




Article

# Neutrosophic $\alpha\psi$ -Homeomorphism in Neutrosophic Topological Spaces

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**Abstract:** In this article, the concept of neutrosophic homeomorphism and neutrosophic  $\alpha\psi$  homeomorphism in neutrosophic topological spaces is introduced. Further, the work is extended as neutrosophic  $\alpha\psi^*$  homeomorphism, neutrosophic  $\alpha\psi$  open and closed mapping and neutrosophic  $T_{\alpha\psi}$  space in neutrosophic topological spaces and establishes some of their related attributes.

**Keywords:** neutrosophic homeomorphism; neutrosophic  $\alpha\psi$  homeomorphism; neutrosophic  $\alpha\psi^*$  homeomorphism; neutrosophic  $\alpha\psi$  open; neutrosophic  $\alpha\psi$  closed mapping; neutrosophic  $T_{\alpha\psi}$  space

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## 1. Introduction

Zadeh [1] introduced the fuzzy set in 1965, the elements used of which in the fuzzy set had only the degree of membership. Later, Atanassav [2] introduced the intuitionistic fuzzy set in 1983. Both the fuzzy set and intuitionistic fuzzy set used all the elements that had the degree of membership and degree of non-membership. Salama and Alblowi [3] introduced the new concept of neutrosophic topological space (NTS) in 2012, which had been investigated recently. In the neutrosophic set, all the elements have the degree of membership, indeterminacy and degree of non-membership. The neutrosophic closed sets and neutrosophic continuous functions were introduced by Salama, Smarandache and Valeri [4] in 2014. Arokiarani et al. [5] introduced the neutrosophic  $\alpha$  closed set in NTS. The neutrosophic  $\omega$  closed sets in NTS were introduced by Santhi et al. [6] in 2016. The intuitionistic homeomorphism was introduced by Lee and Lee [7]. Jeevitha and Parimala [8] studied the concept of minimal  $\alpha\psi$  closed sets in minimal structure spaces also Parimala et al. [9] studied the concept of neutrosophic  $\alpha\psi$  closed sets.

The purpose of this article is to introduce the idea of neutrosophic homeomorphism and neutrosophic  $\alpha\psi$  homeomorphism in neutrosophic topological spaces and establish some of their attributes. It also establishes the notion of neutrosophic  $\alpha\psi^*$  homeomorphism, neutrosophic  $\alpha\psi$  open and closed mapping and neutrosophic  $T_{\alpha\psi}$  space. The present study demonstrates some of the related theorems, results and properties.

## 2. Preliminaries

The neutrosophic set in  $(X, \tau_N)$  has the form  $S = \{ \langle x, M_S(x), I_S(x), N_S(x) \rangle : x \in (X, \tau_N) \}$ , where the functions  $M_S : S \rightarrow [0, 1], I_S : S \rightarrow [0, 1], N_S : S \rightarrow [0, 1]$  denote the degree of membership, indeterminacy and degree of non-membership. The neutrosophic set  $S = \{ \langle x, M_S(x), I_S(x), N_S(x) \rangle : x \in (X, \tau_N) \}$  is called a subset of  $T = \{ \langle x, M_T(x), I_T(x), N_T(x) \rangle : x \in (X, \tau_N) \}$  (in short  $S \subseteq T$ ) if the degree of membership and indeterminacy is minimum in  $S$  and the degree of non-membership is maximum in  $S$  or the degree of membership is minimum and the degree of non-membership and indeterminacy is maximum in  $S$ . The complement to NTS  $S = \{ \langle x, M_S(x), I_S(x), N_S(x) \rangle : x \in (X, \tau_N) \}$  is  $S^c = \{ \langle x, N_S(x), I_S(x), M_S(x) \rangle : x \in (X, \tau_N) \}$ .

**Definition 1 ([10]).** Let  $(X, \tau_N)$  be a non-empty fixed set. A neutrosophic set  $S$  is an object having the form  $S = \{ \langle x, M_S(x), I_S(x), N_S(x) \rangle : x \in (X, \tau_N) \}$ ; where  $M_S(x), I_S(x), N_S(x)$ , which represent the degree of membership, the degree of indeterminacy and the degree of non-membership of each element  $x \in (X, \tau_N)$  to the set  $S$ .

**Definition 2 ([3]).** Let  $S$  and  $T$  be NS of the form  $S = \{ \langle x, M_S(x), I_S(x), N_S(x) \rangle : x \in (X, \tau_N) \}$  and  $T = \{ \langle x, M_T(x), I_T(x), N_T(x) \rangle : x \in (X, \tau_N) \}$ . Then:

- (1)  $S \subseteq T$  if and only if  $M_S(x) \leq M_T(x), I_S(x) \geq I_T(x)$  and  $N_S(x) \geq N_T(x)$  for all  $x \in (X, \tau_N)$  or  $M_S(x) \leq M_T(x), I_S(x) \leq I_T(x)$  and  $N_S(x) \geq N_T(x)$  for all  $x \in (X, \tau_N)$ .
- (2)  $S = T$  if and only if  $S \subseteq T$  and  $T \subseteq S$ .
- (3)  $S^c = \{ \langle x, N_S(x), I_S(x), M_S(x) \rangle : x \in (X, \tau_N) \}$ .
- (4)  $S \cup T = \{ \langle x, M_S(x) \vee M_T(x), I_S(x) \wedge I_T(x), N_S(x) \wedge N_T(x) \rangle : x \in (X, \tau_N) \}$
- (5)  $S \cap T = \{ \langle x, M_S(x) \wedge M_T(x), I_S(x) \vee I_T(x), N_S(x) \vee N_T(x) \rangle : x \in (X, \tau_N) \}$

**Definition 3 ([3]).** A neutrosophic topology on a non-empty set  $X$  is a family  $\tau$  of neutrosophic subsets in  $X$  satisfying the following axioms:

- (1)  $0_N, 1_N \in \tau$
- (2)  $S_1 \cap S_2 \in \tau$ , for  $S_1, S_2 \in \tau$
- (3)  $\cup S_i \in \tau$ , for all  $S_i : i \in J \subseteq \tau$

**Definition 4 ([3]).** Let  $(X, \tau_N)$  be NTS and  $S = \{ \langle x, M_S(x), I_S(x), N_S(x) \rangle : x \in (X, \tau_N) \}$  be an NS in  $X$ . Then, the neutrosophic closer and neutrosophic interior of  $S$  are defined by:

- (1)  $Ncl(S) = \cap \{ K : K \text{ is an neutrosophic closed set in } X \text{ and } S \subseteq K \}$ .
- (2)  $Nint(S) = \cup \{ G : G \text{ is an neutrosophic open set in } X \text{ and } G \subseteq S \}$ .

**Lemma 1 ([3]).** Let  $(X, \tau_N)$  be an NTS and  $A, B$  be two neutrosophic sets in  $X$ . Then, the following properties hold:

- (1)  $Nint(S) \subseteq S$ .
- (2)  $S \subseteq Ncl(S)$ .
- (3)  $S \subseteq T \Rightarrow Nint(S) \subseteq Nint(T)$ .
- (4)  $S \subseteq T \Rightarrow Ncl(S) \subseteq Ncl(T)$ .
- (5)  $Nint(Nint(S)) = Nint(S) \wedge Nint(T)$ .
- (6)  $Ncl(S \cup T) = Ncl(S) \vee Ncl(T)$ .
- (7)  $Nint(1_N) = 1_N$ .
- (8)  $Ncl(0_N) = 0_N$ .

**Definition 5 ([7]).** A bijection  $f : (X, \tau_1) \rightarrow (Y, \tau_1)$  is said to be an intuitionistic homeomorphism if  $f$  is both an intuitionistic continuous and intuitionistic open function in  $X$ .

**Corollary 1** ([4]). Let  $S, \{S_i : i \in J\}$  be NS in  $X$  and  $T, \{T_i : i \in K\}$  and  $f : X \rightarrow Y$  a function. Then:

- (1)  $S_1 \subseteq S_2 \Leftrightarrow g(S_1) \subseteq g(S_2), T_1 \subseteq T_2 \Leftrightarrow g^{-1}(T_1) \subseteq g^{-1}(T_2).$
- (2)  $S \subseteq g^{-1}(g(S)),$  and if  $g$  is injective, then  $S = g^{-1}(g(S)).$
- (3)  $g^{-1}(g(T)) \subseteq T,$  and if  $g$  is surjective, then  $T = g^{-1}(g(T)).$
- (4)  $g^{-1}(\cup T_i) = \cup g^{-1}(T^i)$  and  $g^{-1}(\cap T_i) = \cap g^{-1}(T^i).$
- (5)  $g^{-1}(\cup S_i) = \cup g^{-1}(S^i)$  and  $g^{-1}(\cap S_i) = \cap g^{-1}(S^i),$  and if  $g$  is injective, then  $g(\cap S^i) = \cap g(S^i).$
- (6)  $g^{-1}(1_N) = 1_N$  and  $g^{-1}(0_N) = 0_N.$
- (7)  $g(1_N) = 1_N$  and  $g(0_N) = 0_N$  if  $g$  is surjective.

**Definition 6** ([4]). If  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is a function and  $X$  and  $Y$  a two neutrosophic topological space, then  $g$  is said to be neutrosophic continuous if the preimage of each neutrosophic closed set in  $(Y, \tau_{N2})$  is a neutrosophic set in  $(X, \tau_{N1}).$

**Definition 7** ([6]). A function  $f : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  where  $(X, \tau_{N1})$  and  $(Y, \tau_{N2})$  are two NTS is called  $N\alpha\psi$  continuous if  $f^{-1}(U)$  is an  $N\alpha\psi$  closed set in  $(X, \tau_{N1})$  for every neutrosophic closed set  $U$  of  $(Y, \tau_{N2}).$

**Definition 8** ([4]). If  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is a function and  $X$  and  $Y$  a two neutrosophic topological space, then  $g$  is said to be neutrosophic open if the image of each neutrosophic set in  $(X, \tau_{N1})$  is a neutrosophic set in  $(Y, \tau_{N2}).$

### 3. Neutrosophic $\alpha\psi$ Homeomorphism

We introduce the following definition.

**Definition 9.** A bijection  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is called a neutrosophic homeomorphism if  $g$  and  $g^{-1}$  are neutrosophic continuous.

**Example 1.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X,$  where  $A = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $B = \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$   
 $C = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $D = \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$  and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y,$  where  
 $E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$   
 $F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle.$  Here,  $g(p) = p, g(q) = q, g(r) = r,$  and assume  $F^c = S = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is neutrosophic closed set in  $Y,$  then  $A^c = g^{-1}(S)$  is neutrosophic closed in  $X.$  Hence,  $g$  is neutrosophic continuous, and  $(g^{-1})^{-1} : X \rightarrow Y$  is said to be neutrosophic continuous. If  $A^c$  is a neutrosophic closed set in  $X,$  then the image  $g(A^c) = F^c$  is neutrosophic closed in  $Y.$  Hence,  $g$  and  $g^{-1}$  are neutrosophic continuous; therefore, it is a neutrosophic homeomorphism.

**Definition 10.** A bijection  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is called a neutrosophic  $\alpha\psi$  homeomorphism if  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  continuous.

**Example 2.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X,$  where  $A = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $B = \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$   
 $C = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $D = \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$  and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y,$  where  
 $E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$

$F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle.$  Here,  $g(p) = p, g(q) = q, g(r) = r,$  and assume  $S = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is a neutrosophic closed set in  $Y,$  then  $g^{-1}(S)$  is neutrosophic  $\alpha\psi$  closed in  $X.$  Hence,  $g$  is neutrosophic  $\alpha\psi$  continuous and  $g^{-1}$  is neutrosophic  $\alpha\psi$  continuous if  $A^c$  is a neutrosophic  $\alpha\psi$  closed set in  $X,$  then the image  $g(A^c) = F^c$  is neutrosophic closed in  $Y.$  Hence,  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  continuous, which is therefore a neutrosophic  $\alpha\psi$  homeomorphism.

**Definition 11.** A bijection  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is called a neutrosophic  $\psi$  homeomorphism if  $g$  and  $g^{-1}$  are neutrosophic  $\psi$  continuous.

**Example 3.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X,$  where  $A = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $B = \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$   
 $C = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $D = \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$  and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y,$  where  
 $E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$   
 $F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle.$  Here,  $g(p) = p, g(q) = q, g(r) = r,$  and assume  $S = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is a neutrosophic closed set in  $Y,$  then  $g^{-1}(S)$  is neutrosophic  $\psi$  closed in  $X.$  Hence,  $g$  is neutrosophic  $\psi$  continuous and  $g^{-1}$  is neutrosophic  $\psi$  continuous if  $A^c$  is a neutrosophic  $\psi$  closed set in  $X,$  then the image  $g(A^c) = F^c$  is neutrosophic closed in  $Y.$  Hence,  $g$  and  $g^{-1}$  are neutrosophic  $\psi$  continuous; therefore, it is a neutrosophic  $\psi$  homeomorphism.

**Theorem 1.** Each neutrosophic homeomorphism is a neutrosophic  $\alpha\psi$  homeomorphism.

**Proof.** Let a bijection mapping  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be neutrosophic homeomorphism, in which  $g$  and  $g^{-1}$  are neutrosophic continuous. We know that every neutrosophic continuous function is  $N_{\alpha\psi}$  continuous; hence,  $g$  and  $g^{-1}$  are  $N_{\alpha\psi}$  continuous. Therefore,  $g$  is an  $N_{\alpha\psi}$  homeomorphism.  $\square$

Let a neutrosophic  $\alpha\psi$  homeomorphism be not a neutrosophic homeomorphism by the following example.

**Example 4.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X,$  where  $A = \langle x, (\frac{p}{0.5}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $B = \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$   
 $C = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $D = \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$  and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y,$  where  
 $E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$   
 $F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle.$  Here,  $g(p) = p, g(q) = q, g(r) = r,$  and assume  $S = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is a neutrosophic closed set in  $Y,$  then  $g^{-1}(S)$  is neutrosophic  $\alpha\psi$  closed in  $X.$  Hence,  $g$  is neutrosophic  $\alpha\psi$  continuous and  $g^{-1}$  is neutrosophic  $\alpha\psi$  continuous if  $A^c$  is a neutrosophic  $\alpha\psi$  closed set in  $X,$  then the image  $g(A^c) = F^c$  is neutrosophic closed in  $Y.$  Hence,  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  continuous; therefore, it is a neutrosophic  $\alpha\psi$  homeomorphism. However, here,  $S$  is a neutrosophic closed set in  $Y,$  but it is not a neutrosophic closed set in  $X.$  Therefore, it is not neutrosophic continuous. Therefore, it is not a neutrosophic homeomorphism.

**Theorem 2.** Each neutrosophic  $\psi$  homeomorphism is a neutrosophic  $\alpha\psi$  homeomorphism.

**Proof.** Let a bijection mapping  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be an  $N_\psi$  homeomorphism; in that  $g$  and  $g^{-1}$  are neutrosophic  $\psi$  continuous. We know that every neutrosophic  $\psi$  continuous function is  $N_{\alpha\psi}$  continuous; hence,  $g$  and  $g^{-1}$   $N_{\alpha\psi}$  continuous. Therefore,  $g$  is an  $N_{\alpha\psi}$  homeomorphism.  $\square$

Let a neutrosophic  $\alpha\psi$  homeomorphism be not a neutrosophic  $\psi$  homeomorphism by the following example.

**Example 5.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X$ , where  $A = \langle x, (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.2}), (\frac{p}{0.5}, \frac{q}{0.1}, \frac{r}{0.2}), (\frac{p}{0.2}, \frac{q}{0.5}, \frac{r}{0.6}) \rangle$ ,  
 $B = \langle x, (\frac{p}{0.6}, \frac{q}{0.3}, \frac{r}{0.4}), (\frac{p}{0.1}, \frac{q}{0.5}, \frac{r}{0.1}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.5}) \rangle$ ,  
 $C = \langle x, (\frac{p}{0.6}, \frac{q}{0.4}, \frac{r}{0.4}), (\frac{p}{0.1}, \frac{q}{0.1}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.2}, \frac{r}{0.5}) \rangle$ ,  
 $D = \langle x, (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.6}) \rangle$ , and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y$ , where  
 $E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle$ ,  
 $F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle$ . Here,  $g(p) = p, g(q) = q, g(r) = r$ , and assume  $S = \langle y, (\frac{p}{0.6}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.1}, \frac{r}{0.2}) \rangle$ . Here,  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  continuous; therefore, it is a neutrosophic  $\alpha\psi$  homeomorphism, but it is not a neutrosophic  $\psi$  homeomorphism because  $S$  is neutrosophic  $\psi$  continuous.

**Definition 12.** Let  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be called a neutrosophic  $\alpha\psi$  open mapping if the image  $g(H)$  is neutrosophic  $\alpha\psi$  open in  $Y$  for every neutrosophic open set in  $X$ .

**Example 6.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X$ , where  $A = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $B = \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle$ ,  
 $C = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $D = \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle$ , and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y$ , where  
 $E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle$ ,  
 $F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle$ . Here,  $g(p) = p, g(q) = q, g(r) = r$ , and assume  $S = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is a neutrosophic open set in  $X$ , then  $g(S)$  is neutrosophic  $\alpha\psi$  open in  $Y$ . Therefore,  $g$  is a neutrosophic  $\alpha\psi$  open mapping.

**Definition 13.** Let  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be called a neutrosophic  $\alpha\psi$  closed mapping if the image  $g(H)$  is neutrosophic  $\alpha\psi$  closed in  $Y$  for every neutrosophic closed set in  $X$ .

**Example 7.** Let  $X = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $X$ , where  
 $E = \langle x, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle$ ,  
 $F = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $G = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $H = \langle x, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle$ , and let  $Y = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $Y$ , where  
 $A = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $B = \langle y, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle$ ,  
 $C = \langle y, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,

$D = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle$  Here,  $g(p) = p, g(q) = q, g(r) = r$ , and assume  $S = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is a neutrosophic closed set in  $X$ , then  $g(S)$  is neutrosophic  $\alpha\psi$  closed in  $Y$ . Therefore,  $g$  is a neutrosophic  $\alpha\psi$  closed mapping.

**Theorem 3.** Each neutrosophic closed mapping is a neutrosophic  $\alpha\psi$  closed mapping.

**Proof.** Let us assume that  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is a neutrosophic closed mapping, such that  $H$  is a neutrosophic closed set in  $X$ . Since  $g$  is a neutrosophic closed mapping,  $g(H)$  is neutrosophic closed in  $Y$ . We know that every neutrosophic closed set is a neutrosophic  $\alpha\psi$  closed set. Therefore,  $g(H)$  is an  $N_{\alpha\psi}$  closed set in  $Y$ . Hence,  $g$  is an  $N_{\alpha\psi}$  closed mapping.  $\square$

Let a neutrosophic  $\alpha\psi$  closed mapping be not a neutrosophic closed mapping by the following example.

**Example 8.** Let  $X = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $X$ , where

$E = \langle x, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$   
 $F = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $G = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $H = \langle x, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$  and let  $Y = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $Y$ , where

$A = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $B = \langle y, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$   
 $C = \langle y, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$   
 $D = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle.$  Here,  $g(p) = p, g(q) = q, g(r) = r$ , and assume  $S = \langle y, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is a neutrosophic closed set in  $X$ , then  $g(S)$  is a neutrosophic  $\alpha\psi$  closed in  $Y$ . Therefore,  $g$  is a neutrosophic  $\alpha\psi$  closed mapping. However, it is not a neutrosophic closed mapping because  $g(S)$  is not neutrosophic closed in  $Y$ .

**Theorem 4.** A map  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is a  $N_{\alpha\psi}$  closed mapping if the image of each neutrosophic open set in  $X$  is a  $N_{\alpha\psi}$  open set in  $Y$ .

**Proof.** Let  $H$  be a neutrosophic open set in  $X$ . Hence,  $S^c$  is a neutrosophic closed set in  $X$ . Since  $g$  is an  $N_{\alpha\psi}$  closed mapping,  $g(H^c)$  is an  $N_{\alpha\psi}$  closed set in  $Y$ . Since  $g(H^c) = (g(H))^c, g(H)$  is an  $N_{\alpha\psi}$  open set in  $Y$ .  $\square$

**Theorem 5.** Let  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be a bijective mapping, then the following statements are equivalent:

- (a)  $g$  is a neutrosophic  $\alpha\psi$  open mapping.
- (b)  $g$  is a neutrosophic  $\alpha\psi$  closed mapping.
- (c)  $g^{-1}$  is  $N_{\alpha\psi}$  continuous.

**Proof.** (a)  $\Rightarrow$  (b) Let us assume that  $g$  is a neutrosophic  $\alpha\psi$  open mapping. By definition,  $H$  is a neutrosophic open set in  $X$ , then the image  $g(H)$  is a neutrosophic  $\alpha\psi$  open set in  $Y$ . Here,  $H$  is neutrosophic closed set in  $X$ , then  $X - H$  is a neutrosophic open set in  $X$ . By assumption, we have  $g(X - H)$  is a neutrosophic  $\alpha\psi$  open set in  $Y$ . Hence,  $Y - g(X - H)$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . Therefore,  $g$  is a neutrosophic  $\alpha\psi$  closed mapping.

(b)  $\Rightarrow$  (c) Let  $H$  be a neutrosophic closed set in  $X$ . By (b),  $g(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . Hence,  $g(H) = (g^{-1})^{-1}(H)$ , so  $g^{-1}$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . Hence,  $g^{-1}$  is neutrosophic  $\alpha\psi$  continuous.

(c)  $\Rightarrow$  (a) Let  $H$  be a neutrosophic open set in  $X$ . By (c),  $(g^{-1})^{-1}(H) = g(H)$  is a neutrosophic  $\alpha\psi$  open mapping.  $\square$

**Theorem 6.** Let  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be a bijective mapping. If  $g$  is neutrosophic  $\alpha\psi$  continuous, then the following statements are equivalent:

- (a)  $g$  is a neutrosophic  $\alpha\psi$  closed mapping.
- (b)  $g$  is a neutrosophic  $\alpha\psi$  open mapping.
- (c)  $g^{-1}$  is an  $N_{\alpha\psi}$  homeomorphism.

**Proof.** (a)  $\Rightarrow$  (b) Let us assume that  $g$  is a bijective mapping and a neutrosophic  $\alpha\psi$  closed mapping. Hence,  $g^{-1}$  is a neutrosophic  $\alpha\psi$  continuous mapping. We know that each neutrosophic open set in  $X$  is a neutrosophic  $\alpha\psi$  open set in  $Y$ . Hence,  $g$  is a neutrosophic  $\alpha\psi$  open mapping.

(b)  $\Rightarrow$  (c) Let  $g$  be a bijective and neutrosophic open mapping. Furthermore,  $g^{-1}$  is a neutrosophic  $\alpha\psi$  continuous mapping. Hence,  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  continuous. Therefore,  $g$  is a neutrosophic  $\alpha\psi$  homeomorphism.

(c)  $\Rightarrow$  (a) Let  $g$  be a neutrosophic  $\alpha\psi$  homeomorphism, then  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  continuous. Since each neutrosophic closed set in  $X$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ , hence  $g$  is a neutrosophic  $\alpha\psi$  closed mapping.  $\square$

**Definition 14.** Let  $(X, \tau_{N1})$  be a neutrosophic topological spaces said to be a neutrosophic  $T_{\alpha\psi}$  space if every neutrosophic  $\alpha\psi$  closed set is neutrosophic closed in  $X$ .

**Theorem 7.** Let  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be a neutrosophic  $\alpha\psi$  homeomorphism, then  $g$  is a neutrosophic homeomorphism if  $X$  and  $Y$  are  $NT_{\alpha\psi}$  space.

**Proof.** Through the assumption that  $H$  is a neutrosophic closed set in  $Y$ , then  $g^{-1}(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . Since  $X$  is an  $NT_{\alpha\psi}$  space,  $g^{-1}(H)$  is a neutrosophic closed set in  $X$ . Therefore,  $g$  is neutrosophic continuous. By hypothesis,  $g^{-1}$  is neutrosophic  $\alpha\psi$  continuous. Let  $G$  be a neutrosophic closed set in  $X$ . Then,  $(g^{-1})^{-1}(G) = g(G)$  is a neutrosophic closed set in  $Y$ , by presumption. Since  $Y$  is an  $NT_{\alpha\psi}$  space,  $g(G)$  is a neutrosophic closed set in  $Y$ . Hence,  $g^{-1}$  is neutrosophic continuous. Hence,  $g$  is a neutrosophic homeomorphism.  $\square$

**Theorem 8.** Let  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be a neutrosophic topological space, then the following are equivalent if  $Y$  is an  $NT_{\alpha\psi}$  space:

- (a)  $g$  is neutrosophic  $\alpha\psi$  closed mapping.
- (b) If  $H$  is a neutrosophic open set in  $X$ , then  $g(H)$  is neutrosophic  $\alpha\psi$  open set in  $Y$ .
- (c)  $g(int(H)) \subseteq cl(int(g(H)))$  for every neutrosophic set  $H$  in  $X$ .

**Proof.** (a)  $\Rightarrow$  (b) This is obviously true.

(b)  $\Rightarrow$  (c) Let  $H$  be a neutrosophic set in  $X$ . Then,  $int(H)$  is a neutrosophic open set in  $X$ . Then,  $g(int(H))$  is a neutrosophic  $\alpha\psi$  open set in  $Y$ . Since  $Y$  is an  $NT_{\alpha\psi}$  space, so  $g(int(H))$  is a neutrosophic open set in  $Y$ . Therefore,  $g(int(H)) = int(g(int(H))) \subseteq cl(int(g(H)))$ .

(c)  $\Rightarrow$  (a) Let  $H$  be a neutrosophic closed set in  $X$ . Then,  $H^c$  is a neutrosophic open set in  $X$ . In the supposed way,  $g(int(H^c)) \subseteq cl(int(g(H^c)))$ . Hence,  $g(H^c) \subseteq cl(int(g(H^c)))$ . Therefore,  $g(H^c)$  is neutrosophic  $\alpha\psi$  open set in  $Y$ . Therefore,  $g(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . Hence,  $g$  is a neutrosophic closed mapping.  $\square$

**Theorem 9.** Let  $f : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  and  $g : (Y, \tau_{N2}) \rightarrow (Z, \tau_{N3})$  be neutrosophic  $\alpha\psi$  closed, where  $(X, \tau_{N1})$  and  $(Z, \tau_{N3})$  are two neutrosophic topological spaces and  $(Y, \tau_{N2})$  an  $NT_{\alpha\psi}$  space, then the composition  $g \circ f$  is neutrosophic  $\alpha\psi$  closed.

**Proof.** Let  $H$  be a neutrosophic closed set in  $X$ . Since  $f$  is neutrosophic  $\alpha\psi$  closed and  $f(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ , by assumption,  $f(H)$  is a neutrosophic closed set in  $Y$ . Since  $g$  is

neutrosophic  $\alpha\psi$  closed, then  $g(f(H))$  is neutrosophic  $\alpha\psi$  closed in  $(\tau_{N1}, \tau_{N3})$  and  $g(f(H)) = g \circ f(H)$ . Therefore,  $g \circ f$  is neutrosophic  $\alpha\psi$  closed.  $\square$

**Theorem 10.** Let  $f : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  and  $g : (Y, \tau_{N2}) \rightarrow (Z, \tau_{N3})$  be two neutrosophic topological spaces, then the following hold:

- (a) If  $g \circ f$  is neutrosophic  $\alpha\psi$  open and  $f$  is neutrosophic continuous, then  $g$  is neutrosophic  $\alpha\psi$  open.
- (b) If  $g \circ f$  is neutrosophic open and  $g$  is neutrosophic  $\alpha\psi$  continuous, then  $f$  is neutrosophic  $\alpha\psi$  open.

**Proof.** (a) Let  $H$  be a neutrosophic open set in  $Y$ . Then,  $f^{-1}(H)$  is a neutrosophic open set in  $X$ . Since  $g \circ f$  is a neutrosophic  $\alpha\psi$  open map and  $(g \circ f)f^{-1}(H) = g(f(f^{-1}(H))) = g(H)$  is neutrosophic  $\alpha\psi$  open in  $Z$ , hence  $g$  is neutrosophic  $\alpha\psi$  open.

- (b) Let  $H$  be a neutrosophic open set in  $X$ . Then,  $g(f(H))$  is a neutrosophic open set in  $Z$ . Therefore,  $g^{-1}(g(f(H))) = f(H)$  is a neutrosophic  $\alpha\psi$  open set in  $Y$ . Hence,  $f$  is neutrosophic  $\alpha\psi$  open.  $\square$

#### 4. Neutrosophic $\alpha\psi^*$ Homeomorphism

**Definition 15.** A bijection  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is called a neutrosophic  $\alpha\psi^*$  homeomorphism if  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  irresolute mappings.

**Example 9.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X$ , where

$$A = \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$$

$$B = \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$$

$$C = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$$

$D = \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle$ , and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y$ , where

$$E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$$

$$F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$$

$$G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$$

$$H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle.$$

Here,  $g(p) = p, g(q) = q, g(r) = r$ , and assume  $S = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ , then  $g^{-1}(S)$  is neutrosophic  $\alpha\psi$  closed in  $X$ . Hence,  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  irresolute; therefore, it is a neutrosophic  $\alpha\psi^*$  homeomorphism.

**Theorem 11.** Each neutrosophic  $\alpha\psi^*$  homeomorphism is a neutrosophic  $\alpha\psi$  homeomorphism.

**Proof.** A map  $g$  is a neutrosophic  $\alpha\psi^*$  homeomorphism. Let us assume that  $H$  is a neutrosophic closed set in  $Y$ . This shows that  $H$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . By assumption,  $g^{-1}(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . Hence,  $g$  is a neutrosophic  $\alpha\psi$  continuous mapping. Hence,  $g$  and  $g^{-1}$  are neutrosophic  $\alpha\psi$  continuous mappings. Hence  $g$  is a neutrosophic  $\alpha\psi$  homeomorphism.  $\square$

Let a neutrosophic  $\alpha\psi$  homeomorphism be not a neutrosophic  $\alpha\psi^*$  homeomorphism by the following example.

**Example 10.** Let  $X = \{p, q, r\}$  and  $\tau_{N1} = \{0, A, B, C, D, 1\}$  be a neutrosophic topology on  $X$ , where  $A =$

$$\langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$$

$$B = \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle,$$

$$C = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$$

$D = \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle$ , and let  $Y = \{p, q, r\}$  and  $\tau_{N2} = \{0, E, F, G, H, 1\}$  be a neutrosophic topology on  $Y$ , where

$$E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$$

$$F = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle,$$



$G = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$ ,  
 $H = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle$ . Here,  $g(p) = p, g(q) = q, g(r) = r$ , and assume  
 $S = \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle$  is neutrosophic  $\alpha\psi$  continuous, then it is neutrosophic  $\alpha\psi$  homeomorphism. However, it is not a neutrosophic  $\alpha\psi^*$  homeomorphism, because it is not neutrosophic  $\alpha\psi$  irresolute.

**Theorem 12.** If  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  is a neutrosophic  $\alpha\psi^*$  homeomorphism, then  $N\alpha\psi cl(g^{-1}(H)) \subseteq g^{-1}(N\psi cl(H))$  for each neutrosophic topological space  $H$  in  $Y$ .

**Proof.** Let  $H$  be a neutrosophic topological space in  $Y$ . Then,  $N\psi cl(H)$  is a neutrosophic  $\psi$  closed set in  $Y$ , and every neutrosophic  $\psi$  closed set is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . By assuming the mapping  $g$  is  $N\alpha\psi$  irresolute,  $g^{-1}(N\psi cl(B))$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ , then  $N\alpha\psi cl(g^{-1}(N\psi cl(H))) = g^{-1}(N\psi cl(H))$ . Here,  $N\alpha\psi cl(g^{-1}(H)) \subseteq N\alpha\psi cl(g^{-1}(N\psi cl(H))) = g^{-1}(N\psi cl(H))$ . Therefore,  $N\alpha\psi cl(g^{-1}(H)) \subseteq g^{-1}(N\psi cl(H))$  for every neutrosophic set  $H$  in  $Y$ .  $\square$

**Theorem 13.** Let  $g : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  be a neutrosophic  $\alpha\psi^*$  homeomorphism, then  $N\psi cl(g^{-1}(H)) = g^{-1}(N\psi cl(H))$  for each neutrosophic set  $H$  in  $Y$ .

**Proof.** Since  $g$  is a neutrosophic  $\alpha\psi^*$  homeomorphism, then  $g$  is a neutrosophic  $\alpha\psi$  irresolute mapping. Let  $H$  be a neutrosophic set in  $Y$ . Clearly,  $N\psi cl(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . This shows that  $N\psi cl(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . Since  $g^{-1}(H) \subseteq g^{-1}(N\psi cl(H))$ , then  $N\psi cl(g^{-1}(H)) \subseteq N\psi cl(g^{-1}(N\psi cl(H))) = g^{-1}(N\psi cl(H))$ . Therefore,  $N\psi cl(g^{-1}(H)) \subseteq g^{-1}(N\psi cl(H))$ .  $\square$

Let  $g$  be a neutrosophic  $\alpha\psi^*$  homeomorphism.  $g^{-1}$  is a neutrosophic  $\alpha\psi$  irresolute mapping. Let us consider neutrosophic set  $g^{-1}(H)$  in  $X$ , which brings out that  $N\psi cl(g^{-1}(H))$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . Hence,  $N\alpha\psi cl(g^{-1}(H))$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . This implies that  $(g^{-1})^{-1}(N\psi cl(g^{-1}(H))) = g(N\psi cl(g^{-1}(H)))$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . This proves  $H = (g^{-1})^{-1}(g^{-1}(H)) \subseteq (g^{-1})^{-1}(N\psi cl(g^{-1}(H))) = g(N\psi cl(g^{-1}(H)))$ . Therefore,  $N\psi cl(H) \subseteq N\psi cl(g(N\psi cl(g^{-1}(H)))) = g(N\psi cl(g^{-1}(H)))$ , since  $g^{-1}$  is a neutrosophic  $\alpha\psi$  irresolute mapping. Hence,  $g^{-1}(N\psi cl(H)) \subseteq g^{-1}(g(N\psi cl(g^{-1}(H)))) = N\psi cl(g^{-1}(H))$ . That is,  $g^{-1}(N\psi cl(H)) \subseteq N\psi cl(g^{-1}(H))$ . Hence,  $N\psi cl(g^{-1}(H)) = g^{-1}(N\psi cl(H))$ .

**Theorem 14.** If  $f : (X, \tau_{N1}) \rightarrow (Y, \tau_{N2})$  and  $g : (Y, \tau_{N2}) \rightarrow (Z, \tau_{N3})$  are neutrosophic  $\alpha\psi^*$  homeomorphisms, then the composition  $g \circ f$  is a neutrosophic  $\alpha\psi^*$  homeomorphism.

**Proof.** Let us take  $f$  and  $g$  to be two neutrosophic  $\alpha\psi^*$  homeomorphisms. Assume  $H$  is a neutrosophic  $\alpha\psi$  closed set in  $Z$ . Then, by the supposed way,  $g^{-1}(H)$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . Then, by hypothesis,  $f^{-1}(g^{-1}(H))$  is a neutrosophic  $\alpha\psi$  closed set in  $X$ . Hence,  $g \circ f$  is a neutrosophic  $\alpha\psi$  irresolute mapping. Now, let  $G$  be a neutrosophic  $\alpha\psi$  closed set in  $X$ . Then, by presumption,  $f(G)$  is a neutrosophic  $\alpha\psi$  closed set in  $Y$ . Then, by hypothesis,  $g(f(G))$  is a neutrosophic  $\alpha\psi$  closed set in  $Z$ . This implies that  $g \circ f$  is a neutrosophic  $\alpha\psi$  irresolute mapping. Hence,  $g \circ f$  is a neutrosophic  $\alpha\psi^*$  homeomorphism.  $\square$

### 5. Conclusions

In this paper, the new concept of a neutrosophic homeomorphism and a neutrosophic  $\alpha\psi$  homeomorphism in neutrosophic topological spaces was discussed. Furthermore, the work was extended as the neutrosophic  $\alpha\psi^*$  homeomorphism, neutrosophic  $\alpha\psi$  open and closed mapping and neutrosophic  $T_{\alpha\psi}$  space. Further, the study demonstrated neutrosophic  $\alpha\psi^*$  homeomorphisms and also derived some of their related attributes.

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