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MHD SLIP FLOW PAST AN EXTENDING SURFACE WITH THIRD TYPE BOUNDARY CONDITION AND THERMAL RADIATION EFFECTS

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MHD slip flow past an extending surface with third type (convective) boundary condition and thermal radiation is analysed. The governing momentum and energy equations are converted into set of nonlinear ordinary differential equations using appropriate similarity transformations. The Fourth-Order Runge-Kutta shooting method is applied for obtaining the numerical solution of the resulting nonlinear ordinary differential equations. The numerical results for velocity and temperature distribution are found for different values of the vital parameters, namely: the magnetic interaction factor, slip factor, convective factor, Prandtl number and radiation factor and are presented graphically, and discussed.

Key words: third type (convective) boundary condition, slip flow, thermal radiation, extending surface.

1. Introduction

Forced convection flows with radiation heat transfer effects are of prime importance in space science and high temperature processes. In literature, many studies pertaining to the effect of MHD flow past a body with radiation have been recorded. Further, effects due to thermal radiation play a predominant role in polymer industries for heat process control. Radiative MHD flows with enormous temperature often occur in power generation, re-entry of space vehicle into the earth atmosphere, atomic engineering and other industrial process. Soundalgerkar *et al*. [1] investigated radiation effects past a semi-infinite flat plate for a free convection flow of a gas. Boundary layer effects under thermal radiation for an absorbing and emitting media were explored by Viskanta and Grosh [2]. Combined radiative MHD heat transfer models were discussed by Mosa [3]. Hossain and Takhar [4], investigated the radiation effect of an optically dense fluid with uniform free stream velocity and surface temperature over a heated vertical plate. Thermal radiation due to the Rosseland diffusion approximation was taken into consideration by them for grey gases that emit and absorb, but do not scatter.

Hossain *et al*. [5] discussed the effects of radiation in the dearth of a magnetic field and viscous dissipation for a heat transfer problem under free convection. Radiation effect son heat transfer over a stretching surface were investigated by Elbashbeshy [6]. Duwari and Damesh [7] investigated the MHD effect for a fluid flow past a vertical sheet under radiation-conduction interaction in free and mixed convection. Pop *et al*. [8] studied the flow near the quiescence point of an elongating sheet under radiation effects. The radiation effect in Blasius flow was analysed by Cortell [9]. Heat transfer flow about an inclined plane under the influence of MHD mixed convection was studied by Aydin and Kaya [10].

Mukhopadhy *et al*. [11] examined the effect of a steady boundary layer, heat transfer flow past a porous stimulative plate under the influence of nonlinear radiation. MHD flow with heat transfer over a surface extending with a power law velocity by taking the effects of variable viscosity and nonlinear radiation was analysed by Anjali Devi and Gururaj [12]. Thermal radiation and heat transfer effects on an

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unsteady flow of MHD micropolar fluid subjected to suction over a flat plate were analysed by Shit *et al*. [13]. Sandeep *et al*. [14] investigated the boundary layer flow of a nanofluid over a nonlinearly permeable stretching sheet under radiation effects. Radiation effects on MHD flow past a stretching sheet with variable viscosity and heat source/sink under slip condition were analysed by Devi *et al*. [15].

Recently, investigations pertaining to a boundary layer flow with third type (convective) boundary conditions have gained immense importance. The pioneer in this study was Aziz [16], who considered the convective surface boundary condition in the study of a thermal boundary layer flow over a flat plate in a uniform stream. Similarity solutions for the steady laminar boundary layer flow over a permeable plate with a convective boundary condition were investigated by Ishak [17]. The effect of a convective boundary condition on two dimensional boundary layer flows past an extending sheet in a nanofluid was investigated by Makinde and Aziz [18]. Ashikin *et al*. [19] investigated the boundary layer flow over a stretching sheet with slip and convective boundary condition effect. MHD stagnation point flow of a nanofluid towards an extending surface with convective boundary condition with thermal radiation was analysed by Noreen Sher Akbar *et al*. [20]. Ramesh and Gireesha [21] investigated the influence of heat source/sink on a Maxwell fluid over a stretching surface with contribution of nanoparticles under the effect of convective boundary. Recently, Rahman *et al*. [22] discussed the boundary layer flow of a nanofluid past a permeable exponentially shrinking surface with convective boundary condition employing Buongiorno's model.

No contribution has been made to study radiation effects on MHD boundary layer slip flow past a stretching surface with third type (convective) boundary condition which forms the backbone of the current study.

2. Problem formulation

MHD laminar two dimensional boundary layer slip flow of a viscous, incompressible, electrically conducting and radiating fluid past an extending surface under the third type (convective) boundary condition is considered. A magnetic field is applied in the y direction and the fluid under consideration is taken to be grey. Further, the fluid is an absorbing and emitting and non-scattering medium with constant physical properties. The x-axis is taken along the stretching surface and the y-axis normal to the stretching surface. The stretching surface is stretched with velocity $u_w = ax$, where $a > 0$. The continuity, momentum and energy equations under the usual boundary layer assumptions can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_o^2}{\rho}u\,, \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{2.3}$$

where u and v are the velocity components in the x and y directions, respectively, T is the temperature, υ is the kinematic viscosity, σ - electrical conductivity of the fluid. B_o is the magnetic field strength, ρ - density of the fluid, c_p - specific heat at constant pressure, κ - thermal conductivity of the fluid, L - proportional constant of the velocity slip and q_r - radiative heat flux. For very small magnetic Reynolds number, the induced magnetic field is assumed to be negligible in comparison with the applied magnetic field and hence neglected. The radiative heat flux in the energy expression is given by the expression

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.4}$$

where σ^* and k^* are the Stephen-Boltzmann constant and Rosseland mean absorption coefficient, respectively. For adequately small temperature differences within the flow, T^4 can be expressed as a linear function of temperature. Hence using the Taylor series expansion we have

$$T^4 \approx 4T_\infty^4 T - 3T_\infty{}^4 . \tag{2.5}$$

Using Eq.(2.5) the energy Eq.(2.3) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\left(1+R\right)\frac{\partial^2 T}{\partial y^2} \tag{2.6}$$

where $R = \dfrac{16\sigma^* T_\infty^3}{3\kappa k^*}$ is the radiation factor.

The velocity field boundary conditions are given as

$$\begin{aligned} &u = u_w + L\frac{\partial u}{\partial y}, \quad v = 0 \quad \text{at} \quad y = 0, \\ &u \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{2.7}$$

The heat transfer coefficient h_f is generated due to the heating of the bottom surface by convection due to a hot fluid with temperature T_f , and hence the boundary condition for the energy field can be written as

$$\begin{aligned} &-\kappa\frac{\partial T}{\partial y} = h_f\left(T - T_\infty\right) \quad \text{at} \quad y = 0, \\ &T \to T_\infty \quad \text{as} \quad y \to \infty. \end{aligned} \tag{2.8}$$

The continuity Eq.(2.1) is satisfied by introducing a stream function $\psi\left(x,y\right)$ defined in an usual form as

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x}.$$

The similarity solutions for Eqs (2.1)-(2.3) with the boundary conditions (2.7)-(2.8) are obtained by introducing the following similarity transformation

$$\eta = \left(\frac{a}{\upsilon}\right)^{\frac{1}{2}} y, \quad \psi = \left(\upsilon a\right)^{\frac{1}{2}} xf\left(\eta\right), \quad \theta\left(\eta\right) = \frac{T - T_\infty}{T_f - T_\infty}. \tag{2.9}$$

Introducing (2.9) into Eqs (2.2) and (2.3) subject to boundary conditions (2.7) and (2.8), the following nonlinear ordinary differential equations are obtained

$$f''' + ff'' - f'^2 - Mf' = 0\,, \tag{2.10}$$

$$\theta''(\eta) = -\frac{\Pr f \theta'}{(1+R)} \tag{2.11}$$

where $M = \dfrac{\sigma B_o^2}{\rho a}$ is the magnetic interaction factor and $\Pr = \dfrac{\upsilon \rho c_p}{\kappa}$ is the Prandtl number. The heat transfer coefficient h_f takes the form $h_f = cx^{-\frac{1}{2}}$, where c is a constant. The transformed boundary conditions are given by

$$f'(0) = 1 + Kf''(0), \quad f(0) = 0, \quad \theta'(0) = -\gamma\left[1 - \theta(0)\right],$$
$$f'(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{2.12}$$

where the prime denotes differentiation with respect to η, $K = L\left(\dfrac{a}{\upsilon}\right)^{\frac{1}{2}}$ is the slip factor and $\gamma = \dfrac{c}{\kappa}\sqrt{\dfrac{\upsilon}{U_\infty}}$ is the convective factor.

3. Numerical solutions

Equations (2.10) and (2.11) are coupled nonlinear ordinary differential equations which constitute the required nonlinear boundary value problem that has to be analysed. The resulting nonlinear boundary value problem is reduced to an initial value problem by utilizing the shooting method and is solved numerically using the fourth-order Runge-Kutta technique subject to the boundary conditions given by Eq.(2.12). To kick start the shooting process, initial guesses for the values of $f''(0)$ and $\theta(0)$ are made. Different initial guesses were made for different values of physical factors in accordance with convergence. Numerical results are obtained for several values of the physical factors $M, K, \gamma, \Pr$ and R.

4. Numerical results and discussion

Numerical solutions of the problem concerned with MHD slip flow past an extending surface with third type (convective) boundary condition with thermal radiation are obtained for different values of the physical factors involved in the problem, namely: the magnetic interface factor (M), slip factor(K), convective factor(γ), Prandtl number (Pr) and radiation factor (R) and are displayed graphically.

4.1. Validation result

To validate the current study, the results are compared with those of Nor Ashikin *et al*. [19], which are portrayed through Figs 1 to 4. It is evident from the figures that the results are in good agreement in the absences of the magnetic effect and radiation.

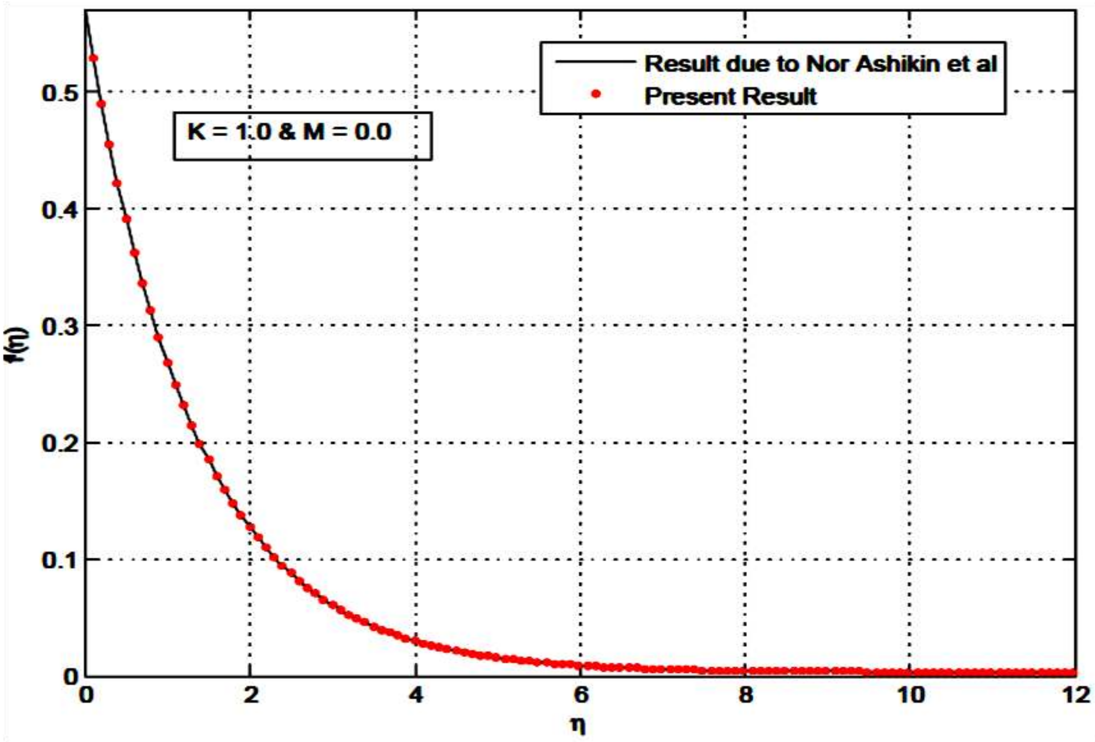

Fig.1. Velocity profile for $K=1.0$ and $M=0.0$.

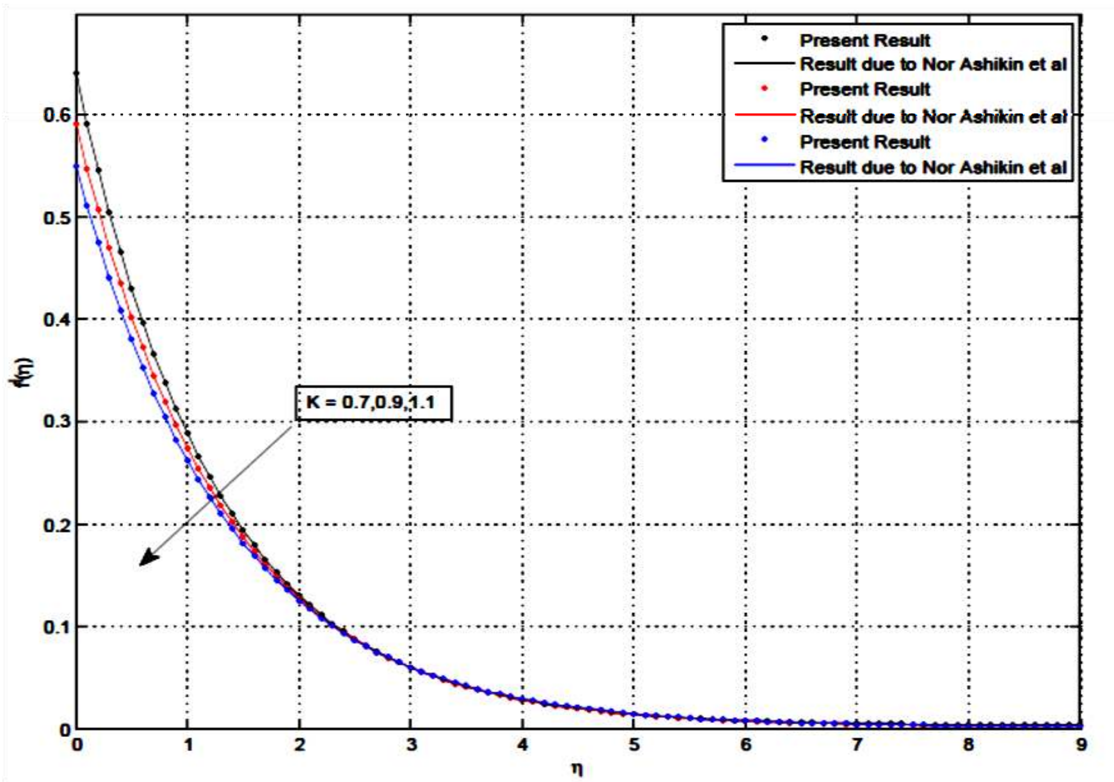

Fig.2. Velocity profile for various K when $M=0.0$.

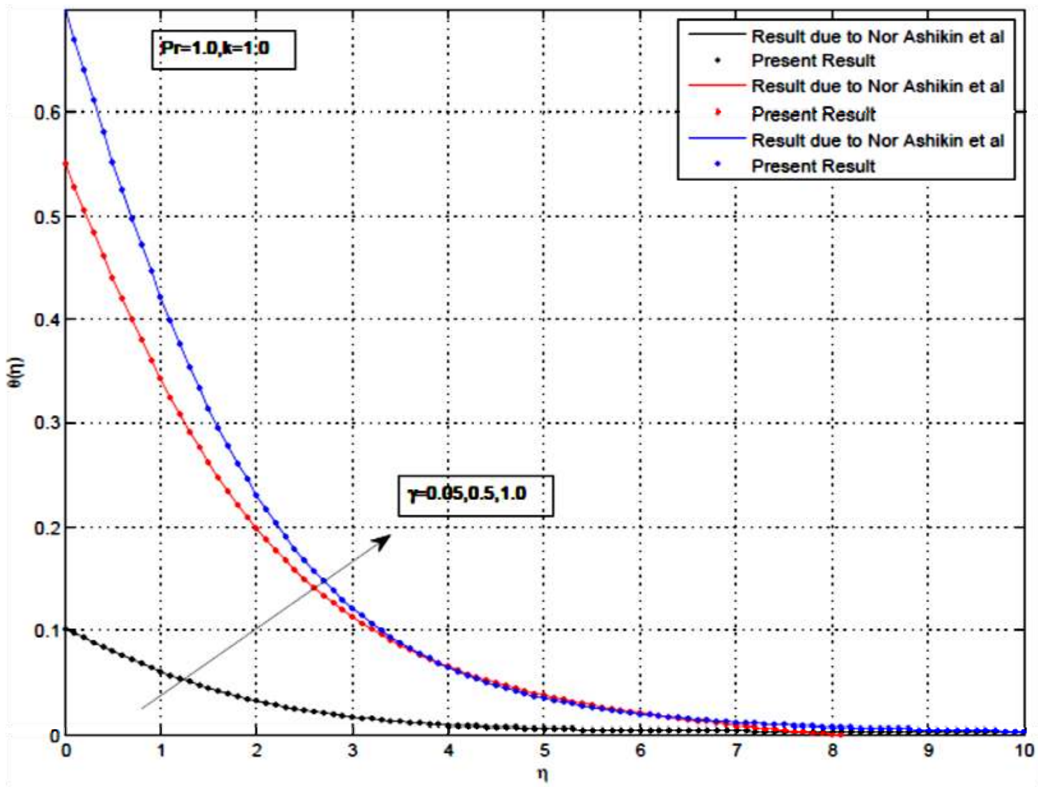

Fig.3. Temperature profiles for different values of γ when Pr=*1.0*, *K=1.0* and *R=0.0*.

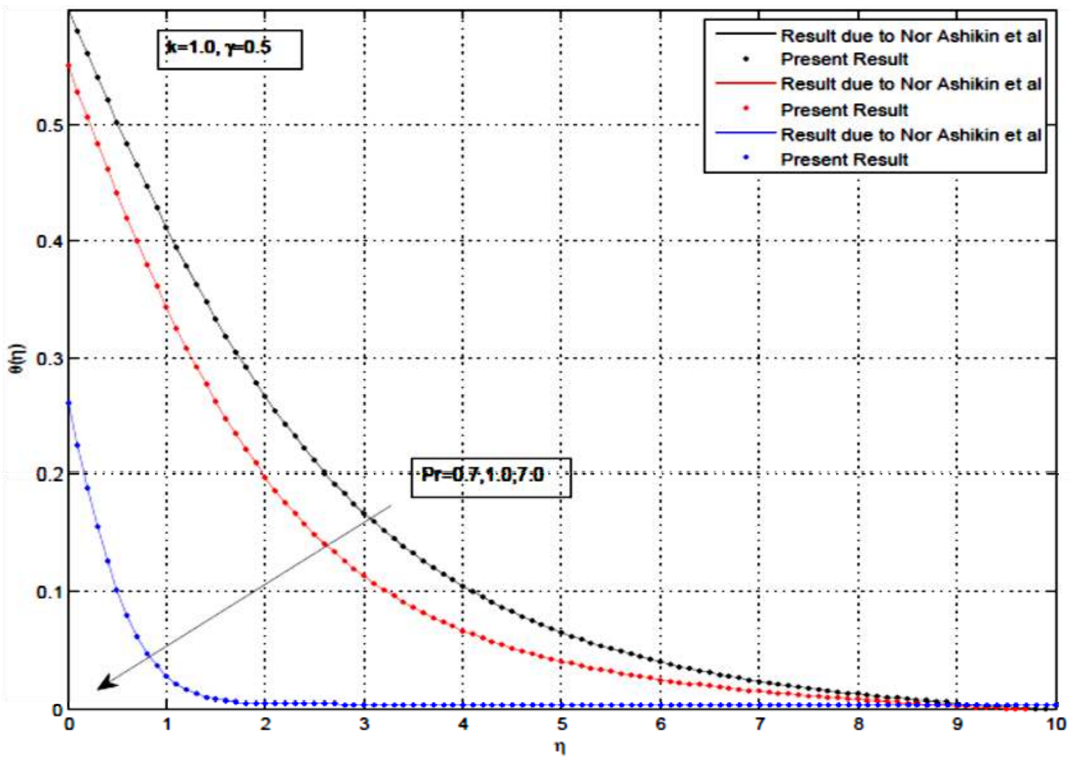

Fig.4. Temperature profiles for different values of Pr with *K=1.0* and $\gamma = 0.5$.

4.2. Impact of magnetic interaction factor on the velocity field

Figure 5 shows that for an increasing *M*, the Lorentz force caused by the magnetic field will act against the motion of the fluid and hence there is a decrease in the velocity. Further for larger values of M , the flow profile starts to become flatter with steep gradients close to the wall which satisfies the no-slip boundary condition $(K = 0.0)$.

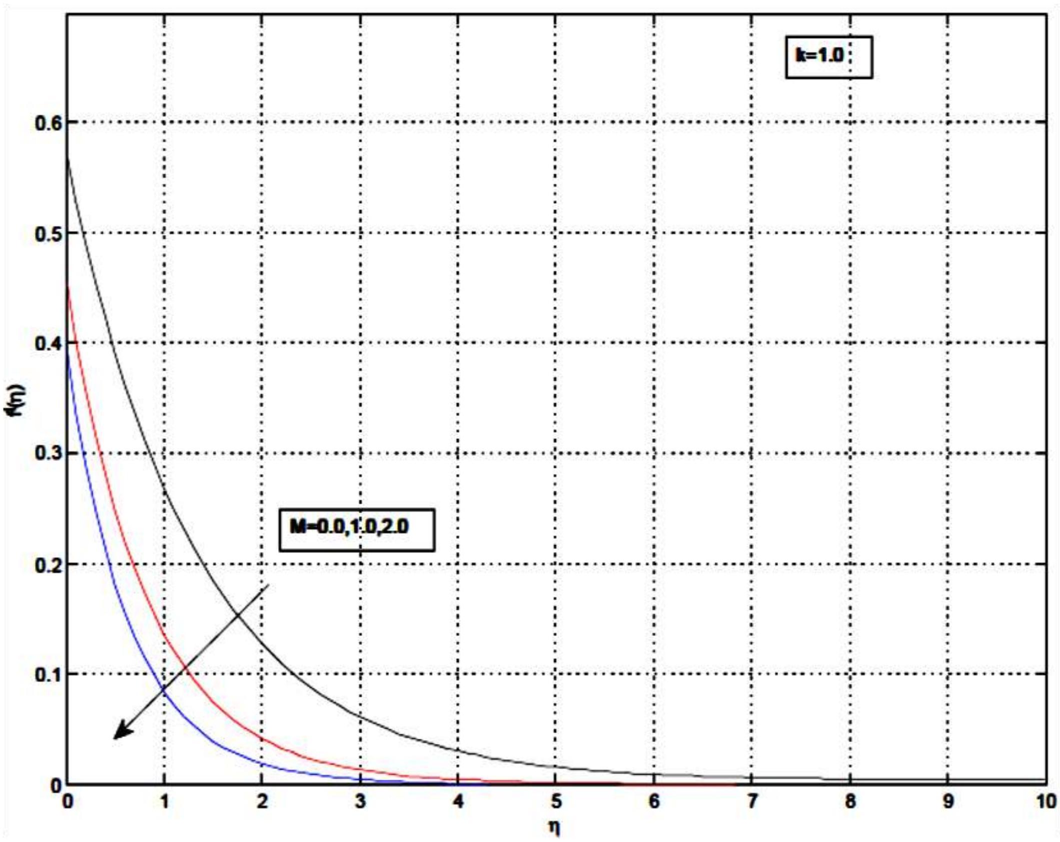

Fig.5. Velocity profiles for various *M* when *K=1.0*.

4.3. Impact of slip factor

It is seen from Fig.6 that as the slip factor K increases, the momentum boundary layer decreases rapidly. Further, it can be inferred that the skin friction coefficient $f''(0)$ drops at the boundary layer and is inversely proportional to the slip factor in magnitude. It is also noted that the presence of the magnetic effect enhances the reduction of the momentum boundary layer for an increasing slip factor K .

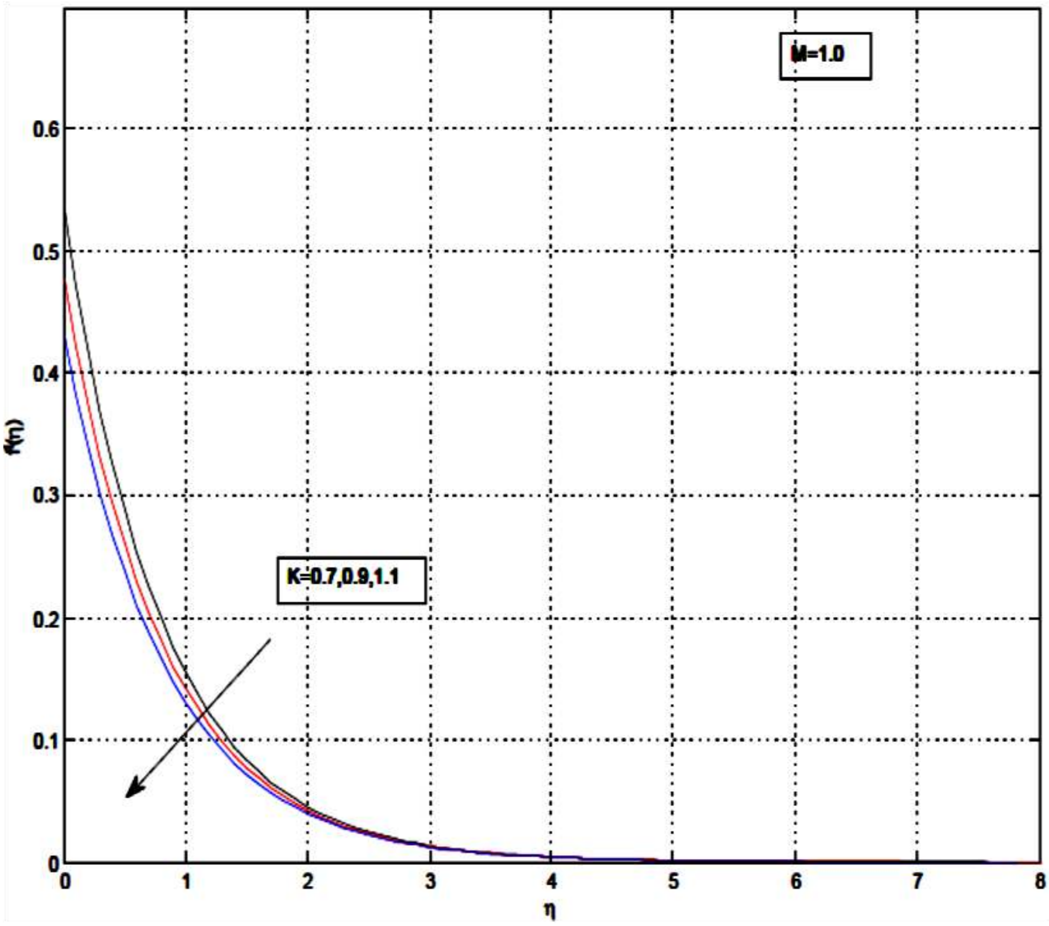

Fig.6. Velocity profiles for various *K* when *M=1.0*.

4.4. Impact of radiation factor

It is interesting to note from Fig.7 that the thermal boundary layer increases rapidly for an increasing radiation factor *R* which has an opposite trend to that found in the literature. This happens due to heating of the bottom surface of the plate by a hot fluid, which is reflected by the presence of the convective factor.

Figure 8 shows the temperature profile when the radiation factor $R = 10^9$. It can be noted that as R tends to become larger, $\theta''(\eta)$ becomes zero. The temperature $\theta(\eta)$ thus becomes a linear function of η which is ascertained from Fig.8. It is also evident that the temperature decreases when we move farther along the surface for larger values of R.

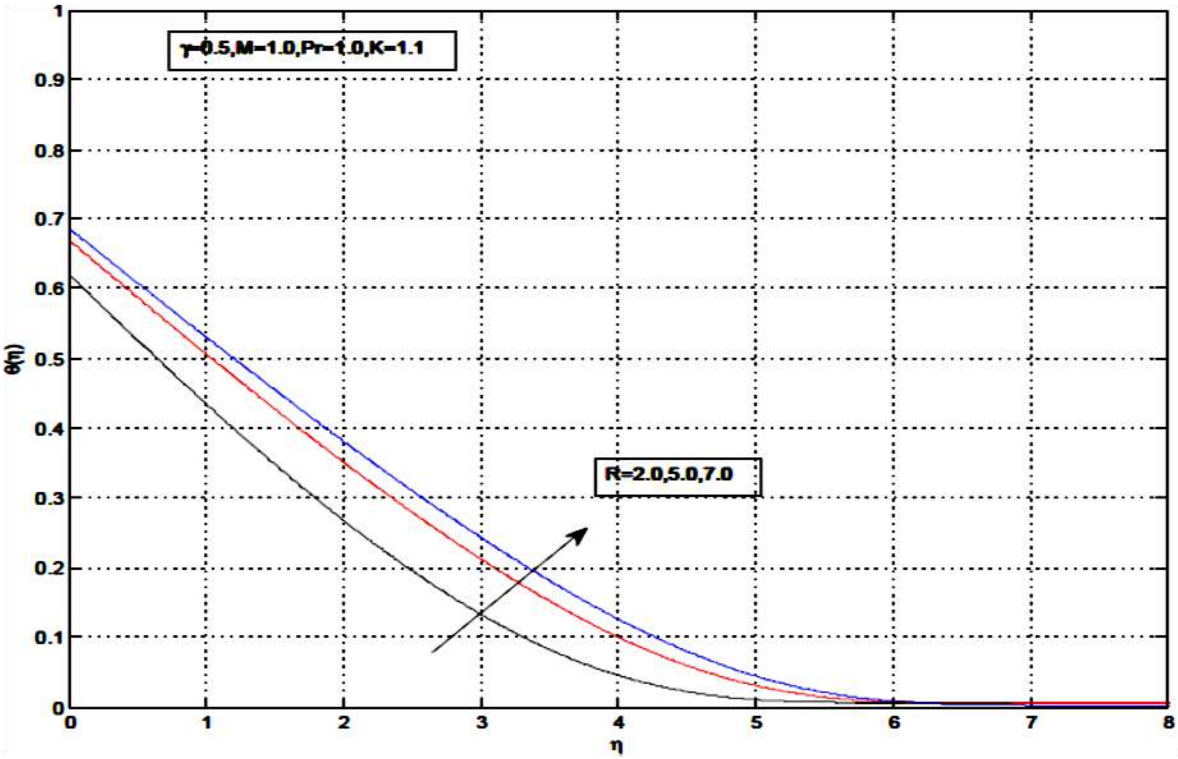

Fig.7. Temperature profile for different values of *R* when Pr=*1.0*, *K=1.1*, *M=1.0* and $\gamma = 0.5$.

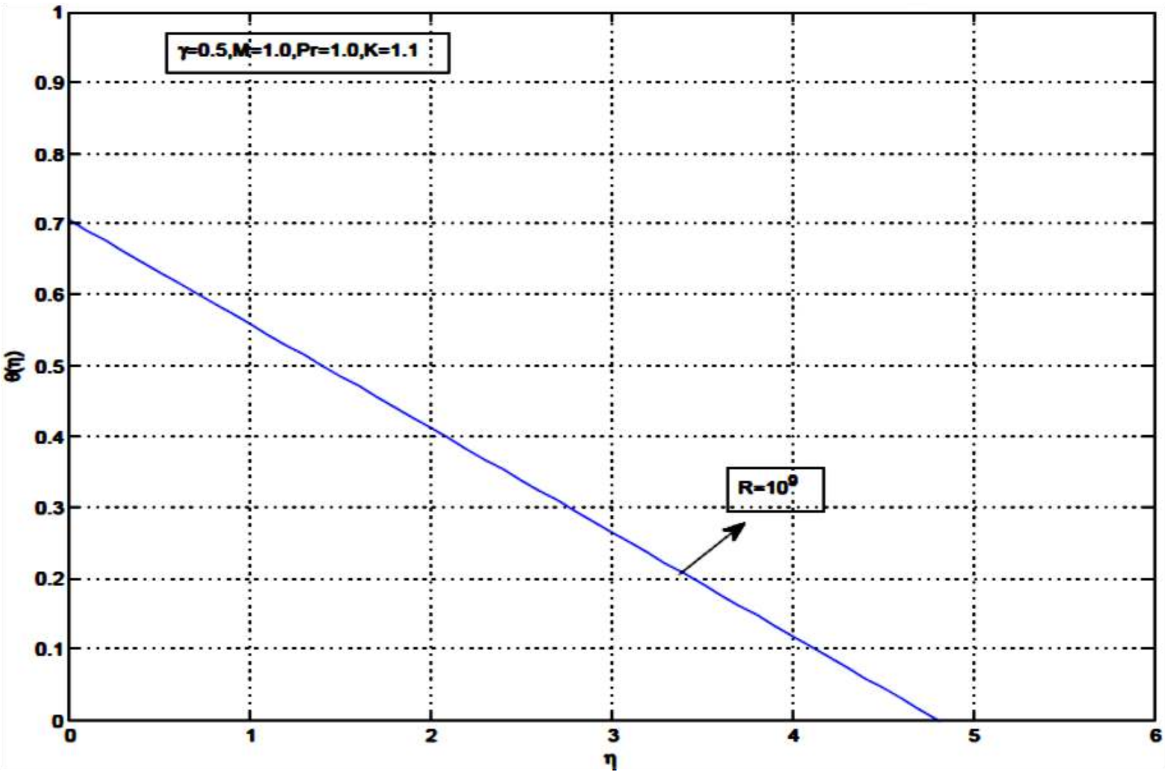

Fig.8. Temperature profile for $R=10^9$ when Pr=*1*, *K=1*, *M=1.0* and $\gamma = 0.5$.

4.5. Impact of convective factor

The impact of the convective factor γ on temperature is elucidated in Fig.9. It is seen that the surface and the fluid temperature increases when γ is increased that leads to the enhancement of the thermal boundary layer thickness. For increasing γ, the strength of convective heating on the surface increases which leads to an increase in the convective heat transfer from the hot fluid on the lower surface to the fluid on the upper surface.

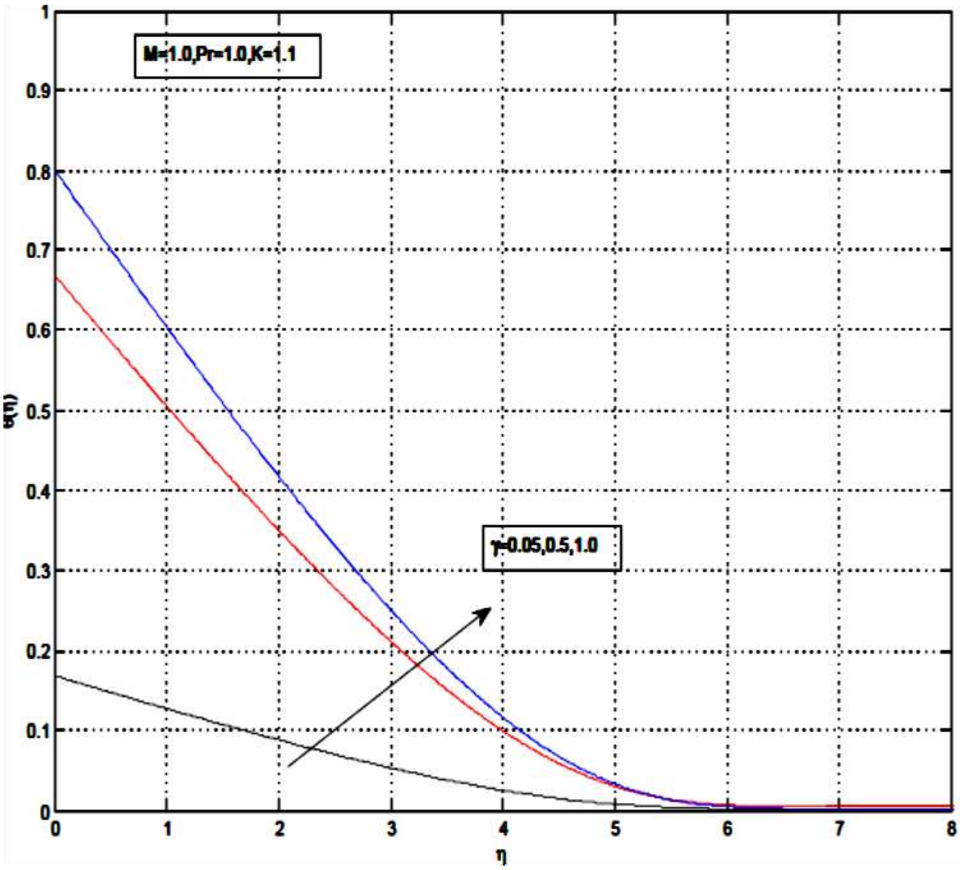

Fig.9. Temperature profile for various values of when *M=1.0*, Pr=*1.0*, *K=1.1* and $R = 5.0$.

4.6. Impact of Prandtl number

Figure10 exhibits the variation in dimensionless temperature for various values of Pr. It is seen that as Pr increases the thermal boundary layer thickness decreases due to the fact that a higher Pr value leads to low thermal conductivity reducing the conduction, which in turn diminishes the boundary layer and temperature. It is also noted that as the thermal gradient at the surface increases, the effect of the Prandtl number is to enhance the heat transfer rate at the surface.

4.7. Impact of magnetic interaction factor on the temperature field

The impact of the magnetic interaction factor M over the temperature field is presented in Fig.11. It can be noted that the heat transfer gets enhanced for increasing M as expected under the influence of radiation and convective heating.

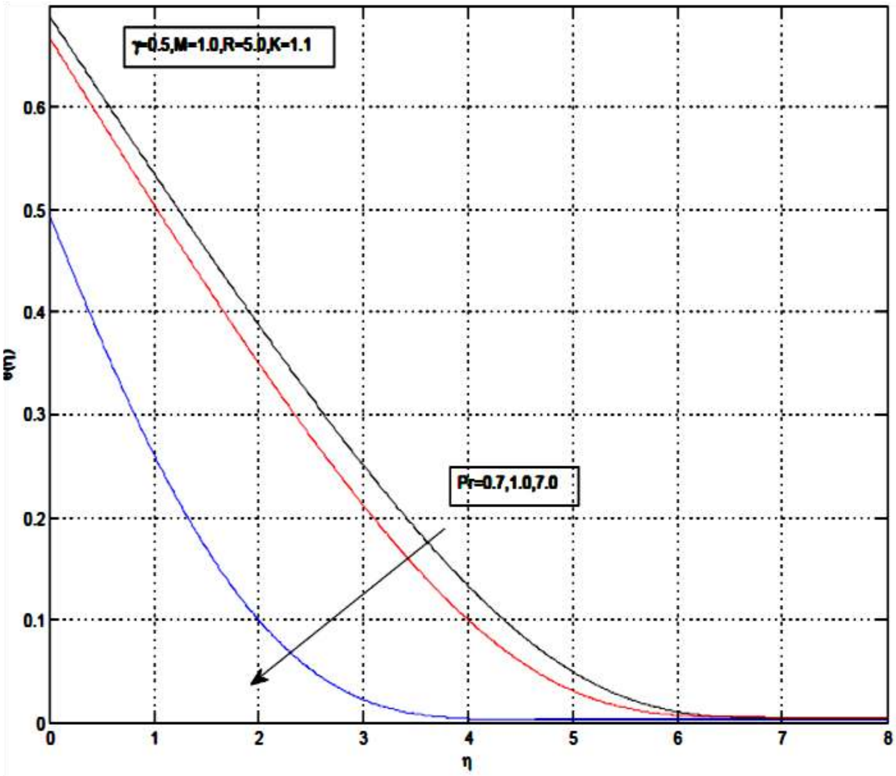

Fig.10. Temperature profile for various values of Pr when $M=1.0$, $K=1.1$, $\gamma = 0.5$ and $R = 5.0$.

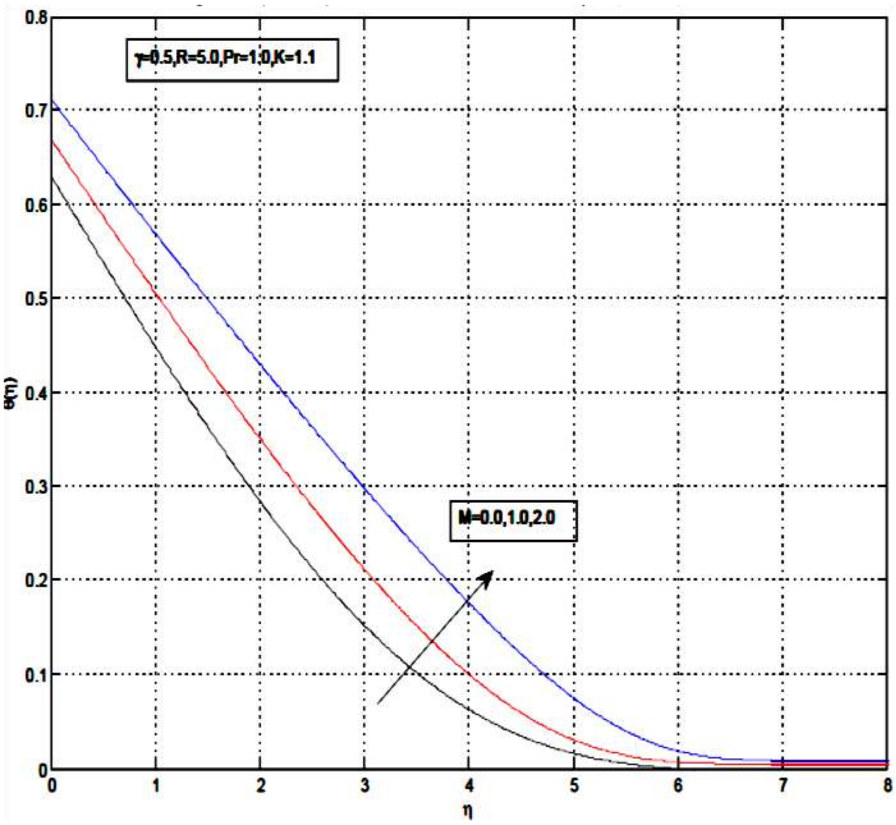

Fig.11. Temperature profiles for various values of M when $\gamma = 0.5$, Pr=1.0, $K=1.1$ and $R = 5.0$.

5. Conclusion

In this study, the problem of thermal radiation MHD boundary layer slip flow past a stretching surface with convective boundary condition is investigated. Graphical results are presented for different physical parameters of the problem, namely the magnetic interaction factor (M), slip factor (K), convective factor (γ), radiation factor (R) and Prandtl number (Pr).

In the non-existence of magnetic field and thermal radiation, the results are in good agreement with those of Nor Ashikin *et al*. [19].

The following conclusions can be drawn from this study:

- The Lorentz force caused by the magnetic field acts against the flow field and hence results in the decrease of velocity, but its effect is to increase the thermal boundary layer.
- The skin-friction coefficient decreases at the boundary layer and is in reverse proportion to the slip parameter.
- The thermal boundary layer increases rapidly for an increasing radiation parameter. This effect is due to hot fluid that heats the bottom surface of the plate.
- As the radiation parameter becomes larger, the temperature becomes a linear function and the temperature decreases along the surface for a larger radiation parameter.
- The strength of convective heating on the surface increases for an increasing convective parameter.
- The Prandtl number enhances the heat transfer rate at the surface due to an increase in the thermal gradient.

Nomenclature

a – extending rate
B_o – magnetic field strength
C_p – specific heat at constant pressure
h_f – heat transfer coefficient
k^* – Rosseland mean absorption coefficient
K – slip factor
L – proportional constant of the velocity slip
M – magnetic interaction factor
Pr – Prandtl number
R – radiation factor
T – temperature of the fluid
T_∞ – temperature of the ambient fluid
u, v – velocity component of the fluid in the x and y direction
u_w – extending surface velocity
u_∞ – free stream velocity
q_r – radiative heat flux
x – dimensional distance along the extending surface
y – dimensional distance normal to the extending surface
γ – convective factor
η – similarity variable
θ – dimensionless temperature
κ – thermal conductivity of the fluid
ν – kinematic viscosity
ρ – density of the fluid

σ – electrical conductivity of the fluid

σ^* – Stephen-Boltzmann constant

$\psi(x, y)$ – stream function

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