Mixed convection effects on heat and mass transfer in a non Newtonian fluid with chemical reaction over a vertical plate

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Abstract This paper studies mixed convection, double dispersion and chemical reaction effects on heat and mass transfer in a non-Darcy non-Newtonian fluid over a vertical surface in a porous medium under the constant temperature and concentration. The governing boundary layer equations, namely, momentum, energy and concentration, are converted to ordinary differential equations by introducing similarity variables and then are solved numerically by means of fourth-order Runge-Kutta method coupled with double-shooting technique. The velocity, temperature concentration, heat and mass transfer profiles are presented graphically for various values of the parameters, and the influence of viscosity index n, thermal and solute dispersion, chemical reaction parameter χ are observed. [doi:10.1063/2.1104207]

Keywords chemical reaction, double dispersion, mixed convection, heat and mass transfer

The process of heat transfer by mixed convection flow over vertical surfaces occurs in many industrial and technical applications which include nuclear reactors cooled during emergency shutdown, electronic devices cooled by fans, solar central receivers exposed to wind currents and heat exchangers placed in a low velocity environment. Mucoglu and Chen[1] have studied the mixed convection flow over an inclined surface for both the assisting and the opposing buoyant forces. Anjalidevi and Kandasamy[2] studied the effects caused by the chemical-diffusion mechanisms and the inclusion of a general chemical reaction of order n on the combined forced and natural convection flows over a semi-infinite vertical plate immersed in an ambient fluid. Amin, Aissa and Salama[3] studied the effects of chemical reaction and double dispersion on non-Darcy free convective heat transfer flows and aimed at analyzing the effects of radiation with chemical reaction on non-Darcy free convective flow from a vertical surface embedded in a porous medium. Nield and Bejan[4] and Pop and Ingham[5] have made comprehensive reviews of the studies of heat transfer in relation to the above applications. Cheng and Minkowycz[6] presented similarity solutions for free convective heat and mass transfer from a vertical plate in a fluid saturated porous medium. The effects of double dispersion on natural convection heat and mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium have been investigated by Murthy et al.[7] Murti and Kameswaran[8] analyzed the effects of radiation, chemical reaction and double dispersion on heat and mass transfer in non-Darcy free convective flow. Murti and Sastry[9] analyzed the effects of mixed convection, double dispersion and chemical reaction effects on heat and mass transfer in non-Darcy flow over a vertical surface. In the present analysis we analyzed the combined effects of chemical reaction and double dispersion on mixed convective flow for different parameters and investigated the changes in velocity, temperature, concentration, heat and mass transfer effects. Consider the mixed convection in a porous medium saturated with a non Newtonian fluid over a vertical plate. The x-coordinate is measured along the surface and the y-coordinate normal to it. The wall is maintained at constant temperature T_w and constant concentration C_w. The ambient conditions for temperature T_∞ and concentration C_∞ are assumed to be constant. The governing equations for the boundary layer flow, heat and mass transfer from the wall $y = 0$ into the fluid saturated porous medium $x \geq 0\,, y > 0$ are given by continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

momentum equation

$$\frac{\partial u^n}{\partial y} + \rho_\infty \frac{bk^*}{\mu^*}\frac{\partial u^2}{\partial y} = \frac{\rho_\infty g k^*}{\mu^*}\left(\beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y}\right), \tag{2}$$

energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha_y \frac{\partial T}{\partial y}\right), \tag{3}$$

concentration equation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(D_y \frac{\partial C}{\partial y}\right) - K_0 \left(C - C_\infty\right)^m, \tag{4}$$

$$\rho = \rho_\infty \left[1 - \beta\left(T - T_\infty\right) - \beta^*\left(C - C_\infty\right)\right], \tag{5}$$

$$y = 0\ ;\ v = 0,\ T_w = \text{constant}\,, C_w = \text{constant}$$

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$$y \to \infty\,; u = u_\infty, T \to T_\infty, C \to C_\infty, \tag{6}$$

where u and v are velocities in the x and y directions respectively, T is the temperature in the thermal boundary layer, K is the permeability constant β^* and β are the coefficients of thermal and solutal expansions, ρ_∞ is the density at some reference point, g is the acceleration due to gravity. The chemical reaction effect is added as the last-term in the right hand side of Eq. (4) where the power $m(=1)$ is the order of reaction. The quantities α_y and D_y, defined as, $\alpha_y = \alpha + \gamma \mathrm{d}\,|\boldsymbol{v}|$ and $D_y = D + \zeta\,\mathrm{d}\,|\boldsymbol{v}|$, represent thermal dispersion and solutal diffusivity, respectively. Introduce the stream function ψ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, similarity variables $\psi = f(\eta)\,\alpha\sqrt{Ra_x}$, $\eta = (y/x)Ra_x^{1/2}$, $\phi(\eta) = (C - C_\infty)/(C_w - C_\infty)$.

The above equations of motion reduce to

$$\left(nf'^{n-1} + 2G_r^* f'\right) f'' = \in (\theta' + N\phi'), \tag{7}$$

$$\theta'' + \frac{1}{2} f\theta' + \gamma Ra_d\,(f'\theta'' + f''\theta') = 0, \tag{8}$$

$$\phi'' + \frac{1}{2} Le f \phi' + \zeta Ra_d\, Le\,(f'\phi'' + f''\phi') - \in Sc\lambda \frac{Gc}{Re_x^2}\phi = 0. \tag{9}$$

where $G_r^* = b\left\{\left(k^{*^2}\rho_\infty^2\,[g\beta_T\,(T_w - T_\infty)]^{2-n}\right)/\mu^{*^2}\right\}^{1/n}$ is Grashof number, $\in = Ra_x/Pe_x$ is the mixed convection parameter, $Ra_x = (x/\alpha)(\rho_\infty k^* K g\beta_T\theta_w/\mu^*)^{1/n}$, $Pe_x = u_\infty x/\alpha$ are the modified pore diameter-dependent Rayleigh number, pore diameter-dependent on Peclet number $N = \beta^*\,(C_w - C_\infty)/[\beta\,(T_w - T_\infty)]$ is the buoyancy ratio parameter, Gc is the modified Grashof number, Re_x is the local Reynolds number, Sc and λ are the Schmidt number and non-dimensional chemical parameters defined as $Gc = \beta^* g\,(C_w - C_\infty)^2\,x^3/\upsilon^2$, $Re_x = u_r x/\upsilon$, $Sc = \upsilon/D$, $\lambda = K_0\alpha d\,(C_w - C_\infty)^{n-3}/(kg\beta^*)$, where the diffusivity ratio Le is the ratio of Schmidt number and Prandtl number and $u_r = \sqrt{g\beta d\,(T_w - T_\infty)}$ is the reference velocity. Now Eq. (9) can be written as

$$\varphi'' + \frac{1}{2} Le f \varphi' + \xi Ra_d\, Le\,(f'\varphi'' + f''\varphi') - \in \chi\varphi = 0. \tag{10}$$

And also χ is defined as $\chi = Sc\lambda\, Gc/Re_x^2$.

The boundary conditions become

$$f(0) = 0, \theta(0) = \phi(0) = 1\ , f'(\infty) = 1, \quad \theta(\infty) = \phi(\infty) = 0, \tag{11}$$

By the definition of stream function, the velocity components become $u = (\alpha Ra_x/x)\ f'$ and $v = -(\alpha Ra_x^{1/2}/2x)\ [f - \eta f']$. The local heat transfer rate from the surface of the plate is given by

$$q_w = -k_e\left[\frac{\partial T}{\partial y}\right]_{y=0}. \tag{12}$$

The Nusselt number is

$$Nu_x = \frac{q_w x}{(T_w - T_\infty)\,k_e}, \tag{13}$$

where k_e is the effective thermal conductivity of the porous medium which is the sum of the molecular and thermal conductivity k and the dispersion thermal conductivity k_d. Substituting the equations $\theta(\eta)$ and Eq. (12) into Eq. (13) we obtain the modified Nusselt number as $Nu_x/(Ra_x)^{1/2} = -[1 + \gamma Ra_d f'(0)]\,\theta'(0)$. Also the local mass flux at the vertical wall is given by $j_w = -D_y\,(\partial C/\partial y)_{y=0}$ defining another dimensionless variable which is the local Sherwood number Sh_x defined as $Sh_x = j_w x/[D\,(c_w - c_\infty)]$. This may also define another dimensionless variable i.e., $Sh_x/(Ra_x)^{1/2}$ which is equal to $-[1 + \zeta\, Ra_d f'(0)]\,\phi'(0)$.

The coupled non-linear ordinary differential Eqs. (7), (8) and (10) along with the boundary conditions (11) are solved numerically by the fourth-order Runge-Kutta method with the double shooting technique. By giving appropriate hypothetical values for $f'(0)$, $\theta'(0)$ and $\phi'(0)$ we get the corresponding boundary conditions for $f'(\infty)$, $\theta(\infty)$, $\phi(\infty)$ respectively. In addition, the boundary condition $\eta \to \infty$ is approximated by $\eta_{\max} = 4$ which is found to be sufficiently large for the velocity and temperature to approach the relevant free stream properties. This choice of $\eta_{\max}$ helps to compare our results with those of earlier works.

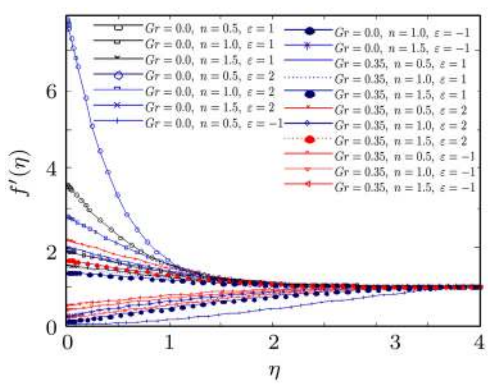

Fig. 1. Velocity f' against similarity space variable η ($N = -0.1, Le = 0.5, \chi = 0.02, Ra_d = 0.7, \gamma = \zeta = 0$).

When mixed convection parameter $\in = Ra_x/Pe_x > 0$, we observe from Figs. 1 and 2 that for fixed values Grashof number, mixed convection i.e., G_r^*, ε velocity decreases and temperature increases with an increase in power law index. For $\varepsilon = -1$ velocity increases and temperature decreases with an increase in power law index. It is also noted that for fixed G_r^*, n the velocity increases and temperature decreases with an increase in the mixed convection parameter. Also it can be observed that for fixed n, ε velocity decreases and temperature increases with an increase in Grashof number. From Fig. 3 it can be noted that for fixed n, ε

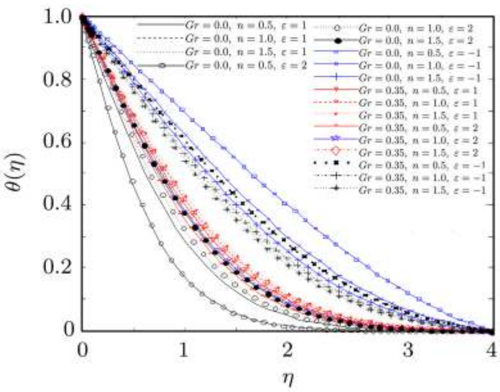

Fig. 2. Temperature θ against similarity space variable η ($N = -0.1, Le = 0.5, \chi = 0.02, Ra_d = 0.7, \gamma = \zeta = 0$).

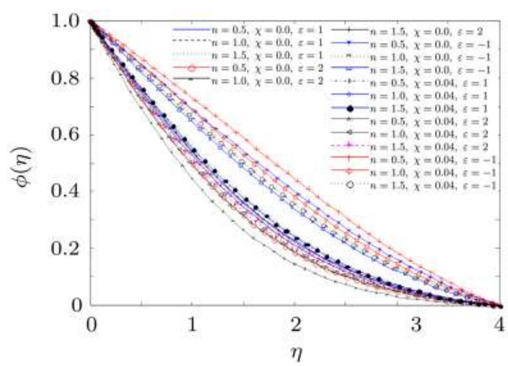

Fig. 3. Concentration ϕ against similarity space variable η ($G_r^* = 0.21, N = -0.1, Le = 0.5, Ra_d = 0.7, \gamma = \zeta = 0$).

concentration decreases with an increase in chemical reaction parameter χ. Also for fixed n, χ concentration decreases with an increase in mixed convection parameter. For fixed values of chemical reaction and mixed convection, concentration increases with an increase in power law index n. For $\varepsilon = -1$ concentration decreases with an increase in chemical reaction parameter and also noted that for fixed n, concentration increases with an increase of chemical reaction. From Figs. 4 and 5 it

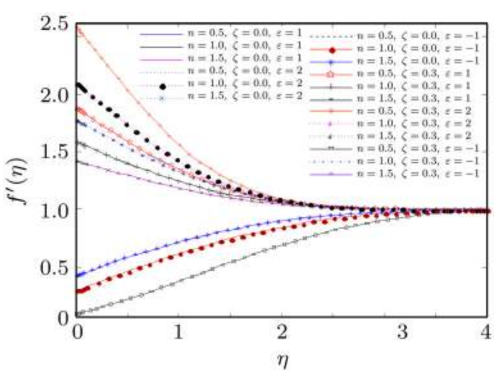

Fig. 4. Velocity f' against similarity space variable η ($G_r^* = 0.21, N = -0.1, Le = 0.5, Ra_d = 0.7, \chi = 0.02\gamma = 0$).

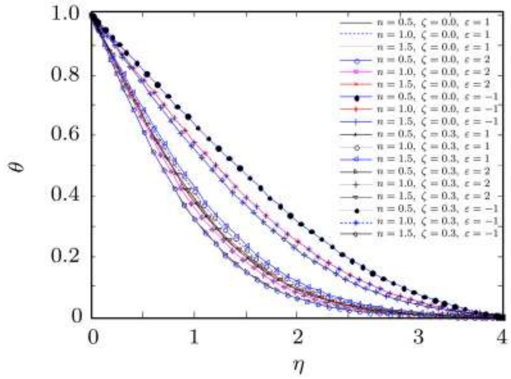

Fig. 5. Temperature against η ($G_r^* = 0.21, N = -0.1, Le = 0.5, Ra_d = 0.7, \chi = 0.02, \gamma = 0$).

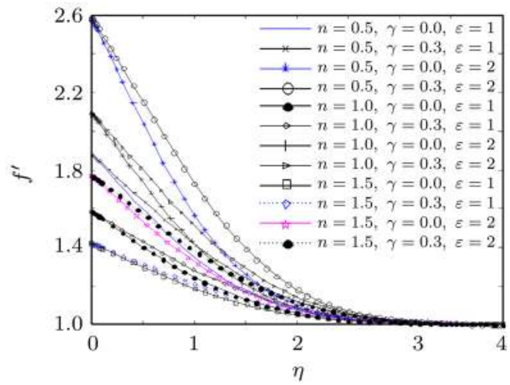

Fig. 6. Velocity against similarity space variable ($G_r^* = 0.21, N = -0.1, Le = 0.5, Ra_d = 0.7, \chi = 0.02, \zeta = 0$).

is clear that velocity decreases with an increase in solutal dispersion coefficient, for fixed values of n and ε temperature also increases with an increase in solute dispersion. For $\varepsilon = -1$ velocity increases with an increase in solute dispersion coefficient, except near the wall for smaller values of $n(= 0.5)$. It is observed from Figs. 6 and 7 that velocity and temperature increase

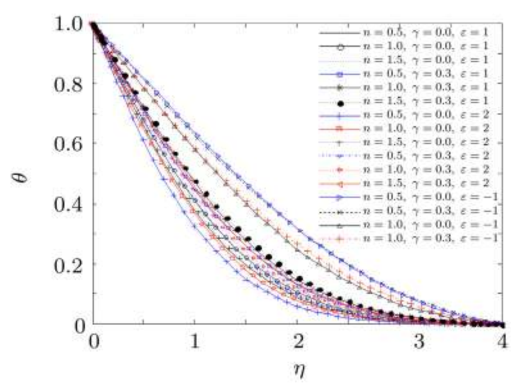

Fig. 7. Temperature θ against similarity space variable η ($G_r^* = 0.21, N = -0.1, Le = 0.5, Ra_d = 0.7, \chi = 0.02, \zeta = 0$).

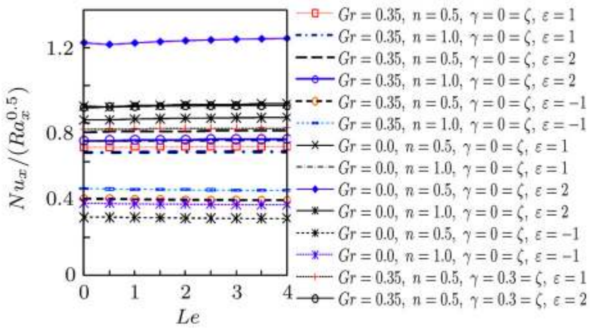

Fig. 8. Heat transfer rate against Lewis number ($\chi = 0.02, N = -0.1, Ra_d = 0.7$).

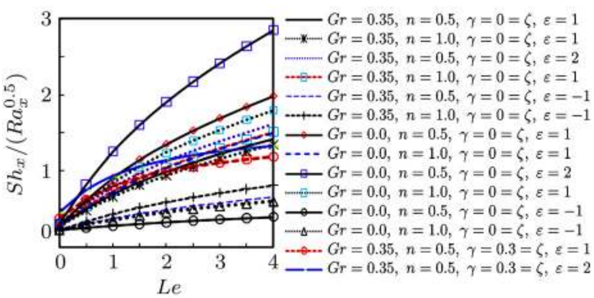

Fig. 9. Mass transfer rate against Lewis number ($\chi = 0.02, N = -0.1, Ra_d = 0.7$).

with an increase in thermal dispersion coefficient. For $\varepsilon = -1$ we observe that near the wall for small values of n (=0.5) temperature decreases with an increase in the thermal dispersion coefficient. From Figs. 8 and 9 we observe that heat transfer rates do not change significantly with the increase in Lewis number. Mixed convection parameter enhances both the heat mass transfer rates. Also it is observed that with the increase in the Grashof number the heat and mass transfer rates reduces. It can be seen that the increase in the value of n reduce the heat and mass transfer rates. Increase in the dispersion enhances the mass transfer rate near the wall and diminishes away from the wall.

Increase in the Grashof number reduces the heat and mass transfer rates. It can be seen that increase in the value of n reduces the heat and mass transfer rates. Increase in the dispersion enhances the mass transfer rate near the wall and diminishes away from the wall.

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