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MHD Flow of a Nanofluid Embedded with Dust Particles Due to Cone with Volume Fraction of Dust and Nano Particles

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Abstract

This paper devotedly analyze the momentum and heat transfer behaviour of MHD nanofluid flow embedded with the conducting dust particles past a cone in the presence of non-uniform heat source/sink, volume fractions of dust and nano particles. The governing partial differential equations of the flow and heat transfer are transformed into nonlinear ordinary differential equations by using self similarity transformation and solved numerically. The effects of various non-dimensional governing parameters on velocity and temperature profiles are discussed with the help of graphs. Further, the effects of these parameters on skin friction coefficient and Nusselt number are also discussed and presented through tables. Moreover it is found that an increase in the mass concentration of dust particles depreciates the velocity profiles of the fluid and dust phases.

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1. Introduction

The suspension of nano sized metallic or non-metallic particles in the base fluids give the nanofluid. Generally, the base fluids like water, kerosene and ethylene glycol have low thermal conductivity. But the suspension of above mentioned metals or non-metals in the form of nano size, helps to enhance the thermal conductivity of the

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basefluids. Masuda et al. [1] was one of the researchers to observe an enhancement in the thermal conductivity of the base fluid by the dispersion of ultra-fine particles in the base fluids. Further, the researchers like Awad et al. [2] and Raju et al. [3] have investigated the effects of different physical parameters on the flow of nanofluids over different channels with the help of their investigations. The dusty fluids have wide range of industrial applications as well as in science and technology. So, several researchers analyzed the flow characteristics of dusty fluids. Among them Ramana Reddy et al. [4] is one to analyze the heat generation or absorption effects on dusty viscous flow under the influence of aligned magnetic field. Giressha et al. [5] discussed the MHD heat transfer effects on dusty fluid flow over a stretching sheet. The combined heat and mass transfer flow along with MHD effects have tremendous applications in power transformer electronics solar energy systems etc. Owing to this fact Mohan Krishna et al. [6] analyzed the influence of radiation and chemical reaction on MHD convective flow with suction and heat generation effects. The flow of viscous Ag-water and Cu-water nanofluids past a stretching surface has been analyzed by Vajravelu et al. [7]. On the other hand the flow of Jeffrey nanofluid with nano particles was analyzed by Hayat et al. [8]. To understand the mechanism of convective heat transfer, it is important to study the flow behaviour past an axisymmetric structure such as vertical and horizontal cylinders along with the Cone and spheres. So, Roy et al. [9], Khan and Sultan [10] concentrated their research on the flow of a fluid through different channels with axisymmetric structures. Nadeem and Saleem [11] discussed the unsteady Eyring Powell nanofluid flow in a rotating cone. Since the both the nano and dusty fluids are useful to enhance the thermal conductivity, Sandeep and Sulochana [12] studied the MHD flow of dusty nanofluids over a stretching surface. By making use of all the above cited articles, we make an attempt to analyze the heat transfer in MHD dusty nanofluids past a cone under the influence of non uniform heat source/sink along with the volume fractions of nano and dust particles. The reduced governing equations of the flow are solved numerically. Further, the effects of various parameters involved in the governing equations are discussed through graphs and tables, which are drawn with the help of MATLAB package

2. Mathematical Analysis

Consider a steady, laminar, incompressible boundary layer flow of a dusty nanofluid over a vertical cone pointing downward with half angle γ and radius r . The x -axis varies along the surface of the cone and y -axis is normal to the surface of the cone. The origin is located at the vertex of the cone. The flow field is exposed by the influence of external magnetic field strength B_0 along x -axis. The temperature distribution T_w will vary along the surface of the cone. The fluid has a uniform ambient temperature T_∞ . The Boundary layer equations, which govern the flow of the present problem per the above made assumptions, are given by (See Khan and Sultan [10], Sandeep and Sulochana [12])

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \quad (1)$$

$$\frac{\partial(ru_p)}{\partial x} + \frac{\partial(rv_p)}{\partial y} = 0, \quad (2)$$

$$\rho_{nf}(1-\phi_d) \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = (1-\phi_d) \left(\mu_{nf} \frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf}(T-T_\infty)\cos\gamma \right) + KN(u_p - u) - \sigma B^2 u, \quad (3)$$

$$Nm \left(u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) = KN(u - u_p), \quad (4)$$

$$(\rho c_p)_{nf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{N_1}{\tau_v} (u_p - u)^2 + q''' \quad (5)$$

The appropriate boundary conditions of the problem are given by

$$\left. \begin{aligned} u = U_w(x) = \frac{v_f}{x}(Gr)^{\frac{1}{2}}, v = 0, T = T_w(x) = ax, & \quad \text{at } y = 0, \\ u \rightarrow 0, u_p \rightarrow 0, v_p \rightarrow v, T \rightarrow T_\infty & \quad \text{as } y \rightarrow \infty, \end{aligned} \right\} \tag{6}$$

Where u, v are the velocities of the nanofluid along the x and y directions respectively. Similarly u_p, v_p stands for the velocities of dust phase along the x and y directions respectively. $r = x \sin \gamma$ is the local radius of the cone, a is the constant, ϕ_d is the volume fraction of the dust particles, g is the acceleration due to gravity, ρ_{nf} is the density of the nanofluid, β_{nf} is the coefficient of thermal expansion of the nanofluid due to temperature difference, T is the temperature, T_w is the temperature of the fluid far away from the surface of the cone, T_∞ ambient temperature of the fluid. K is the stokes resistance, N is the number density of dust particles, σ is the electrical conductivity of the fluid, m is the mass of the dust particle, $(\rho C_p)_{nf}$ is the heat capacitance of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, $N_1 = Nm$ is the density of the particle phase, τ_v is the relaxation time of the dust particle.

The space and temperature dependent heat generation/absorption (non uniform heat source/sink) q''' is defined as (See Pal [13]),

$$q''' = \left(\frac{k_f U_w(x, t)}{x v_f} \right) \left(A^* (T_w - T_\infty) f' + B^* (T - T_\infty) \right), \tag{7}$$

In Eqn. (7), A^* and B^* are parameters of the space and temperature dependent internal heat generation/absorption.

The nanofluid constants are given by (See Kalidas [14], Raju et al. [3])

$$\left. \begin{aligned} (\rho\beta)_{nf} &= (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s, \quad (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \\ \frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \end{aligned} \right\} \tag{8}$$

Now, we introduce the following similarity transformations to make the governing Eqs. (1) to (5) subject to the boundary conditions represented in Eq. (6), into coupled non linear ordinary differential equations.

$$\left. \begin{aligned} u = \frac{v_f}{x}(Gr)^{\frac{1}{2}} f'(\eta), v = \frac{v_f}{x}(Gr)^{\frac{1}{4}} \left(\frac{\eta}{4} f'(\eta) - \frac{1}{2} f(\eta) \right), \eta = \frac{y}{x}(Gr)^{\frac{1}{4}}, u_p = \frac{v_f}{x}(Gr)^{\frac{1}{2}} F'(\eta), \\ v_p = \frac{v_f}{x}(Gr)^{\frac{1}{4}} \left(\frac{\eta}{4} F'(\eta) - \frac{1}{2} F(\eta) \right), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{\beta_f g x^3 \cos \gamma (T_w - T_\infty)}{v_f^2}, \end{aligned} \right\} \tag{9}$$

Now define the dimensionless stream function by $ru = \partial \psi / \partial y$ and $rv = -\partial \psi / \partial x$,

Where $\psi = vx \sin \gamma (Gr)^{1/4} f(\eta)$.

Using the equations (7)-(9), then Eqs. (3) to (6) will reduce the following form, and Eqs. (1) and (2) identically satisfies.

$$\begin{aligned} f''' - (1-\phi)^{2.5} (1-\phi + \phi(\rho_s / \rho_f)) \left(\frac{f'^2}{2} - \frac{ff''}{2} \right) + (1-\phi + \phi((\rho\beta)_s / (\rho\beta)_f)) \theta + \\ + \frac{(1-\phi)^{2.5}}{(1-\phi_d)} (\Lambda \alpha \beta (f' - F') - M f'') = 0, \end{aligned} \tag{10}$$

$$\left(\frac{F'^2}{2} - \frac{FF''}{2}\right) - \beta(f' - F') = 0, \tag{11}$$

$$\frac{1}{\text{Pr}} \frac{1}{(1 - \phi + \phi(\rho_{c_p})_s / (\rho_{c_p})_f)} \left[\frac{k_{nf}}{k_f} \theta'' + \text{Pr} \alpha \beta \text{Ec} (F' - f')^2 + (A^* f' + B^* \theta) \right] + \frac{1}{2} f \theta' = 0, \tag{12}$$

Subject to boundary conditions

$$\left. \begin{aligned} f(\eta) = S, f'(\eta) = 1, \theta(\eta) = 1, & \quad \text{at } \eta = 0, \\ f'(\eta) = 0, F'(\eta) = 0, F(\eta) = f(\eta), \theta(\eta) = 0, & \quad \text{as } \eta \rightarrow \infty, \end{aligned} \right\} \tag{13}$$

where α is the mass concentration of the dust particles, β is the fluid particle interaction parameter for the velocity with τ_v , Λ is the fluid parameter, M is the magnetic field parameter, Pr is the Prandtl number, Ec is the Eckert Number, which are given by

$$\alpha = (Nm) / \rho_f, \beta = 1 / \tau_v, \tau_v = m / K, \Lambda = (x^2 \text{Re}) / (v_f Gr), M = (\sigma B_0^2 x^2) / \rho_f v_f (Gr)^{1/2}, \text{Pr} = v_f / k_f, \text{Ec} = (v_f (Gr)^{1/2}) / (C_p)_f$$

For engineering interest the local skin friction coefficient C_f and local Nusselt number Nu_x are given by

$$C_f \text{Re}_x^{1/2} = f''(0), Nu_x \text{Re}_x^{-1/2} = -\theta'(0), \tag{14}$$

Where Re_x is the local Reynolds number defined by $\text{Re}_x = \frac{xU_w(x)}{v_f}$.

3. Results and discussion

In order to solve the coupled non-linear ordinary differential Eqs. (10) to (12) with the help of boundary conditions of the flow represented by Eq. (13), we make use of Runge-Kutta and Newton’s method presented by Mallikarjuna et al. [15]. Further the effects of various parameters on velocity and temperature distribution involved in the Eqs. (10) to (12) are studied through graphs. For numerical results we have considered $M = 1, \phi_d = 0.1, \alpha = 0.2, \beta = 0.5, \text{Pr} = 6.2, \text{Ec} = 0.2, \phi = 0.15, A^* = B^* = 0.1$.

Table 1 Thermophysical properties of water, Cu and Al₂O₃ nano particles.

	ρ (Kg m ⁻³)	c_p (J Kg ⁻¹ K ⁻¹)	k (W m ⁻¹ K ⁻¹)	β (10 ⁻⁵ K ⁻¹)
H ₂ O	997.1	4179	0.613	21
Cu	8933	385	401	1.67
Al ₂ O ₃	3970	765	46	0.63

The influence of magnetic field parameter (M) can be seen in Fig. 1. It is observed that an increase in the magnetic field parameter reduces the velocity profiles of the both fluid and dust phases. This is due to the fact that an increase in the magnetic field parameter develops the opposite force to the flow, is called the Lorentz force.

Figs. 2 and 3 illustrate the effect of volume fraction of nano particles (ϕ) on velocity and temperature profiles respectively. It is observed that an increase in the volume fraction of the nano particles helps to improve the velocity of the fluid as well as dust phases over the boundary layer of the cone. But reverse action takes place in

temperature profiles due to the fact that improvement in the velocity profiles enhances the thermal conductivity.

Fig. 4 depicts the velocity profiles of the fluid and dust phases for different values of volume fraction of dust particles. It is noticed that an increase in the volume fraction of dust particles reduces the velocity profiles for both fluid and dust phases, because the volume occupied by the dust particles per unit volume of the fluid is high then the dust concentration in the fluid will increase.

It is observed from Fig. 5 that the velocity profiles of fluid and dust phases are reduced with an enhancement in α . But from Fig. 6, we noticed an opposite result in temperature profiles. Fig. 7 depicts the velocity profiles of fluid and dust phases for different values of fluid-particle interaction parameter (β). We have observed an interesting result that an increase in the fluid-particle interaction parameter enhances the velocity of dust phase, but depreciates the velocity of the fluid phase.

Fig. 8 shows the effect of space dependent non-uniform heat source/sink parameter on temperature profiles. It is evident that increase in A^* causes an increase in the temperature field. The reason behind this is the thermal boundary layer generates the energy for positive values of A^* .

Table 2 shows the validation of the present results with the existed literature under some special limited cases. We found an excellent agreement of the present results with the existed results. This proves the validity of the present results along with the accuracy of the numerical technique we used in this study. It can be seen from Table 3 that an enhancement in the physical parameters M, ϕ_d, α, β depreciates the profiles of Skin-friction coefficient, Nusselt and Sherwood numbers.

Table 2 comparison of the present results for $n = 1$ (power of f' in present problem it is 1), when $\phi = \phi_d = A^* = B^* = \alpha = \beta = 0$.

Grosan et al.[16]	Yih [17]	Present Results
0.7686	0.7686	0.768613

Table 3. Variation in friction factor, local Nusselt and Sherwood numbers.

M	ϕ	ϕ_d	α	β	A^*	$Cu - H_2O$		$Al_2O_3 - H_2O$	
						C_f	Nu_x	C_f	Nu_x
1						0.3997	1.1037	0.4809	1.1284
2						0.2298	1.0837	0.2922	1.1061
3						0.0735	1.0647	0.1218	1.0852
	0.1					0.4093	1.1141	0.5832	1.1401
	0.2					0.5664	0.7513	0.6157	0.7956
	0.3					0.7029	0.6146	0.6563	0.6599
		0.1				0.4903	1.1141	0.5832	1.1401
		0.3				0.4395	1.1082	0.5264	1.1335
		0.5				0.3504	1.0978	0.4276	1.1219
			1			0.4469	1.1016	0.5346	1.1268
			2			0.2396	1.0426	0.3057	1.0641
			3			0.0470	0.9882	0.0965	1.0069
				0.5		0.4562	1.1096	0.5451	1.1353
				0.8		0.3620	1.0982	0.4407	1.1229
				1.1		0.2758	1.0880	0.3457	1.1117
					-0.1	0.3716	1.2290	0.4540	1.2600
					0.0	0.3816	1.1646	0.4647	1.1930
					0.1	0.3916	1.0998	0.4754	1.1255

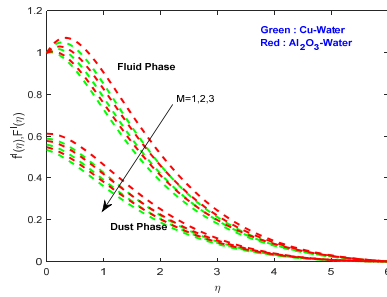


Fig.1. Velocity profiles for different values of magnetic field parameter M .

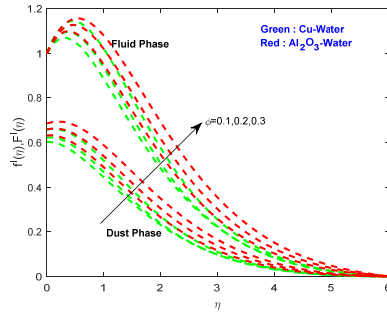


Fig.2. Velocity profiles for different values of volume fraction of nanoparticles ϕ .

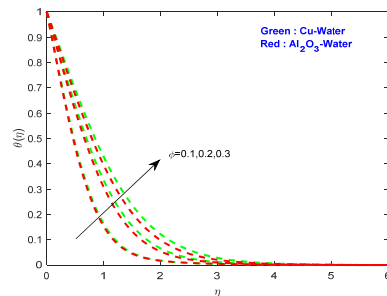


Fig.3. Temperature profiles for different values of volume fraction of nanoparticles ϕ .

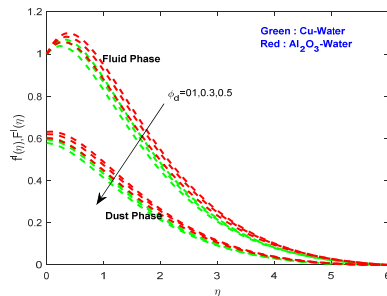


Fig.4. Velocity profiles for different values of volume fraction of dust particles ϕ_d .

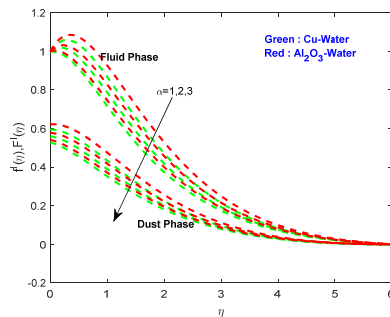


Fig.5. Velocity profiles for different values of mass concentration of dust particles α .

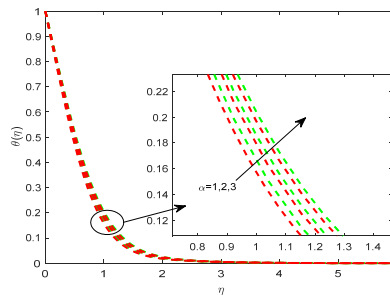


Fig. 6. Temperature profiles for different values of mass concentration of dust particles α .

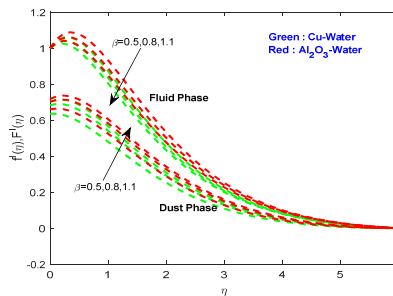


Fig.7. Velocity profiles for different values of fluid-particle interaction parameter β .

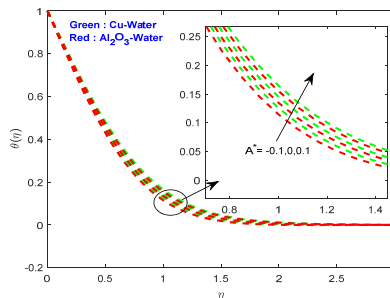


Fig.8. Temperature profiles for different values of non-uniform heat source/sink parameter A^* .

4. Conclusions

- An increase in fluid particle interaction parameter enhances the heat transfer rate.
- An increase in volume fraction of nano particles enhances the thermal conductivity of the flow.
- A raise in the volume fraction of dust particles reduces the momentum boundary layer thickness.
- Cu-water dusty nanofluid shows better heat transfer performance compared with Al_2O_3 -water dusty nanofluid.
- Embedding the conducting dust particles in to nanofluids enhances the heat transfer rate.

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