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On the Cognitive Process of Abstraction

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Abstract

Concepts are the basic elements of propositions. Concepts can be best understood as constituted by its subset of objects (Extent) and subset of attributes (Intent). Psychological capacities of human mind for example, learning, thinking, memorizing can be performed by concepts and their association. In this paper, we will explain how human will be able to generalize concrete concepts of Formal Concept Analysis into abstract concepts. In particular, we model the functionalities of concept algebra by making use of Formal Concept Analysis; we illustrate the proposed model with experiments on sample context. This model simulates the thinking process of human mind.

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Keywords: Abstraction; Cognitive Informatics; Concept Algebra; Formal Concept Analysis.

1. Introduction

Nowadays, thinking in abstraction is thought to be one of the key attributes in human behaviours. The legitimate model of the brain is the highest level of abstraction for demonstrating cognitive mechanisms[1]. In the process of thinking, human concentrates on different levels which lead us to abstraction. Abstraction is the way of thinking that depends on human capacity to recognize the real world under different levels. It is another intricate part in the field of Cognitive Informatics (CI) which explores on human information processing techniques.

Cognitive Informatics gives a reasonable structure for clarifying human cognitive procedures for example, thinking, learning, abstraction, decision making and so on[2]. The most interesting disclosures in CI are the diversity of sciences and engineering areas such as informatics, processing, software engineering. When individuals confront a disorganized and unlabelled data, they used to abstract a lot of complex data into some basic ideas with a specific end goal to break down and comprehend them unmistakably. Abstraction describes human cognition to get an organized thinking pattern[3] and is regarded as a thought process[4]. It is one of the main qualities of the human mind recognized in Cognitive Informatics. To show that human perceive things from various perspectives and levels, some cognition model has been proposed for example, cognition psychology model[5], cognitive informatics model[2], cognition model on Granular Computing[6] and ontology cognition model[7].

Meaning of "Abstraction" in linguistic is the concept or representation that interfaces any related concepts as a gathering field or classification. It is evident from the writing that the concepts are the essential elements of cognition

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which convey the meaning of thinking, learning, reasoning and comprehension[8]. Particularly, in cognition concept identifies and model the existent world and abstract subject's solid unit into hypothetical world[9]. Significance of a concept is already given in all aspects of cognition. Within few decades, several models have been developed based on concepts[2,6]. Concrete and abstract are the two types of concept[9]. Concrete concept is a solid concept that can be straightforwardly portrayed into a real world entity such as table, pen while the abstract concept is a virtual concept that cannot be portrayed into a real world entity for example idea, mind.

Wille presented the idea of concept which results in Formal Concept Analysis[10]. A few researchers consideration is gotten by FCA[11] in different fields, for example, data mining, information retrieval, software engineering[12–14]. FCA may be understood as "Applied abstract algebra", which offers conceptual knowledge representation in a levelled structure[10]. It is a mathematical framework, which connotes intension and extension of a concept and the relationship between them. The intension of the concept is described as the group of attribute or properties shared by objects while extension of a concept is the group of objects which shares the collection of attributes identified for concept. FCA takes data as input and produces output as concept lattice, attribute implication and formal concepts. Formal concept in formal context is the pair of object and attribute closed by Galois connection[15].

Strict modelling and formal treatment of concepts plays a major role for knowledge representation and manipulation[16]. For the formal treatments of concepts and their control, another numerical structure is presented known as "Concept Algebra". Concept algebra is a denotational mathematics used for rigorous manipulation of formal concepts[9]. It is demanded in many disciplines such as cognitive linguistics, cognitive informatics, formal semantic analysis, machine learning and many more. Concepts in concept algebra are abstract concept that contains significance in cognitive processes of thinking, learning, and reasoning[9].

This paper introduces a conceptual structure to model the functionalities of Concept algebra namely extension, tailoring, substitution, generalization, independent and related operators by using Formal concept analysis (FCA). We also present a conceptual framework of creating abstraction or thought process based on the unified view with contribution from Formal Concept Analysis (FCA) and Concept Algebra (CA). We have divided the abstraction process into two sub-processes generalization related to human memory and another one is abstraction related to human thinking. The network of abstraction and the descriptively of abstract means at various levels has been efficiently displayed. Another important aspect of this paper is that, on of the cognitive processed of thinking can be modelled by linking FCA and CA to understand how human thinks when we present the real world example. Rest of this paper, Section 2 describes related work developed in Cognitive Informatics, Formal Concept Analysis (FCA) & Concept Algebra (CA) followed by Section 3, where proposed model is developed and explained. Section 4, talks about illustrative experiments that were performed. Conclusion is given in Section 5.

2. Related Works

In various order of human learning or thinking, the majority of the difficult issues yet to be unravelled, provides the origin in the notion of the components of natural science and the cognitive process of brain. In this manner, Cognitive Informatics (CI) is the regulation that produces connection between normal sciences and life sciences disciplines with informatics processing sciences[2,17]. It provides contemporary hypotheses for clarifying human psychological procedures for example thinking, problem solving, learning, decision making. Radical hypothesis developed in CI incorporates Information-Matter-Energy-Intelligence (IME-I) model[3], the Layered Reference Model of the Brain (LRMB)[18], Object-Attribute-Relation (OAR) model of information representation in the brain[19], cognitive informatics model of the brain[4], Natural Intelligence (NI) and neuroinformatics[1].

IME-I model[3], uncover that the natural world (NW) is a double world enveloping both concrete world and the dynamic world and says data assumes a noteworthy part in connecting the concrete world with the abstract world. According to the functional model of the brain[3], brain can be modelled by 37 cognitive processes at six layers namely sensation, memory, perception, action, Meta and higher cognitive layers. On the basis of LRMB, an assortment of Cognitive processes has been thoroughly portrayed. The model exhibits that the memory is the establishment of any natural intelligence. Examination concerning the cognitive models of data and information representation in the brain is seen to be one of the basic exploration zones that uncover the mechanism of the brain. The Object-Attribute-Relation (OAR) Model[19] describes the long term memory of human brain using relative analogy rather than the conventional

analogy in information science and phycology. The OAR model demonstrates that human memory and information is spoken to by relations, i.e., associations of neurotransmitters between neurons, as opposed to by the neurons itself as the conventional analogy. Neural informatics[2] is a new integrative inquiry regarding the primitive and psychological representation of information and knowledge in the brain. Recent studies in cognitive informatics give rise to a new field called "Denotational mathematics[20]" have been introduced for the mathematical need in natural, abstract and artificial intelligence. The development in Denotational mathematics is driven by the reasonable needs in CI, Intelligence sciences, software science and information science. These advanced orders study complicated human and machine practices and their treatments. Therefore, Denotational mathematics is an arrangement of contemporary scientific structures for managing abstract intelligence and cognitive informatics. The upsides of embracing denotation mathematics are that abstract intelligence can be empowered by programming and computational knowledge. In particular, an arrangement of novel denotation mathematical structural forms given by[21,22], are evolved known as Concept Algebra[8], System Algebra[23], Real Time Process Algebra (RTPA)[24] and Visual Semantic Algebra[25]. Among them, Concept Algebra is intended to handle the abstract numeric structure of concepts and their representation and control.

Formal concepts in concept algebra are abstract structure that conveys special semantics in every single psychological process, for example, information representation, learning and thinking[8]. In the Concept Algebra, formal concepts can be defined as 5-tuple (A, O, R^c, R^i, R^o) in the universe of discourse U, where A and O is the collection of attributes and objects of a concept, (R^c, R^i, R^o) is a set of internal, input and output relations of a concept[26]. When it comes to knowledge representation and formalization of "concept" a new mathematical framework is observed named "Formal Concept Analysis", which connotes intension and extension of a concept and the relationship between them. FCA can be defined as (O, A, R) where O, A and R is object, attribute and relationship between them. Within few decades several model based on FCA has been developed which includes an algorithm using Shannon entropy[15] for generating interval -valued fuzzy formal concepts or the demonstration of attribute exploration in order to understand the dependencies among the attributes in the data[4,27], Non Negative Matrix factorization[28] in order to take the advantage of relation between matrix decomposition and Singular Matrix Decomposition[29] to diminish the lattice size. A group of researches revealed that FCA can be used in cognitive informatics as well. Very recently, modelled the functionalities of Bidirectional associative memory using FCA[1], their work concentrated on the functionalities such as learning, memorizing, recalling the memorized. Cognitive Space Mark-up Language (CSML) is an after effect of modelling people's thinking and cognitive abilities and has established a novel cognitive system through FCA[5].

It is obvious from the literature that the concepts in Concept algebra extends the concept in FCA from triple to a pair of five tuple[26]. This observation forms the basis of connection between the FCA with concept algebra. Motivated by the study[18], where they have described the 39 cognitive processes of the LRMB model in order to model brain and mind, we propose one of the cognitive processes of abstraction which is present in the Meta cognitive layer of the LRMB model by linking the FCA and CA. It establishes an abstract model for a concrete entity or real world. Modelling of abstract world is still of particular interest that is yet to be explored. From the reference to the above study, we show a conceptual framework for creating abstraction or thought process based on the unified view with contribution from Formal Concept Analysis (FCA) and Concept Algebra (CA).

3. Proposed Model

The proposed work models the functionalities of Concept Algebra (CA) using Formal Concept Analysis (FCA). A concept defined in FCA is the formal concept which constitutes the set of objects, their attributes and the binary relation between them as given by Wille[30]. A concept given by Wang[26] is an abstract structure that conveys the meaning of some cognitive process such as thinking, learning, reasoning, knowledge representation. Therefore, the abstract concept is the arrangement of objects, their attribute and the internal and external relation respectively[23]. In the proposed work, we take a formal context sample which can be defined by its objects, sub-object or attributes to characterize the object. From the given formal context we will first find the list of generated formal concepts. The list of n number of formal concepts can be regarded as the concrete concepts of memory[10]. This memory is analogous to concept structured human memory[31]. Abstraction creator given in the proposed model shown in Fig. 1 will perform Concept algebra operations on this memory when provided with the compound cues[31] and creates one of the Meta cognitive processes "Abstraction" known as thinking engine of human[18].

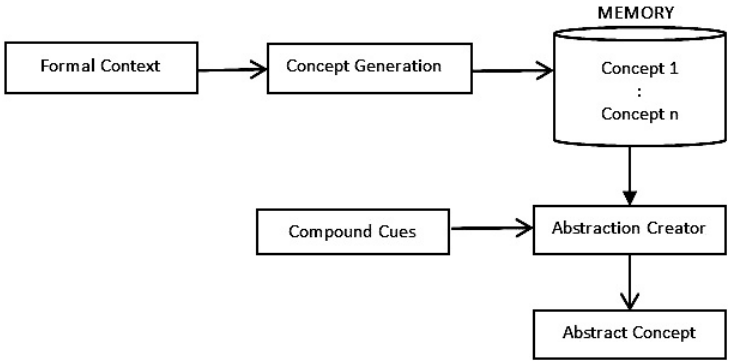

Fig. 1. Proposed Model.

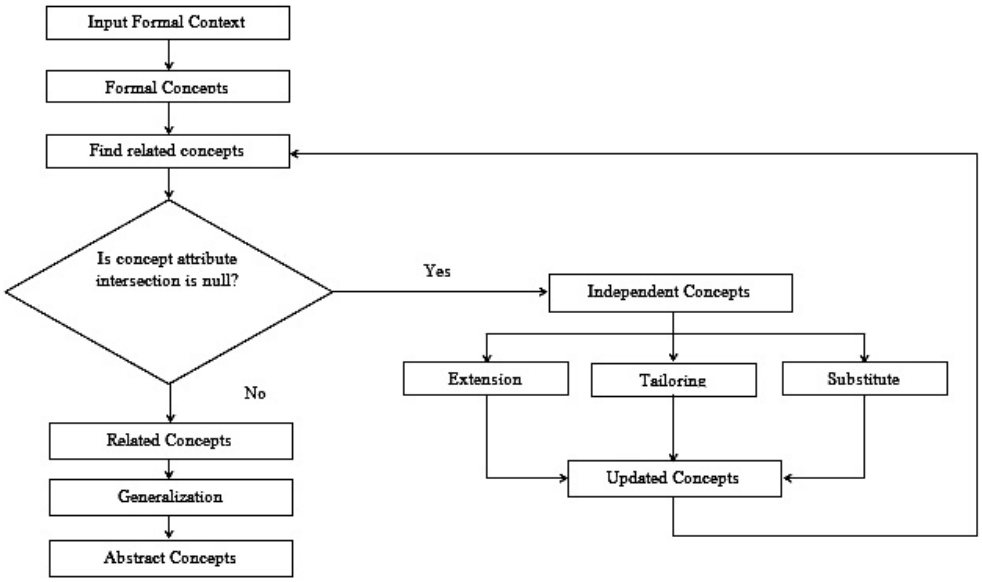

Fig. 2. Detailed Descriptions.

As a result we end up with the abstract concept. Hence, we model the formal concepts or concrete concepts as abstract concepts or more generalized concepts.

Another important aspect of this proposal is that, one of the cognitive processes of thinking can be modelled by linking the two concepts defined above to know how human thinks when we present the real world example. The list of formal concepts will be regarded as concrete concept using FCA[10]. We manipulate the formal concepts through the properties of concept algebra (CA) to form abstract concept given by Wang from the concrete concept. This explain how human will be able to generalize concrete or formal concepts in FCA into abstract concept. Figure 2 explains the process followed by abstraction creator on the memory to form the abstract concept for the given compound cue. The detailed description of the proposed model Fig. 2 is explained below.

3.1 Building formal concepts

Concepts are the fundamental unit of belief that underlie human insight and communication. In order to obtain the list of formal concepts, we identify the set of objects, set of attributes or sub-object used to indicate the properties

of an object and a relation between them. This together with a binary relationship forms a formal context. From this formal context we derive the collection of all the formal concepts. Various concept generation algorithms have been used previously. To obtain the list of formal concepts, we are using Next Neighbor concept generation algorithm[32]. In this algorithm, every object-attribute set the comparing lower neighbors are found. After finding the corresponding lower neighbor, maximal general candidate (concept) is returned. The discovered lower neighbor is then assigned as a present level for which lower neighbors are further found. This procedure is repeated until we locate all the concepts.

3.2 Related concepts

We are regarding the list of formal concepts as memory. Now in this memory, the abstraction finder receives a set of concepts as a compound cue to find whether they are related to each other. It is perceived that the relationship between concepts is exclusively dictated by the connection of both their intensions and extensions[9]. To find the related concepts the abstraction creator will follow the following procedure:

1. The abstraction creator will find whether the intersection of attributes of compound cues is null or not.
2. When the intersection of attributes of compound cues is null, then it is regarded as an independent concepts.
3. When the intersection of attributes of compound cues is not equal to null, then it is a related concepts.

3.3 Generalization

Concept generalization helps to organize information into categories. It is noteworthy that generalization is always applied to related concepts[8]. To perform the concept generalization, the abstraction creator will perform the intersection of attributes and the union of the objects of the related compound cues. As a result, we will be having an abstract concept.

3.4 Independent concepts

These section discuses on how we will manipulate the compound cues to make it a related concept to form abstract concept. For a given compound cues, when the abstraction creator cannot find the related concepts (intersection of attributes is null), it will return the compound cues as independent concepts. Upon that we apply concept algebra to manipulate the concepts according to suitability of the concepts which includes extension, tailoring and substitute to make it a related concept. The concept manipulation is handled as follows:

Case 1: Extension

Extension of concept can be defined as the creation of the new concept based on old concept by adding additional objects and attributes which is not a subset of old concept[26]. We can extend the concept by adding new objects attributes to the formal concept where the new object/attribute should not be a subset of given compound cues of formal concepts in memory shown in Fig. 3.

Case 2: Substitute

Substitute of a concept from parent concept creates the new concept by replacing the objects and attributes of parent concepts which is not a subset[26]. We can replace the attributes/objects to the formal concepts where the new object/attribute should be a subset of given compound cues of formal concepts in memory as in Fig. 4.

Case 3: Tailoring

Tailoring of a concept from a parent concept is described as removal of objects and attributes which is not a subset of parent concept to create a new concept[26]. Tailoring of a formal concept is a strategy to remove the specific subset of object/attribute from the given compound cues. The process of tailoring operation is described in Fig. 5.

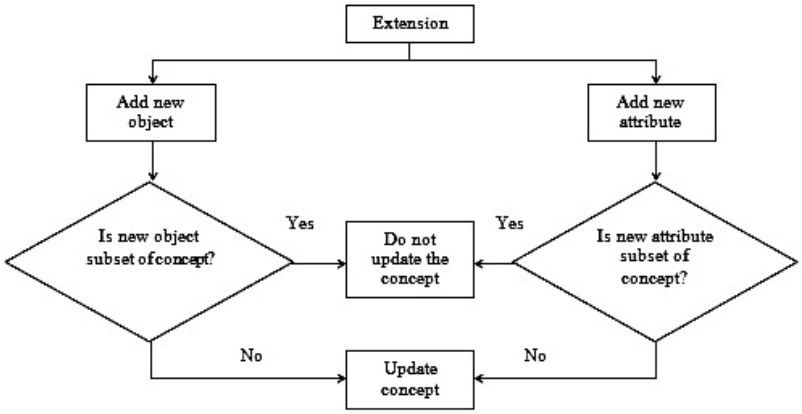

Fig. 3. Extension of a Concept.

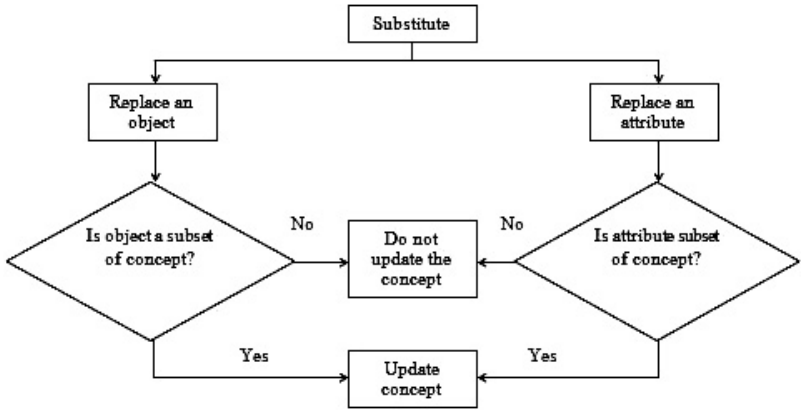

Fig. 4. Substitution of a Concept.

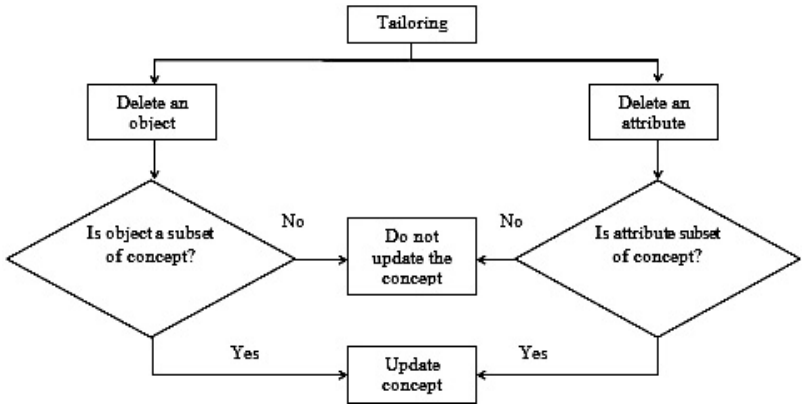

Fig. 5. Tailoring of a Concept.

Table 1. Formal Context Representing the Knowledge.

	A1	A2	A3	A4	A5	A6	A7	A8	A9
O1	×	×							
O2		×	×						
O3	×		×	×	×				
O4	×		×		×	×			
O5							×	×	
O6								×	×

Table 2. Formal Concepts generated from Table 1.

1	{O1, O2, O3, O4, O5, O6}, {empty}
2	{O4}, {A1, A3, A5, A6}
3	{O3}, {A1, A3, A4, A5}
4	{O1}, {A1, A2}
5	{O2}, {A2, A3}
6	{O6}, {A8, A9}
7	{O5}, {A7, A8}
8	{O1, O3, O4}, {A1}
9	{O2, O3, O4}, {A3}
10	{O1, O2}, {A2}
11	{O5, O6}, {A8}
12	{O3, O4}, {A1, A3, A5}
13	{empty}, {A1, A2, A3, A4, A5, A6, A7, A8, A9}

4. Experimental Analysis

This section talks about illustrative experiments that were performed on a sample context to provide the better understanding of the model.

4.1 Illustration

To provide a better insight of the model, we elaborate the proposed work using illustrative context taken from the real world scenario. Let us consider the scenario by taking a formal context containing 6 objects as {car, truck, ordinary train, express train, dugout, liner} and 9 attributes as {passenger, road run, cargo, high speed, rail run, slow speed, engine powered, operation in water, hollowed out log}. In the formal context all the objects having attributes are marked as ("×") which shows the relationship between them as shown in Table 1. To our convenience we are representing the set of objects as {O1, O2, O3, O4, O5, O6} and set of attributes as {A1, A2, A3, A4, A5, A6, A7, A8, A9}. Acquired list of formal concepts from this context is shown in Table 2. We have conducted the test to explain the generalization of concrete or formal concepts into abstract concept as shown in Fig. 1. In order to explain the proposed model, we are going to illustrate the functioning of the proposed model, when the compound cues are related concepts and when it is independent concepts. The two circumstances are handled as follow.

Case 1: When the compound cues are related concepts.

Let us consider the compound cues as concepts 2 and 3 from the Table 2 of formal concepts. Now, step-by-step demonstration of generating abstract concepts is described below:

Step 1. The model receives a compound cues, when the intersection of the concepts intent is not equal to null then the compound cues are regarded as the related concepts.

For example, formal concepts 2 and 3, intersection of concepts attributes are not equal to null as they share common attributes. Hence, they are regarded as the related concepts.

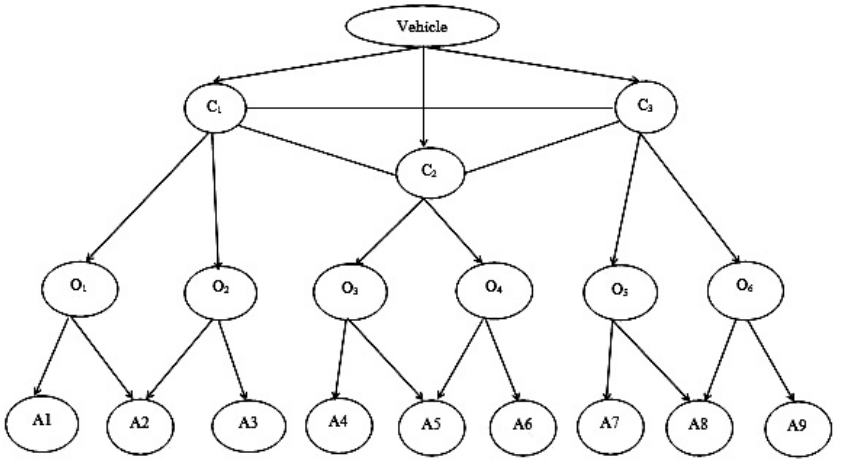

Fig. 6. Concept Network of "Vehicles".

Step 2. After finding the related concept, we generalize it as described in the section of proposed work to form abstract concepts. Concepts 2 and 3 can be generalized as a new abstract concept as "train". Similarly, concepts 4 and 5 are generalized as new concepts "automobile" and concepts 6 and 7 are generalized as "ship".

Case 2: When the compound cues are independent concepts.

When the concepts are not related then they are independent concepts i.e. the intension of the concepts is disjoint. The model receives a compound cues, when the intersection of the concepts intent is equal to null then the compound cues are regarded as independent concepts. To make it a related concept, we will perform operations of concept algebra according to suitability as follows:

1. **Substitute** or replace the attribute or object of a concept using algorithm in Fig. 4 to make it a related concepts. For example, formal concept 2 and 6 are independent concepts as they do not share a common attribute. So, we replace the attribute A5 "run on rail" as "driven by human" to make it related. After, the concepts become related; again generalization will be performed to make it an abstract concept as "vehicle"
2. **Extension** of a concept is performed using algorithm in Fig. 3. For example, compound cues 2 and 10 are independent. To make it related we add new attribute "Cooling system" to the concept. Objects O4, O1 & O2 will share the new attribute. Hence Concepts 2 & 10 becomes related concepts.
3. **Tailoring** of a concept is performed using algorithm in Fig. 5. For example, if we remove an object "liner" from the subset of objects. The concepts will become related and again generalized using generalization process as "vehicle".

It is evident from the experimental analysis of the proposed model that we are able to generalize the concrete or formal concepts as abstract concept in order to imitate the thinking of human mind. Using the abstract concept model and OAR, human thinking can be modelled as concept network shown in Fig. 6. Concept network can be utilized for information representation which regards the concepts as a fundamental unit of knowledge representation and thinking[9]. Figure 6 is displayed as concept network made out of of nodes and edges. Concepts, objects and attributes are abstracted as nodes. The relations between them are abstracted as edges. Concept network empowers concepts to be logically accessed and manipulated. In the hierarchy, Concepts C_1, C_2 & C_3 are generalized as "Automobiles", "Train" & "Ship".

5. Conclusions

Main objective of this work is to use the mathematical framework, Formal Concept analysis to model the functionalities of Concept Algebra in order to impersonate the human thinking process. The proposed model is divided

into two sub-processes defined as the generalization related to human memory and another one is abstraction related to human thinking. This study explains on how human generalizes the particular concept in mind or shows the abstract intelligence of human and shows that the integration of concept algebra and formal concept analysis is possible at a conceptual level. In the proposed model, we have considered the generated formal concept as memory. This memory is analogous to the human memory. As we present the compound cues to the system, it is efficiently performing the manipulation of formal concept in order to generalize it to abstract concepts.

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