Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 57 (2015) 587 – 591

Procedia Computer Science

3rd International Conference on Recent Trends in Computing 2015 (ICRTC-2015)

# Minimum Connected Dominating set for Certain Circulant Networks

N. Parthiban[a], Indra Rajasingh[b,*], R. Sundara Rajan[b]

[a]*School of Computing Science and Engineering, VIT University, Chennai, India*
[b]*School of Advanced Sciences, VIT University, Chennai, India*

**Abstract**

A Minimum Connected Dominating Set is a minimum set of connected nodes such that every other node in the network is one hop connected with a node in this set. In general, the problem is proved to be NP-hard. In this paper we find a Minimum Connected Dominating Set for certain Circulant Networks.

*Keywords*: *Circulant Network, Dominating Set, Connected Dominating set, Maximum Leaf Spanning Tree.*

| **Nomenclature** | | **Greek symbols** | |
|---|---|---|---|
| $G$ | Graph | $\gamma(G)$ | Minimum Dominating set (MDS) |
| $G(n,\pm\{1,2,3...j\})$ | Circulant Network | $\gamma_c(G)$ | Minimum Connected Dominating set (MCDS) |
| $S$ | Dominating Set | $\tau_l(G)$ | Maximum Leaf Spanning Tree (MLST) |
| $R$ | Connected Dominating Set | | |
| $N[v]$ | Closed neighbourhood of $v$ | | |

## 1. Introduction

A graph is a collection of nodes interconnected by edges. Thus 'networks' are nothing but graphs in which nodes represent processors or processes and edges represent communication links between them. The domination problem is a fundamental problem in graph theory. Given a graph *G*, a dominating set of the graph is a set of nodes such that every node in *G* is either in the set or has a direct neighbouring node in the set. This problem, along with its variations, such as the connected dominating set or the *k*-dominating set, play significant role in wireless sensor networks [1], ad hoc sensor networks [2], peer-to-peer networks, Interconnection Networks [3] etc. A connected dominating set (CDS) of a graph *G* is a dominating set whose induced graph is connected. A connected dominating set serves as a virtual backbone of a network which can help with routing. Any vertex outside the virtual backbone can send message or signal to another vertex through the virtual backbone. A virtual backbone supports shortest path routing [4], fault-tolerant routing [4], multi-casting [4], radio broadcasting [4], clustering [5] and so on. Furthermore, a virtual backbone of a wireless network may reduce communication overhead, increase bandwidth efficiency, and decrease energy consumption [6].

A spanning tree of a connected graph *G* is defined as a maximal set of edges of *G* that contains no cycle, or as a minimal set of edges that connect all vertices. The Maximum Leaf Spanning Tree (MLST) problem is to find a spanning tree of a graph with as many leaves as possible. A leaf in a graph *G* of a spanning tree is a vertex in *G* of degree one in the spanning tree. Given a graph *G*, the Maximum Leaf Spanning Tree problem is to find the maximum number of vertices of degree one in a spanning tree over all spanning trees of *G*.

Corresponding author.
*E-mail address:* parthiban24589@gmail.com

*This work is supported by Project No. SR/S4/MS: 846/13, Department of Science and Technology, SERB, Government of India.


Peer-review under responsibility of organizing committee of the 3rd International Conference on Recent Trends in Computing 2015 (ICRTC-2015)
doi:10.1016/j.procs.2015.07.401

Some broadcasting problems in network design ask to minimize the number of broadcasting nodes, which must be connected to a single root. This translates the problem into finding a spanning tree with many leaves and few internal nodes. Ongoing research on this topic is motivated by the fact that variants of this problem occur frequently in real life applications [7,8,9]. The Maximum Leaf Spanning Tree problem can be found in the area of communication networks and circuit layouts [7]. In communication networks where the vertices correspond to terminals, the problem on message routing is to design a tree-like layout in the network where "leaf terminals" may have lighter workloads than "intermediate terminals" of degree at least two. Hence, in this case, the solution of MLST problem provides a reasonable layout [9].

Minimum Connected Dominating Set and Maximum Leaf Spanning Tree problem are known to be NP-hard [10] and are equivalent [17]. Many literature references discussed the MCDS and MLST problems by approximation algorithms [11,13,14,16,17]. The problems are discussed for some generalized trapezoid graphs [18], grid graphs [19], Hypercube and Star networks [20]. Even though there are numerous results and discussions on MCDS and MLST problems, most of them deal with only approximate results.

In this paper we exhibit a Minimum Connected Dominating Set for the circulant network $G(n,\pm\{1,2,3\ldots j\})$ , thereby solving the Maximum Leaf Spanning Tree problems also.

## 2. Circulant Network

The circulant network is a natural generalization of double loop network, which was first considered by Wong and Coppersmith [21]. Circulant graphs have been used for decades in the design of computer and telecommunication networks due to their optimal fault-tolerance and routing capabilities [22]. It is also used in VLSI design and distributed computation Theoretical properties of circulant graphs have been studied extensively and surveyed by Bermond et al. [23]. Every circulant graph is a vertex transitive graph and a Cayley graph [3]. Most of the earlier research concentrated on using the circulant graphs to build interconnection networks for distributed and parallel systems [22,23]. Classes of graphs that are circulant graphs include the complete graphs, complete bipartite graphs, Paley graphs of prime order, prism graphs, möbius ladder graph, tetrahedral graph and torus grid graphs.

**Definition 1 [24]:** A circulant graph denoted by $G(n,\pm\{1,2,\ldots,j\})$, $1 < j \le \lfloor n/2 \rfloor, n \ge 3$ is defined as a graph consisting of the vertex set $V = \{0,1,2,3,\ldots,n-1\}$ and the edge set $E = \{(i,j) : |j-i| \equiv s(\mathrm{mod}\, n), s \in \{1,2,\ldots,j\}\}$. See Figure 1.

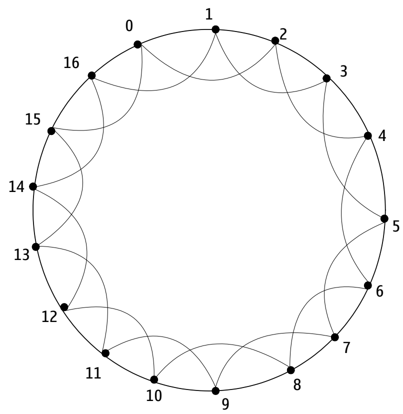


Figure 1. Circulant Network $G(17,\pm(1,2))$

## 3. Main Results

The following problems have been considered in the literature and are NP-complete [10].

**Problem 1:** Given a connected graph $G(V,E)$ and a vertex set $S \subseteq V$, $S$ is a dominating set if each vertex in $G$ is either in $S$ or has at least one neighbour in $S$. A dominating set with minimum cardinality is called Minimum Dominating Set. Minimum Domination Problem is to determine $\gamma(G)$ defined as $\gamma(G) = \min |S|$ where the minimum is taken over all dominating sets of $G$.

**Problem 2:** *R* is a connected dominating set if *R* is a dominating set and the induced subgraph $G[R]$ is connected. A connected dominating set with minimum cardinality is called a minimum connected dominating set. Minimum Connected domination Problem is to determine $\gamma_c(G)$ defined as $\gamma_c(G) = m \quad |R|$ where the minimum is taken over all connected dominating sets of *G*.

**Problem 3:** Given an undirected graph $G(V, E), |V| = m, |E| = n$, find a spanning tree *T* of *G* with a maximum number of leaves $\imath_l(G)$ among all spanning trees of *G*.

The following result is easy to observe.

**Lemma 1:** Let *G* be an *r* - regular graph on *n* vertices. Then $\gamma(G) \geq \lceil n/r + 1 \rceil$

**Domination Algorithm $\boldsymbol{G(n, \pm\{1, 2, \dots, j\})}$**

**Input** : Circulant Graph $G(n, \pm\{1,2, \dots, j\})$
**Algorithm :** Label the vertices of graph $G(n, \pm\{1,2, \dots, j\})$ as 0,1,2…,*n*-1 in the clockwise sense. Select the vertices $S = \{0, 2j + 1, 2(2j + 1), 3(2j + 1), \dots, (k - 1)(2j + 1)\}$ where $k = \lceil n/2j + 1 \rceil$.
**Output** : Minimum Dominating Set $S$ for $G(n, \pm\{1,2, \dots, j\})$. See Figure 2.

**Proof of correctness:** Clearly $N[k_1(2j + 1)] \cap N[k_2(2j + 1)] = \emptyset, k_1 \leq k_2 \leq \lceil n/2j + 1 \rceil$ and *S* dominates all the vertices in *G*.

**Theorem 1**: Let *G* be the circulant graph $G(n, \pm\{1,2, \dots, j\})$. Then $\gamma(G) = \lceil n/2j + 1 \rceil$

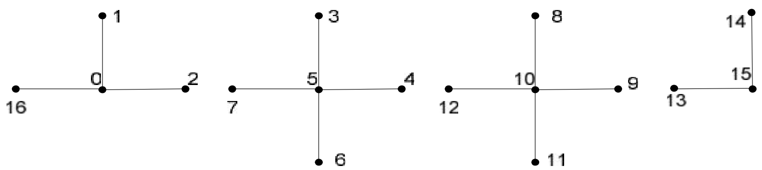


Figure 2. Minimum Dominating Set *S* for $G(17, \pm(1,2))$

**Lemma 2:** Let *G* be graph and $V = \{v_1, v_2, v_3, \dots, v_n\}$ be an ordered set of vertices in *G*. Let $R = \{u_1, u_2, u_3, \dots, u_k\} \subseteq V$ be a dominating set with $u_1 < u_2 < u_3 \dots < u_k$ satisfying the following conditions:

i) $N[u_i] \cap N[u_{i+2}] = \{u_{i+1}\}$, and $N[u_i] \cap N[u_j] = \emptyset, i < j, j \neq i + 1, i + 2$.
ii) $N[u_{i+1}] \subset N[u_i] \cup N[u_{i+2}]$.

Then *R* is a minimum connected dominating set of *G*.

**Proof :**
By condition (i), *R* is a connected dominating set.
By condition (ii), the closed neighborhood of $u_{i+1}$ is contained in $N[u_i] \cup N[u_{i+2}]$. Thus $u_{i+1}$ will not dominate any vertex in $V \backslash N[u_i] \cap N[u_{i+2}]$. Hence we need at least *k* elements to dominate *V*. Thus *R* is minimum.

**Connected Domination Algorithm $\boldsymbol{G(n, \pm\{1, 2, \dots, j\})}$**

**Input** : Circulant Graph $G(n, \pm\{1,2, \dots, j\})$
**Algorithm :** Label the vertices of $G(n, \pm\{1,2, \dots, j\})$ as 0,1,2…,*n*-1 in the clockwise sense. Select the vertices $R = \{0, j, 2j, 3j, \dots \dots, k.j\}$ where $k = \lfloor n - (j + 2)/j \rfloor$.
**Output** : Minimum Connected Dominating Set *R* for $(n, \pm\{1,2, \dots, j\})$ . See Figure 3.

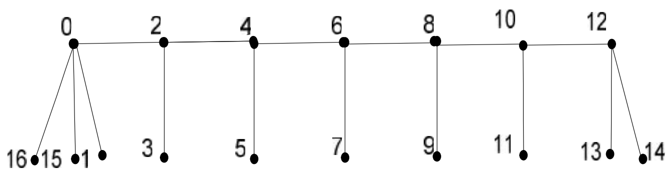


Figure 3. Minimum Connected Dominating Set *R* for $G(17, \pm(1,2))$

**Proof of Correctness:** *R* satisfies the conditions of Lemma 2. Hence *R* is a Minimum Connected Dominating Set of the circulant graph $G(n, \pm\{1,2, \dots, j\})$.

The proofs of the following theorems are easy consequences of Lemma 2 and the Connected Domination Algorithm $G(n, \pm\{1,2, \dots, j\})$.

**Theorem 2:** Let *G* be circulant graph $G(n, \pm\{1,2, \dots, j\})$. Then $\gamma_c(G) = 1 + \lceil n - (2j+1)/j \rceil$.

**Theorem 3:** Let *G* be circulant graph $G(n, \pm\{1,2, \dots, j\})$. Then $\iota_l\,(G) = n - \gamma_c(G)$.

Next we consider another class of circulant network, namely $G(n, \pm(1,3))$.

**Connected Domination Algorithm $\boldsymbol{G(n, \pm(1,3))}$**

**Input** : Circulant Graph $G(n, \pm(1,3))$
**Algorithm :** Label the vertices of $G(n, \pm(1,3))$ as 0,1,2…,*n*-1 in the clockwise sense. Select the labelled vertices $R = \{0,3,6, \dots, (l+1).3\}$ if $n \not\equiv 1\ m\quad 5$ and $R = \{0,3,6, \dots . l.3\}$ otherwise, where $l = \lfloor n/5 \rfloor$.
**Output** : Minimum Connected Dominating Set *R* for $G(n, \pm(1,3))$.

**Proof of Correctness:** *R* satisfies the conditions of Lemma 2. Hence *R* is a Minimum Connected Dominating Set of the circulant graph $G(n, \pm(1,3))$.

The proof of the following theorem is an easy consequence of Connected Domination Algorithm $G(n, \pm(1,3))$.

**Theorem 4:** Let *G* be $G(n, \pm(1,3))$. Then $\gamma_c(G) = l + 2$, if $n \equiv 1\ m\quad 5$, $\gamma_c(G) = l + 1$ otherwise. where $l = \lfloor n\ /\ 5 \rfloor$.

## 4. Conclusion

In this paper we calculated the exact value for Minimum Dominating Set and Minimum Connected Dominating Set, thereby solving Maximum Leaf Spanning Tree problem for certain Circulant Networks.

## References

[1]. F. Dai, J. Wu, "An extended localized algorithm for connected dominating set formation in ad hoc wireless networks", IEEE Transactions on Parallel and Distributed Systems, 2004, 15, 908-920.
[2]. R. Misra, C. Mandal, "Minimum connected dominating set using a collaborative cover heuristic for ad hoc sensor networks", IEEE Transactions on Parallel and Distributed Systems, 2010, 21, 292-302.
[3]. J. Xu, "Topological Structure and Analysis of Interconnection Networks", Kluwer Academic Publishers, 2001.
[4]. D. Kim, Y. Wu, Y. Li, F. Zou, D. Z. Du, "Constructing minimum connected dominating sets with bounded diameters in wireless networks", IEEE Transactions on Parallel and Distributed Systems, 2009, 20, 147-157.
[5]. Dimokas, D. Katsaros, Y. Manolopoulos, "Energy-efficient distributed clustering in wireless sensor networks", Journal of Parallel & Distributed Computing, 2010, 70, 371-383.
[6]. M. Rai, S. Verma, S. Tapaswi, "A Power Aware Minimum Connected Dominating Set for Wireless Sensor Networks", Journal of networks, 2009, 4, 511-519.
[7]. J. A. Storer, "Constructing full spanning trees for cubic graphs", Information Processing Letters, 1981, 13, 8-11.
[8]. D. J. Kleitman, D. B. West, "Spanning trees with many leaves", SIAM Journal on Discrete Mathematics, 1991, 4, 99-106.
[9]. L. M. Fernandes, L. Gouveia, "Minimal spanning trees with a constraint on the number of leaves", European Journal of Operational Research, 1998, 104, 250-261.
[10]. M. R. Garey, D. S. Johnson, "Computers and Intractability: A guide to the theory of NP-completeness", New York, W. H.Freeman & Co., 1979.
[11]. S. Guha, S. Khuller, "Approximation algorithms for connected dominating sets", Algorithmica, 1998, 20, 374-387.
[12]. J. Daligault, G. Gutin, E. J. Kim, A. Yeo, "FPT algorithms and kernels for the directed k-leaf problem", Journal of Computer and System Sciences, 2010, 76, 144-152.
[13]. F. V. Fomin, F. Grandoni, D. Kratsch, "Solving connected dominating set faster than $2^n$", Algorithmica, 2008, 52, 153-166.
[14]. H. Lu, R. Ravi, "Approximating maximum leaf spanning trees in almost linear time", Journal of Algorithms, 1998, 29, 132-141.
[15]. Oba, "2-approximation algorithm for finding a spanning tree with maximum number of leaves", Lecture notes in computer science, 1988, 1461, 441-452.
[16]. T. Fujie, "An exact algorithm for the maximum leaf spanning tree problem", Computers and Operations Research, 2003, 30, 1931-1944.
[17]. H. Fernau, J. Kneis, D. Kratsch, A. Langer, M. Liedloff, D. Raible, P. Rossmanith, "An exact algorithm for the Maximum Leaf Spanning Tree problem", Theoretical Computer Science, 2011, 412, 6290-6302.
[18]. Y. T. Tsai, Y. L. Lin, F. R. Hsu, "Efficient algorithms for the minimum connected domination on trapezoid graphs", Information Sciences. 2007, 177 , 2405-2417.
[19]. P. C. Li, M. Toulouse, "Maximum Leaf Spanning Trees for Grid Graphs", Journal of Combinatorial Mathematics and Combinatorial Computing, 2010, 73, 181-193.

[20]. Y.C.Chen, Y. L. Syu, "Connected Dominating Set of Hypercubes and Star Graphs", International Conference on Software and Computer Applications, 2012, 41.
[21]. G. K. Wong, D. A. Coppersmith, "A combinatorial problem related to multi module memory organization", Journal of Association for Computing Machinery, 1974, 21, 392-401.
[22]. F. T. Boesch, J. Wang, "Reliable circulant networks with minimum transmission delay", IEEE Transactions on Circuit and Systems, 1985, 32, 1286-1291.
[23]. J. C. Bermond, F. Comellas, D. F. Hsu, "Distributed loop computer networks, A survey", Journal of Parallel and Distributed Computing, 1995, 24, 2-10.
[24]. I. Rajasingh, B. Rajan, R. S. Rajan, "Combinatorial Properties of Circulant Networks", International Journal of Applied Mathematics, 2011, 41, 349-351.