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To cite this article: M Kaviyarasu and K Indhira 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **263** 042142

IOP Conf. Series: Materials Science and Engineering **263** (2017) 042142 doi:10.1088/1757-899X/263/4/042142

On intuitionistic fuzzy INK-ideals of INK-algebras

M Kaviyarasu and K Indhira
Department of Mathematics, School of Advanced Sciences, VIT University, Vellore 632 014, India

E-mail: kindhira@vit.ac.in

Abstract. In this paper, first we define the notions of INK-algebra, INK-ideals, fuzzy sets, intuitionistic fuzzy INK-ideals and intuitionistic fuzzy closed INK-ideals. Using the concept of level subsets, we prove some theorems which show that there are some relationships between these notions. Finally we define the homomorphism of INK-algebras and then we give related theorem about the relationship between their images and intuitionistic fuzzy INK-ideals.

1. Introduction

The fuzzy set cerebration was introduced L.A Zadeh in 1965 [15]. After some authors an extension of the absicht of the fuzzy set, then the concept of fuzzy set has been development rapid. In Imai and Iseki introduced two logical algebras from the abstract algebras namely BCI/BCK-algebras [8] and [9]. In this paper, we establish the concept of INK-algebraic structure which is generalization of TM/Q/BCK/BCI algebras, and also we dispute of intuitionistic Fuzzy INK-ideal in INK-algebras and studied some definitions, properties, theorems and give some examples.

2. INK-algebras

Let us start this section with some known basic definitions of INK-algebra, intuitionistic fuzzy set on INK-algebras from [21] and [22], which we are used frequently in our next section.

Definition 2.1.Let * be a binary operation on A with constant 0, Then a nonempty set (X ,*,0) is called a INK- algebra, if

(INK-I) $x * x = 0$,
(INK-I I) $x * 0 = x, \forall x \in X$.
(INK-I I I)$0 * x = x$,
(INK-I V) $(y * x) * (y * z) = (x * z)$ for all x, y, z $\in$ X.

Remark 2.2.A binary relation $\leq$ by $x \leq y$ if and only if $x * y = 0$.

Definition2.3.Let A be a INK- algebra and S be a subset of A . Then S is denoted by ideal of A .if

ID-1) $0 \in S$,
ID-2) $a * b \in S$ and $b \in S \Rightarrow a \in S$, $\forall a, b \in A$.

Definition 2.4. An ideal B of a INK-algebra A is said to be closed if $0 * a \in A, \forall\ a \in A$.

Definition2.5. A fuzzy (sub) set βof the set A (nonempty set) is a function $\beta: A \rightarrow [0,1]$ and the complement of β is denoted by $\beta^c(a) = 1- \beta(a)$ for all $a \in A$.

Definition2.6. A fuzzy (sub) set βof the set A (nonempty set) is called a fuzzy sub algebra of INK-algebras A if,

$$\beta(a* b) \geq \min \{\beta (a), \beta (b)\} \ \forall\ a, b \in A .$$

Definition 2.7. A fuzzy (sub)set β in a INK-algebra X is called a fuzzy ideal of A , if

FID-1) $\beta (0) \geq \beta(a)$,
FID-2) $\beta (a) \geq \min\{\beta (a* b), \beta (b) \},\forall\ a ,b \in A$.

Definition 2.8. An IFS B in a (nonempty) set A is an object having the form

$$B = \{(A, \beta_B(a),\delta_B(b)) / a\in A\},$$

where the function β_B: $A \rightarrow [0,1]$ and $\delta_B : A \rightarrow [0,1]$ denote the degree of membership ($\beta_B(a)$) and non-membership ($\delta_B(a)$) , $\forall\ a \in A$ to the set A respectively, and $0 \leq \beta_B(a) + \delta_B(a) \leq 1, \forall\ a \in A$. so written as $B= (A , \beta_B, \delta_B)$ for the intuitionistic fuzzy set $B = \{(A, \beta_B(a), \delta_B(a)) /a\in A \}$.

Definition 2.9. Let $B = (A, \beta_{B,B})$ be an IFS in A. Then $B = (A, \beta_{B,}\bar{\beta}_B)$ and $B= (A, \delta_{B,}\bar{\delta}_B)$.

Definition2.10.An IFS $B = (A, \beta_{B,B})$ is called an intuitionistic fuzzy sub algebra of A if it satisfies,

IFS-I) $\beta_B (a* b) \geq \min \{ \beta_B (a), \beta_B (b)\}$,
IFS-II) $\delta_B(a* b) \leq \max \{ \delta_A (a), \delta_A (b)\}, \forall\ a ,b\in A$.

3. Intuitionistic fuzzy INK-ideal

We start with the definition of INK-ideal from [22].

Definition 3.1. Let S be a nonempty (sub)set of a INK –algebra X is called INK-ideal of A , if

INK-1) $0 \in S$,
INK-2) $(c * a) * (c * b) \in S$ and $c \in S \Rightarrow x \in I , \forall\ a ,b ,c \in A$.

Definition 3.2. A fuzzy subset β in a INK-algebra X is called a fuzzy INK- ideal of X, if

FINK-a) $\beta (0) \geq \beta (a)$,
FINK-b) $\beta (x) \geq \min\{ \beta (c * a) * (c * b) , \beta (b) \}, \forall\ a ,b ,c \in A$.

Definition 3.3. An IFS $B = (A, \beta_B,\delta_B)$ in a INK-algebra X (nonempty) is called an intuitionistic fuzzy INK-ideal of A , if

IFINK-I) $\beta_B (0) \geq \beta_B (a)$ and $\delta_B(0) \leq \delta (a)$,
IFINK-II) $\beta_B (x) \geq \min\{ \beta_B (c * a) * (c * b) , \beta_B (b) \}$,
IFINK-III) $\delta_B(x) \leq \max\{\delta_B (c * a) * (c * b) , \delta_B (b) \} \ \forall\ a ,b ,c \in A$.

Example3.4 .Let $B= \{0, l , m , n\}$ with binary operation * given by

IOP Conf. Series: Materials Science and Engineering **263** (2017) 042142 doi:10.1088/1757-899X/263/4/042142

*	0	l	m	n
0	0	l	m	n
l	l	0	n	m
m	m	n	0	l
n	n	m	l	0

Take IFS B =(A ,β_B, δ_B) in A as by β_B (0) = 0.6 ,β_B (l) = 0.4, β_B (m) = β_B (n) = 0.3.and δ_B (0) = 0.3, δ_B (l) = δ_B (m) = 0.5, δ_B(n) = 0.4. Then it is easy to verify that B= (A, $\beta_{B,\ B}$) is anintuitionistic fuzzy INK-ideal of A .

Definition 3.5.An IFS B = (A,β_B, δ_B) in a INK-algebra A is called an intuitionistic fuzzy closed INK-ideal of A, if

IFCINK-I) β_B (0 * a) $\geq \beta_B$ (a) and δ_B(0 * a) $\leq$ δ_B (a) ,
IFCINK-II)β_B(a) $\geq$ min{ β_B (c*a)*(c*b) , β_B (b) },
IFCINK-III) δ_B (a) $\leq$ max{δ_B (c*a)*(c*b), δ_B (b) }, $\forall$ a ,b ,c $\in$ A .

Definition3.6.Let B be a IFS in a nonempty set A .The upper s-level of β_B is

U(β_B; s) ={ a∈A / β_B (a) $\geq$ s}

and thelower t-level of δ_B is

L (δ_B ; t) = { a ∈ A / δ_B (a) $\leq$ t } .

Theorem 3.7.Let B be a(nonempty) subset and IFINK-ideal of a INK-algebra B. Then so is B = (A, $\beta_B, \bar{\beta}_B$) and B = (A, $\delta_B, \bar{\delta}_B$).

Proof.We know that,$\beta_B(0) \geq \beta_B(a)$. This implies that 1-$\bar{\beta}_B(0) \geq 1- \bar{\beta}_B(a)$ and hence

$$\bar{\beta}_B(0) \leq \bar{\beta}_B(a) \text{ for every } a \in A .$$

Let us consider∀a ,b ,c ∈ A. Then

β_B (a) $\geq$ min{ β_B (c*a)*(c*b) , β_B (b) }
$\Rightarrow 1-\bar{\beta}_A(a) \geq \min\{ 1-(\bar{\beta}_A(c*a)*(c*b)) , 1-\bar{\beta}_A (b) \}$
$\Rightarrow \bar{\beta}_B(a) \leq 1-\min\{ (\bar{\beta}_B(c*a)*(c*b)), \bar{\beta}_B (b) \}$
$\Rightarrow \bar{\beta}_B(a) \leq \max\{ (\bar{\beta}_B(c*a)*(c*b)), \bar{\beta}_B (b) \}$.

Hence B =(A, $\beta_B, \bar{\beta}_B$) is an intuitionistic fuzzy INK-ideal of A. To prove the second part, letBbe an intuitionistic fuzzy INK-ideal of a INK-algebra A.Now, we have

$\delta_B(0) \leq \delta_B$ (a)
$\Rightarrow 1-\bar{\delta}_B(0) \leq 1- \delta_B(a)$
$\Rightarrow \bar{\delta}_B(0) \geq \bar{\delta}_B(x)$, ∀ a∈ A

Now consider, ∀ a, b, c∈ A. Then, we have

$\delta_B (a) \leq \max\{\delta_B (c * a)* (c * b) , \delta_B(b) \}$
$\Rightarrow 1-\bar{\delta}_B(x) \leq \max\{ 1-(\bar{\delta}_B(c * a)* (c * b)) , 1-\bar{\delta}_B(b) \}$
$\Rightarrow \bar{\delta}_B(x) \geq 1-\max\{ (\bar{\delta}_B(c * a)* (c * b)), \bar{\delta}_B(b) \}$
$\Rightarrow \bar{\delta}_B(x) \geq \min\{ (\bar{\delta}_B(c * a)* (c * b)), \bar{\delta}_B(b) \}$.

Hence B=(A, δ_B ,$\bar{\delta}_B$) is an intuitionistic fuzzy INK-ideal of A.

Theorem. 3.8.LetB (non-empty set) be an IFCINK-ideal of a INK-algebra A. Then B = (A, δ_B ,$\bar{\delta}_B$) and B= (x, β_B,$\bar{\beta}_B$) is also a closed IFCINK-ideal.
Proof.First we prove *B* =(A, δ_B ,$\bar{\delta}_B$) is closed IFCINK-ideal. For all a ∈ A, we have

$\delta_B(0* a) \leq \delta_B (a)$
$\Rightarrow 1-\bar{\delta}_B(0* a) \leq 1- \bar{\delta}_B(a)$
$\Rightarrow \bar{\delta}_B(0 * a) \geq \bar{\delta}_B(a)$.

Therefore B =(A, δ_B ,$\bar{\delta}_B$) is an IFCINK-ideal of A. To prove the second part let a ∈ A,thenwe have
$\beta_B (0 * a) \geq \beta_B (a)$
$\Rightarrow 1-\bar{\beta}_B (0 * a) \geq 1- \bar{\beta}_B(a)$
$\Rightarrow \bar{\beta}_B(0 * a) \leq \bar{\beta}_B(a)$ for any ∀ a, b, c ∈ A.

Hence B =(A, β_B,$\bar{\beta}_B$) is an intuitionistic closed K-ideal of A .

Theorem 3.9. Let B (nonempty set) be an IFINK-ideal of a INK-algebra A if and only if the non-empty upper s-level cut U (β_B ; s) and the non-empty lower t-level cut L (δ_B; t) are INK-ideal of A , ∀ s, t ∈ [0,1].

Proof.Suppose B= (A,β_B,λ_B) be an IFINK-ideal of a INK-algebra A. For alls,t∈ [0,1],we define the set

$$U(\beta_B; s) =\{a\in A / \beta_B(a) \geq s\}$$

and

$$L (\delta_B; t) =\{ a\in A /\delta_B (a) \leq t \}.$$

BecauseL (δ_B; t) ≠ φ, ∀ a ∈ L (δ_B; t) $\Rightarrow \delta_B(a) \leq t \Rightarrow \delta_B (0) \leq t \Rightarrow$ 0 ∈L (δ_B; t).Let
(c* a) *(c * b) ∈L (δ_B; t) and b∈L (δ_B; t) then $\delta_B((c * a)* (c * b)) \leq t$and $\delta_B (b) \leq t$ }. Since for all a, b, c ∈ A,
$\delta_B(a) \leq \max\{\delta_B ((c * a) *(c * b)) , \delta_B (b) \} \leq \max \{t, t\}= t \Rightarrow \delta_B(a) \leq t$.

Therefore a ∈L (δ_B; t) and hence L (δ_B; t) is an INK-ideal of A.Similarly, we can prove U (β_B ; s) is an INK-ideal of A .

Conversely, suppose U(β_B; s) and L (δ_B ; t) are INK-ideal of A . ∀s ,t ∈ [0,1].Let assume that a_0, b_0 ∈ A such that $\beta_B (0) < \beta_B (a_0)$ and $\delta_B(0) >\delta_A(b_0)$.Put

$s_0=\frac{1}{2}[\beta_B(0)+ \beta_B(a_0)] \Rightarrow s_0< \beta_B(a_0)$,

$0 \leq \beta_B (0) < s_0< 1 \Rightarrow a_0\in U(\beta_B; s_0)$.
Since U(β_B; s_0) is an INK-ideal of A , we have 0 ∈ U(β_B; s_0) implies that $\beta_B(0) \geq s_0$,which is a contradiction. Therefore $\beta_B(0) \geq \beta_B(a)$, ∀ a∈ A . By taking

$$t_0 =\frac{1}{2}[\delta_B(0) + \delta_B(b_0)]$$

we can show that

$$\delta_B(0) \le \delta_B(b), \forall b \in A.$$

Assumethat $a_0, b_0, c_0 \in A$ .such that $\beta_B(a_0) < \min\{\beta_B(c_0 * a_0) * (c_0 * b_0), \beta_B(b_0)\}$,

$$s_0 = \frac{1}{2}[\beta_B(a_0) + \min\{\beta_B(c_0 * a_0) * (c_0 * b_0), \beta_B(b_0)\}]$$

Put $s_0 > \beta_B(a_0)$ and $s_0 < \min\{\beta_B(c_0 * a_0) * (c_0 * b_0), \beta_B(b_0)\}$

$$\Rightarrow s_0 > \beta_B(a_0), s < \beta_B(c_0 * a_0) * (c_0 * b_0) \text{ and } s_0 < \beta_B(b_0)$$
$$\Rightarrow a_0 \notin U(\beta_B; s_0), (c_0 * a_0) * (c_0 * b_0) \in U(\beta_B; s_0) \text{ and } b_0 \in U(\beta_B; s_0),$$

which is acontradiction to INK-ideal $U(\beta_B; s_0)$. Therefore,

$$\beta_B(a) \ge \min\{\beta_B(c*a)*(c*b), \beta_B(b)\}, \forall a,b,c \in A.$$

Similarly ,we can prove , $\delta_B(a) \le \max\{\delta_B(c*a)*(c*b), \delta_B(b)\}$, $\forall a,b,c \in A$. Hence $B = (A,\beta_B,\delta_B)$ be an intuitionistic fuzzy INK-ideal of an INK-algebra A.

Theorem3.10.Let $B = (A, \beta_B, \delta_B)$ be an IFCINK-ideal of a INK-algebra A if and only if the non-empty upper s-level cut $U(\beta_B; s)$ and the non-empty Lower t-level cut $L(\delta_B; t)$ are closed INK-ideal of A , $\forall s, t \in [0,1]$.

Proof.If$B = (A, \beta_B,\delta_B)$ be an IFCINK-ideal of a INK-algebra A.We have $\beta_B(0*a) \ge \beta_B(a)$ and $\delta_B(0*a) \le \delta_B(a)$ for all $a \in A$.For $a \in U(\beta_B; s) \Rightarrow a \in A$. and $\beta_B(a) \ge s \Rightarrow \beta_B(0*a) \ge s \Rightarrow 0*a \in U(\beta_B; s)$ and $a \in L(\delta_B; t) \Rightarrow a \in A$ and $\delta_B(a) \le t \Rightarrow \delta_B(0*a) \le t \Rightarrow 0*a \in L(\delta_B; t)$.Therefore $U(\beta_B; s)$ and $L(\delta_B; t)$ are closed INK-ideal of A. Conversely, we show that $\beta_B(0*a) \ge \beta_B(a)$ and $\delta_B(0*a) \le \delta_B(a)$,for all $a \in A$ .If possible, assume $a_0 \in A$. such that $\beta_B(0*a_0) < \beta_B(a_0)$ then we take

$$s_0 = \frac{1}{2}[\beta_B(0*a_0) + \beta_B(a_0)],$$
$$\beta_B(0*a_0) < s_0 < \beta_B(a_0)$$

$\Rightarrow a_0 \in (\beta_B; s_0)$

but $0*a_0 \notin U(\beta_B; s_0)$,which is contradiction to closed INK-ideal. Hence $\beta_B(0*a) \ge \beta_B(a)$, for every $a \in A$. Similarly we can prove that,$_B(0*a) \le \delta_B(a)$, $\forall a \in A$.

Definition 3.11.A mapping ψ from A to B of INK-algebra is called a homomorphism

$\psi(a*b) = \psi(a)*\psi(b)$, $\forall a,b,c \in A$.

Note that $\psi : A \to B$ is a homomorphism of INK-algebra then $\psi(0) = 0$.Let $\psi: A \to B$ is a homomorphism of INK-algebra for any IFS $B = (\beta_B,\delta_B)$ in B, we define a new intuitionistic fuzzy set$B^\psi = (\beta_B^\psi,\delta_B^\psi)$ in A by IFS $\beta_B^\psi(a) = \beta_A(\psi(a))$, $\delta_B^\psi(a) = \delta_B(\psi(a))$, for all $a \in A$.

Theorem 3.12.Let ψ be a mapping from A to B is a homomorphism of INK-algebra. Suppose an IFS $B = (\beta_B,\delta_B)$ is an intuitionistic fuzzy INK-ideal of B, then an IFS $B^\psi = (\beta_B^\psi,\delta_B^\psi)$ in B is a IFINK-ideal of A.

Proof. First we prove that,

$$\beta_B^\psi(a) = \beta_B(\psi(a)) \le \beta_B(0) = \beta_B(\psi(0)) = \beta_B^\psi(0)$$
$$\delta_B^\psi(a) = \delta_B(\psi(x)) \ge \delta_B(0) = \delta_B(\psi(0)) = \delta_B^\psi(0), \forall a,b,c \in A.$$

Consider

$\min\{\beta_B^\psi((c*a)*(c*b)), \beta_B^\psi(b)\} = \min\{\beta_B((\psi(c)*\psi(a))*(\psi(c)*\psi(b)), \beta_B\psi(b)\}$

$= \min\{\beta_B((\psi(c*a))*\psi(a*b)), \beta_B(\psi(b))\}$

= min{ β_B (ψ ((c * a) * (c * b)) , β_B (ψ (b)) }
= min {$\beta_B{}^{\Psi}$((c* a)* (c * b)), $\beta_B{}^{\Psi}$(b))}.
≤ β_B(ψ (a)) = $\beta_B{}^{\Psi}$(a) .
implies that min{$\beta_B{}^{\Psi}$((c * a) * (a * b)),$\beta_B{}^{\Psi}$(b) } = $\beta_B{}^{\Psi}$(a) and
max{$\delta_B{}^{\Psi}$((c * a) * (c * b)) , $\delta_B{}^{\Psi}$(b) }
= max {$_B$ ((ψ (c) * ψ (a)) * ψ (c) * ψ (b))) ,$_B$(ψ(b)) }
= max { δ_A ((ψ (c*a)) * ψ (c*b)) , δ_A (ψ (b)) }
= max { δ_A (ψ ((c*a) * (c*b)) ,δ_A (ψ (b)) }
= max {$_B{}^{\Psi}$((c*a) * (c*b)) ,$\delta_B{}^{\Psi}$(b))}.
≥ $_B$(ψ (a)) = $\delta_B{}^{\Psi}$(a)
implies that max {$\delta_B{}^{\Psi}$((c*a) * (c*b)), $\delta_B{}^{\Psi}$(b) } = $\delta_B{}^{\Psi}$(b).

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