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Multi control scheme with modified Smith predictor for unstable first order plus time delay system

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ABSTRACT

A multi control scheme is proposed for unstable first order plus time delay (UFOPTD) system. The proposed control structure consists of two controllers which are intended for two distinct roles, set point tracking and disturbance rejection. A PI controller cascaded with a first order filter is employed as servo controller and a PID controller in series with a lead/lag filter is employed for disturbance rejection. The controller parameters are derived systematically using polynomial approach. The proposed scheme decouples the servo response from the regulatory response under nominal conditions which facilitates to tune the controllers independently. Analytical rules for the tuning of controllers are proposed based on maximum sensitivity (MS) which is a measure of robust stability. Set point weighting is employed to reduce the overshoot and settling time in the servo response. Several examples are included to show the effectiveness of the proposed structure.

1. Introduction

In industrial and chemical practice, open loop unstable and integrating systems are difficult to control. Continuous stirred tank reactor (CSTR) and polymerization furnaces are typical examples of open loop unstable processes. A good example of integrating process is an evaporation process in the food industry. Lot of academic research has been devoted to achieve effective control of unstable and integrating processes.

The proportional-integral-derivative (PID) controller is widely used in the process industries because of its well understood structure and robust performance in rejecting load changes. A detailed study of tuning of PI/PID controller is presented in [1] for various class of systems. The tuning techniques for unstable and integrating processes are complex and difficult to be dealt with when compared to that of stable processes. In practice, the complexity becomes high with the association of time delay. The time delays are unavoidable and inherent in the chemical process control loops due to measurement lag, process lag, recycle loops etc. Smith predictor is well known dead time compensator for a class of stable time delay systems. The concept of Smith predictor is successfully applied to unstable systems by De Paor [2] with a modified Smith predictor. Later several researchers extended the work of De Paor [3–7]. Simultaneously, internal model control (IMC) based methods also have been reported to achieve effective control of unstable time delay processes [8,9]. In these methods, attention is given to improve the closed loop performance evaluated by the error criterion. But these methods involve complex calculations which require user defined parameters and do not reduce overshoot satisfactorily in the closed loop responses. A modified IMC based controller design is proposed in [10] for integrating and unstable first order plus time delay (UFOPTD) systems. A set point filter is employed in this design and a PID controller with lead/lag filter is derived. The lead/lag filter is of higher order and involves complicated calculations in the controller design. A double two degree of freedom control structure with four controllers is proposed in [11] in which servo and regulatory responses are decoupled. A new modified Smith predictor with three controllers is designed in [12] with decoupled servo response and regulatory responses. The developed double two degree of freedom control and modified Smith predictor structures are complex as they involve more number of controllers. Later a control strategy with two tuning parameters for first order and second order unstable systems is proposed in [13]. Cascade control scheme augmented by the modified Smith predictor is reported for unstable time delay processes [14] and integrating processes [15]. An enhanced modified Smith predictor for a class of second order unstable systems is reported in [16] in which a PID controller with lead/lag filter is used for disturbance rejection. One of the lead/lag filter parameters is selected as a

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function of time delay. A direct synthesis based PID controller design for integrating systems where a lead/lag filter is cascaded to the controller is proposed in [17]. An IMC based PID controller is proposed by Panda [18] for unstable and integrating time delay systems. The derived tuning rules are based on Laurent series expansion and have shown good robust performance. A parallel control structure with direct synthesis based tuning is reported [19] for integrating systems where tuning parameters are selected to achieve maximum sensitivity (MS) equal to 2. A modified parallel control structure for a class of unstable systems with small time delay is proposed in [20]. In case of UFOPTD, the disturbance rejection controller is derived in PI form with lead/lag filter. One of the filter's parameters is taken as a function of time delay. This method is applicable only for the systems with small time delay to time constant ratios. A two degree of freedom (TDF) IMC method is proposed by Tan [21] which employs four controllers. Out of these four controllers, one is designed to stabilize the process, the second is for set point tracking, the third and fourth controllers are employed for disturbance rejection. These controllers are not derived in the form of PI(D) and the selection of their tuning parameters are inadequately justified. One of the controllers employed for disturbance rejection is of higher order and complex in nature in case of delay dominant UFOPTD process. Recently, Wang et al. [22] have developed PID tuning rules through IMC for stable and unstable processes with time delay. It is found that this method fails to control the delay dominant UFOPTD processes based on the analysis of developed tuning rules.

Review of the literature reveals that though there are many methods available to design PID controllers, still there is a scope to improve the performance and robustness of the control structure for UFOPTD processes. Recently proposed parallel control structure [20] and IMC based control [22] structures are effective for UFOPTD processes with small time delay to time constant ratios only. Therefore in the present work, design of modified Smith predictor with multiple controllers for both lag/delay time dominant UFOPTD systems is considered to enhance the robust performance. The proposed scheme consists of two controllers. A PI controller cascaded with a first order filter is employed for set point tracking and a PID controller cascaded with a lead/lag filter is designed for disturbance rejection. The parameters of the controllers are derived as a solution of Diophantine equations [23] using polynomial approach. The superiority of polynomial approach lies in its flexibility to select the controller form which makes it easier in deriving controller parameters. The main contribution of the present work is the design of new multi control scheme based on polynomial approach with improved robust stability for lag/delay time dominant UFOPTD systems. Simple analytical tuning rules are reported based on MS which is a measure of robust stability.

For clear illustration, the paper is organised as follows: proposed control structure is described in Section 2 whereas derivation of controller parameters and the set point weighting phenomena is elucidated in Section 3. Furthermore robust stability analysis is presented in Section 4. Simulation results and comparison are carried out in Section 5 followed by conclusion in Section 6.

2. Proposed control structure

Typical transfer function of a UFOPTD system is

$$G(s) = G_p(s)e^{-s\theta} = \frac{ke^{-s\theta}}{(\tau s - 1)} \tag{1}$$

The block diagram of the proposed control scheme is shown in Fig. 1. Here, G_p is the transfer function of unstable plant, θ_p is the plant time delay, G_m is the transfer function of the plant model and θ_m is the model time delay. G_{cs} is set point tracking controller and G_{cd} is disturbance rejection controller and K_1 is a constant (proportional controller), which is meant for stabilizing the delay free unstable process. The closed loop transfer functions for servo and regulatory responses under nominal conditions $(G_p e^{-s\theta_p} = G_m e^{-s\theta_m})$ are shown in (2) and (3) respectively.

$$\frac{y}{r} = \frac{K_1 G_{cs} G_m e^{-s\theta_m}}{(1 + K_1 G_m + K_1 G_{cs} G_m)} \tag{2}$$

$$\frac{y}{d} = \frac{K_1 G_m e^{-s\theta_m}}{1 + K_1 G_{cd} G_m e^{-s\theta_m}} \tag{3}$$

From (2) and (3), it is clearly evident that servo and regulatory responses are decoupled from each other, thus making it easy for independent tuning of the controllers.

3. Controller design

In the present work, the controllers of the proposed structure are designed using the polynomial approach.

3.1. Design of G_{cs}

G_{cs} is considered as PI controller with a first order filter as shown in (4). Addition of filter to the PI controller improves the total variation (TV) of manipulated input.

$$G_{cs} = \frac{q}{p} = \left(k_p + \frac{k_i}{s}\right)\frac{1}{\tau_f s + 1} \tag{4}$$

where $q = k_p s + k_i$ and $p = s(\tau_f s + 1)$

The delay free process is considered as ratio of two polynomials as shown in (5).

$$G_m = \frac{b}{a} \tag{5}$$

where $a = \tau s - 1$ and $b = k$

with the help of (4) and (5), (2) is modified as

$$\frac{y}{r} = \frac{bqK_1}{ap + bq + bpK_1} e^{-s\theta_m} \tag{6}$$

From (6), it is clear that degree of denominator is greater than or equal to that of numerator. The characteristic equation (CE) of servo response is rewritten as shown in (7) using (4) and (5) and $K_1 = 2/k$. The reason for selecting particular value for K_1 is explained in the next section.

$$CE = s^3 + \frac{\tau_f + \tau}{\tau_f \tau}s^2 + \frac{2k_p + 1}{\tau_f \tau}s + \frac{2k_i}{\tau_f \tau} = 0 \tag{7}$$

The CE is solved to have three poles at λ_s as in (8).

$$s^3 + \frac{\tau_f + \tau}{\tau_f \tau}s^2 + \frac{2k_p + 1}{\tau_f \tau}s + \frac{2k_i}{\tau_f \tau} = (s + \lambda_s)^3 \tag{8}$$

G_{cs} parameters are derived as shown in (9) by solving (8)

$$k_p = \frac{3{\lambda_s}^2\tau^2 - 3\lambda_s\tau + 1}{(6\lambda_s\tau - 2)} \tag{9a}$$

$$k_i = \frac{{\lambda_s}^3\tau^2}{(6\lambda_s\tau - 2)} \tag{9b}$$

$$\tau_f = \frac{\tau}{3\lambda_s\tau - 1} \tag{9c}$$

Here λ_s is the tuning parameter which is to be selected properly to obtain good servo performance. More emphasis on selection of λ_s is presented in Section 3.4.1.

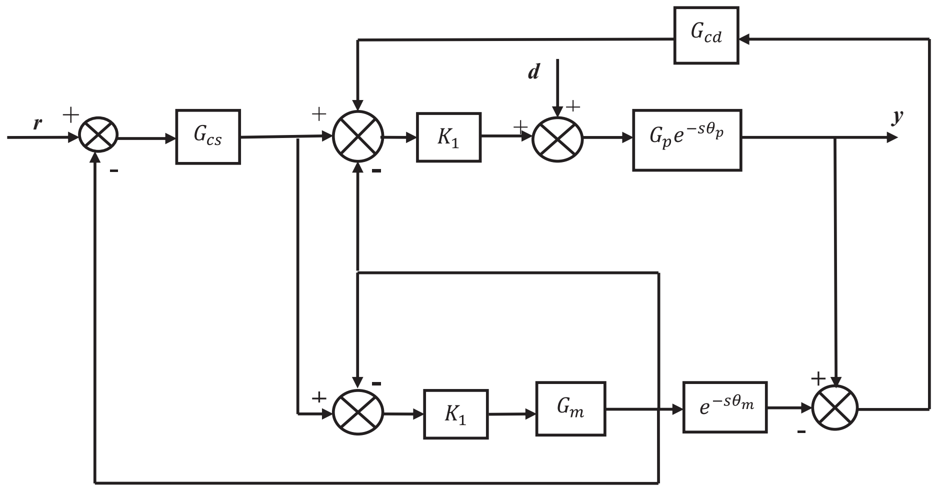

Fig. 1. Proposed control scheme.

3.2. Selection of K_1

The block diagram representation of servo response (2) is shown in Fig. 2. The constant K_1 is selected such that the inner loop dynamics, i.e., $G^* = K_1G_m/(1 + K_1G_m)$ is stabilized. Simple analysis leads to a constraint mentioned in (4) to obtain a stable G^*.

$$K_1 > \frac{1}{k} \tag{10}$$

The value of K_1 should be selected in order to make the closed loop response faster, at the same time smoothness in the manipulated variable should be preserved. As there exist trade-offs between speed of response and smoothness in the manipulated variable (control effort), the K_1 value should be selected properly. The procedure used to select K_1 is as follows.

The variations in integral absolute error (IAE) and TV are plotted against $K_1 > 1/k$ at a fixed MS value of 1.2 for the servo loop using a normalized UFOPTD system. Fig. 3 shows the graph drawn between normalized performance indices IAE, TV and K_1.

The steps involved in the construction of Fig. 3 are as follows

1. The λ_s value is calculated to achieve MS = 1.2 for servo loop in the range $\frac{1.5}{k} \le K_1 \le \frac{5}{k}$.
2. The $\boldsymbol{G_{cs}}$ parameters are determined for various values of λ_s calculated in step 1.
3. The servo loop shown in Fig. 2 is simulated using $\boldsymbol{G_{cs}}$ parameters obtained in step 2 and then IAE and TV are calculated for the range $\frac{1.5}{k} \le K_1 \le \frac{5}{k}$.
4. The graph between K_1 and normalised IAE, TV is plotted.

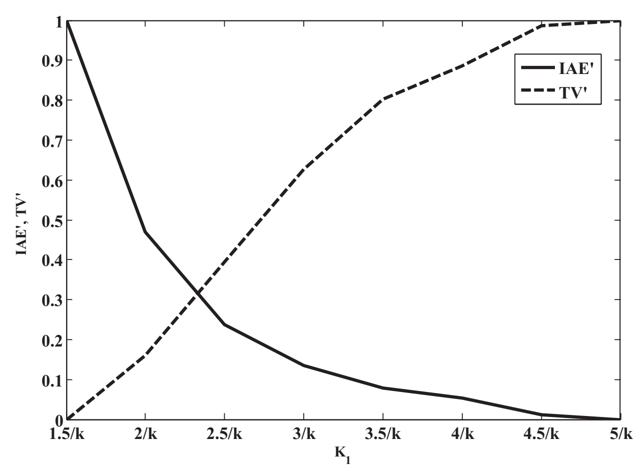

Fig. 3. Variation of IAE, TV with K_1.

From the Fig. 3, it is clear that the higher values of K_1 provide good IAE but they introduce large variations in the manipulated variable, i.e., large TV. Similarly, the lower values of K_1 provide good TV, but they cause large IAE. Thus, the authors suggest $2/k \le K_1 \le 2.5/k$ as a trade-off between IAE and TV. The authors considered the value of K_1 as 2/k in the present design.

$$K_1 = \frac{2}{k} \tag{11}$$

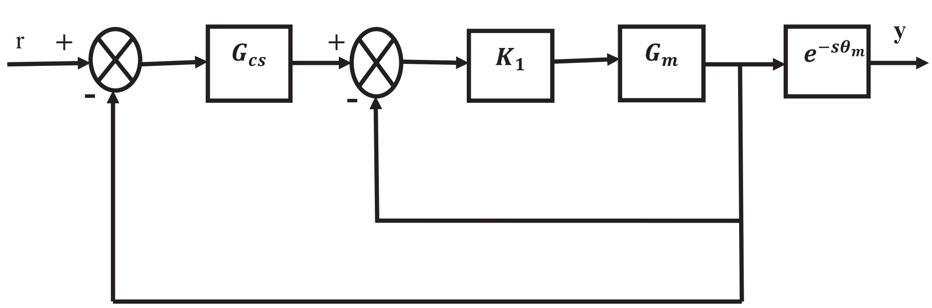

Fig. 2. Block diagram representation of servo response.

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3.3. Design of G_{cd}

In literature, researchers [10,16,17,19,20] employed PI/PID controller with lead/lag filter for effective disturbance rejection. In the present work, G_{cd} is considered as a PID controller with lead lag filter given in (12).

$$G_{cd} = \frac{u}{v} = \left(k_{pd} + \frac{k_{id}}{s} + k_{dd}s\right)\frac{\alpha s+1}{\beta s+1} \tag{12}$$

where $u = (k_{dd}s^2 + k_{pd}s + k_{id})(\alpha s+1)$ and $v = s(\beta s+1)$

Using (5) and (12), (3) is rewritten as

$$\frac{y}{d} = \frac{K_1 b v e^{-s\theta_m}}{a v + b u K_1 e^{-s\theta_m}} \tag{13}$$

By using first order Pade's approximation of delay term in the denominator, (13) is modified as shown in (14) with the help (5) and (12).

$$\frac{y}{d} = \frac{K_1 ks(\beta s+1)(1+s\frac{\theta_m}{2})}{s(\tau s-1)(\beta s+1)(1+s\frac{\theta_m}{2}) + kK_1(k_{dd}s^2+k_{pd}s+k_{id})(\alpha s+1)(1-s\frac{\theta_m}{2})}e^{-s\theta_m} \tag{14}$$

The term $(1 - s\theta_m/2)$ in the denominator of (14) which is an approximation of $e^{-(s\theta_m)/2}$ is again modified as $e^{-s\theta_m/2} = (1 - s\theta_m/4)/(1 + s\theta_m/4)$ using first order Pade's approximation. By selecting $\alpha = \theta_m/4$, CE of (14) is rewritten as

$$CE = s^4 + d_3s^3 + d_2s^2 + d_1s + d_0 = 0 \tag{15}$$

where

$$d_3 = \frac{2\beta\tau - k_{dd}\theta_m - \theta_m(\beta - \tau)}{\beta\tau\theta_m} \tag{15a}$$

$$d_2 = \frac{4k_{dd} - 2\beta + 2\tau - \theta_m - k_{pd}\theta_m}{\beta\tau\theta_m} \tag{15b}$$

$$d_1 = \frac{4k_{pd} - k_{id}\theta_m - 2}{\beta\tau\theta_m} \tag{15c}$$

$$d_0 = \frac{4k_{id}}{\beta\tau\theta_m} \tag{15d}$$

From (14), it is observed that the controller is introducing a zero in the regulatory response at $-2/\theta_m$. This zero may cause undesired overshoot/undershoot in the response. So, CE is solved to have three poles at λ_d, and one pole at $-4/\theta_m$. The pole placed at $-4/\theta_m$ is meant for compensating the effect of zero. If the pole is placed far from the actual zero on left side, overshoot/undershoot is not attenuated to a great extent where as if the pole is placed far from the zero on right side close to imaginary axis, overshoot/undershoot is minimized but the speed of response is reduced. So as a trade-off, pole is placed at $-4/\theta_m$. This is a compromise between speed of response and overshoot/undershoot.

$$s^4 + d_3s^3 + d_2s^2 + d_1s + d_0 = (s+\lambda_d)^3\left(s + \frac{4}{\theta_m}\right) \tag{16}$$

The derived disturbance rejection controller parameters are

$$k_{pd} = \frac{6\lambda_d^3\tau^2\theta_m^2 - \lambda_d^3\tau\theta_m^3 + 36\lambda_d^2\tau^2\theta_m - 3\lambda_d^2\tau\theta_m^2 + 24\lambda_d\tau\theta_m + 8\tau + 6\theta_m}{\tau\lambda_d^3\theta_m^3 + 12\tau\lambda_d^2\theta_m^2 + 48\lambda_d\tau\theta_m + 12\theta_m + 16\tau} \tag{16a}$$

$$k_{id} = \frac{3\lambda_d(4\lambda_d^2\tau^2\theta_m - \lambda_d^2\tau\theta_m^2)}{\tau\lambda_d^3\theta_m^3 + 12\tau\lambda_d^2\theta_m^2 + 48\lambda_d\tau\theta_m + 12\theta_m + 16\tau} \tag{16b}$$

$$k_{dd} = \frac{\lambda_d^3\tau^2\theta_m^3 + 12\tau^2\lambda_d^2\theta_m^2 + 12\lambda_d\tau^2\theta_m + 9\lambda_d\tau\theta_m^2 - 8\tau^2 + 6\tau\theta_m + 3\theta_m^2}{\tau\lambda_d^3\theta_m^3 + 12\tau\lambda_d^2\theta_m^2 + 48\lambda_d\tau\theta_m + 12\theta_m + 16\tau} \tag{16c}$$

$$\beta = \frac{k_{id}}{\lambda_d^3\tau} \tag{16d}$$

$$\alpha = \frac{\theta_m}{4} \tag{16e}$$

Here λ_d is the tuning parameter which is to be selected as a trade-off between robust performance and nominal performance. More emphasis on selection of λ_d is presented in Section 3.4.2.

3.4. Selection of tuning parameters λ_s, λ_d and set point weighting parameter

Proper selection of tuning parameters is very important as it is directly related to the performance of the control structure. In literature, some of the proposed methods [6,7,12,13] selected tuning parameter as a function of time delay. A more analytical way of selection of tuning parameters is addressed in [10,17,19,20] where MS based tuning parameter selection is considered. MS is defined as the inverse of shortest distance to the critical point (−1, 0) from the Nyquist curve of the loop transfer function or in other words MS is the maximum magnitude of sensitivity function S = 1/(1 + L) where L is loop transfer function.

$$MS = max\left|\frac{1}{1+L}\right| \tag{17}$$

The proposed control structure comprises different loop transfer functions for servo and regulatory responses. The loop transfer function given in (18) for servo response is derived using (2) and Fig. 2.

$$L_s = \frac{K_1G_mG_{cs}}{(1 + K_1G_m)} \tag{18}$$

Similarly, the loop transfer function for regulatory response is derived as

$$L_d = K_1G_{cd}G_me^{-s\theta_m} \tag{19}$$

A control loop with a lower MS value is more stable for uncertainties in the process and a loop with a higher MS value is more susceptible to uncertainties of the process. In the present work, MS based tuning parameter selection is proposed to achieve desired performance.

3.4.1. Selection of λ_s

The value of λ_s should be selected in order to make the servo loop response faster, at the same time smoothness in the manipulated variable should be preserved. The $\lambda_s$value should be selected properly as there exist trade-off between speed of response and smoothness in the manipulated variable (control effort). The authors used the following procedure to select λ_s.

The variation in MS of the servo loop is plotted against λ_s using a normalized UFOPTD system and is shown in Fig. 4. From the Fig. 4, it is observed that almost similar values of MS are obtained over a wide range of λ_s. In order to choose the optimum value of λ_s from this wide range, the variation in IAE and TV are plotted against a range $0.5 \le \lambda_s\tau \le 4$. Fig. 5 shows the relationship between the normalized performance indices IAE, TV and λ_s.

It can be observed that the higher values of $\lambda_s\tau$ ensure good IAE and result large, rapidly varying TV. Similarly the lower values of $\lambda_s\tau$ ensure good TV but result in large IAE. So, the range $1.5 \le \lambda_s\tau \le 2.5$ is suggested by authors as a trade-off between

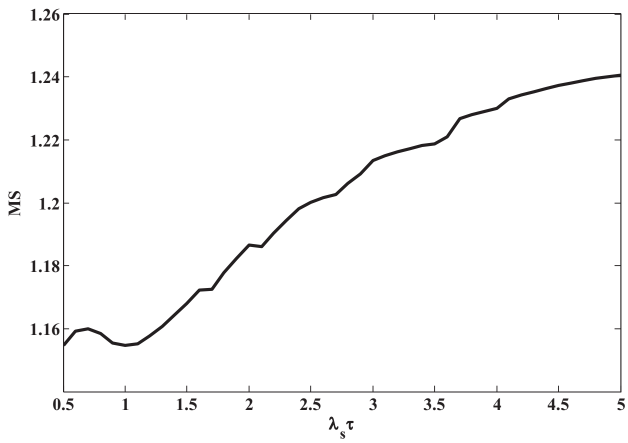

Fig. 4. Variation of MS of servo response loop with respect to λ_s.

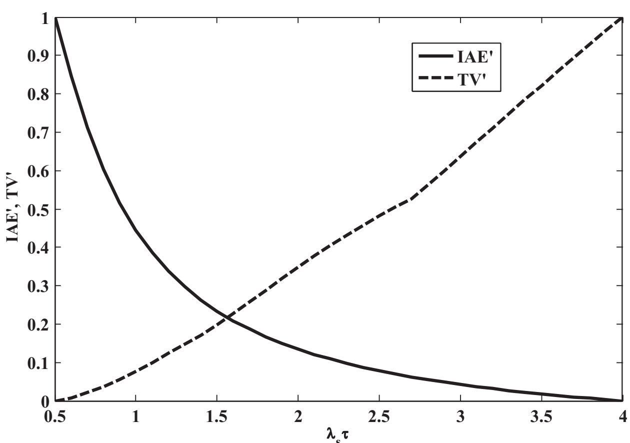

Fig. 5. Variation of IAE and TV for servo loop with respect to λ_s.

IAE and TV. The authors considered the value of $\lambda_s\tau$ as 2.5 in the present work.

$$\lambda_s = \frac{2.5}{\tau} \tag{20}$$

3.4.2. Selection of λ_d

In literature [10,17,19,20], MS values greater than 2 are used to achieve required performance because it is not possible to achieve lower MS values in case of unstable systems with time delay. In the proposed method, initial guess of λ_d is that which corresponds to the lowest possible MS value. Fig. 6 shows the relationship between tuning parameter λ_d and MS for regulatory response loop for two different time delay to time constant ratio values.

From Fig. 6 it can be observed that for lower values of θ/τ, author has more freedom to select λ_d from wide range of values as the variation of MS is not aggressive around the minimum possible MS with respect to λ_d but not in the case of higher θ/τ values. The expression for λ_d corrsponding to the lowest possible MS and the corresponding expression for MS are mathematically given in (21) and (22) respectively. These are derived by calculating the MS value for each θ/τ and then curve fitting tool is used to generate a relationship.

$$\lambda_d = \frac{1}{\tau}\left(\frac{-0.1714\left(\frac{\theta}{\tau}\right)^2 + 0.166\left(\frac{\theta}{\tau}\right) + 0.4714}{\left(\frac{\theta}{\tau}\right) - 0.008003}\right) \tag{21}$$

$$MS = \frac{23.52}{\left(\frac{\theta}{\tau}\right)^2 - 11.75\left(\frac{\theta}{\tau}\right) + 15.5} \tag{22}$$

3.4.3. Set point weighting

From (2), it can be understood that the G_{cs} introduces a zero in the servo response which causes overshoot. Moving the zero far from the imaginary axis reduces the overshoot and the settling time. This can be achieved by implementing set point weighted PID controller as shown in (23).

$$u(t) = k_c e_p(t) + k_i e(t) + k_d \frac{de(t)}{dt} \tag{23}$$

where $e_p(t) = \varepsilon y_{sp} - y$ and $e(t) = y_{sp} - y$.

Here $\mathcal{E}$ is the set point weighting parameter lies between 0 and 1. The values of $\mathcal{E}$ close to 0 reduce the overshoot significantly at the cost of speed. The values of $\mathcal{E}$ close to 1 offer good speed of response but cause high overshoot. So the selection of $\mathcal{E}$ is a trade-off between the speed of response and over shoot. In the present work, $\mathcal{E}$ value is taken as 0.4 in the simulated examples. For clear understanding, the proposed controller tuning rules given in Section 3 are summarized as follows.

Step 1: For a fixed MS value and the set point weighting parameter of servo loop, select K_1 value such that the inner loop of servo response (2) is stabilized and also to provide good compromise between speed of response and control effort. The recommended values of MS are typically in the range $1.2 \le \text{MS} \le 2$. The authors have considered required MS as 1.2, $K_1 = 2/k$. The set point weighting parameter is selected $\varepsilon = 0.4$ as a trade-off between the speed of response and overshoot.
Step 2: Select the range of λ_s to achieve the desired MS value taken in step 1. If the MS value is not varying/slowly varying for a specific range of λ_s values, choose a λ_s value which does provide a good compromise between IAE and TV. The authors recommend the value of λ_s as $1.5/\tau \le \lambda_s \le 2.5/\tau$. In the present work, the simulations are carried out with $\lambda_s = 2.5/\tau$.
Step 3: Select a λ_d value which gives the lowest possible MS value for the regulatory loop as an initial choice using (21). For the processes with lower values of $/\tau$, one can vary this initially selected λ_d to get the required performance but the value should be kept close to initially selected λ_d for the processes with higher values of θ/τ.
Step 4: Use the tuning parameters λ_s and λ_d obtained in step 2 and step 3 respectively to calculate the set point, disturbance rejection controller parameters using (9) and (16) for the UFOPTD process of interest.

4. Robust stability analysis

The controller parameters are derived using the approximate model of true dynamics of the actual system. It is therefore necessary to analyze the robust stability of the control system in the presence of uncertainties. The types of uncertainties considered are: the parametric uncertainties such as uncertainty in the process gain, time constant, and time delay. The robust stability analysis for the proposed method is carried out using well known and widely used small gain theorem for multiplicative uncertainty represented by M-Δ structure [24]. The proposed structure consists of two major loops for two distinct goals that are set point tracking and disturbance rejection.

4.1. Robust stability analysis of disturbance rejection loop

The closed loop system is robustly stable if and only if the constraint given in (24) is met based on the small gain theorem.

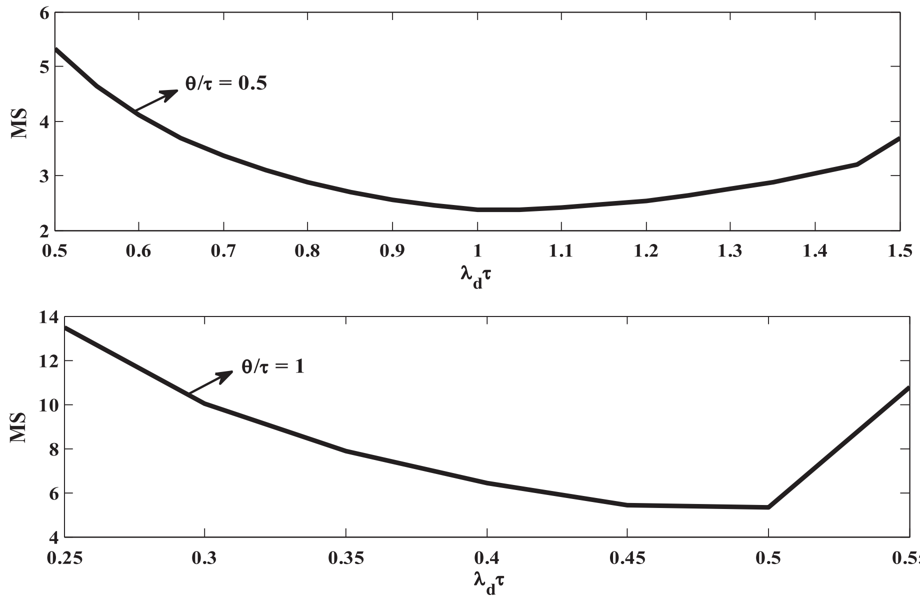

Fig. 6. Variation of MS for regulatory response loop with respect to λ_d.

$$\|l_m(j\omega)T_d(j\omega)\| < 1 \qquad \forall\omega(-\infty,\infty) \tag{24}$$

where $T_d(j\omega)$ is the complimentary sensitivity function. Complimentary sensitivity function is expressed as $L_d(j\omega)/(1 + L_d(j\omega))$ where $L_d(j\omega)$ is the loop transfer function of regulatory response (3).

$$L_d(j\omega) = K_1 G_{cd}(j\omega)G_m(j\omega)e^{-s\theta_m} \tag{25}$$

and

$$T_d(j\omega) = \frac{K_1 G_{cd}(j\omega)G_m(j\omega)e^{-s\theta_m}}{1 + K_1 G_{cd}(j\omega)G_m(j\omega)e^{-s\theta_m}} \tag{26}$$

Using Eqs. (5), (11), (12) and (16) along with Pade's approximation of delay

$$T_d(j\omega) = \frac{(k_{id} - k_{dd}\omega^2 + jk_{pd}\omega)\left(1 - \frac{j\omega\theta}{2}\right)}{\beta\tau(j\omega + \lambda_d)^3} \tag{27}$$

Bound on complimentary sensitivity function $l_m(j\omega)$ is

$$l_m(j\omega) = \frac{G_p(j\omega)e^{-s\theta_p} - G_m(j\omega)e^{-s\theta_m}}{G_m(j\omega)e^{-s\theta_m}} \tag{28}$$

where $G_p(j\omega)e^{-s\theta_p}$ is the actual process and $G_m(j\omega)e^{-s\theta_m}$ is the assumed process.

If the uncertainty exists in the time delay θ_m, say $\theta_p = \theta_m + \Delta\theta_p$

$$l_m(j\omega) = \frac{G_m(j\omega)e^{-s(\theta_m+\Delta\theta_p)} - G_m(j\omega)e^{-s\theta_m}}{G_m(j\omega)e^{-s\theta_m}} = \frac{e^{-s(\theta_m+\Delta\theta_p)} - e^{-s\theta_m}}{e^{-s\theta_m}} \simeq \frac{-j\omega\Delta\theta_p}{j\omega\frac{\Delta\theta_p}{2} + 1} \tag{29}$$

Using (24) and (29)

$$\|T_d(j\omega)\|_\infty < \frac{1}{\left|\frac{-j\omega\Delta\theta_p}{j\omega\frac{\Delta\theta_p}{2}+1}\right|} \tag{30}$$

Similarly, if the uncertainty exists in the parameter gain k, selection of λ_d should meet the constraint

$$\|T_d(j\omega)\|_\infty < \frac{1}{\left|\frac{\Delta k}{k}\right|} \tag{31}$$

If the uncertainty exists in the parameter unstable time constant τ, selection of λ_d should meet the constraint

$$\|T_d(j\omega)\|_\infty < \frac{1}{\left|\frac{-\Delta\tau j\omega}{(\tau+\Delta\tau j\omega)-1}\right|} \tag{32}$$

Similarly, the conditions for robust stability of servo response loop also can be verified.

5. Simulation results

In this section, the effectiveness of the proposed method is compared with the recently reported methods in the literature. The performance of the proposed method is evaluated in terms of IAE, integral square error (ISE), TV and setting time (t_s). Mathematical description of various performance indices are given through (33)–(36).

$$\text{IAE} = \int_0^\infty |e| \; dt \tag{33}$$

$$\text{ISE} = \int_0^\infty e^2 \; dt \tag{34}$$

$$\text{ITAE} = \int_0^\infty t|e| \; dt \tag{35}$$

where e is the error. From the mathematical description, it can be understood that IAE treats all the errors equally, ISE penalizes large errors and integral time absolute error (ITAE) rejects long lasting errors.

$$\text{TV} = \sum_{i=0}^{\infty} |u_{i+1} - u_i| \tag{36}$$

where u_i and u_{i+1} are the process inputs at ith and $(i + 1)$th instants respectively. TV given in (36) is a measure of smoothness of the manipulated variable. The smaller value of TV ensures the smooth variations in the manipulated variable causing less wear and tear of process equipment.

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Example 1. In this example a lag time dominant UFOPTD process with θ/τ = 0.2 is considered.

$$G(s) = \frac{1}{s-1}e^{-0.2s} \tag{37}$$

The proposed method is compared against a recently reported method by Ajmeri & Ali [20] in which a parallel control structure is proposed with two controllers. The set point weighting or set point filter is not used in the method reported in [20] because of low overshoot in servo response. Ajmeri & Ali [20] have suggested the set point tracking controller tuning parameter is to be equal to time delay and the disturbance rejection controller tuning parameter is suggested to be taken as equal to 1.3 times of time delay. The proposed method suggests λ_s as 2.5 according to (20). λ_d is selected as 2.6 using (21) and adjusted to 3.5 for achieving better performance according to the discussion made in Section 3.4.2. At this value of tuning parameter, MS is 2. The controller parameters are mentioned in Table 1. With these settings both the methods are tested against a unit step set point change at t = 0 s and negative unit step disturbance at t = 10 s. Nominal performance is presented in Fig. 7.

To show the effect of set point weighting employed in proposed method, the proposed method is worked out for various set point weighting parameter values and presented in Fig. 8. To analyse the robust performance a +20% error in time delay and −10% error in time constant are assumed. With these perturbations simulations are carried out and the results are presented in Fig. 9 and Table 2. From Figs. 7, 9, Tables 1 and 2, though the proposed method's performance is not superior in servo response, the proposed method offers considerable improvement in the disturbance rejection under both nominal and perturbed conditions when compared to the method proposed by [20]. From Fig. 9 it can be understood that the proposed method is more robust when compared to [20].

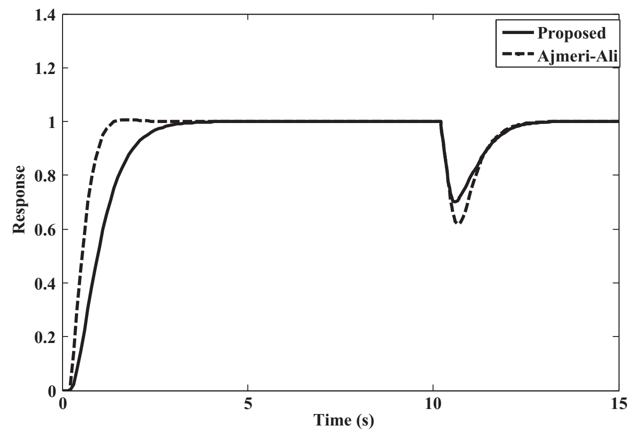

Fig. 7. Nominal response of Example 1.

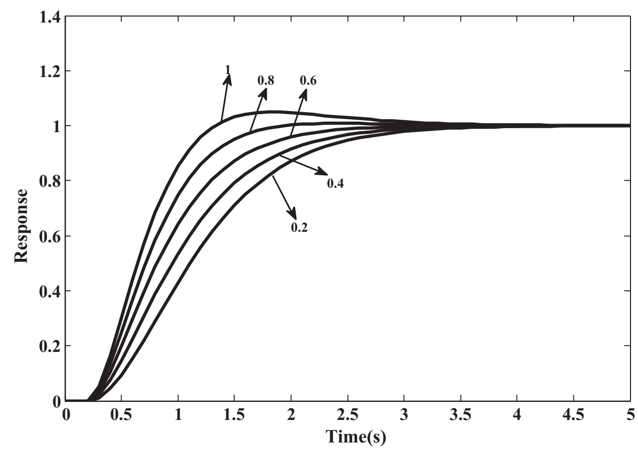

Fig. 8. Servo response of Example 1 for various set point weighting parameters.

Example 2. In this example the UFOPTD process with θ/τ = 0.5 is considered.

$$G(s) = \frac{1}{s-1}e^{-0.5s} \tag{38}$$

With the help of (20) λ_s is derived as 2.5 which gives a MS of 1.2 for servo response loop. λ_d is selected around the value specified by (21) as 1.2 which corresponds to a MS of 2.544. The controller parameters are mentioned in Table 1 along with the controller parameters derived for the method proposed by Ajmeri-Ali [20]. The performance under nominal conditions for a step change in the set point at t = 0 s and a negative step change in the disturbance at t = 15 s is presented in Figs. 10, 11 and Table 2. A +20% perturbation is considered in time delay to analyse the performance under perturbed conditions and results are presented in Figs. 12, 13 and Table 3. From Figs. 10–13, Tables 2 and 3, it can be under stood that both the methods offer equal performance under nominal conditions but under perturbed conditions there is significant improvement with the proposed method especially in the case of disturbance rejection.

Table 1
Details of controller parameters.

Process	Method	Set point tracking controller	Disturbance rejection Controller
$\frac{1}{s-1}e^{-0.2s}$	Proposed[a]	$\left(0.9423+\frac{1.2019}{s}\right)\frac{1}{0.1538s+1}$	$\left(2.107+\frac{1.6790}{s}+0.1581s\right)\frac{0.05s+1}{0.0392s+1}$
	Ajmeri-Ali[b]	$\left(1.25+\frac{12.5}{s}\right)$	$\left(3.5683+\frac{2.9185}{s}\right)\frac{0.1s+1}{0.051s+1}$
$\frac{1}{s-1}e^{-0.5s}$	Proposed[a]	$\left(0.9423+\frac{1.2019}{s}\right)\frac{1}{0.1538s+1}$	$\left(0.950+\frac{0.164}{s}+0.1841s\right)\frac{0.125s+1}{0.095s+1}$
	Ajmeri-Ali[c]	$\left(0.5+\frac{2}{s}\right)$	$\left(1.7887+\frac{0.3585}{s}\right)\frac{0.25s+1}{0.0984s+1}$
$\frac{1}{s-1}e^{-1.2s}$	Proposed[a]	$\left(0.9423+\frac{1.2019}{s}\right)\frac{1}{0.1538s+1}$	$\left(0.5786+\frac{0.0088}{s}+0.293s\right)\frac{0.3s+1}{0.189s+1}$
	Shamsuzzoha[d]	$\left(0.0317+\frac{0.0396}{s}+0.0095s\right)\frac{29.748s+1}{0.271s+1}$	–
	Tan[g]	$K_1=\frac{s+1}{2s+1}$	$K_2=\frac{0.02821s^3+0.1828s^2+0.7374s+1.135}{0.00552s^3+0.0511s^2+0.04532s+1}K_3=\frac{s+1}{2s+1}$
$\frac{3.433}{103.1s-1}e^{-20s}$	Proposed[e]	$\left(2.5+\frac{0.0202}{s}\right)\frac{1}{13.7461s+1}$	$\left(2.162+\frac{0.0174}{s}+16.173s\right)\frac{5s+1}{3.927s+1}$
	Ajmeri-Ali[f]	$\left(0.3754+\frac{0.03754}{s}\right)$	$\left(1.0665+\frac{0.0088}{s}\right)\frac{10s+1}{5.1556s+1}$

[a] K_1 = 2.
[b] k_c = 8.5.
[c] k_c = 4.
[d] F(s) = 1/(29.7476s + 1).
[e] K_1 = 0.5826.
[f] $k_c = 2.5437$.
[g] K_0 = 2.

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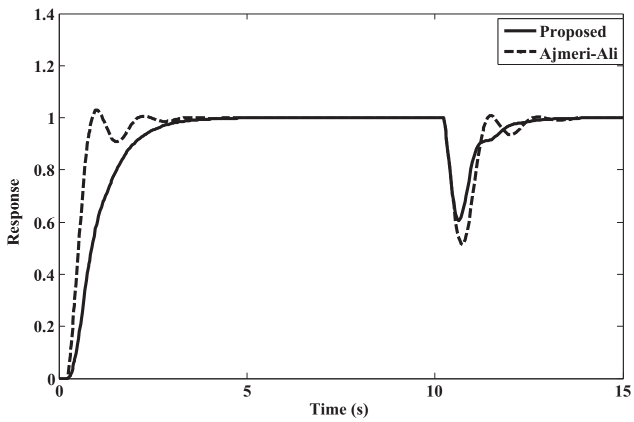

Fig. 9. Perturbed response of Example 1.

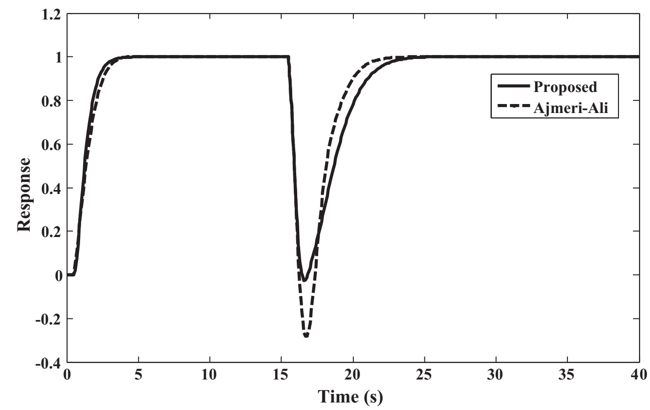

Fig. 10. Nominal response of Example 2.

A perturbation of +20% (0.2 units) is considered in the process gain (k) and +50% perturbation (0.25 units) in time delay θ_p are considered to analyse the robust stability of the regulatory response loop of the proposed method. Using (29), corresponding bounds are derived as

$$l_m(j\omega)_{for\ \ \Delta k=0.2} = 0.2 \tag{39}$$

$$l_m(j\omega)_{for\ \ \Delta\theta_p=0.25} = \frac{-j0.25\omega}{j0.125\omega+1} \tag{40}$$

Fig. 14 shows the magnitude plots of complementary sensitivity function (27) and the bounds. From Fig. 14, it can be understood that the proposed method is stable for a perturbation of 0.2 units in the process gain or 0.25 units perturbation in the time delay. Similarly, the robust performance is further analysed for a perturbation of -0.5 units in the process time constant. This corresponds to a bound

$$l_m(j\omega) = \frac{j0.5\omega}{j0.5\omega-1} \tag{41}$$

The magnitude plot corresponding to the bound in (41) is shown in Fig. 15. It is evident from the Fig. 15 that the system is not stable for the selected tuning parameter for a perturbation of -0.5 units in the time constant. To further verify this, the regulatory response is verified for the proposed perturbances and the results are presented in Figs. 16 and 17. So, the robust stability constraints (30)–(32) are verified by the Figs. 14–17.

Example 3. In this example a delay time dominant process with $\theta/\tau = 1.2$ is considered.

$$G(s) = \frac{1}{s-1}e^{-1.2s} \tag{42}$$

Here, the proposed method is compared with the methods proposed by Shamsuzzoha [10] and Tan [21]. Shamsuzzoha [10] proposed an IMC controller with set point filter and derived a PID

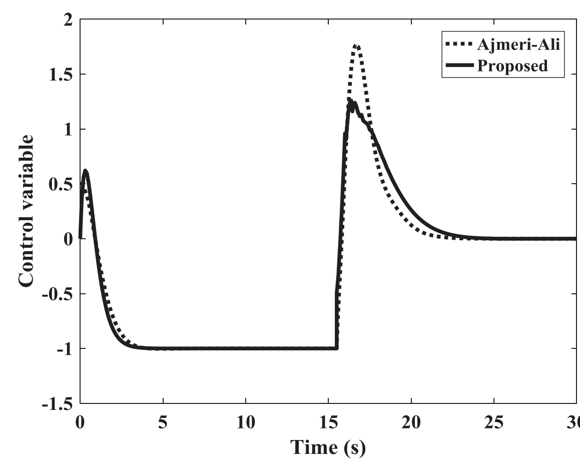

Fig. 11. Control signal of Example 2 for nominal response.

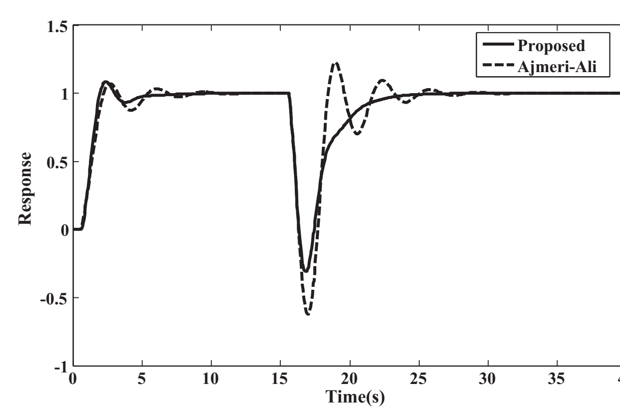

Fig. 12. Perturbed response of Example 2.

Table 2
Performance comparison under nominal conditions.

Process	Method	Set point tracking				Disturbance rejection			
		$t_s(s)$	IAE	ISE	TV	$t_s(s)$	IAE	ISE	TV
$\frac{1}{s-1}e^{-0.2s}$	Proposed	2.7513	1.087	0.7571	2.192	2.6825	0.298	0.06	1.996
	Ajmeri-Ali	1.3463	0.604	0.4411	3.577	2.4721	0.342	0.088	3.5038
$\frac{1}{s-1}e^{-0.5s}$	Proposed	3.0534	1.387	1.057	2.191	7.8	3.052	2.104	3.4767
	Ajmeri-Ali	3.370	1.508	1.104	1.507	6.26	2.79	2.363	4.5056
$\frac{1}{s-1}e^{-1.2s}$	Proposed	3.753	2.087	1.757	2.192	24.136	5.70	2.115	1.675
	Shamsuzzoha	7.836	4.578	3.543	1.920	12.875	3.343	1.057	4.470
	Tan	9.0245	3.2	2.2	2.0	15.7743	2.37	0.678	1.0823

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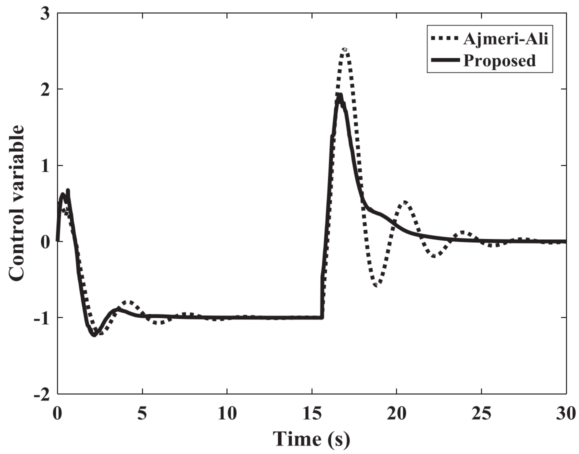

Fig. 13. Control signal of Example 2 under perturbed conditions.

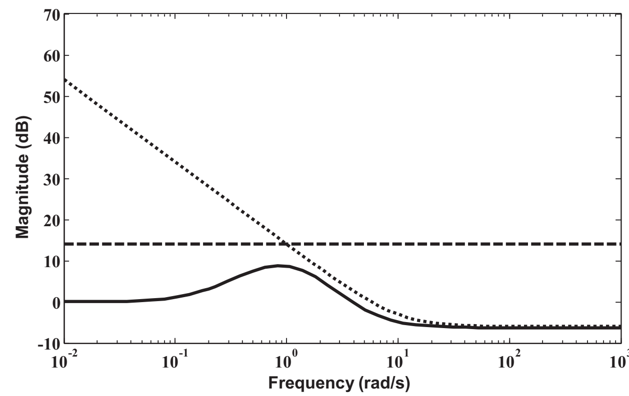

Fig. 14. Magnitude plot of complementary sensitivity function $T_d(j\omega)$ (solid), $1/l_m(j\omega)$for $\Delta\theta_p = 0.25$ (dot), $1/l_m(j\omega)$for Δk= 0.2 (dash).

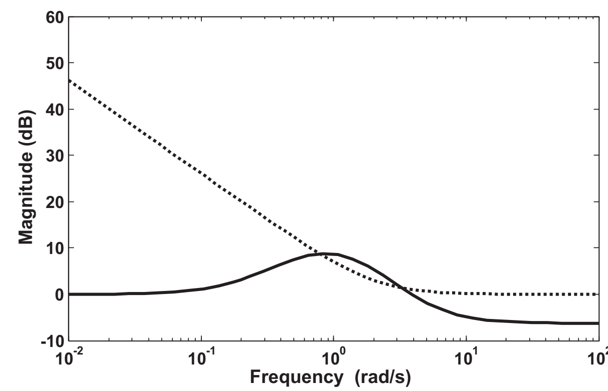

Fig. 15. Magnitude plot of complementary sensitivity function $T_d(j\omega)$ (solid), $1/l_m(j\omega)$for $\Delta\tau = -0.5$ (dot).

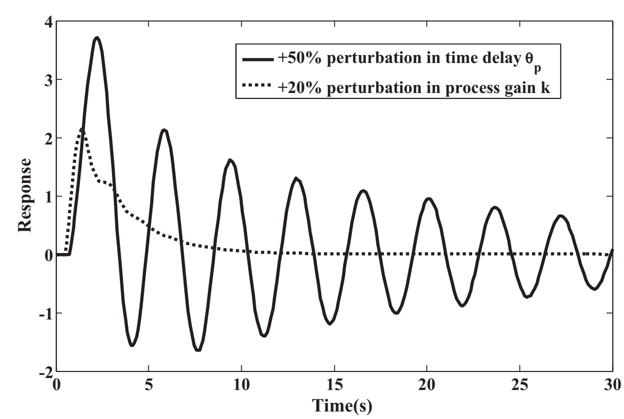

Fig. 16. Robust performance analysis of Example 2 for perturbations in process gain and time delay.

controller with lead /lag filter. Tan [21] proposed a TDF IMC with set point filter to reduce over shoot in servo response.

For the proposed method, λ_s is selected as 2.5 according to (20) and λ_d is derived as 0.336 using (21) which corresponds to a MS value of 8.2729. λ_d is adjusted to 0.36 to achieve desired performance. As per the discussion made in Section 3.4.2, the author does not take much freedom to vary this parameter. The details of controller parameters are given in Table 1. A unit step change in the set point is introduced at t = 0 s and a step change of 0.1 units disturbance is introduced at t = 40 s to analyse the performance. The nominal performance is analysis is shown in Fig. 18 and Table 2. Simulations are carried out for a +10% perturbation in the time delay to analyse the robust performance. A set point change of unit step is considered at t = 0 s and a step disturbance of 0.1 units is considered at t = 100 s. The performance is presented in Fig. 19 and Table 3.

From Figs. 18, 19, Tables 2 and 3, it can be concluded that the proposed method offers superior performance only in the servo response under nominal conditions and Tan's method has better disturbance rejection. But under perturbed conditions, the proposed method is much superior to the methods proposed by Shamsuzzoha [10] and Tan [21]. Tan's method is not shown in the perturbed response (Fig. 19) as it is becoming unstable.

Example 4. The open loop behaviour of chemical reactor with non-ideal mixing is described by the nonlinear equation mentioned in (43).

$$\frac{dC}{dt} = \frac{F(t)}{V}(C_i(t) - C(t)) - \frac{k_1 C(t)}{(k_2 C(t) + 1)^2} \quad (43)$$

where $C_i(t)$ is input concentration, $C(t)$ is output concentration, $F(t)$ is the input flow, V is reactor volume having the parameter values k_1 = 10 l/s, k_2 = 10 l/mol, V = 1 l. This model is previously studied

Table 3
Performance comparison under perturbed conditions.

Perturbed process	Method	Set point tracking				Disturbance rejection			
		$t_s(s)$	IAE	ISE	TV	$t_s(s)$	IAE	ISE	TV
$\frac{1}{0.9s-1}e^{-0.24s}$	Proposed	3.041	1.086	0.7394	2.35	2.79	0.298	0.069	3.2082
	Ajmeri-Ali	1.97	0.6038	0.4424	4.7451	2.487	0.343	0.112	5.2871
$\frac{1}{s-1}e^{-0.6s}$	Proposed	5.279	1.5	1.117	2.920	8.14	3.05	2.428	4.769
	Ajmeri-Ali	8.130	1.68	1.156	2.247	9.66	3.387	3.331	9.036
$\frac{1}{s-1}e^{-1.32s}$	Proposed	22.21	4.458	2.43	6.211	26.156	5.682	2.437	2.400
	Shamsuzzoha	53.88	7.53	3.814	4.27	65.53	8.463	2.696	7.40
	Tan (**unstable**)	–	–	–	–	–	–	–	–

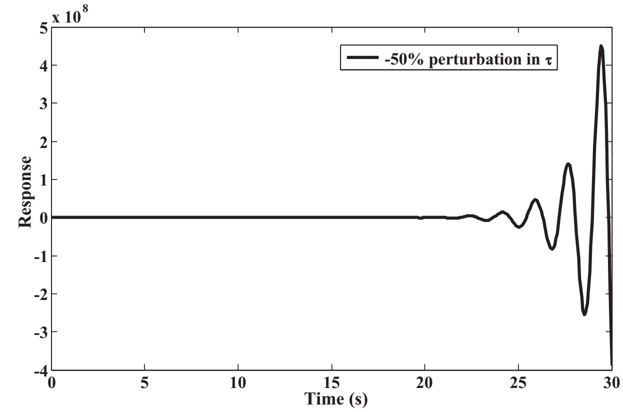

Fig. 17. Robust performance analysis of Example 2 for perturbation in time constant.

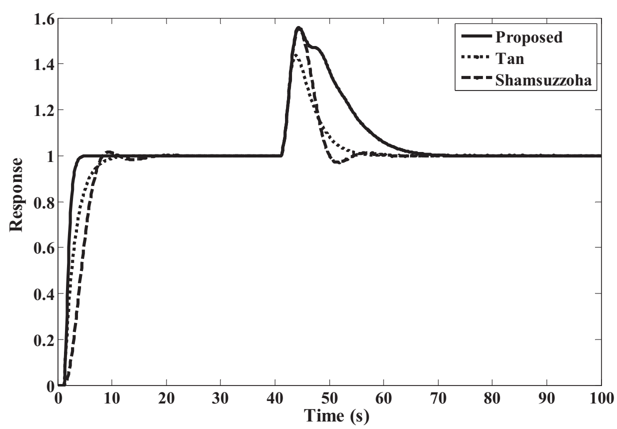

Fig. 18. Nominal Response of Example 3.

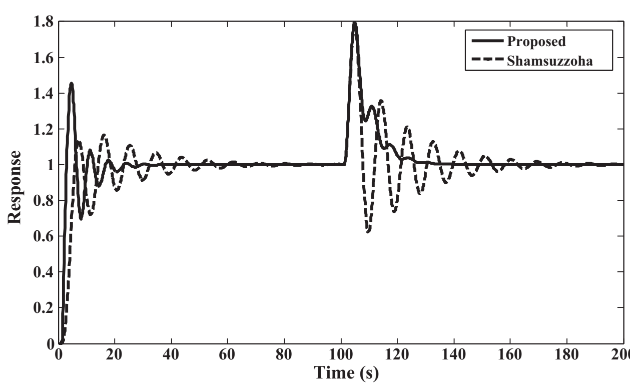

Fig. 19. Perturbed response of Example 3.

in the literature [20,23] at an operating point $F = 0.0333$ l/s, $C_i = 3.288$ mol/l, $C = 1.316$ mol/l. A dead time of 20 s is assumed due to concentration transducer. The linearized model at this operating point is

$$G(s) = G_p(s)e^{-s\theta} = \frac{3.433e^{-20s}}{103.1s - 1} \tag{44}$$

$\lambda_s = 0.02425$ is considered according to (20). Similarly λ_d is selected as 0.035 using (21). The derived controller parameters are shown in Table 1. The comparison is made with recently reported method by Ajmeri-Ali [20]. Ajmeri-Ali [20] have

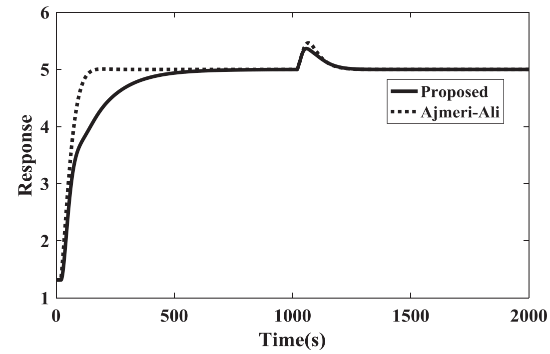

Fig. 20. Nominal response of Example 4.

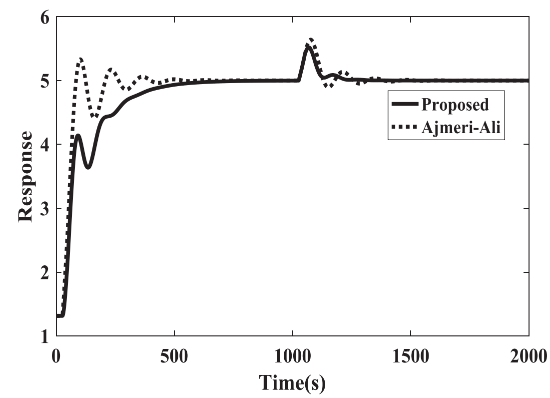

Fig. 21. Perturbed response of Example 4.

compared their method with Rao and Chidambaram [25] and reported enhanced robust performance. A modified Smith predictor controller is proposed in [25] for UFOPTD processes in which the disturbance rejection controller is PD controller for which the parameters are derived based on empirical relations. A set point change from 1.316 to 5 is given at t = 0 s and a disturbance change from 0.0333 to 0.4 at t = 1000 s is considered to check the performance of the proposed method around the operating point. Simulation results for nominal and perturbed case are presented in Figs. 20 and 21 respectively. A +30% perturbation in time delay is considered. From Figs. 20 and 21, one can conclude that the proposed method offers better robust performance.

6. Conclusion

A new control structure is proposed with two controllers for lag/delay time dominant UFOPTD systems. The controllers are intended for two distinct tasks, set point tracking and disturbance rejection. Set point tracking controller is a PI controller with a filter and the disturbance rejection controller is a PID controller with a lead/lag filter. Analytical tuning rules are provided separately for the controllers which are derived based on the MS value which is a measure of robust stability, IAE and TV. The benchmark examples which are widely studied by various researchers in the past are considered in the present work to prove the effectiveness of the designed control structure. The performance of the proposed structure is compared with the recently reported methods in terms of IAE, ISE, TV and settling time. Promising results are obtained with significant contrast in the disturbance rejection and robust performance compared to the existing methods.

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