layer.
- $Gr, Du, Sr, Ec, Bi_1, Bi_2$ are enhances the thermal boundary layer.
- The heat and mass transfer rate are higher in $n = 0$ when compared with $n = 1$ and $n = 0.5$.
- The heat source parameter improves the local Nusselt number and minimizes the local Sherwood number for $n = 1$, $n = 0$, and $n = 0.5$.

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