# On Solving Bottleneck Bi-Criteria Fuzzy Transportation Problems 

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#### Abstract

A fuzzy block-dripping method (FBDM) has been proposed to find the best compromise solution and efficient solutions of the bottleneck bi-criteria transportation problem under uncertainty. The procedure of the proposed method is illustrated by numerical example.


Keywords: Compromise solution; Efficient solution; Fuzzy block-dripping method.

## 1. Introduction

Transportation problem (TP) is one of the applications of linear programming problems. In general, TPs may be designed more gainfully with the concurrent consideration of multiple objectives because a transportation system decision-making person normally pursues multiple goals. A new algorithm for the criteria space and the bottleneck bi-criteria space was discussed by Aneja and Nair [2]. Rita Malhotra [15] proposed an algorithm to identify the efficient solutions in the bi-objective and in the bottleneck biobjective TP. Many researchers $[5,6,7,8,16,17,18]$ developed different algorithms for solving time minimizing TPs. Pandian and Natarajan [13] found the optimal solution to the FTP using fuzzy zero point method (ZPM).Pandian and Anuradha [12] proposed dripping method to find the set of all solutions which is efficient to the BTP. To identify the set of efficient solutions and best compromise solution of bottleneck bi-criteria TP, Anuradha and Pandian [4] proposed a block-dripping method (BDM). Abirami [1] found the fuzzy optimal solution for time minimizing FTP using generalized fuzzy non-normal/-norm trapezoidal fuzzy numbers. Naresh kumar and Kumaraghuru [10] have obtained an optimal solution to fuzzy bottleneck TP and all efficient points of a bottleneck cost TP using blocking method and blocking ZPM.
In this paper, we propose fuzzy BDM for finding the set of all solutions of a bottleneck bi-criteria fuzzy transportation problem (BBFTP). A numerical example is chosen to illustrate the proposed procedure.

## 2. Bottleneck Bi-criteria Fuzzy Transportation Problem (BBFTP)

Mathematical model of a BBFTP is given as follows
(P) Minimize $\tilde{\mathrm{z}}_{1}=\sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \tilde{\mathrm{c}}_{\mathrm{ij}} \tilde{x}_{\mathrm{ij}}$

Minimize $\tilde{\mathbf{z}}_{2}=\sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \tilde{d}_{\mathrm{ij}} \tilde{x}_{i j}$

Minimize $\tilde{\mathrm{T}}=\left[\underset{(\mathrm{i}, \mathrm{j})}{\operatorname{Maximize}} \tilde{\mathrm{t}}_{\mathrm{ij}} / \tilde{x}_{\mathrm{ij}}>0\right]$
Subject to
$\sum_{\mathrm{j}=1}^{\mathrm{n}} \tilde{x}_{\mathrm{ij}}=\tilde{\mathrm{a}}_{\mathrm{i}}, \quad \mathrm{i}=1,2, \ldots, \mathrm{~m}$
$\sum_{\mathrm{i}=1}^{\mathrm{m}} \tilde{x}_{\mathrm{ij}}=\tilde{b}_{\mathrm{j}}, \mathrm{j}=1,2, \ldots, \mathrm{n}$
$\tilde{x}_{\mathrm{ij}} \geq 0$, forall i and j are integers
where $\tilde{c}_{\mathrm{ij}}$ be the unit fuzzy transportation cost from origin $i$ to the destination $j, \tilde{d}_{\mathrm{ij}}$ is the unit fuzzy deterioration cost from origin $i$ to destination $j, \tilde{\mathrm{t}}_{\mathrm{ij}}$ be the unit fuzzy transportation time from origin $i$ to the destination $j$ and $\tilde{x}_{i j}$ is the fuzzy amount shipped from $i$ th origin to $j$ th destination. $\tilde{a}_{i}$ is the quantity available at $i$ th origin and $\tilde{b}_{j}$ is the demand at $j$ th destination.

The following three fuzzy TPs can be constructed from the problem (P) are first criteria FTP (FCFTP), second criteria FTP (SCFTP) and the bottleneck FTP (BFTP) of the given problem (P):
(FCFTP) Minimize $\tilde{\mathrm{z}}_{1}=\sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \tilde{\mathrm{c}}_{\mathrm{ij}} \tilde{x}_{i j}$
Subject to (2.1), (2.2) and (2.3).
(SCFTP) Minimize $\tilde{\mathrm{z}}_{2}=\sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \tilde{d}_{\mathrm{ij}} \tilde{x}_{i j}$
Subject to (2.1), (2.2) and (2.3).
and (BFTP) Minimize $\tilde{T}=\left[\underset{(\mathrm{i}, \mathrm{j})}{\operatorname{Maximize}} \tilde{\mathrm{t}}_{\mathrm{ij}} / \tilde{x}_{\mathrm{ij}}>0\right]$
Subject to (2.1), (2.2) and (2.3).
Let $\tilde{Z}_{1}(\mathrm{X})$ and $\tilde{Z}_{2}(\mathrm{X})$ denote the values of $\tilde{Z}_{1}$ and $\tilde{Z}_{2}$ related to $\tilde{X}=\left\{\tilde{x}_{i j}, \mathrm{i}=1,2, \ldots, \mathrm{~m} ; \mathrm{j}=1,2, \ldots, \mathrm{n}\right\}$ respectively.

The definitions of the following fuzzy set, fuzzy number, triangular fuzzy number and arithmetic operations on the triangular fuzzy number which can be found in $[11,19]$.

Definition 2.1: A set $\left(\tilde{\mathrm{X}}^{0}, \tilde{\mathrm{~T}}^{0}\right)$ where $\tilde{X}^{0}=\left\{\tilde{x}_{i j}^{0}, \mathrm{i}=1,2, \ldots, m\right.$; $\mathrm{j}=1,2, \ldots, \mathrm{n}\}$ and $\tilde{\mathrm{T}}^{0}$ is a fuzzy time, is said to be feasible to the problem (P) if $\tilde{\mathrm{X}}^{0}$ satisfies the conditions (2.1) to (2.3) within the total fuzzy time transportation $\tilde{\mathrm{T}}^{0}$.

Definition 2.2: A feasible point ( $\tilde{\mathrm{X}}^{0}, \tilde{\mathrm{~T}}^{0}$ ) of the problem ( P ) is said to be an efficient (non-dominated) solution to the problem (P) if there exist no other feasible point ( $\tilde{\mathrm{X}}, \tilde{\mathrm{T}}$ ) of (P) such that $\left(\tilde{\mathrm{Z}}_{1}(\tilde{\mathrm{X}}), \tilde{\mathrm{Z}}_{2}(\tilde{\mathrm{X}}), \tilde{T}\right) \leq\left(\tilde{\mathrm{Z}}_{1}\left(\tilde{\mathrm{X}}^{0}\right), \tilde{\mathrm{Z}}_{2}\left(\tilde{\mathrm{X}}^{0}\right), \tilde{T}^{0}\right)$. Otherwise, it is called non-efficient (dominated) solution for (P).
For simplicity, a triplet $\left(\tilde{\mathrm{Z}}_{1}\left(\tilde{\mathrm{X}}^{0}\right), \tilde{\mathrm{Z}}_{2}\left(\tilde{\mathrm{X}}^{0}\right), \tilde{T}^{0}\right)$ is called a solution (an efficient / a non-efficient) to the problem (P) if ( $\tilde{\mathrm{X}}^{0}, \tilde{\mathrm{~T}}^{0}$ ) is a solution (efficient / non-efficient solution) to the problem (P).

## 3. Fuzzy Block-Dripping Method

We now introduce a fuzzy block dripping method for finding all the solutions to the BBFTP.

The fuzzy block-dripping method proceeds as follows:

Step 2: Using blocking method [14], solve the BFTP. Let $\tilde{\mathrm{T}}^{0}$ be an optimal solution of BFTP.

Step 3: Solve the FCFTP and SCFTP of the BBFTP separately by fuzzy ZPM [13] and also, find their related fuzzy transportation time. Let it be $\tilde{\mathrm{T}}_{\bar{m} 1}$ and $\tilde{\mathrm{T}}_{m 2}$ respectively. Let $\tilde{\mathrm{T}}^{\bar{n}}=\max \left\{\tilde{\mathrm{T}}_{\bar{m} 1}, \tilde{T}_{m 2}\right\}$. Step 4: After blocking the cells having more than the time $\tilde{\mathrm{M}}$, construct the bi-criteria FTP for each time $\tilde{\mathrm{M}}$ in $\left[\tilde{\mathrm{T}}^{0}, \tilde{\mathrm{~T}}^{\tilde{m}}\right]$ which is found from BBFTP.

Step 5: Using the dripping method [12], find all the solutions to the bi-criteria FTP obtained in step 4.

Step 6: A solution to BBFTP will be obtained for each time $\tilde{M}$ in [ $\left.\tilde{\mathrm{T}}^{0}, \tilde{\mathrm{~T}}^{\bar{m}}\right]$ together with the solution of the bi-criteria FTP related to it.

Step 7: Combine all the solutions to BBFTP found in Step 6.
The FBDM for solving a BBFTP is illustrated by the following example.

## 4. Numerical Example

Consider the following bottleneck bi-criteria FTP:

Step 1: Construct the BFTP from the BBFTP.

|  |  | Warehouses j |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{W}_{1}$ | $\mathrm{W}_{2}$ | $\mathrm{W}_{3}$ | $\mathrm{W}_{4}$ |  |
|  | $\mathrm{F}_{1}$ | $\begin{aligned} & (19,20,21) \\ & ((1,2,3),(7,8,9)) \end{aligned}$ | $\begin{aligned} & (189,190,191) \\ & ((3,4,5),(7,8,9)) \end{aligned}$ | $\begin{aligned} & (145,146,147) \\ & ((13,14,15),(5,6,7)) \end{aligned}$ | $\begin{aligned} & (103,104,105) \\ & ((13,14,15),(7,8,9)) \end{aligned}$ | $(7,8,9)$ |
|  | $\mathrm{F}_{2}$ | $\begin{aligned} & (131,132,133) \\ & ((1,2,3),(9,10,11)) \end{aligned}$ | $\begin{aligned} & (135,136,137) \\ & ((17,18,19),(15,16,17)) \end{aligned}$ | $\begin{aligned} & (59,60,61) \\ & ((5,6,7),(17,18,19)) \end{aligned}$ | $\begin{aligned} & (41,42,43) \\ & ((7,8,9),(19,20,21)) \end{aligned}$ | $(18,19,20)$ |
|  | $\mathrm{F}_{3}$ | $\begin{aligned} & (193,194,195) \\ & ((15,16,17),(11,12,13)) \end{aligned}$ | $\begin{aligned} & (125,126,127) \\ & ((17,18,19),(3,4,5)) \end{aligned}$ | $\begin{aligned} & (37,38,39) \\ & ((7,8,9),(9,10,11)) \end{aligned}$ | $\begin{aligned} & (45,46,47) \\ & ((11,12,13),(1,2,3)) \end{aligned}$ | (16,17,18) |
| Demand |  | $(10,11,12)$ | $(2,3,4)$ | $(13,14,15)$ | $(15,16,17)$ |  |

The aim is to find the best compromise solution and efficient solutions for the BBFTP.
Now, the BFTP of BBFTP is given below:

|  |  | Warehouses j |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{W}_{1}$ | $\mathrm{W}_{2}$ | $\mathrm{W}_{3}$ | $\mathrm{W}_{4}$ |  |
|  | $\mathrm{F}_{1}$ | (19,20,21) | $(189,190,191)$ | (145,146,147) | (103,104,105) | $(7,8,9)$ |
|  | $\mathrm{F}_{2}$ | $(131,132,133)$ | $(135,136,137)$ | (59,60,61) | $(41,42,43)$ | $(18,19,20)$ |
|  | $\mathrm{F}_{3}$ | $(193,194,195)$ | $(125,126,127)$ | $(37,38,39)$ | $(45,46,47)$ | $(16,17,18)$ |
| Demand |  | (10,11,12) | $(2,3,4)$ | $(13,14,15)$ | $(15,16,17)$ |  |

Now, the optimal solution of the above BFTP of BBFTP is ( $131,132,133$ ) using the blocking method [14].

Now, solving the FCFTP and SCFTP separately by fuzzy ZPM [13], we obtain the following optimal solutions:

Fuzzy Bottleneck Bi-criteria Value (FBBV)

| Optimal solution |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FCFTP |  |  |  | SCFTP |  |  |  |
| $(3,5,7)$ | $(2,3,4)$ | $(0,0,0)$ | $(0,0,0)$ | $(0,0,0)$ | (0,0,0) | $(7,8,9)$ | $(0,0,0)$ |
| $(3,6,9)$ | $(0,0,0)$ | $(0,0,0)$ | $(9,13,17)$ | $(10,11,12)$ | $(-2,2,6)$ | $(4,6,8)$ | $(0,0,0)$ |
| $(0,0,0)$ | $(0,0,0)$ | $(13,14,15)$ | $(-1,3,5)$ | $(0,0,0)$ | $(-4,1,6)$ | $(0,0,0)$ | $(15,16,17)$ |
| ((242, 286, 330), (382, 530,678), (189, 190, 191)) |  |  |  | $((372,416,458),(275,334,393),(145,146,147))$ |  |  |  |

Thus, we have $\tilde{T}_{m 1}=(189,190,191)$ and $\tilde{T}_{m 2}=(145,146,147)$.
Therefore, $\tilde{T}^{m}=\max \{(189,190,191),(145,146,147)\}$.

Now, since the overall time range is between $(131,132,133)$ and (189,190,191), we have
$\left[\tilde{\mathbf{T}}^{0}, \tilde{\mathbf{T}}^{m}\right]=\{(131,132,133),(135,136,137),(145,146,147),(189,190,191)\}$

Now, the ideal solution to the problem is $((242,286,330)$, (275,334,393),(131,132,133)).
Case 1: $\tilde{\mathrm{M}}=(131,132,133)$
The bi-criteria FTP for $\tilde{\mathrm{M}}=(131,132,133)$ is given below:

|  | Warehouses j |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{W}_{1}$ | $\mathrm{W}_{2}$ | $\mathrm{W}_{3}$ | $\mathrm{W}_{4}$ |  |
| - $\mathrm{F}_{1}$ | ((1,2,3),(7,8,9)) | - | - | ((13,14,15),(7,8,9)) | $(7,8,9)$ |
| \% ${ }^{\circ}$ | ((1,2,3),(9,10,11)) | - | ((5,6,7),(17,18,19)) | ((7,8,9),(19,20,21)) | $(18,19,20)$ |
| ${ }^{\text {L }}=\mathrm{F}_{3}$ | - | ((17,18,19),(3,4,5)) | ((7,8,9),(9,10,11)) | ((11,12,13),(1,2,3)) | $(16,17,18)$ |
| Demand | $(10,11,12)$ | $(2,3,4)$ | $(13,14,15)$ | $(15,16,17)$ |  |

Now, by dripping method [12], the set of all solutions S of the BBFTP for $\tilde{\mathrm{M}}=(131,132,133)$ is given below:

| No | The set of all solutions S of the BBFTP |  |  |
| :--- | :--- | :--- | :--- |
|  |  | No |  |
| 1 | $((276,316,356),(504,566,628),(131,132,133))$ | 10 | $((294,334,374),(414,476,538),(131,132,133))$ |
| 2 | $((278,318,358),(494,556,618),(131,132,133))$ | 11 | $((296,336,376),(404,466,528),(131,132,133))$ |
| 3 | $((280,320,360),(484,546,608),(131,132,133))$ | 12 | $((298,338,378),(394,456,518),(131,132,133))$ |
| 4 | $((282,322,362),(474,536,598),(131,132,133))$ | 13 | $((300,340,380),(384,446,508),(131,132,133))$ |
| 5 | $((284,324,364),(464,526,588),(131,132,133))$ | 14 | $((302,342,382),(374,436,498),(131,132,133))$ |
| 6 | $((286,326,366),(454,516,578),(131,132,133))$ | 15 | $((304,344,384),(364,426,488),(131,132,133))$ |
| 7 | $((288,328,368),(444,506,568),(131,132,133))$ | 16 | $((306,350,394),(364,416,468),(131,132,133))$ |
| 8 | $((290,330,370),(434,496,558),(131,132,133))$ | 17 | $((312,356,400),(354,406,458),(131,132,133))$ |
| 9 | $((292,332,372),(424,486,548),(131,132,133))$ |  |  |

Thus, for the time $(131,132,133)$, we obtain 17 efficient solutions to the problem and the best compromise solution to the problem is ((304,344,384),(364,426,488),(131, 132,133)).

Case 2: $\tilde{\mathrm{M}}=(135,136,137)$
The bi-criteria FTP for $\tilde{M}=(135,136,137)$ is given below:

|  | Warehouses j |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{W}_{1}$ | $\mathrm{W}_{2}$ | $\mathrm{W}_{3}$ | $\mathrm{W}_{4}$ |  |
| $\mathrm{F}_{1}$ | ((1,2,3),(7,8,9)) | - | - | ((13,14,15),(7,8,9)) | $(7,8,9)$ |
| - | ((1,2,3),(9,10,11)) | ((17,18,19),(15,16,17)) | ((5,6,7),(17,18,19)) | ((7,8,9),(19,20,21)) | $(18,19,20)$ |
| $\mathrm{L}^{-1} \mathrm{~F}_{3}$ | - | ((17,18,19),(3,4,5)) | ((7,8,9),(9,10,11)) | ((11,12,13),(1,2,3)) | $(16,17,18)$ |
| Demand | $(10,11,12)$ | $(2,3,4)$ | $(13,14,15)$ | $(15,16,17)$ |  |

Now, by dripping method [12], the set of all solutions S of the BBFTP for $\tilde{\mathrm{M}}=(135,136,137)$ is given below:

| No | The set of all solutions S of the BBFTP |  |  |
| :--- | :--- | :--- | :--- |
|  |  | No |  |
| 1 | $((312,356,400),(354,406,458),(135,136,137))$ | 10 | $((290,330,370),(434,496,558),(135,136,137))$ |
| 2 | $((306,350,394),(364,416,468),(135,136,137))$ | 11 | $((288,328,368),(444,506,568),(135,136,137))$ |
| 3 | $((304,344,384),(364,426,488),(135,136,137))$ | 12 | $((286,326,366),(454,516,578),(135,136,137))$ |
| 4 | $((302,342,382),(374,436,498),(135,136,137))$ | 13 | $((284,324,364),(464,526,588),(135,136,137))$ |
| 5 | $((300,340,380),(384,446,508),(135,136,137))$ | 14 | $((282,322,362),(474,536,598),(135,136,137))$ |
| 6 | $((298,338,378),(394,456,518),(135,136,137))$ | 15 | $((280,320,360),(484,546,608),(135,136,137))$ |
| 7 | $((296,336,376),(404,466,528),(135,136,137))$ | 16 | $((278,318,358),(494,556,618),(135,136,137))$ |
| 8 | $((294,334,374),(414,476,538),(135,136,137))$ | 17 | $((276,316,356),(504,566,628),(135,136,137))$ |
| 9 | $((292,332,372),(424,486,548),(135,136,137))$ |  |  |

Thus, for the time $(135,136,137)$, we obtain 17 efficient solutions to the problem and the best compromise solution to the problem is $((304,344,384),(364,426,488),(135,136,137))$.

Case 3: $\tilde{\mathrm{M}}=(145,146,177)$
The bi-criteria FTP for $\tilde{\mathrm{M}}=(145,146,177)$ is given below:

|  | Warehouses j |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{W}_{1}$ | $\mathrm{W}_{2}$ | $\mathrm{W}_{3}$ | $\mathrm{W}_{4}$ |  |
| - $\mathrm{F}_{1}$ | ((1,2,3),(7,8,9)) | - | ((13,14,15),(5,6,7)) | ((13,14,15),(7,8,9)) | $(7,8,9)$ |
| - $\mathrm{F}_{2}$ | ((1,2,3),(9,10,11)) | ((17,18,19),(15,16,17)) | ((5,6,7),(17,18,19)) | ((7,8,9),(19,20,21)) | $(18,19,20)$ |
| $\mathrm{L}^{\sim} \mathrm{F}_{3}$ | - | ((17,18,19),(3,4,5)) | ((7,8,9),(9,10,11)) | ((11,12,13),(1,2,3)) | $(16,17,18)$ |
| Demand | $(10,11,12)$ | $(2,3,4)$ | $(13,14,15)$ | $(15,16,17)$ |  |

Now, by dripping method [12], the set of all solutions S of the BBFTP for $\tilde{\mathrm{M}}=(145,146,177)$ is given below:

| No | The set of all solutions S of the BBFTP |  |  |
| :--- | :--- | :--- | :--- |
|  |  | No |  |
| 1 | $((272,316,360),(522,566,610),(145,146,147))$ | 18 | $(((332,356,428),(362,406,450),(145,146,147))$ |
| 2 | $((274,318,362),(512,556,600),(145,146,147))$ | 19 | $((316,360,404),(362,406,450),(145,146,147))$ |
| 3 | $((276,320,364),(502,546,590),(145,146,147))$ | 20 | $((312,364,416),(352,396,440),(145,146,147))$ |
| 4 | $((278,322,366),(492,536,580),(145,146,147))$ | 21 | $((324,368,412),(352,396,440),(145,146,147))$ |
| 5 | $((280,324,368),(482,526,570),(145,146,147))$ | 22 | $((320,372,424),(342,386,430),(145,146,147))$ |
| 6 | $((282,326,370),(472,516,560),(145,146,147))$ | 23 | $((332,376,420),(342,386,430),(145,146,147))$ |
| 7 | $((284,328,372),(462,506,550),(145,146,147))$ | 24 | $((328,380,432),(332,376,420),(145,146,147))$ |
| 8 | $((286,330,374),(452,496,540),(145,146,147))$ | 25 | $((340,384,428),(332,376,420),(145,146,147))$ |
| 9 | $((288,332,376),(442,486,530),(145,146,147))$ | 26 | $((336,388,440),(322,366,410),(145,146,147))$ |
| 10 | $((290,334,378),(432,476,520),(145,146,147))$ | 27 | $((348,392,436),(322,366,410),(145,146,147))$ |
| 11 | $((292,336,380),(422,466,510),(145,146,147))$ | 28 | $((344,396,464),(312,356,400),(145,146,147))$ |
| 12 | $((294,338,382),(412,456,500),(145,146,147))$ | 29 | $((356,400,444),(312,356,400),(145,146,147))$ |
| 13 | $((296,340,384),(402,446,490),(145,146,147))$ | 30 | $((352,404,456),(302,346,390),(145,146,147))$ |


| 14 | $((298,342,386),(392,436,480),(145,146,147))$ | 31 | $((354,406,458),(302,346,390),(145,146,147))$ |
| :--- | :--- | :--- | :--- |
| 15 | $((300,344,388),(382,426,470),(145,146,147))$ | 32 | $((356,408,450),(302,346,390),(145,146,147))$ |
| 16 | $((326,350,422),(372,416,460),(145,146,147))$ | 33 | $((360,412,454),(296,412,384),(145,146,147))$ |
| 17 | $((308,352,396),(372,416,460),(145,146,147))$ | 34 | $((364,416,458),(292,416,298),(145,146,147))$ |

Thus, for the time $(145,146,177)$, we obtain 34 efficient solutions to the problem and the best compromise solution to the problem is $((320,372,424),(342,386,430),(145,146,147))$.

Case 4: $\tilde{\mathrm{M}}=(189,190,191)$
The bi-criteria FTP for $\tilde{M}=(189,190,191)$ is given below:

|  | Warehouses j |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{W}_{1}$ | $\mathrm{W}_{2}$ | $\mathrm{W}_{3}$ | $\mathrm{W}_{4}$ |  |
| - $\mathrm{F}_{1}$ | ((1,2,3),(7,8,9)) | ((3,4,5),(7,8,9)) | ((13,14,15),(5,6,7)) | ((13,14,15),(7,8,9)) | $(7,8,9)$ |
| - $\mathrm{F}_{2}$ | ((1,2,3),(9,10,11)) | ((17,18,19),(15,16,17)) | ((5,6,7),(17,18,19)) | ((7,8,9),(19,20,21)) | $(18,19,20)$ |
| L- $\mathrm{F}_{3}$ | - | ((17,18,19),(3,4,5)) | ((7,8,9),(9,10,11)) | ((11,12,13),(1,2,3)) | $(16,17,18)$ |
| Demand | $(10,11,12)$ | $(2,3,4)$ | $(13,14,15)$ | $(15,16,17)$ |  |

Now, by dripping method [12], the set of all solutions $S$ of the BBFTP for $\tilde{\mathrm{M}}=(189,190,191)$ is given below:

| No | The set of all solutions S of the BBFTP |  |  |
| :--- | :--- | :--- | :--- |
|  |  | No |  |
| 1 | $((242,286,330),(486,530,574),(189,190,191))$ | 14 | $((268,312,356),(356,400,444),(189,190,191))$ |
| 2 | $((244,288,332),(476,520,564),(189,190,191))$ | 15 | $((268,320,374),(356,390,434),(189,190,191))$ |
| 3 | $((246,290,334),((466,510,554),(189,190,191))$ | 16 | $((276,328,382),(346,380,424),(189,190,191))$ |
| 4 | $((248,292,336),(456,500,544),(189,190,191))$ | 17 | $((284,336,390),(336,370,414),(189,190,191))$ |
| 5 | $((250,294,338),(446,490,534),(189,190,191))$ | 18 | $((292,344,398),(326,360,404),(189,190,191))$ |
| 6 | $((252,296,340),(436,480,524),(189,190,191))$ | 19 | $((300,352,406),(316,350,394),(189,190,191))$ |
| 7 | $((254,298,342),(426,470,514),(189,190,191))$ | 20 | $((350,372,394),(294,342,386),(189,190,191))$ |
| 8 | $((256,300,344),(416,460,504),(189,190,191))$ | 21 | $((300,374,450),(326,346,366),(189,190,191))$ |
| 9 | $((258,302,346),(406,450,494),(189,190,191))$ | 22 | $((372,394,416),(290,338,382),(189,190,191))$ |
| 10 | $((260,304,348),(396,440,484),(189,190,191))$ | 23 | $((322,396,472),(322,342,362),(189,190,191))$ |
| 11 | $((262,306,350),(386,430,474),(189,190,191))$ | 24 | $((344,418,494),(318,338,358),(189,190,191))$ |
| 12 | $((264,308,352),(376,420,464),(189,190,191))$ | $((372,416,460),(290,334,378),(189,190,191))$ |  |
| 13 | $((266,310,354),(366,410,454),(189,190,191))$ |  |  |

Thus, for the time $(189,190,191)$, we obtain 22 efficient solutions and 3 non-efficient solutions and the best compromise solution to the problem is $((284,336,390),(336,370,414),(189,190,191))$.

Now, over all efficient solutions, P to the given problem is given below:

| No | The set of all solutions P of the BBFTP |  |  |
| :--- | :--- | :--- | :--- |
|  |  | No |  |
| 1 | $((276,316,356),(504,566,628),(131,132,133))$ | 24 | $((360,412,454),(296,412,384),(145,146,147))$ |
| 2 | $((278,318,358),(494,556,618),(131,132,133))$ | 25 | $((364,416,458),(292,416,298),(145,146,147))$ |
| 3 | $((280,320,360),(484,546,608),(131,132,133))$ | 26 | $((242,286,330),(486,530,574),(189,190,191))$ |
| 4 | $((282,322,362),(474,536,598),(131,132,133))$ | 27 | $((244,288,332),(476,520,564),(189,190,191))$ |
| 5 | $((284,324,364),(464,526,588),(131,132,133))$ | 28 | $((246,290,334),((466,510,554),(189,190,191))$ |
| 6 | $((286,326,366),(454,516,578),(131,132,133))$ | 29 | $((248,292,336),(456,500,544),(189,190,191))$ |
| 7 | $((288,328,368),(444,506,568),(131,132,133))$ | 30 | $((250,294,338),(446,490,534),(189,190,191))$ |
| 8 | $((290,330,370),(434,496,558),(131,132,133))$ | 31 | $((252,296,340),(436,480,524),(189,190,191))$ |
| 9 | $((292,332,372),(424,486,548),(131,132,133))$ | 32 | $((254,298,342),(426,470,514),(189,190,191))$ |
| 10 | $((294,334,374),(414,476,538),(131,132,133))$ | 33 | $((256,300,344),(416,460,504),(189,190,191))$ |
| 11 | $((296,336,376),(404,466,528),(131,132,133))$ | 34 | $((258,302,346),(406,450,494),(189,190,191))$ |
| 12 | $((298,338,378),(394,456,518),(131,132,133))$ | 35 | $((260,304,348),(396,440,484),(189,190,191))$ |
| 13 | $((300,340,380),(384,446,508),(131,132,133))$ | 36 | $((262,306,350),(386,430,474),(189,190,191))$ |
| 14 | $((302,342,382),(374,436,498),(131,132,133))$ | 37 | $((264,308,352),(376,420,464),(189,190,191))$ |
| 15 | $((304,344,384),(364,426,488),(131,132,133))$ | 38 | $((266,310,354),(366,410,454),(189,190,191))$ |
| 16 | $((306,350,394),(364,416,468),(131,132,133))$ | 39 | $((268,312,356),(356,400,444),(189,190,191))$ |
| 17 | $((312,356,400),(354,406,458),(131,132,133))$ | 40 | $((268,320,374),(356,390,434),(189,190,191))$ |
| 18 | $((312,364,416),(352,396,440),(145,146,147))$ | 41 | $((276,328,382),(346,380,424),(189,190,191))$ |
| 19 | $((320,372,424),(342,386,430),(145,146,147))$ | 42 | $((284,336,390),(336,370,414),(189,190,191))$ |
| 20 | $((328,380,432),(332,376,420),(145,146,147))$ | 43 | $((292,344,398),(326,360,404),(189,190,191))$ |
| 21 | $((336,388,440),(322,366,410),(145,146,147))$ | 44 | $((300,352,406),(316,350,394),(189,190,191))$ |
| 22 | $((344,396,464),(312,356,400),(145,146,147))$ | 45 | $((350,372,394),(294,342,386),(189,190,191))$ |
| 23 | $((352,404,456),(302,346,390),(145,146,147))$ |  |  |

The overall best compromise solution to the given problem is ( $(312,356,400)$,
(354,406,458),
$(131,132,133))$
where $\tilde{\mathrm{Z}}_{1}\left(\tilde{\mathrm{X}}^{0}\right)=(312,356,400), \tilde{\mathrm{Z}}_{2}\left(\tilde{\mathrm{X}}^{0}\right)=(354,406,458), \quad \tilde{\mathrm{X}}^{0}=$ $\left\{\tilde{x}_{11}=(3,6,9), \tilde{x}_{14}=(-2,2,6), \quad \tilde{x}_{21}=(3,5,7), \quad \tilde{x}_{23}=(13,14,15)\right.$, $\left.\tilde{x}_{32}=(2,3,4), \quad \tilde{x}_{34}=(12,14,16)\right\}$ and time $\tilde{\mathrm{T}}=(131,132,133)$.

## 5. Conclusion

The time of transport might be significant factor in many TPs. In this paper, we presented a fuzzy block-dripping method for BBFTP. The proposed method provides the set of efficient solutions and best compromise solutions for BBFTP for each time in the specified time interval.

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