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Non - domination subdivision stable graphs

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Abstract. Subdividing an edge in the graph may increase the domination number or remains the same. In this paper, we introduce a new kind of graph called non - domination subdivision stable graph (NDSS). We obtain a necessary and sufficient condition for a graph to be NDSS. We provide a constructive characterization of NDSS trees and a MATLAB program for identifying NDSS graphs.

1. Introduction

Dominating sets has been used in graph theory for characterizing graphs based on various properties. In [1], B. Sharada et.al have provided the problem of domination subdivision number of grid graphs $P_{m,n}$ and determine the domination subdivision numbers of grid graphs $P_{m,\, n}$ for m = 2, 3 and n ≥ 2. In [2], Magda Dettlaff, Joanna Raczek and Jerzy Topp have proved that the decision problem of the domination subdivision number is NP - complete even for bipartite graphs In [3], Yamuna and Karthika provided a constructive procedure to generate a spanning tree for any graph from its dominating set, γ – set and introduced a new kind of minimum dominating set and hence generate a minimum weighted spanning tree from a γ – set for G.

In [4], Prosenjit Bose et al provided the characterization yields a linear - time algorithm for recognizing and realizing degree sequences of 2 – trees. In [5], Gunasekaran and Nagarajan have provided the model by using Unified Relationship Matrix, which improves the movement of groups. In [6], Pushpalakshmi, Vincent Antony Kumar have presented a routing protocol based on distributed dominating set based clustering algorithm. In [7], Hsu andShan have proposed algorithms for finding the minimum connected domination set of interval and circular - arc graphs. In [8], Balaji et al provided a new approach for constructing the CDS, based on the idea of total dominating set and bipartite theory of graphs.

In [9], Yamuna and Karthika have obtained the domatic number of the subdivision graph of a just excellent graph and proved the following result.

R1. If u is an up vertex for a graph in G, then u must be included in every possible γ – set.

2. Materials and methods

We consider only simple connected undirected graphs G = (V, E) with n vertices and m edges. The open neighborhood of $v \in V(G)$ is defined by $N(v) = \{ u \in V(G) \mid uv \in E(G) \}$, while its closed neighborhood is $N[v] = N(v) \cup \{ v \}$. H is a subgraph of G, if $V(H) \subseteq V(G)$ and $uv \in E(H)$ implies $uv \in E(G)$. If H satisfies the added property that for every $uv \in E(H)$ if and only if $uv \in E(G)$, then H is said to be an induced subgraph of G and is denoted by $\langle H_i \rangle$. Two graphs are homeomorphic if one can be obtained from the other by the creation of edges in series or by the merging the edges in series. In graph theory, K_5 and $K_{3,3}$ are called Kuratowski′s graph. A path is a trail in which all vertices (except perhaps the first and last ones) are distinct, P_n denotes the path with n vertices. A cycle is a circuit in which no vertex except the first (which is also the last) appears more than once. C_n is a cycle with n vertices. K_n is a complete graph with n vertices. A star S_n is the complete bipartite graph $K_{1,n}$: a tree with one internal node and n leaves (but, no internal nodes and n + 1 leaves when $n \leq 1$). The complement of a graph G is a graph $\overline{G}$ on the same vertices ∍ two distinct vertices of $\overline{G}$are adjacent if and only if they are not adjacent in G. For the properties related to graph theory we refer to F. Harary[10].

A set of vertices D in G is said to be a dominating set if for every vertex of V – D is ⊥ to some vertex of D. The smallest possible cardinality of any dominating set D of G is called a minimum dominating set – abbreviated MDS. The cardinality of any MDS for G is called the domination number of G and it is denoted by γ (G).The private neighborhood of $v \in D$ is defined by pn [v, D] = N (v) – N (D – { v}). A vertex v is said to be selfish in the MDS D, if v is required only to dominate itself. A vertex of degree one is called pendant vertex and its neighbor is called a support vertex. If there is a γ – set of G containing v, the v is said to be good. If v does not belongs to any of the γ – set of G , then v is said to be a bad vertex. A vertex v is known to be a down vertex if γ(G - u) <γ (G). A vertex v is known to be a level vertex if γ(G - u) = γ (G). A vertex v is said to be a up vertex if γ(G - u) >γ (G). For the properties related to domination we refer to Haynes, Hedetniemi, and Slater [11].

A subdivision of a graph G is a graph obtained from the subdivision of edges in G. The subdivision of some edge e with end vertices { u , v } generate a graph with one new vertex w, and with an edge set replacing e by two new edges, { u, w } and { w, v } and it is denoted by G_{sd}uv. Let w be the vertex introduced by subdividing uv. We shall denote this by G_{sd}uv = w. If G is any graph and D is a γ – set for G, then D ∪ { w } is a γ – set for G_{sd}uv implies γ (G_{sd}uv) ≥γ (G), ∀ u, v ∈V (G), u ⊥ v. A graph G is defined as DSS, if γ (G_{sd}uv) = γ (G), ∀ u, v ∈V (G), u ⊥ v [12]. In [12], the following result is proved.

R2. A graph G is domination subdivision stable if and only if ∀ u, v ∈V(G), either ∃ a γ – set containing u and v or ∃γ – set D such that

1. pn (u, D) = { v } or
2. v is 2 – dominated.

In this paper we consider graphs for which γ(G_{sd}uv) = γ (G) + 1.

3. Results and Discussion

In this section we introduce a new kind of graph called NDSS graph. We provide a necessary and sufficient condition for a graph to be NDSS and prove some results satisfied by NDSS graphs.

3.1. Non - domination subdivision stable graph

A graph G is said to be non - domination subdivision stable (NDSS) if γ (G_{sd}uv) = γ (G) + 1 for all u, v ∈V (G), u adjacent to v.

Example of NDDS graphs

1. P_{3n} is NDSS.
2. C_{3n} is NDSS.
3. Complete graph K_n.
4. Star graph S_n.
5. The graph G in Fig. 1 is NDSS.

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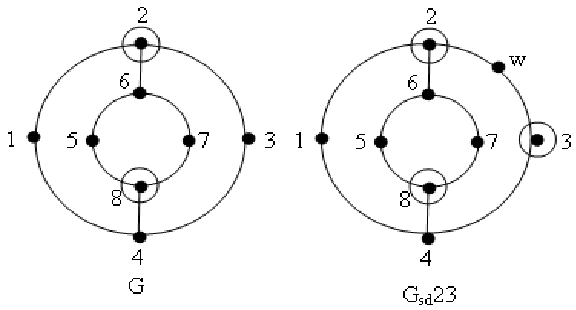

Figure 1.

In Fig. 1 $\gamma(G) = 2$, $\gamma(G_{sd}23) = 3$. This is true $\forall$ u, v $\in V(G)$, u $\perp$ v.

Theorem 1

A graph G is NDSS if and only if for every possible γ – set D for G, N (u, D), N (v, X) $\in$ V – D for all u $\in$ D, v $\in$ D where X = B (D).

Proof

Let G be NDSS graph. Let D { $u_1, u_2, \ldots, u_k$ }be a γ – set for G, X = { $x_1, x_2, \ldots, x_p$} = B (D), Y = { $y_1, y_2, \ldots, y_q$ } = B (X).

1. If there exist some xi $\in$ N (u_j), i = 1 to p, j = 1 to k such that x_i adjacent to u_j, x_j, $u_j \in$ D. $\gamma(G_{sd}x_iy_j) = \gamma(G)$ since, x_i, u_j dominates w.
2. If there exist some u_i, x_j, y_l such that u_i adjacent to x_j, x_j adjacent to y_l, u_i ,$y_l \in$ D. $\gamma(G_{sd}u_ix_j) = \gamma(G)$ since u_i dominates w and y_l dominates x_j. Also $\gamma(G_{sd}x_jy_l) = \gamma(G)$ since y_l dominates w and $u_i$dominates x_j.

In both cases, we get a contradiction to assumption G is an NDSS graph.

Conversely, assume that for every γ – set D of G, N (u, D), N (v, x) $\in$ V – D where X = B (D).

We have to prove G is NDSS. If possible assume that G is NDSS. This means that G is not NDSS.

This implies that $\gamma(G_{sd}uv) = \gamma(G)$.

By DSS, $\gamma(G_{sd}uv) = \gamma(G)$ if and only if

- u, v $\in$ D.
- if u $\in$ D, v is 2 – dominated.
- pn (u, D) = { v }.

If u, v $\in$ D, u adjacent to v is not possible since N (u, D) $\in$ V – D by our assumption.

If u $\in$D , v is 2 – dominated is not possible since N (v, x) $\in$ D by our assumption.

If pn (u, D) = { v }, let D′ = D – { u } $\cup$ { v }. Let Z = N (v) = { $z_1, z_2, \ldots, z_s$ }. In D there exist one b $\in$ D such that b is adjacent to some $z_i \in$ Z (since pn (u, D) = v, z_i is dominated by b) v, b $\in$ D, a contradiction to our assumption that for any u $\in$ D, N (v, x) $\in$ V – D where X = B (D).

In all these cases, we get a contradiction to our assumption, implies G is NDSS.

Remark

1. Since for any u $\in$ D, N (u, D) $\in$ V – D, we conclude that if G is NDSS then every γ – set of G is independent.
2. Since N (u, D), N (v, X) $\in$ V – D, where x $\in$ B (D), we conclude that if G is NDSS then no vertex in V – D is 2 – dominated.

NG - type result

Theorem 2

If G is a NDSS graph, then

$\gamma(G) + \gamma(\overline{G}) \leq \lfloor \frac{n}{3} \rfloor + 2$

$\gamma(G) . \gamma(\overline{G}) \leq 2 \lfloor \frac{n}{3} \rfloor$

Proof

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Let G be a NDSS graph. Let D = { u_1, u_2, …., u_k } be a γ – set for G. pn (u , D) ≥ 2 ∀ u ∈ D, implies n = D + pn (u, D) + k, k a non – negative integer. n = 3D + k, implies D $\leq \frac{(n-k)}{3}$. If pn (u_i, D) = 2 for all u_i∈ D then k = 0, implies | D | $\leq \frac{n}{3}$. In G, u_i dominates V (G) – pn (u_i, D). In $\overline{G}$ any u_j , j ≠ i dominates pn (u_i , D), implies γ ($\overline{G}$) = { u_i, u_j } = 2.

$\gamma(G) + \gamma (\overline{G}) \leq \lfloor \frac{n}{3} \rfloor + 2$

$\gamma(G) . \gamma(\overline{G}) \leq 2 \lfloor \frac{n}{3} \rfloor$.

Remark

By Theorem 2, for any u_i, u_j∈ D, γ ($\overline{G}$) = { u_i, u_j }. Also every γ – set of D is independent, implies u_i is adjacent to u_j in G, implies G is not NDSS. So if G is a NDSS graph, G is never NDSS.

Theorem 3

If a graph G has a unique independent γ – set,∍ every v ∈V – D ∈pn (u, D) ∀ u ∈ D, then γ (G_{sd}uv) = γ (G) + 1 ∀ u, v ∈V (G), u ⊥ v.

Proof

Let G be a graph having a unique independent γ – set D, ∍ every v ∈V– D∈pn (u, D) for some u ∈ D. If possible let D′ = γ(G_{sd}uv) = γ (G) for some u, v ∈V (G), u ⊥ v. Let γ(G_{sd}uv) = w. We consider the following cases.

Case 1: u ∈ D′, w, v ∉ D′

Since v ∉ D′ there exist one x ∈ V (G) that dominates v, implies D′ is a γ - set for G ∍ v is two dominated , a contradiction.

Case 2: v ∈ D′, u, w ∉ D′

We get a contradiction similar to case 1.

Case 3: w ∈ D′, u, v∉ D′

In this case D′′ = D′ – { w } ∪ { u }, D′ – { w } ∪ { u } are two possible γ - sets for G, a contradiction to our assumption that G has a unique γ - set.

Case 4: u, w ∈ D′, v ∉ D′

w dominates only v. D′′ = D′ – { w } is a γ - set for G (since in D′′, u dominates v), a contradiction as | D′′ | < | D |.

Case 5: w, v ∈ D′, u ∉ D′

We get a contradiction as in case 4.

Case 6: u, v ∈ D′ , w ∉ D′

D′ is a γ - set for G a contradiction as D′ is not independent.

By the above cases we conclude that γ (G_{sd}uv) = γ (G) + 1 ∀ u, v ∈V (G), u ⊥ v.

Remark

1. If G has no unique independent γ - set, then γ(G_{sd}uv) may be equal to γ (G).

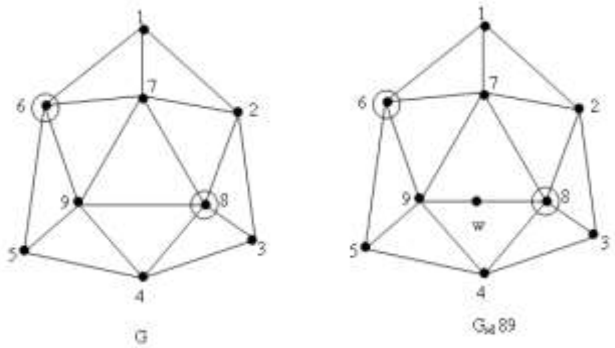

Figure 2.

For the graph G in Fig. 2 { 6, 8 }, { 7, 4 } are 2 γ - sets for G. Also γ (G_{sd}89) = 2.

2. If every $v \in V - D \notin pn(u, D)$, for some $u \in D$, then $\gamma(G_{sd}uv)$ may be equal to $\gamma(G)$.

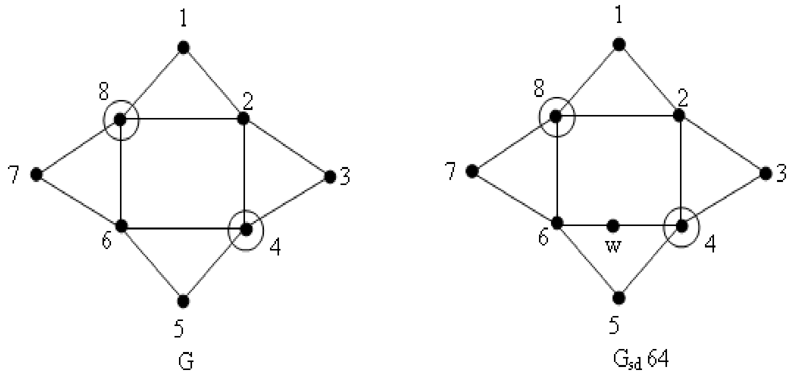

Figure 3.

For the graph G in Fig. 3 $\gamma(G) = \{2, 6\}$, $\{8; 4\} \notin pn(4, D)$ or $pn(8, 4)$. $\gamma(G_{sd}64) = 2$.

Theorem 4

Every graph is an induced subgraph of a NDSS graph.

Proof

Let G be any graph with n vertices. If G is NDSS then there is nothing to prove. Assume that G is not NDSS. Consider a cycle C_n. Label the vertices of C_n as $u_1, u_2, \ldots, u_n$. In C_n we add edges u_iu_j if and only if v_iv_j is an edge in G (retaining the graph simple). Consider n copies of P_3. Label the vertices in P_3 as v_i, w_i, z_i , $i = 1$ to n. Obtain a new graph H by merging u_i, $v_i \forall$ $i = 1$ to n. Label the merged vertices u_iv_i as x_i, $i = 1$ to n. $D = \{w_i,\ i = 1, 2, \ldots, n\}$ is a unique γ - set for H, implies $\gamma(H) = n$.

In graph H, D is a γ - set such that

1. D is unique.
2. D is independent.
3. every $v \in V - D$ is private neighbor of some $u \in D$, implies $\gamma(H_{sd}uv) = \gamma(H) = \gamma(G) + 1$ $\forall$ $u, v \in H$, $u \perp v$, (by Theorem 3) implies H is NDSS.

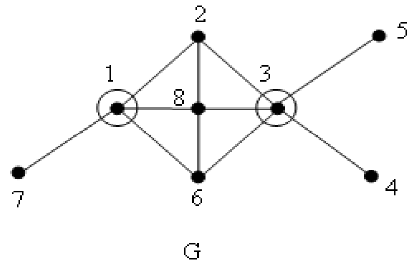

Fig. 4

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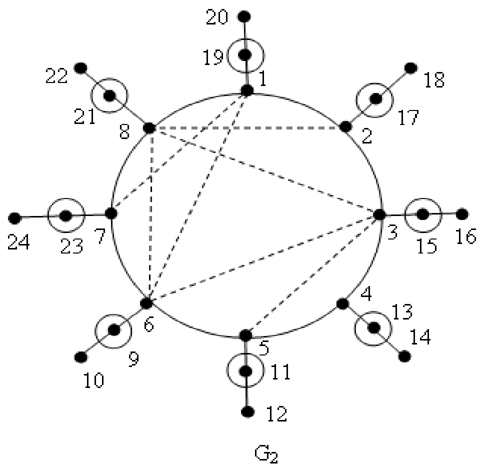


Figure 5.

The graph G in Fig. 4 is not NDSS. We see that this is an induced subgraph in Fig. 5. Also the graph in Fig. 5 is NDSS.

3.2. Tree Characterization

In this section we prove that any NDSS tree has a unique γ - set. We also provide a constructive characterization of NDSS trees.

Theorem 5

If T is a NDSS tree, then G has a unique γ - set.

Proof

Since pn (u , D) $\geq$ 2, $\forall$ u $\in$ D, for a NDSS graph there exist no γ - set for G including a pendant vertex, implies every support vertex is included in every γ - set. If possible assume that the γ - set of T is not unique. Since every support vertex is in every γ - set there exists an internal vertex u such that there exist a γ - set D including u and a γ - set D′ not including u.

Claim 1

u is an up vertex with respect to D.

Proof

Since u cannot be a down vertex (If G is a NDSS graph, then G has no down vertices) u is either a level or an up vertex. If possible let us assume that u is a level vertex.

T – { u } is a disconnected graph with at least two components. Without loss of generality assume that T – { u } is a disconnected graph with two components T_1, T_2. $\gamma(T_1) + \gamma(T_2) = \gamma (T)$. Also pn(u, D) $\geq$ 2 . Assume that pn(u, D) = 2 = { u_1, u_2 }(say). Let us assume that $u_1 \in V (T_1)$, $u_2 \in V (T_2)$. Let $D_1 \subseteq D$ be the set of all vertices in D $\in V (T_1)$. Let $D_2 \subseteq D$ be the set of all vertices in D $\in V (T_2)$. Let D_1' be a γ - set for T_1 and D_2' be a γ - set for T_2. Since $| D_1' + D_2' | = | D |$, either $D_1' = | D_1 |+ 1$ or $D_2' = | D_2 |+ 1$. Assume that D_1 = | D1 | + 1. D_1 dominates $T_1 – \{ u_1\}$. $D_1 \cup \{ u_1 \}$ dominates T_1. $D_3 = D_1 \cup \{ u_1 \} \cup D_2'$ is a γ - set for T such that pn (u_1, D_3) = u, a contradiction as T is NDSS implies u is not a level vertex. Hence u is an up vertex. By claim 1 we know that any internal vertex in D is an up vertex. Also any support vertex is an up vertex in D, no pendant vertex belongs to D, implies every vertex in D is an up vertex and hence D is unique.

Theorem 6

Let G be a NDSS graph, u $\in$ V (G). Let H be the graph obtained by attaching P_1 to u. If H is NDSS then $\gamma(H) = \gamma (G)$.

Proof

Suppose $\gamma (H) \neq \gamma (G)$, then $| \gamma (G) | < | \gamma (H) |$. Let D′ be a γ- set for G such that $| D' | < | \gamma(H) |$. $u \notin D'$ (since if $u \in D'$, $| D' | = | \gamma (H) |$. as D′ dominates H also). Since $u \not\in D'$, there exist atleast one $x \in D'$ such that $x \in N (u)$ to dominate u. Then $D'' = D' \cup \{ u \}$ is a γ - set for H such that u, x $\in$

D′′such that u is adjacent to x, a contradiction as H is NDSS implies, γ (H) = γ (G). Hence D is a γ - set for H, G ∋ u ∈ D.

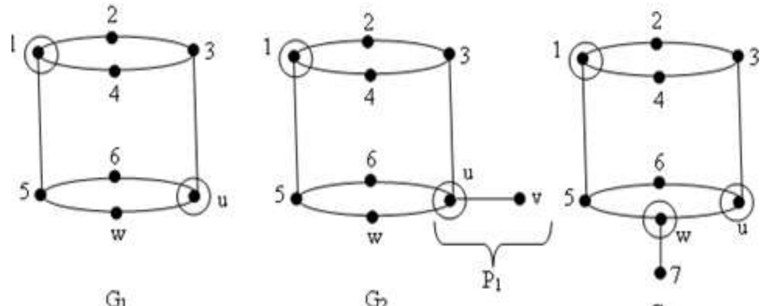


Figure 6.

Let D = { 1, u} be a γ - set for G_1. $\gamma(G_1) = \gamma(G_2)$. $\gamma(G_3) = \gamma(G_1) + 1$. Also u ∈ D, while w ∉ D. We generalize this observation in Theorem 7.

Theorem 7

Let G be a NDSS graph, u ∈ V (G). Let D be a γ - set for G. Let H be the graph obtained by attaching P_1 to u. H is NDSS if and only if u∈ D.

Proof

Let us label the new pendant vertex in H as v. Assume that H is NDSS. Since pn(u, D) ≥ 2 for a NDSS graph, there exist no γ - set for H including v, implies u is included in every γ - set for H. Conversely assume that u is a γ - set for G such that u ∈ D. Since γ(G) = γ (H), D dominates H also. Every γ - set of H is independent Suppose that D is not independent. Since deg (v) = 1, both u, v ∉ D, implies there exist one v_1, v_2∈ V (G) such that v_1, v_2∈ D , v_1 is adjacent to v_2. D is a γ - set for G also, a contradiction as G is NDSS. H has no 2 - dominated vertex. If possible assume that H has a 2 - dominated vertex. Since v is pendant, v is never 2 - dominated, implies there exist one v_1∈ V (G) such that v_1 is 2 - dominated with respect to D, a contradiction as D is a γ - set for G also, G NDSS. From the discussions, we conclude that H is NDSS [by remark of Theorem 1].

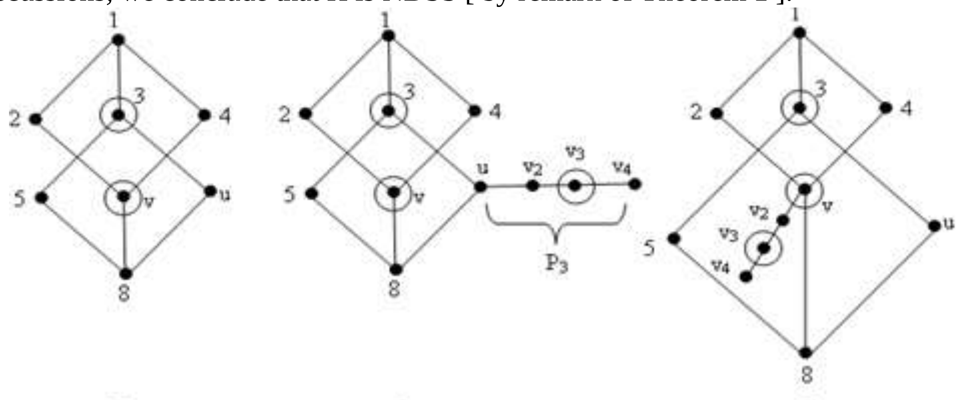


Figure 7.

Let D = { 3, v } be a γ - set for G_1. $\gamma(G_2) = \gamma(G_3) = \gamma (G_1) + 1$. We observe that if P_3 is attached to a good vertex then, H is not NDSS, while if P_3 is attached to a bad vertex, then H is NDSS. Also u ∈ D, while w ∉ D. We generalize this result in Theorem 8.

Theorem 8

Let G be a NDSS graph and let u ∈ V (G). Let H be the graph obtained by attaching $P_3$to u. H is NDSS if and only if u is a bad vertex with respect to G.

Proof

Let us assume that G is NDSS, u ∈ V (G). Let H be the graph generated by attaching a path P_3 to u. Let v_1, v_2, v_3, v_4 be the attached path. Let v_1 be joined to u. $\gamma (H) = \gamma (G) \cup \{ v_3\} = \gamma (G) + 1$. If there exist a γ - set D for G containing u, then $D' = D \cup \{ v_3\}$ is a γ - set for H $\ni$ v_2 is 2 - dominated, a contradiction as H is NDSS, implies u is a bad vertex in G.

Conversely, assume that G is a NDSS graph, $u \in V(G)$, u a bad vertex with respect to G.

Every γ - set of H is independent

If possible assume that $\exists$ a γ - set D for H that is not independent. If $v_2, v_3 \in D$ then $D - \{ v_2\} \cup f\{ v\}$ is a γ - set for G. If $v_3, v_4 \in D$ then $D - \{ v_4\} \cup \{ v\}$ is a γ - set for G. In both cases u is a good vertex, a contradiction to our assumption that u is bad. If there exist some $u_i, u_j \in V(G)$, u_i adjacent to u_j, $u_i, u_j \in D$, then since u is a bad vertex $D - \{ v_3\}$ is a γ - set for G such that u_i adjacent to u_j, a contradiction as G is NDSS, implies every γ - set of H is independent.

H has no 2 - dominated vertex

If possible assume that H has 2 - dominated vertex. If v_2 is 2 – dominated then u, $v_3 \in D$, a contradiction as u is a bad vertex. If v_3 is 2 - dominated then $v_2, v_4 \in D$. $D' = D - \{ v_4\} \cup \{ v\}$ is a γ - set for H containing u, a contradiction as u is a bad vertex. If there exist some $u_i, u_j \in D$, $u_i, u_j \in V(G)$, x adjacent to u_i, u_j, then $D - \{ v_3\}$ is a γ - set for G $\ni$ x is 2 - dominated, a contradiction as G is NDSS. From the above discussion, we conclude that H is NDSS.

By attaching a path P to a vertex v in T, we mean that adding the path P and attaching v to a pendant of P.

Operation O_1 Attach a path P_1 to good vertex v of T.

Operation O_2 Attach a tree path P_3 to a bad vertex v of T.

Let τ be the family defined by $\tau = \{ T / T$ is generated from P_2 by a finite sequence of operations O_1 or $O_2 \}$.

From Theorem 7 and Theorem 8 we know that if $T \in \tau$, then T is a NDSS tree.

Theorem 9

If T is a NDSS tree, then $T \in \tau$.

Proof

We proceed by induction on the order $n \geq 3$ of a NDSS tree. If T is a star, then T can be generated from P_2 by frequent application of operation O_1. Hence we assume that diam(T) ≥ 3. Assume that the lemma is true for all tree T' of order $n' < n$. Let T be rooted at a leaf r of a longest path P. let D be a γ - set for T. Let P be a r - u path. Let v be the neighbor of u. Let w represent the parent of v, x and y are the parent of w and x respectively. By T_x we denote the subtree induced by vertex x and its descendants in the rooted tree T. Since T is NDSS $d_T(v) \geq 2$. Let $T' = T - T_u$. If $d_T(v) = 2$, then pn(v, D) = { u, w } and $v \in D$, since v is support vertex. v is a pendant with respect to T'. Let D' be a γ - set for T' that contains all the support vertices, implies $w \in D'$. Also $v \in D'$. Since $\gamma(T) = \gamma(T')$ [by Theorem 6] , D, D' are two distinct γ - sets of T', a contradiction as T' NDSS [Theorem 5]. Hence $d_T(v) \geq 3$, implies v $\perp$ to atleast two leaves. Let D' be a γ - set that contains all the support vertices for T', implies $v \in D'$. Hence T can be generated from T' by operation O_1.

Suppose $d_T(w) \geq 3$. $T_w - \{ w \}$ is either K_1 or K_2, since P is the longest path. Assume that $T_w - \{ w \}$ contains K_1. If D is the γ - set for T containing all the support vertices, then w, $v \in D$, a contradiction as G is NDSS. Assume that $T_w - \{ w \}$ contains only K_2. Since $d_T(w) \geq 3$, $T_w - \{ w \}$ contains atleast two components, each component K_2. One component is uv. Label the other component as v_1, u_1. v_1 adjacent to w. Let D be a γ - set that contains all the support vertices for T. v, $v_1 \in D$, implies w is 2 - dominated, a contradiction. Hence $d_T(w) = 2$. Let $T' = T - T_w$. Let D' be a γ - set for T' such that $x \in D'$. Then $D' \cup \{ v \}$ is a γ - set for T $\ni$ w is two dominated, a contradiction as T is NDSS. So there exist no γ - set for T' containing x. Since x is a bad vertex T can be obtained from T' by Operation O_2.

As a consequences of Theorem 7 and Theorem 8, we have the following characterization for NDSS trees.

Theorem 10

A tree T is NDSS if and only if $T \in \tau$.

3.3. Matrix representations

Let G be a graph with n – vertices. Let A and N denotethe adjacency matrix and n ×n matrix of G, where

$N = [\, n_{ij}]_{n\times n} = \begin{cases} 1, \text{ if } i = j \\ a_{ij}, \text{the } (i, j)^{th} \text{ entry in the adjacency matrix.} \end{cases}$

Let $x = \langle x(v_1), x(v_2), \ldots, x(v_n) \rangle^T$ be a { 0, 1 } vector. If x represents any dominating set, then $Nx \geq 1$.

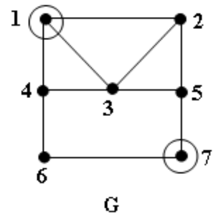


Figure 8.

The corresponding vector x = ⟨ 0, 0, 1, 0, 0, 1, 0 ⟩. We see that $Nx \geq 1$.

$$N = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \quad x = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad Nx = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Nx is a column matrix. In any row of matrix N, the number of non zero entries represents N [v_i] and x represents a dominating set. Every entry in Nx represents the number of vertices dominating any vertex v_i. If row entryv_i in Nx is 1, then $v_i \in V - D$ is a private neighbor. Similarly if row entry v_i in $Nx \geq 2$, then vertex $v_i \in V - D$ is k – dominated by x.

Finding a dominating set using matrix method can be used to characterize graphs satisfying a given domination parameter. Graph characterization based on dominating set focus on γ – set and all possible γ – sets satisfying the defined property. For this purpose, since we are more focused in all possible γ – sets than all possible dominating set, we use the following notation.

Notation

1. Let G be any graph with n vertices $v_1, v_2, \ldots, v_n$. Let γ(G) = k. Label the all possible subsets with k vertices as $S_1, S_2, \ldots, S_p$, where p = nC_k. Let X = { $x_1, x_2, \ldots, x_p$} be a set of { 0, 1 } vectors given by $x_i = \langle x(v_1), x(v_2), \ldots, x(v_n) \rangle^T$, where $x(v_i) = \begin{cases} 1 & \text{if } v_i \in S_i \\ 0 & \text{otherwise.} \end{cases}$ Using the above notation if γ (G) = 2, n = 5, S_1 = then S_1= { v_1, v_2 }, S_2= { v_1, v_3 }, S_3= { v_1, v_4 }, S_4= { v_1, v_5 }, S_5= { v_2, v_3 }, S_6= { v_2, v_4 }, S_7= { v_2, v_4 }, S_8= { v_3, v_4 }, S_9= { v_3, v_5 }, S_{10} = { v_4, v_5 }. So, $x_1 = \langle 1, 1, 0, 0, 0 \rangle^T$, $x_2 = \langle 1, 0, 1, 0, 0 \rangle^T$, $x_3 = \langle 1, 0, 0, 1, 0 \rangle^T$, $x_4 = \langle 1, 0, 0, 0, 1 \rangle^T$, $x_5 = \langle 0, 1, 1, 0, 0 \rangle^T$, $x_6 = \langle 0, 1, 0, 1, 0 \rangle^T$, $x_7 = \langle 0, 1, 0, 0, 1 \rangle^T$, $x_8 = \langle 0, 0, 1, 1, 0 \rangle^T$, $x_9 = \langle 0, 0, 1, 0, 1 \rangle^T$, $x_{10} = \langle 0, 0, 0, 1, 1 \rangle^T$,.
2. Nx_i is a column matrix. Let us denote this as vector, $nx_{i} = \langle nx_i(v_1), nx_i(v_2), \ldots, nx_i(v_n) \rangle^T$.
3. Define a matrix of vectors V as V = $[\, v_{ij}]_{n\times p} = [\, x_1, x_2, \ldots, x_p]$, each x_i, i = 1, 2, …, p represents a vector defined in notation 1. Determine NV, where each column represents vector x_i, that is the columns represents vector $nx_1, nx_2, \ldots, nx_p$.

If x is any vector representing a γ - set then, each entry in matrix Nx represents the number of vertices dominating any vertex in G i.e., if an entry value in Nx is 4, then it is dominated by 4 vertices. Let G be a NDSS graph. By remark 1 of Theorem 1 we know that G is NDSS if every γ - set of G is independent, G has no two dominated vertices. If D is a dominating set and x_i is any vector representing D then $Nx_i = [\ 1, 1,\ldots,1\ ]^T$. Consider NV. If NV contains no zero entry, then every x_i, i = 1, 2, ...,p areγ - set for G, implies there is at least one non independent γ - set for G, implies G is not NDSS. If NV has atleast one zero entry, then consider the non zero column of NV. Let $S \subseteq x$ be the set of all vectors$\exists Nx_i \geq 1$, that is $NS \geq 1$. Let $|\ S\ | = q$, $q < p$. Consider the i^{th} column of NS, that is vector nx_i. We know that $nx_i = \langle nx_i\ (v_1)\ ,nx_i\ (\ v_2\ )\ ,\ \ldots,\ nx_i\ (\ v_n\ )\ \rangle^T$. If there exist atleast one v_j , j = 1 to n such that $nx_i\ (\ v_j) \geq 2$, then vertex v_j is two dominated. So for every x_i, i = 1 to q. If $nx_i = \langle\ 1, 1,\ldots, 1\ \rangle^T$, then every vertex in G is single dominated, implies D is independent and every $v \in V - D$ is a private neighbor of some $u \in D$. That is if in matrix NS all entries are 1, then G is NDSS.

Example

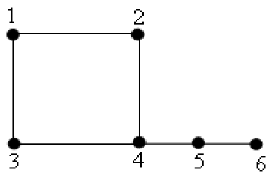


Figure 9.

Consider all possible subsets with two vertices and label them as $\{\ S_1, S_2, S_3, \ldots, S_{15}\ \} = \{\{\ v_1, v_2\}, \{\ v_1, v_3\}, \{\ v_1, v_4\}, \{\ v_1, v_5\}, \{\ v_1, v_6\ \}, \{\ v_2, v_3\ \}, \{\ v_2, v_4\ \}, \{\ v_2, v_5\ \}, \{\ v_2, v_6\ \}, \{\ v_3, v_4\ \}, \{\ v_3, v_5\ \}, \{\ v_3, v_6\ \}, \{\ v_4, v_5\ \}, \{\ v_4, v_6\ \}, \{\ v_5, v_6\ \}\ \}$.

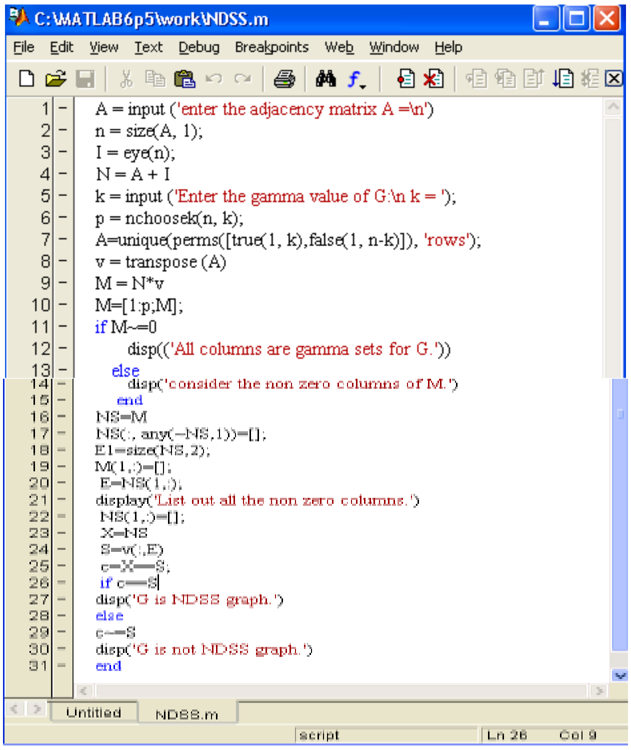
$$NV = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 00 & 0 & 0 & 0 & 1 & 11 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 01 & 1 & 1 & 1 & 0 & 00 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 10 & 0 & 0 & 1 & 0 & 00 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 10 & 0 & 1 & 0 & 0 & 01 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 00 & 1 & 0 & 0 & 0 & 10 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 01 & 0 & 0 & 0 & 1 & 00 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 11 & 1 & 1 & 2 & 1 & 11 & 2 & 2 \\ 0 & 1 & 1 & 0 & 0 & 11 & 1 & 2 & 1 & 1 & 12 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 & 20 & 0 & 1 & 1 & 1 & 12 & 2 & 1 \\ 1 & 1 & 2 & 1 & 2 & 21 & 2 & 2 & 2 & 0 & 11 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 & 11 & 1 & 1 & 0 & 1 & 11 & 0 & 0 \\ 2 & 1 & 1 & 1 & 1 & 01 & 1 & 0 & 0 & 1 & 10 & 0 & 0 \end{pmatrix}$$

From the matrix NV the only non – zero column corresponds to the vector $x_i = \langle\ 1, 0, 0, 0,\ 1, 0\ \rangle$. The corresponding γ - set is $\{\ v_3,\ v_6\ \}$. In matrix NV this column corresponding to Nx_i is 1′s. $\langle\ 1, 1, 1, 1, 1, 1\ \rangle^T$. Hence G is NDSS.

3.4. MAT Lab program for NDSS graphs

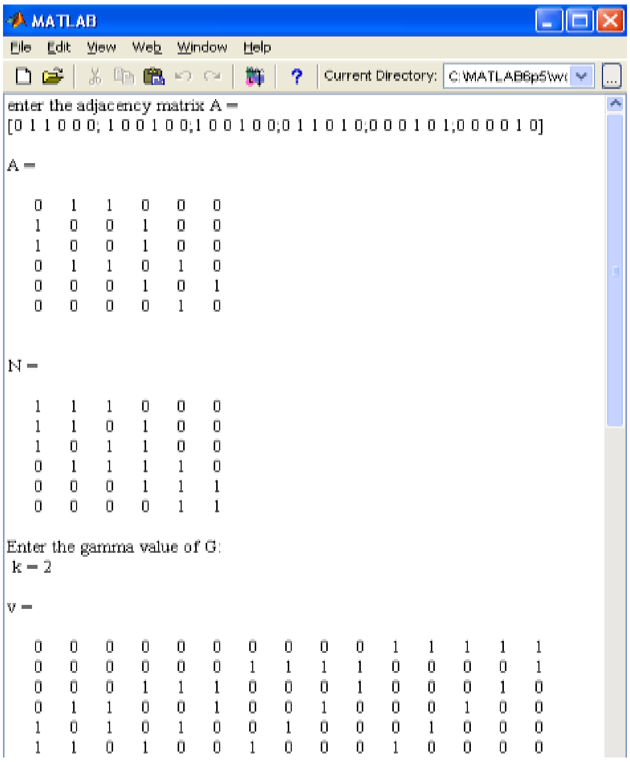
Based on the above discussion snapshot - 1 provides a MATLA code for identifying NDSS graphs. Snapshot - 2 provides the output for the graph in Fig. 9. We see that the output matches the discussion for the graph in Fig. 9.



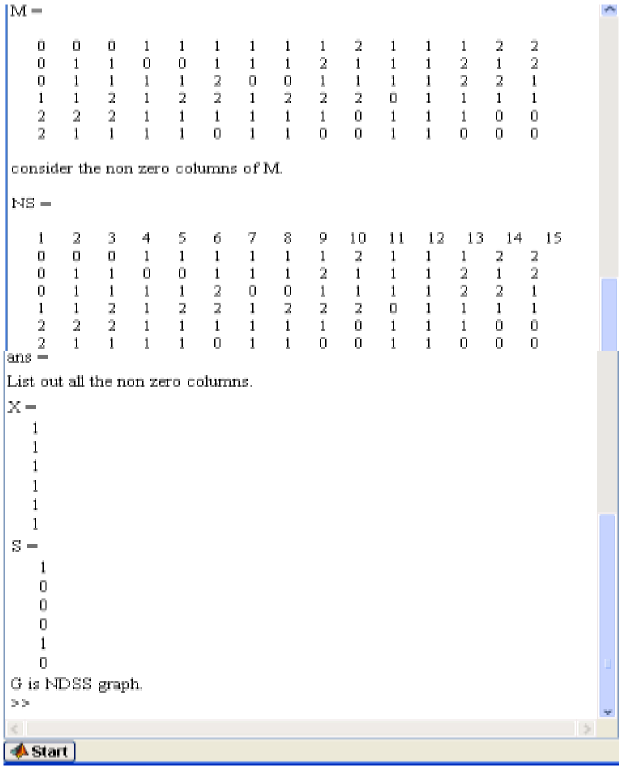
```
A = input ('enter the adjacency matrix A =\n')
n = size(A, 1);
I = eye(n);
N = A + I
k = input ('Enter the gamma value of G:\n k = ');
p = nchoosek(n, k);
A=unique(perms([true(1, k),false(1, n-k)]), 'rows');
v = transpose (A)
M = N*v
M=[1:p;M];
if M~=0
     disp(('All columns are gamma sets for G.'))
  else
     disp('consider the non zero columns of M.')
  end
NS=M
NS(:, any(~NS,1))=[];
E1=size(NS,2);
M(1,:)=[];
 E=NS(1,:);
display('List out all the non zero columns.')
 NS(1,:)=[];
 X=NS
 S=v(:,E)
 c=X==S;
 if c==S
disp('G is NDSS graph.')
else
c~=S
disp('G is not NDSS graph.')
end
```

Snapshot 1.

Output



```
enter the adjacency matrix A =
[0 1 1 0 0 0; 1 0 0 1 0 0;1 0 0 1 0 0;0 1 1 0 1 0;0 0 0 1 0 1;0 0 0 0 1 0]

A =

     0     1     1     0     0     0
     1     0     0     1     0     0
     1     0     0     1     0     0
     0     1     1     0     1     0
     0     0     0     1     0     1
     0     0     0     0     1     0


N =

     1     1     1     0     0     0
     1     1     0     1     0     0
     1     0     1     1     0     0
     0     1     1     1     1     0
     0     0     0     1     1     1
     0     0     0     0     1     1

Enter the gamma value of G:
 k = 2

v =

     0     0     0     0     0     0     0     0     0     0     1     1     1     1     1
     0     0     0     0     0     0     1     1     1     1     0     0     0     0     1
     0     0     0     1     1     1     0     0     0     1     0     0     0     1     0
     0     1     1     0     0     1     0     0     1     0     0     0     1     0     0
     1     0     1     0     1     0     0     1     0     0     0     1     0     0     0
     1     1     0     1     0     0     1     0     0     0     1     0     0     0     0
```



```
M =

     0     0     0     1     1     1     1     1     1     2     1     1     1     2     2
     0     1     1     0     0     1     1     1     2     1     1     1     2     1     2
     0     1     1     1     1     2     0     0     1     1     1     1     2     2     1
     1     1     2     1     2     2     1     2     2     2     0     1     1     1     1
     2     2     2     1     1     1     1     1     1     0     1     1     1     0     0
     2     1     1     1     1     0     1     1     0     0     1     1     0     0     0

consider the non zero columns of M.

NS =

     1     2     3     4     5     6     7     8     9    10    11    12    13    14    15
     0     0     0     1     1     1     1     1     1     2     1     1     1     2     2
     0     1     1     0     0     1     1     1     2     1     1     1     2     1     2
     0     1     1     1     1     2     0     0     1     1     1     1     2     2     1
     1     1     2     1     2     2     1     2     2     2     0     1     1     1     1
     2     2     2     1     1     1     1     1     1     0     1     1     1     0     0
     2     1     1     1     1     0     1     1     0     0     1     1     0     0     0
ans =
List out all the non zero columns.
X =
    1
    1
    1
    1
    1
    1
S =
     1
     0
     0
     0
     1
     0
G is NDSS graph.
>>
```


Snapshot 2.

4. Conclusion

This paper contributes the necessary and sufficient condition, tree characterization of an NDSS graph and also provides a method of identifying NDSS graphs using MATLAB program.

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