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ORIGINAL ARTICLE

Numerical study of magnetohydrodynamics (MHD) boundary layer slip flow of a Maxwell nanofluid over an exponentially stretching surface with convective boundary condition

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Brownian motion;
Thermophoresis parameter

Abstract This paper focuses on a theoretical analysis of a steady two-dimensional magnetohydrodynamic boundary layer flow of a Maxwell fluid over an exponentially stretching surface in the presence of velocity slip and convective boundary condition. This model is used for a nanofluid, which incorporates the effects of Brownian motion and thermophoresis. The resulting non-linear partial differential equations of the governing flow field are converted into a system of coupled non-linear ordinary differential equations by using suitable similarity transformations, and the resultant equations are then solved numerically by using Runge-Kutta fourth order method along with shooting technique. A parametric study is conducted to illustrate the behavior of the velocity, temperature and concentration. The influence of significant parameters on velocity, temperature, concentration, skin friction coefficient and Nusselt number has been studied and numerical results are presented graphically and in tabular form. The reported numerical results are compared with previously published works on various special cases and are found to be an in excellent agreement. It is found that momentum boundary layer thickness decreases with the increase of magnetic

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parameter. It can also be found that the thermal boundary layer thickness increases with Brownian motion and thermophoresis parameters.

1. Introduction

The flow of a non-Newtonian fluid over a stretching sheet has attracted considerable attention during the last two decades due to its vast applications in industrial manufacturing such as hot rolling, wire drawing, glass fiber and paper production, drawing of plastic films, polymer extrusion of plastic sheets and manufacturing of polymeric sheets. For the production of glass fiber/plastic sheets, thermo-fluid problem involves significant heat transfer between the sheet and the surrounding fluid. Sheet production process starts solidifying molten polymers as soon as it exits from the slit die. The sheet is then collected by a wind-up roll upon solidification. To improve the mechanical properties of the fiber/plastic sheet we use two ways, the extensibility of the sheet and the rate of cooling. Crane [1] was the first who reported the analytical solution for the laminar boundary layer flow past a stretching sheet. Several researchers viz. Gupta and Gupta [2], Dutta et al. [3], Chen and Char [4] extended the work of Crane by including the effects of heat and mass transfer under different situations.

In almost all investigations on the flow past a stretching sheet, the flow occurs because of the linear stretching velocity of the flat sheet. However, the boundary layer flow induced by an exponentially stretching/shrinking sheet is not studied much, though it is very important and realistic flow frequently appears in many engineering processes. However, all these studies are restricted to linear stretching of the sheet. It is worth mentioning that the stretching need not necessarily be linear. In view of this, Ali [5] has investigated the thermal boundary layer. The heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet was considered by Magyari and Keller [6]. Elbashbeshy [7] added a new dimension to the study of Ali, by considering exponentially continuous stretching surface. Partha et al. [8] reported a similarity solution for mixed convection flow past an exponentially stretching surface. Srinivas et al. [9] studied on non-Darcian unsteady flow of a micro polar fluid over a porous stretching sheet with thermal radiation and chemical reaction.

In recent years, investigations on the boundary layer flow problem with a convective surface boundary condition have gained much interest among researchers, since first introduced by Aziz [10], who considered the thermal boundary layer flow over a flat plate in a uniform free stream with a convective surface boundary condition. Ishak [11] obtained the similarity solutions for the steady laminar boundary layer flow over a permeable plate with a convective boundary condition. Makinde and Aziz [12] investigated numerically the effect of a convective boundary condition on the two dimensional boundary layer flows past a stretching sheet in a nano fluid.

There has been a renewed interest is studying magnetohydrodynamic flows and heat transfer due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems involving electrically conductive flows. In addition, this type of flow finds applications in many engineering problems such as magnetohydrodynamics (MHD) generators, Plasma studies, Nuclear reactors, and Geothermal energy extractions. Al-Odat et al. [13] analyzed the thermal boundary layer on an exponentially stretching continuous surface in the presence of magnetic field effect. Bhattacharyya and Pop [14] showed the effect of external magnetic field on the flow over an exponentially shrinking sheet. Recently, Nadeem and Lee [15] obtained analytic solutions of boundary layer flow of nanofluid over an exponentially stretching surface using homotopy analysis method (HAM). The buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet was discussed by Makinde et al. [16]. Nadeem et al. [17] investigated the numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles. Akbar et al. [18] analyzed the numerical solutions of Magneto hydrodynamic boundary layer flow of tangent hyperbolic fluid towards a stretching sheet. Akbar et al. [19] studied the radiation effects on MHD stagnation point flow of nano fluid towards a stretching surface with convective boundary condition. The numerical solutions for the g-jitter induced magnetohydrodynamic boundary layer flow of water based nanofluid were discussed by Uddin et al. [20]. Uddin et al. [21] studied the two dimensional magneto hydrodynamics viscous incompressible free convective boundary layer flow of an electrically conducting, chemically reacting nanofluid from a convectively heated permeable vertical surface. The magnetohydrodynamic flow and heat transfer for Maxwell fluid over an exponentially stretching sheet through a porous medium in the presence of non-uniform heat source/sink with variable thermal conductivity was analyzed by Vijendra Singh and Shweta Agarwal [22].

Nomenclature

B	constant
C	the local nanoparticle volume fraction (unit: mol/m^3)
C_f	skin friction coefficient
C_∞	nanoparticle volume fraction (unit: mol/m^3)
D_B	Brownian diffusion coefficient (unit: cm^2/s)
D_T	thermophoretic diffusion coefficient
h_f	the heat transfer coefficient (unit: W/(m^2·K))
L	reference length (unit: m)
M	magnetic parameter
N	velocity slip factor
Nb	Brownian motion parameter
N_1	constant
Nt	thermophoresis parameter number (unit: mol/m^3)
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	surface heat flux (unit: w/m^2)
Re_x	local Reynolds number
Sc	Schmidt number
S_v	non-dimensional velocity slip
T	thermodynamic temperature of the fluid (unit: K)
T_f	temperature of the hot fluid (unit: K)
T_∞	temperature of the ambient fluid (unit: K)
u	velocity component in the x direction (unit: m/s)
U_0	reference velocity (unit: m/s)
U_w	velocity (unit: m/s)
v	velocity component in the y direction (unit: m/s)
x, y	coordinates along and normal to the stretching surface (unit: m)

Greek letters

α	thermal diffusivity (unit: m^2/s)
γ	Biot number
ρ	density of the fluid (unit: kg/m^3)
σ	electrical conductivity (unit: s^{3}A^2/(m^3·kg))
η	similarity variable
θ	dimensionless temperature
ϕ	dimensionless concentration
λ	local Deborah number
λ_1	relaxation time (unit: s)
τ	heat capacity ratio (constant)
τ_w	surface shear stress (unit: N/m^2)

Superscripts

$'$	differentiation with respect to η

Subscripts

F	fluid
w	wall
x	local

The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances. It is a well-known fact that, a viscous fluid normally sticks to the boundary. But, there are many fluids, e.g. particulate fluids, rarefied gas etc., where there may be a slip between the fluid and the boundary. Beavers and Joseph [23] proposed a slip flow condition at the boundary. Of late, there has been a revival of interest in the flow problems with partial slip (Andersson [24]; Ariel [25]). Wang [26] undertook the study of the flow of a Newtonian fluid past a stretching sheet with partial slip and purportedly gave an exact solution. He reported that the partial slip between the fluid and the moving surface may occur in particulate fluid situations such as emulsions, suspensions, foams and polymer solutions. Ibrahim and Shankar [27] analyzed MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with slip conditions. Srinivas et al. [28] studied Effects of chemical reaction and thermal radiation on MHD flow over an inclined permeable stretching surface with non-uniform heat source/sink: An application to the dynamics of blood flow. The steady two-dimensional magnetohydrodynamic laminar free convective boundary layer slip flow of an electrically conducting Newtonian nanofluid from a translating stretching/shrinking sheet in a quiescent fluid is discussed by Uddin et al. [29]. Uddin et al. [30] analyzed the steady two-dimensional laminar mixed convective boundary layer slip nanofluid flow in a Darcian porous medium due to a stretching/ shrinking sheet.

The thermal conductivity of fluids plays an important role in the heat transfer. However, it is observed that enhancement of the thermal conductivity of poor heat transfer fluids is possible in view of the addition of nanoparticles in the base fluids. The nano particles can be found in metals such as (Cu, Ag), oxides (Al_2O_3), carbides (SiC), nitrides (AlN, SiN) or nonmetals (graphite, carbon nanotubes). Nanofluids have novel properties that make them potentially useful in many applications in heat transfer including microelectronics, fuel cells, pharmaceutical processes and hybrid-powered engines. Nanoparticles provide a bridge between bulk materials and molecular structure. The term "nanofluid" was first introduced by Choi [31]. The thermal conductivity of nanofluids depends on factors such as particle size, volume fraction of particles in the suspension and intrinsic thermal conductivities of the base fluid and particles. To account for these factors, the traditional Maxwell model for the effective thermal conductivity of a mixture is modified to account for nonlocal heat transport that can arise due to the small characteristic length in a nanofluid – the particle size thus naturally appears in the modified model. The resulting model is calibrated and validated with experimental measurements of alumina nanofluids: overall, good agreement is found. Furthermore, the modified model is shown to be consistent in the limits of only the base liquid as well as for large particle sizes. Rizwan Ul Haq et al. [32] analyzed buoyancy and radiation effect on stagnation point flow of micropolar nanofluid along a vertically convective stretching surface.

Thermal conductivity of nanofluid is established by coupling the heat transport mechanisms like particle shape, nano layer thick ness in the particle fluid interface and Brownian motion which are expected to enhance the thermal conductivity of nanofluid. Brownian motion by which particles move through liquid and possibly collide, thereby enabling direct solid-solid transport of heat from one to another particle can be expected to increase the thermal conductivity of the nano fluids. It is believed that the Brownian motion contribution to thermal conductivity increases with rising temperature and decreasing particle size. Obviously we use more realistic fluid to make the theoretical results to be of industrial significance. Maxwell model is one of such a fluid of industrial importance. Maxwell model has a viscosity that is constant with shear rate so that the fully developed flow field is quadratic in position for Poiseuille flow. Boundary layer theory is more appropriate for Maxwell fluids in comparison to other viscoelastic fluids. Aliakbar et al. [33] investigate the influence of thermal radiation on MHD flow and heat transfer of Maxwell fluid over a linear stretching sheet.

Maxwell was one of the first to investigate conduction properties analytically for a mixture consisting of particles embedded in a base medium following the effective medium theory. (Effective Medium Theory or Effective Medium Approximation denoted as EMT or EMA). The effective medium theory describes the macroscopic properties of a medium based on the properties of the components and their relative volume fractions. The Maxwell model is one of the first models which predict the thermal conductivity of nano fluids. Mukhopadhyay and Gorla [34] studied unsteady MHD Boundary Layer Flow of an Upper Convected Maxwell Fluid Past a Stretching Sheet with First Order Constructive/Destructive Chemical Reaction. Recently, the boundary layer flow and heat transfer analysis in a Maxwell fluid over an exponentially continuous moving sheet is discussed by Abbas et al. [35]. Very recently, Mustafa et al. [36] analyzed the steady flow of Maxwell nano fluid induced by an exponentially stretching sheet subject to convective heating. Optimal designs and optimization methods under different conditions was studied by several authors (Ko and Ting [37], Hajmohammadi et al. [38], Hajmohammadi et al. [39], Hajmohammadi et al. [40], Sadegh Poozesh [41], Hajmohammadi and Nourazar [42] and Najafi et al. [43]). Bilal Ashraf et.al [44] analyzed the radiative flow of Maxwell fluid over an inclined stretching surface with convective boundary condition.

To the best of author's knowledge, no investigation has been made yet to analyze the magnetohydrodynamic boundary layer slip flow of a Maxwell fluid over an exponentially stretching surface with convective boundary condition. Hence, to fill the gap in the existing literature, a mathematical model is presented here to understand the MHD boundary layer slip flow of Maxwell fluid over an exponentially stretching surface. The coupled partial differential equations of the governing flow are transformed into non-linear coupled ordinary differential equations by a Similarity transformation. The resulting nonlinear coupled differential equations are solved numerically by using fourth order Runge-Kutta scheme together with shooting method. This paper has been arranged as follows: Section II deals with the mathematical formulation of the problem. The results and discussion is given in Section III. Section IV contains the concluding remarks.

2. Mathematical analysis

Consider the study two dimensional flow of an incompressible viscous electrically conducting Maxwell nano fluid over an exponentially stretching sheet. The x-axis is taken along the stretching surface in the direction of the motion while the y-axis is perpendicular to the surface. The stretching surface has the velocity $U_w(x) = U_0\, e^{\frac{x}{L}}$ where U_0 is the reference velocity, L is the reference length. The sheet's temperature is controlled by convective process which is characterized by the heat transfer coefficient $h_f(x)$ and temperature of the hot fluid T_f below the surface. The mass flux of the nano particle at the wall is assumed to be zero.

A variable magnetic field $B = B_0 e^{\frac{x}{2L}}$ is applied normal to the sheet, B_0 is a constant. Under these assumptions, the governing equations for the continuity, momentum, energy and nanoparticle diffusion equations for nanofluid in vector form are given by

$$\Delta \cdot \vec{V} = 0$$

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\cdot\nabla\right)\vec{V}\right] = -\nabla\cdot P + j\tilde{n}B$$

$$\rho C_p\left[\frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T\right] = \nabla\cdot k\nabla T + \tau\left(D_B\nabla\phi\cdot\nabla T + D_T\frac{\nabla T\cdot\nabla T}{T_\infty}\right)$$

$$\frac{\partial C}{\partial t} + \vec{V}\cdot\nabla C = \nabla\cdot\left(D_B\nabla C + D_T\frac{\nabla T}{T_\infty}\right)$$

Where $\vec{V}$- velocity vector.

The two-dimensional boundary layer equations governing the flow can be written as (Mustafa et al. [36])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \lambda_1\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right] - \frac{\sigma B^2}{\rho}u \tag{2}$$

Table 1 Comparison of $\theta'(0)$ with the previous results.

λ	Pr	Magyari and Keller [6]	Mustafa et al. [35]	Present
0	0.5	−0.330493	−0.330493	−0.330493
0	1	−0.549643	−0.549642	−0.549643
0	3	−1.122188	−1.122177	−1.122185
0	5	−1.521243	−1.521222	−1.521241
0	8	−1.991847	−1.991805	−1.991839
0	10	−2.257429	−2.257381	−2.257422

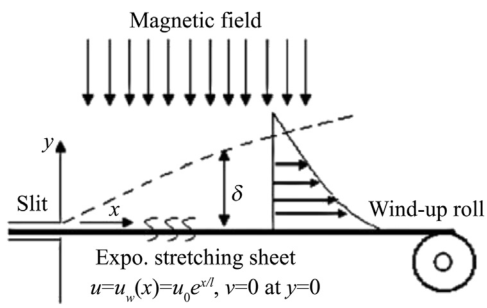

Figure 1 Physical configuration of the problem.

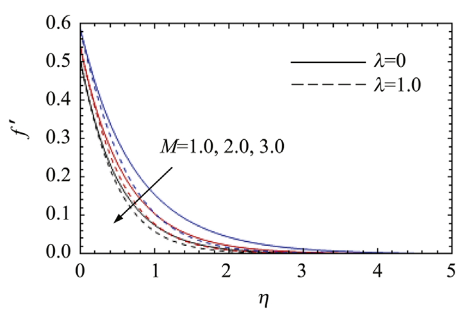

Figure 2 Velocity profiles for some values of M.

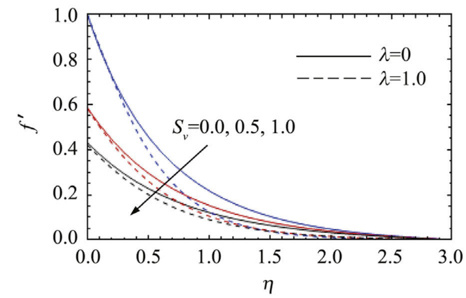

Figure 3 Velocity profiles for some values of S_v.

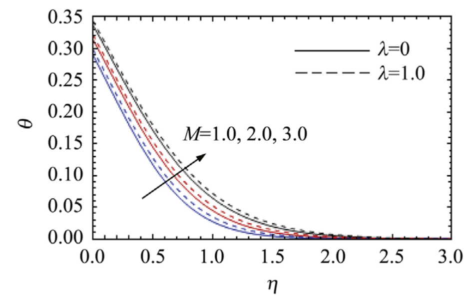

Figure 4 Temperature profiles for some values of M.

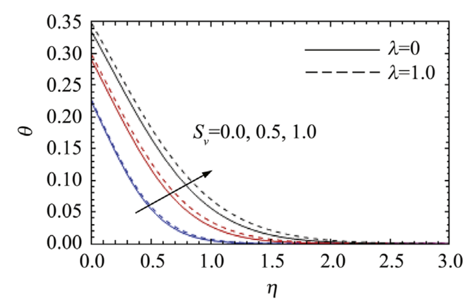

Figure 5 Temperature profiles for some values of S_v.

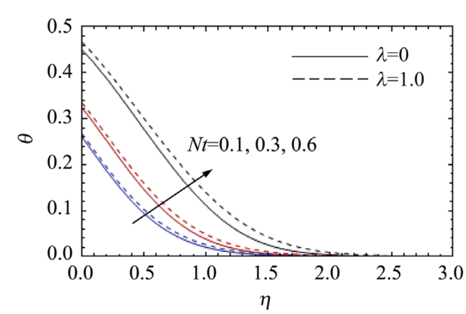

Figure 6 Temperature profiles for some values of Nt.

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

Subject to the boundary conditions:

$$u=U_w(x)+N\mu\frac{\partial u}{\partial y},\ v=0,\ -k\frac{\partial T}{\partial y}=h_f\left(T_f-T\right),$$
$$D_B\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0 \text{ at } y=0$$
$$u\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty \quad (5)$$

where u and v are the velocity components in the x and y directions respectively, υ is the kinematic viscosity, λ_1 is the relaxation time, σ is the electrical conductivity, ρ is the density of the fluid, T is the fluid temperature, α is the thermal diffusivity, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, T_∞ is the temperature of the ambient fluid, C is the local nanoparticle volume fraction, C_∞ is the nanoparticle volume fraction, $N=N_1e^{\frac{-x}{L}}$ is the velocity slip factor, k is the thermal conductivity and $h_f=he^{\frac{x}{2L}}$ is the heat transfer coefficient.

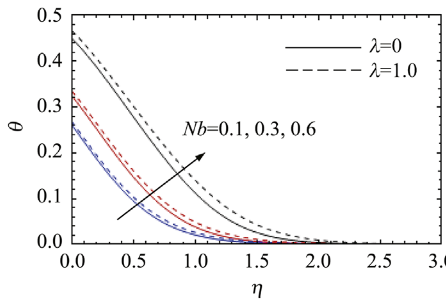

Figure 7 Temperature profiles for some values of *Nb*.

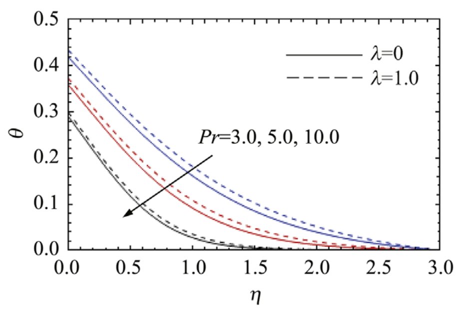

Figure 8 Temperature profiles for some values of *Pr*.

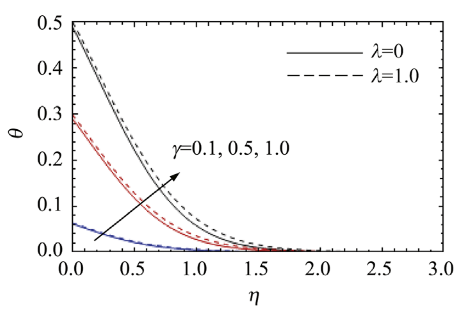

Figure 9 Temperature profiles for some values of γ.

We introduce the similarity variables as

$$\eta = \left(\frac{U_0}{2\upsilon L}\right)^{\frac{1}{2}} e^{\frac{x}{2L}} y,\ U = U_0 e^{\frac{x}{L}} f'(\eta),$$

$$v = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} (f(\eta) + \ \eta f'(\eta)),\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C}{C_\infty} \quad (6)$$

Now substituting Eq. (6) into the Eqs. (2)–(4), we get the following set of ordinary differential equations

$$f''' + ff'' - 2\,(f')^2 + \lambda\left(3ff'f'' + \frac{\eta}{2}(f')^2 - \frac{1}{2}f^2 f''' - 2(f')^3\right) - Mf' = 0 \quad (7)$$

$$\frac{1}{Pr}\theta'' + f\theta' + Nb\,\theta'\phi' + Nt(\theta')^2 = 0 \quad (8)$$

$$\phi'' + Scf\phi' + \frac{Nt}{Nb}\theta'' = 0 \quad (9)$$

with the boundary conditions

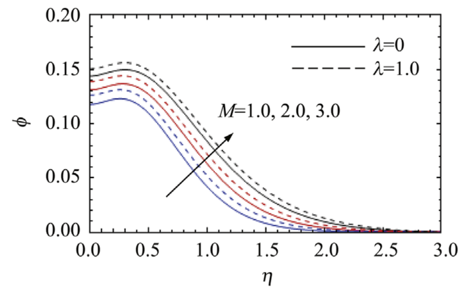

Figure 10 Concentration profiles for some values of *M*.

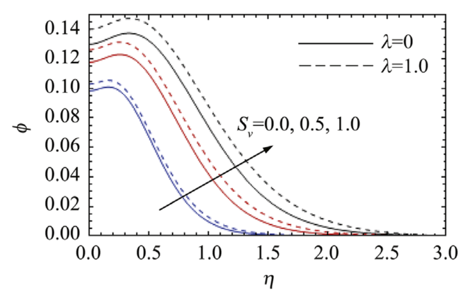

Figure 11 Concentration profiles for some values of S_v.

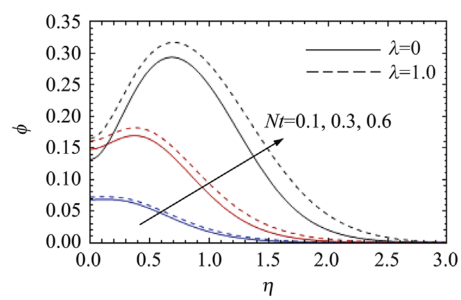

Figure 12 Concentration profiles for some values of *Nt*.

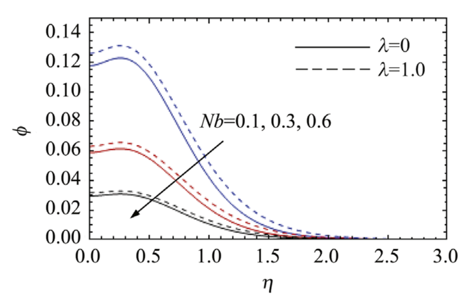

Figure 13 Concentration profiles for some values of *Nb*.

$$\begin{aligned} & f = 0,\ f' = 1 + S_v f''(0),\ \theta' = -\gamma(1 - \theta(0)), \\ & Nb\,\phi' + Nt\phi' = 0 \text{ at } \eta = 0 \\ & f' \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } \eta \to \infty \end{aligned} \quad (10)$$

Where the prime denotes differentiation with respect to η, $\lambda = \frac{Re_x \lambda_1 \upsilon}{2L^2}$ is the local Deborah number, $M = \frac{2\sigma B_0{}^2 L}{\rho U_0}$ is the magnetic parameter, $Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $Nb = \frac{\tau D_B C_\infty}{\upsilon}$ is the Brownian motion parameter,

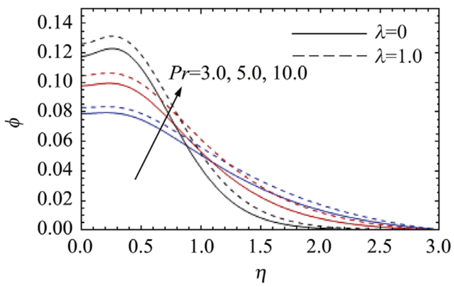

Figure 14 Concentration profiles for some values of Pr.

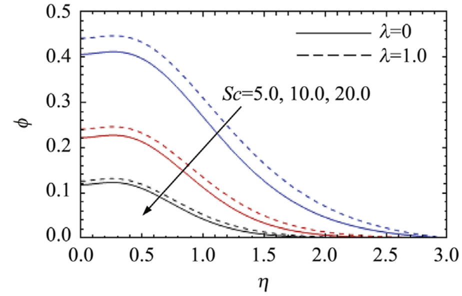

Figure 15 Concentration profiles for some values of Sc.

Table 2 Comparison of $\theta'(0)$ with the previous results.

λ	Pr	Abbas et al. [35]	Mustafa et al. [36]	Present
0.5	0.5	−0.175942	−0.301698	−0.175921
0.5	1	−0.512337	−0.512078	−0.512320
0.5	3	−1.074513	−1.074501	−1.074513
0.5	5	−1.470621	−1.47060	−1.470621
0.5	8	−1.939103	−1.93907	−1.939103
0.5	10	−2.203874	−2.20383	−2.203874

$Nt = \frac{\tau D_T(T_f - T_\infty)}{T_\infty \upsilon}$ is the thermophoresis parameter, $Sc = \frac{\nu}{D_B}$ is the Schmidt number and $\gamma = \frac{h}{k}\sqrt{\frac{2\upsilon L}{U}}$ is the Biot number.

The non-dimensional velocity slip S_v is defined as

$$S_v = N_1 \rho \sqrt{\frac{\upsilon U}{2L}}. \tag{11}$$

The quantities of physical interest in this problem are the skin-friction coefficient and heat transfer, which are defined as

$$C_f = \frac{2\tau_w}{\rho U^2 e^{\frac{2x}{L}}} \quad \text{and} \quad Nu_x = \frac{xq_w}{k(T_f - T_\infty)} \tag{12}$$

respectively. Where the surface shear stress τ_w and surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{13}$$

Substituting Eqs. (6) and (13) into Eq. (12), we get

$$C_f\sqrt{Re_x/2} = f''(0) \text{ and } \frac{Nu_x}{\sqrt{Re_x/2}\sqrt{x/L}} = -\theta'(0) \tag{14}$$

where $Re_x = \frac{xU_w(x)}{\upsilon}$ is the local Reynolds number.

The above skin-friction coefficient and local Nusselt number shows that its variation depends on the variation of the factors $f''(0)$ and $-\theta'(0)$ respectively. It should be noted that reduced Sherwood number which is the dimensionless mass flux is zero.

3. Results and discussion

The system of coupled nonlinear differential Eqs. (7)–(9) subject to the boundary conditions (10) are solved numerically by using the Runge-Kutta algorithm with a systematic guessing of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ by the shooting method until the boundary conditions at infinity $f(0)$, $\theta(0)$ and $\phi(0)$ decay exponentially to zero. In order to judge the validity and accuracy of this method, the obtained numerical solutions for the heat transfer coefficient are compared with the available literature and are found to be in good agreement, as shown in Table 1. In this study, the default values of the parameters are chosen as $M = 1.0$, $Pr = 10$, $Nb = 0.1$, $Nt = 0.2$, $Sc = 20$, $S_v = 0.5$ and $\gamma = 0.5$, unless, otherwise specified.

The effects of the pertinent parameters, namely, magnetic parameter, velocity slip, Prandtl number, Schmidt number, Brownian motion parameter, thermophoresis parameter, and the Biot number on the dimensionless velocity, temperature and concentration for the cases of regular and Maxwell fluid are shown in Figures 1–14. Also, the influences of the skin friction coefficient, Nusselt number and the Sherwood number are shown in Table 2.

The effect of the magnetic parameter (M) on the velocity is presented in Figure 2. It reveals that the velocity as well as the momentum boundary layer thickness decrease when the magnetic parameter increases. This is due to the fact that an increase in M signifies an enhancement of Lorentz force, thereby reducing the magnitude of the velocity. The influence of the velocity slip (S_v) on the velocity is shown in Figure 3. It is seen that the velocity of the fluid decreases with increase of the velocity slip. Figure 4 displays the effect of magnetic parameter on the temperature. It is found that the temperature of the boundary layer of the fluid increase with an increase in magnetic parameter. This is due to fact that the transverse magnetic field to an electrically conducting fluid gives rise to a resistive type of force called the Lorentz force. This force has the tendency to slow down the motion of the fluid and to increases its temperature of the fluid. The temperature profiles for various values of velocity slip are presented in Figure 5. It is noticed that temperature increases as the velocity slip increases. Figure 6 illustrates the variation of temperature with Brownian motion parameter. This is due to fact that the Brownian motion generates micro-mixing which enhances the thermal conductivity of the nanofluid. This higher nanofluid thermal conductivity in effect causes the increase of temperature. The temperature in the boundary layer increases with the increase in the Brownian motion parameter for both cases. The effect of thermophoresis parameter on the temperature distribution is demonstrated in Figure 7. It is clearly shown

Table 3 The values of skin friction coefficient and the Nusselt number for various values of λ, M and S_v.

λ	M	S_v	$f''(0)$	$-\theta'(0)$
1.0	1.0	0.5	−0.836106	0.350509
2.0	1.0	0.5	−0.844972	0.345950
1.0	2.0	0.5	−0.921916	0.339055
1.0	1.0	1.0	−0.578779	0.325829
0.0	1.0	0.5	−0.825789	0.354934
0.0	2.0	0.5	−0.919641	0.342726
0.0	1.0	1.0	−0.618795	0.313673

Table 4 The values of skin friction coefficient and the Nusselt number for various values of λ, Pr, Nb and Nt.

λ	Pr	Nb	Nt	$f''(0)$	$-\theta'(0)$
1.0	10.0	0.1	0.2	−0.836106	0.350509
1.0	11.0	0.1	0.2	−0.836106	0.354658
1.0	10.0	0.5	0.2	−0.836106	0.350509
1.0	10.0	0.1	0.5	−0.836106	0.290285
0.0	10.0	0.1	0.2	−0.825789	0.354934
0.0	11.0	0.1	0.2	−0.825789	0.358794
0.0	10.0	0.5	0.2	−0.825789	0.354934
0.0	10.0	0.1	0.5	−0.825789	0.297715

that an increase in the thermophoresis parameter leads to increase for both cases. Figure 8 illuminates a very important effect of Prandtl number on the temperature profile. An increase in Prandtl number reduces the temperature and the thermal boundary layer thickness for both cases. This is due to increase in Prandtl number the thermal conductivity of the fluid reduces and consequently, thermal boundary layer thickness decreases. The effects of the Biot number on the temperature are displayed in Figure 9. It can be seen that the temperature increases with an increase in Biot number. Figure 10 displays the effect of magnetic parameter on the concentration. It is found that the concentration of the fluid increase with an increase in magnetic parameter. The concentration profiles for various values of velocity slip are presented in Figure 11. It is noticed that concentration increases as the velocity slip increases.

Figure 12 illustrates the variation of concentration with thermophoresis parameter. The concentration of the boundary layer increases with the increase in the thermophoresis parameter for both cases. The effect of Brownian motion parameter on the concentration distribution is demonstrated in Figure 13. It is found that the concentration decreases as the Brownian motion parameter increases. Figure 14 illustrates a very important effect of Prandtl number on the concentration. The concentration increases initially with the Prandtl number. But it decreases after a certain distance η from the sheet. The effects of the Schmidt number on the concentration are displayed in Figure 15. It is observed that the velocity decreases as Schmidt number increases.

The values of skin friction coefficient and the Nusselt number for various values of the involved pertinent parameters are shown in Table 3 and Table 4. It can be noted that the skin friction coefficient decreases with increasing values of local Deborah number and the magnetic parameter, whereas the reverse trend is observed in the case of velocity slip for the cases of regular and Maxwell fluid. It is analyzed that the Nusselt number decreases with increasing values of local Deborah number, magnetic parameter and the thermophoresis parameter, whereas the reverse trend is observed in the case of velocity slip and the Prandtl number for the cases of regular and Maxwell fluid.

4. Conclusions

The present investigation is a worthwhile attempt to study the magnetohydrodynamic boundary layer slip flow of a Maxwell fluid over an exponentially stretching surface with convective boundary condition. Numerical method is used to solve the resulting system of coupled nonlinear ordinary differential equations with prescribed boundary conditions. A parametric study is conducted and dominant effects of various material parameters on the physical quantities for both Maxwell fluid and real fluid have been shown and analyzed in detail. It is observed that the skin friction coefficient decreases with increasing values of local Deborah number and the magnetic parameter, whereas the reverse trend is observed in the case of velocity slip for the cases of regular and Maxwell fluid. It can also be found that the Nusselt number decreases with increasing values of local Deborah number, magnetic parameter and the thermophoresis parameter, whereas the reverse trend is observed in the case of velocity slip and the Prandtl number for the cases of regular and Maxwell fluid.

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