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Multi stage unreliable retrial Queueing system with Bernoulli vacation

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Abstract.In this work we considered the Bernoulli vacation in group arrival retrial queues with unreliable server. Here, a server providing service in k stages. Any arriving group of units finds the server free, one from the group entering the first stage of service and the rest are joining into the orbit. After completion of the i^{th} , ($i=1,2,\dots,k$) stage of service, the customer may go to $(i+1)^{\text{th}}$ stage with probability θ_i , or leave the system with probability $q_i = 1 - \theta_i$, ($i = 1, 2, \dots, k - 1$) and $q_i = 1$, ($i = k$). The server may enjoy vacation (orbit is empty or not) with probability v after finishing the service or continuing the service with probability $1-v$. After finishing the vacation, the server search for the customer in the orbit with probability θ or remains idle for new arrival with probability $1-\theta$. We analyzed the system using the method of supplementary variable.

1.Introduction

Queues with number of unsatisfied attempts (retrial) have widely used to provide stochastic modeling of many problems arising in telecommunication and computer network. Artalejo [1], Artalejo et al. [2] and Choudhury [4] analyzed, the retrial policy in queueing systems.

Service has many stages in nature. Here a server gives the multi optional stages of service. Authors like, Bagyam et al. [3], Chen et al. [5] and Salehurad et al. [12] are surveyed the multi stage, two phase and multi phase service in queueing system. Service station breakdowns are very common in queueing systems. Keet al. [9] discussed about two phases of service batch retrial queueing pattern and delaying repair. Recently Wang et al. [13] discussed, the repairable queueing system.

In this work, server takes a vacations if the orbit has units or not. Chang et al. [4], Chen et al. [6] and Ke et al. [8] are discussed the J vacations queueing models. Wang et al. [14] and Zhang et al. [16] discussed the vacations in queueing system. Doshi [7] presented a survey on



queuing systems with vacations. Radha et al. [11] analyzed about retrial queues with modified vacation policy. Neutset al.[10] discussed about the server's search for customers.

2. Model description

2.1 Arrival process

Units arriving the system in batches with Poisson arrival rate λ . Let X_k , the number of units in the k^{th} batch, where $k=1,2,3,\dots$ with common distribution $\Pr[X_k = n] = \chi_n$, $n=1,2,3,\dots$. The PGF (probability generating function) of X is $X(z)$. The first and second moments are $E(X)$ and $E(X(X-1))$.

2.2 Retrial process

If there is no space to wait, one from the arriving unit begins service (if the server is free) and rest are waiting in the orbit. If an arriving group finds the server either busy or on vacation or breakdown, then the group joins into an orbit. Here the time interval between two continuous arrivals has the distribution $R(x)$ with Laplace-Stieltjes transform (LST) $R^*(s)$.

2.3 Service process

Here a server gives k stages of service. The First Stage Service (FSS) is followed by i stages of service. After completion of the i^{th} , ($i=1,2,\dots,k$) stage of service, the customer go to $(i+1)^{\text{th}}$ stage with probability θ_i , or leave the system with probability $q_i = 1 - \theta_i$, ($i=1,2,\dots,k-1$) and $q_i = 1$, ($i=k$). The service time S_i for $i=1,2,\dots,k$ has a distribution (general) function $S_i(x)$ having LST $S_i^*(s)$ and first and second moments are $E(S_i)$ and $E(S_i^2)$, ($i=1,2,\dots,k$).

2.4 Vacation process

If the orbit is empty or not, the server may enjoy vacation (simply taking break or secondary job etc.) of random length V with probability v after finishing the service. Otherwise continuing the service with probability $1-v$. After finishing the vacation, the server searching the customer in the orbit with probability θ or waiting for new arrival with probability $1-\theta$. Here the distribution function $V(x)$ and LST $V^*(s)$ with moments $E(V)$ and $E(V^2)$.

2.5 Breakdown and repair

The service station may down at any time with Poisson rate α_i where $i=1,2,\dots,k$ during service. The unit on service has to wait to complete the remaining service. This waiting time is taken as delay time. The server continues the service for this unit after the repair process.

Here the waiting time is defined as delay time. The delay time D_i has density function $D_i(y)$, Laplace- Stieltjes Transform $D_i^*(s)$ and finite k^{th} moment $E(D_i^k)$ ($i=1,2,\dots,k$ and $k=1,2$). The repair time G_i has the distributions function $G_i(y)$ and LST $G_i^*(s)$ for ($i=1,2,\dots,k$). Consider various Probability processes involved in the system are mutually exclusive.

$$\text{Let the server state random variable, } C(t) = \begin{cases} 0, & \text{idle} \\ 1, & \text{busy on } i^{\text{th}} \text{ stage} \\ 2, & \text{repair on } i^{\text{th}} \text{ stage} \\ 3, & \text{delay in repair on } i^{\text{th}} \text{ stage,} \\ 4, & \text{on vacation} \end{cases}$$

The Markov process $\{C(t), N(t); t \geq 0\}$ describes the system state, where $C(t)$ - the server state and $N(t)$ - the number in orbit at time t . Then define $B_i^* = S_1^* S_2^* \dots S_i^*$ and $B_0^* = 1$. The first moment M_{1i} and second moment M_{2i} of B_i^* are given by

$$M_{1i} = \lim_{z \rightarrow 1} dB_i^*[A_i(z)]/dz = \sum_{j=1}^i \lambda E(X) S_j^{(1)} (1 + \alpha_j [g_j^{(1)} + d_j^{(1)}])$$

$$M_{2i} = \lim_{z \rightarrow 1} d^2 B_i^*[A_i(z)]/dz^2 = \sum_{j=1}^i \left[-M_{1i} \left\{ \begin{array}{l} \lambda E(X(X-1)) + \alpha_j [\lambda E(X(X-1)) [g_j^{(1)} + d_j^{(1)}]] \\ -\alpha_j (\lambda E(X))^2 [g_j^{(2)} + d_j^{(2)}] \end{array} \right\} \right. \\ \left. + (\lambda E(X) S_j^{(1)})^2 S_j^{(2)} (1 + \alpha_j [g_j^{(1)} + d_j^{(1)}]) \right]$$

where

$$A_i(z) = \alpha_i (1 - G_i^*(b(z)) D_i^*(b(z))) + b(z) \text{ and } b(z) = (1 - X(z)) \lambda$$

Let $\{t_n; n = 1, 2, \dots\}$ be the sequence of time either a service period or repair period ends. In this system, $Z_n = \{C(t_n+), N(t_n+)\}$ forms an embedded Markov chain which is ergodic $\Leftrightarrow \rho < 1$, where

$$\rho = E(X) \left((1 - R^*(\lambda))(1 - \nu\theta) + \lambda \nu V^{(1)} \right) + \sum_{i=1}^k \Theta_{i-1} M_{1i} + \sum_{i=1}^{k-1} \Theta_i M_{1i}.$$

3. Steady state distribution

For $\{N(t), t \geq 0\}$, we define the probability functions at time t ,

$P_0(t)$ - Pr (the system is empty),

At time t and n customers in the orbit,

$P_n(x, t)$ - Pr (an elapsed retrial time x of the retrial customers),

$\Pi_{i,n}(x, t), (1 \leq i \leq k)$ - Pr (elapsed service time x on i^{th} stage of the customer under service),

$V_n(x, t)$ - Pr (elapsed vacation time x of the customer on vacation),

$R_{i,n}(x, y, t), (1 \leq i \leq k)$ - Pr (an elapsed times for service is x and repair is y on i^{th} stage),

$D_{i,n}(x, y, t), (1 \leq i \leq k)$ - Pr (elapsed times for service is x and delay in repair is y on i^{th} stage).

The stability condition exists for $t \geq 0, x \geq 0, y \geq 0, n \geq 0$ for $i = 1, 2, \dots, k$.

$$P_0 = \lim_{t \rightarrow \infty} P_0(t), P_n(x) = \lim_{t \rightarrow \infty} P_n(x, t), \Pi_{i,n}(x) = \lim_{t \rightarrow \infty} \Pi_{i,n}(x, t),$$

$$V_n(x) = \lim_{t \rightarrow \infty} V_n(x, t), \Omega_{i,n}(x, y) = \lim_{t \rightarrow \infty} \Omega_{i,n}(x, y, t), \text{ for } t \geq 0. R_{i,n}(x, y) = \lim_{t \rightarrow \infty} R_{i,n}(x, y, t), \text{ for } t \geq 0.$$

3.1 Steady state equations

The following equations are obtained by the supplementary variable technique for $(i=1, 2, \dots, k)$.

$$\lambda P_0 = \bar{\theta} \int_0^\infty \gamma(x) V_0(x) dx + (1 - \nu) \sum_{i=1}^{k-1} q_i \int_0^\infty \mu_i(x) \Pi_{i,n}(x) dx + (1 - \nu) \int_0^\infty \mu_k(x) \Pi_{k,n}(x) dx. \tag{1}$$

$$\frac{dP_n(x)}{dx} = -P_n(x) [\lambda + a(x)], \text{ where } n \geq 1. \tag{2}$$

$$\frac{d\Pi_{i,n}(x)}{dx} = -\Pi_{i,n}(x) [\lambda + \alpha_i + \mu_i(x)] + \int_0^\infty \xi_i(y) R_{i,n}(x, y) dy + \lambda \sum_{k=1}^n \chi_k \Pi_{i,n-k}(x), n \geq 1. \tag{3}$$

$$\frac{dV_n(x)}{dx} = -V_n(x)\lambda + \gamma(x) + \lambda \sum_{k=1}^n \chi_k V_{n-k}(x), \text{ where } n \geq 1, \quad (4)$$

$$\frac{d\Omega_{i,n}(x, y)}{dy} = -\Omega_{i,n}(x, y)[\lambda + \xi_i(y)] + \lambda \sum_{k=1}^n \chi_k \Omega_{i,n-k}(x, y), \text{ where } n \geq 1. \quad (5)$$

$$\frac{dR_{i,n}(x, y)}{dy} = -R_{i,n}(x, y)[\lambda + \xi_i(y)] + \lambda \sum_{k=1}^n \chi_k R_{i,n-k}(x, y), \text{ where } n \geq 1. \quad (6)$$

Boundary conditions at $x = 0$ and $y = 0$ are

$$P_n(0) = (1 - v) \sum_{i=1}^{k-1} q_i \int_0^{\infty} \mu_i(x) \Pi_{i,n}(x) dx + \bar{\theta} \int_0^{\infty} \gamma(x) V_n(x) dx, \text{ where } n \geq 1. \quad (7)$$

$$\Pi_{1,n}(0) = \theta \int_0^{\infty} \gamma(x) V_{n+1}(x) dx + \int_0^{\infty} a(x) P_{n+1}(x) dx + \lambda \left(\chi_{n+1} P_0 + \sum_{k=1}^n \chi_k \int_0^{\infty} P_{n-k+1}(x) dx \right), n \geq 1. \quad (8)$$

$$\Pi_{i,n}(0) = \theta_{i-1} \int_0^{\infty} \mu_{i-1}(x) \Pi_{i-1,n}(x) dx, \text{ where } n \geq 1, (2 \leq i \leq k). \quad (9)$$

$$V_n(0) = v \left\{ \sum_{i=1}^{k-1} q_i \int_0^{\infty} \Pi_{i,n}(x) \mu_i(x) dx \right\}, n \geq 0. \quad (10)$$

$$\Omega_{i,n}(x, 0) = [\Pi_{i,n}(x)] \alpha_i, n \geq 0. \quad (11)$$

$$R_{i,n}(x, 0) = \int_0^{\infty} \eta_i(y) \Omega_{i,n}(x, y) dy, n \geq 0. \quad (12)$$

The normalizing condition is

$$P_0 + \sum_{n=1}^{\infty} \int_0^{\infty} P_n(x) dx + \sum_{n=0}^{\infty} \left(\sum_{i=1}^k \left(\int_0^{\infty} \Pi_{i,n}(x) dx + \int_0^{\infty} \int_0^{\infty} R_{i,n}(x, y) dx dy + \int_0^{\infty} \int_0^{\infty} \Omega_{i,n}(x, y) dx dy \right) + \int_0^{\infty} V_n(x) dx \right) = 1. \quad (13)$$

3.2 Steady state solutions

The above equations are solved by using the method of generating functions. Multiplying Eqns. (2) to Eqn. (12) by $\sum_{n=0}^{\infty} z^n$ then,

$$\frac{\partial P(x, z)}{\partial x} = -P(x, z)[\lambda + a(x)] \quad (14)$$

$$\frac{\partial \Pi_i(x, z)}{\partial x} = -\Pi_i(x, z)[\lambda(1 - X(z)) + \alpha_i + \mu_i(x)] + \int_0^{\infty} \xi_i(y) R_i(x, y, z) dy. \quad (15)$$

$$\frac{\partial V(x, z)}{\partial x} = -V(x, z)[\lambda(1 - X(z)) + \gamma(x)] \quad (16)$$

$$\frac{d\Omega_i(x, y, z)}{dy} = -\Omega_i(x, y, z)[\lambda(1 - X(z)) + \xi_i(y)]. \quad (17)$$

$$\frac{dR_i(x, y, z)}{dy} = -R_i(x, y, z)[\lambda(1 - X(z)) + \xi_i(y)] = 0. \quad (18)$$

The boundary conditions at $x = 0$ and $y = 0$ are

$$P(0, z) = (1 - \nu) \sum_{i=1}^k \left\{ (q_i) \int_0^{\infty} \mu_i(x) \Pi_i(x, z) dx \right\} + -\lambda P_0 + \bar{\theta} \int_0^{\infty} \gamma(x) V(x, z) dx \quad (19)$$

$$\Pi_1(0, z) = \frac{\theta}{z} \int_0^{\infty} \gamma(x) V(x, z) dx + \frac{1}{z} \int_0^{\infty} a(x) P(x, z) dx + \lambda \frac{X(z)}{z} \left(P_0 + \int_0^{\infty} P(x, z) dx \right) \quad (20)$$

$$\Pi_i(0, z) = \theta_{i-1} \int_0^{\infty} \Pi_{i-1,n}(x) \mu_{i-1}(x) dx, \quad (2 \leq i \leq k). \quad (21)$$

$$V(0, z) = \nu \left\{ \sum_{i=1}^{k-1} q_i \int_0^{\infty} \mu_i(x) \Pi_{i,n}(x) dx \right\}, \quad n \geq 0. \quad (22)$$

$$\Omega_i(x, 0, z) = [\Pi_i(x, z)] \alpha_i \quad (23)$$

$$R_i(x, 0, z) = \int_0^{\infty} \eta_i(y) \Omega_i(x, y, z) dy, \quad n \geq 0. \quad (24)$$

Solving the equations (20) to (24), it follows that for $(1 \leq i \leq k)$

$$P(x, z) = P(0, z) e^{-\lambda x} [1 - R(x)] \quad (25)$$

$$\Pi_i(x, z) = \Pi_i(0, z) e^{-A_i(z)x} [1 - S_i(x)] \quad (26)$$

$$Q(x, z) = Q(0, z) e^{-b(z)x} [1 - V(x)] \quad (27)$$

$$\Omega_i(x, y, z) = \Omega_i(x, 0, z) e^{-b(z)y} [1 - D_i(y)] \quad (28)$$

$$R_i(x, y, z) = R_i(x, 0, z) e^{-b(z)y} [1 - G_i(y)] \quad (29)$$

Theorem 3.1. Under $\rho < 1$, the stationary distributions of the numbers in the system when server being idle, busy during i^{th} stage, on j^{th} stage of vacation and repair on i^{th} stage are given by

$$P(z) = P_0 \left(1 - R^*(\lambda) \right) \left\{ \frac{\Sigma \left((1 - \nu) X(z) + \nu V^*(b(z)) (\bar{\theta} X(z) + \theta) \right) - z}{z - \Sigma \left(\left[R^*(\lambda) + X(z) (1 - R^*(\lambda)) \right] (1 - \nu + \nu V^*(b(z)) \bar{\theta}) + \theta \nu V^*(b(z)) \right)} \right\}. \quad (30)$$

$$\Pi_i(z) = \lambda P_0 \frac{(1 - \delta_i^*(A_i(z)))}{A_i(z)} \left\{ \frac{\Theta_{i-1} R^*(\lambda) (X(z) - 1) (B_{i-1}^*[A_{i-1}(z)])}{z - \Sigma \left(\left[R^*(\lambda) + X(z) (1 - R^*(\lambda)) \right] (1 - \nu + \nu V^*(b(z)) \bar{\theta}) + \theta \nu V^*(b(z)) \right)} \right\}. \quad (31)$$

$$V(z) = P_0 \frac{(1-V^*(b(z)))}{(1-X(z))} \left\{ \frac{v\Sigma R^*(\lambda)(X(z)-1)}{z-\Sigma \left(\left[R^*(\lambda) + X(z)(1-R^*(\lambda)) \right] (1-v+vV^*(b(z))\bar{\theta}) + \theta vV^*(b(z)) \right)} \right\}. \tag{32}$$

$$\Omega_i(z) = \frac{\lambda P_0 \alpha_i \Theta_{i-1}}{A_i(z)b(z)} \left\{ \frac{R^*(\lambda)(X(z)-1)(B_{i-1}^*(A_{i-1}(z)))(1-S_i^*(A_i(z)))(1-D_i^*(b(z)))}{z-\Sigma \left(\left[R^*(\lambda) + X(z)(1-R^*(\lambda)) \right] (1-v+vV^*(b(z))\bar{\theta}) + \theta vV^*(b(z)) \right)} \right\} \tag{33}$$

$$R_i(z) = \frac{\lambda P_0 \alpha_i \Theta_{i-1}}{A_i(z)b(z)} \left\{ \frac{R^*(\lambda)(X(z)-1)(B_{i-1}^*(A_{i-1}(z)))D_i^*(b(z))(1-S_i^*(A_i(z)))(1-G_i^*(b(z)))}{z-\Sigma \left(\left[R^*(\lambda) + X(z)(1-R^*(\lambda)) \right] (1-v+vV^*(b(z))\bar{\theta}) + \theta vV^*(b(z)) \right)} \right\} \tag{34}$$

where,

$$P_0 = \frac{\left\{ 1-E(X)\left((1-R^*(\lambda))(1-v\theta) + \lambda vV^{(1)} \right) - \omega \right\}}{R^*(\lambda) \left\{ 1-\omega + E(X) \sum_{i=1}^k \lambda \Theta_{i-1} S_i^{(1)} \left(1+\alpha_j [g^{(1)} + d^{(1)}] \right) \right\}}, \tag{35}$$

and $\Sigma = \sum_{i=1}^k q_i \Theta_{i-1} \left(B_i^*[A_i(z)] \right)$.

Proof. The statement is obtained by using

$$P(z) = \int_0^\infty P(x,z)dx, \Pi_i(z) = \int_0^\infty \Pi_i(x,z)dx, V(z) = \int_0^\infty V(x,z)dx, R_i(x,z) = \int_0^\infty R_i(x,y,z)dy, R_i(z) = \int_0^\infty R_i(x,z)dx,$$

$$\int_0^\infty \Omega_i(x,z) = \int_0^\infty \Omega_i(x,y,z)dy, \Omega_i(z) = \int_0^\infty \Omega_i(x,z)dx. \text{ and } P_0 + P(1) + V(1) + \sum_{i=1}^k (\Pi_i(1) + \Omega_i(1) + R_i(1)) = 1$$

Theorem 3.2. Under $\rho < 1$, probability generating function of the system size and orbit size distribution at stationary point of time is

$$S(z) = \frac{P_0 R^*(\lambda) \left(z - \Sigma - z \sum_{i=1}^k \Theta_{i-1} \left(B_{i-1}^*[A_{i-1}(z)] \right) \left(1 - S_i^*(A_i(z)) \right) \right)}{z - \Sigma \left(\left[R^*(\lambda) + X(z)(1-R^*(\lambda)) \right] (1-v+vV^*(b(z))\bar{\theta}) + \theta vV^*(b(z)) \right)} \tag{36}$$

Also,

$$Q(z) = \frac{P_0 R^*(\lambda) \left(z - \Sigma - \sum_{i=1}^k \Theta_{i-1} \left(B_{i-1}^*[A_{i-1}(z)] \right) \left(1 - S_i^*(A_i(z)) \right) \right)}{z - \Sigma \left(\left[R^*(\lambda) + X(z)(1-R^*(\lambda)) \right] (1-v+vV^*(b(z))\bar{\theta}) + \theta vV^*(b(z)) \right)}, \tag{37}$$

where $\Sigma = \sum_{i=1}^k q_i \Theta_{i-1} \left(B_i^*[A_i(z)] \right)$.

Proof. The statement is obtained by using $S(z) = P_0 + P(z) + V(z) + z \sum_{i=1}^k \Pi_i(z) + z \sum_{i=1}^k \Omega_i(z) + z \sum_{i=1}^k R_i(z)$ and

$$Q(z) = P_0 + P(z) + V(z) + \sum_{i=1}^k \Pi_i(z) + \sum_{i=1}^k \Omega_i(z) + \sum_{i=1}^k R_i(z).$$

4. Performance measures

Theorem 4.1. If the system satisfies $\rho < 1$, then the following probabilities of the server state, that is the server is idle during the retrial, busy during i^{th} stage, on vacation, delaying repair during i^{th} stage and under repair on i^{th} stage respectively are obtained.

$$P = \frac{P_0(1 - R^*(\lambda))(E(X)(1 - \theta v + \lambda v V^{(1)}) + \omega - 1)}{1 - E(X)((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)}) - \omega}.$$

$$\Pi_i = \sum_{i=1}^k \Pi_i = \frac{\lambda P_0 \sum_{i=1}^k \Theta_{i-1} E(X) S_i^{(1)} R^*(\lambda)}{1 - E(X)((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)}) - \omega}.$$

$$V = \frac{P_0 \lambda v V^{(1)} E(X) R^*(\lambda)}{1 - E(X)((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)}) - \omega}.$$

$$\Omega_i = \sum_{i=1}^k \Omega_i = \frac{\lambda P_0 \sum_{i=1}^k \alpha_i \Theta_{i-1} E(X) R^*(\lambda) S_i^{(1)} d^{(1)}}{1 - E(X)((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)}) - \omega}.$$

$$R_i = \sum_{i=1}^k R_i = \frac{\lambda P_0 \sum_{i=1}^k \alpha_i \Theta_{i-1} E(X) R^*(\lambda) S_i^{(1)} g^{(1)}}{1 - E(X)((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)}) - \omega}.$$

Proof. The statement followed by using

$$P = \lim_{z \rightarrow 1} P(z), \sum_{i=1}^k \Pi_i = \lim_{z \rightarrow 1} \sum_{i=1}^k \Pi_i(z), V = \lim_{z \rightarrow 1} V(z), \sum_{i=1}^k \Omega_i = \lim_{z \rightarrow 1} \sum_{i=1}^k \Omega_i(z) \text{ and } \sum_{i=1}^k R_i = \lim_{z \rightarrow 1} \sum_{i=1}^k R_i(z).$$

Theorem 4.2. Let L_s, L_q, W_s and W_q be the average system size, average orbit size, average waiting time in the system and average waiting time in the orbit respectively, then under $\rho < 1$,

$$L_q = P_0 R^*(\lambda) \left[\frac{L_2(\omega - 1) - \tau L_1 + (\tau + L_2) E(X) ((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)})}{2(1 - E(X)((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)}) - \omega)^2} + (1 - \omega - L_1) \left(\frac{E(X^2 - X)(1 - R^*(\lambda))(1 - v\theta) + 2E(X)((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)})}{(E(X)(1 - R^*(\lambda)) + \omega) - \lambda v (\lambda (E(X))^2 V^{(2)} + E(X^2 - X) V^{(1)})} \right) \right]$$

$$\omega = \sum_{i=1}^k \Theta_{i-1} M_{1i} + \sum_{i=1}^{k-1} \Theta_i M_{1i}, \quad \tau = \sum_{i=1}^k \Theta_{i-1} M_{2i} - \sum_{i=1}^{k-1} \Theta_i M_{2i},$$

$$L_1 = -\sum_{i=1}^k \Theta_{i-1} M_{1i} \quad \text{and} \quad L_2 = -\sum_{i=1}^k \Theta_{i-1} (M_{2i} + M_{1i} M_{1i-1}).$$

$$L_s = P_0 R^*(\lambda) \frac{\left[\begin{aligned} &(L_2 - 2L_1)(\omega - 1) - \tau L_1 + (\tau + L_2 + 2L_1)E(X) \left((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)} \right) \\ &+ (1 - \omega - L_1) \left(\begin{aligned} &E(X^2 - X)(1 - R^*(\lambda))(1 - v\theta) + 2E(X) \left((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)} \right) \\ &(E(X)(1 - R^*(\lambda)) + \omega) - \lambda v \left(\lambda (E(X))^2 V^{(2)} + E(X^2 - X) V^{(1)} \right) \end{aligned} \right) \end{aligned} \right]}{2 \left(1 - E(X) \left((1 - R^*(\lambda))(1 - v\theta) + \lambda v V^{(1)} \right) - \omega \right)^2}$$

$$W_s = \frac{L_s}{\lambda E(X)} \quad \text{and} \quad W_q = \frac{L_q}{\lambda E(X)}.$$

Proof: Under $\rho < 1$, L_q is obtained from (37), $L_q = \lim_{z \rightarrow 1} \frac{d}{dz} Q(z) = Q'(1)$. And L_s is obtained from (6)

$$L_s = \lim_{z \rightarrow 1} \frac{d}{dz} S(z) = S'(1). \quad W_s \text{ and } W_q \text{ are obtained by Little's formula,}$$

$$L_s = \lambda W_s \quad \text{and} \quad L_q = \lambda W_q.$$

5. Conclusion

In this paper, the Bernoulli vacation in group arrival retrial queues with unreliable server provides service in k stages are meticulously studied. The PGF of the numbers in the system and orbit are found. The performance measures were obtained. L_s, L_q, W_s and W_q are obtained.

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